Generation of optical phonons in semiconductors following intraband absorption of electromagnetic radiation

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The generation of optical phonons as a result of intraband absorption of electromagnetic radiation by the conduction electrons of a semiconductor is investigated theoretically. The electron–phonon coupling constant is assumed to be small. Single phonon processes involving the emission of an arbitrary number of electromagnetic field quanta are considered, i.e., a theory nonlinear in the field intensity is constructed. Expressions for the rate of phonon generation are derived at various ratios of the electromagnetic field frequency to the limiting phonon frequency, and at different field intensities. The expressions are analyzed in detail for the case in which the conduction electron distribution is described by a Boltzmann function that depend on the erature. It is indicated that can appear in the optical-phonon system upon irradiation of the semiconductor by an intense electromagnetic field. This instability manifests itself in a change in the sign of the coefficient of the linear damping of the phonons.

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As is known, the laws of conservation of energy and quasimomentum forbid processes of direct intraband absorption of a phonon by an electron. These processes can take place only with participation of a third body. This latter is usually either a defect of the crystal lattice or a phonon (acoustic or optical). The process of infrared absorption with participation of an optical phonon was studied by Lang, Firsov, and one of us[1] for optically isotropic semiconductors and also by Lang and the authors[2] for semiconductors possessing optical anisotropy. In these papers the infrared absorption was considered as a second-order process, in which an electron with energy $\varepsilon(p)$ absorbs a photon with energy $\hbar\Omega$ (its quasimomentum can be assued to be zero within the limits of accuracy of our calculation) and absorbs or emits an optical phonon with energy $\hbar\Omega_0(q)$, where q is the wave vector of the phonon.

The purpose of the present work was to trace the "fate" of the produced optical phonons. The basic process that impedes their accumulation is the decay of the optical phonon into two acoustic phonons. The corresponding characteristic decay time amounts, according to rough estimates, to 10^{-11} sec, i.e., it is already not so small. And this means that, at a sufficient intensity of electromagnetic radiation in the semiconductor, a considerable number of optical phonons can accumulate.

For definiteness, we shall consider the interaction of conduction electrons with polar longitudinal optical phonons is optically isotropic (i.e., cubic) semiconductors. Then the qualitative picture depends weakly either on the character of the electron-phonon interaction or the isotropy assumption (cf. Ref. 2). We assume the electron–phonon coupling constant to be sufficiently small and limit ourselves to single-phonon processes only.

On the other hand, there exist very intense sources of electromagnetic radiation at the present time. Taking this into account, we shall investigate processes with participation of an arbitrary number l of electromagnetic quanta. As a result of such a process, an electron with initial energy ε goes over into a state with energy $\varepsilon \pm \hbar\Omega_0 + \hbar l\Omega$, where Ω_0 is the limiting frequency of the phonon,[1] Ω is the frequency of electromagnetic radiation, and l is an arbitrary integer, positive (absorption) or negative (emission). We shall be interested in the case of Boltzmann statistics and sufficiently low temperature, when $\exp(-\hbar\Omega_0/T_0) \ll 1$ and the probability of absorption of an optical phonon is $\exp(\hbar\Omega_0/T_0)$ times smaller than the probability of its emission. Here T_0 is the lattice temperature. For this reason we shall consider chiefly processes taking place with emission of an optical phonon.

We distinguish between three cases, depending on the ratio of the frequencies Ω and Ω_0.

1) Resonance: $\Omega = \Omega_0$ with sufficient accuracy. As a result of the process of second order, the final state of the electron is identical with the initial. Therefore, the heating of the electrons by the electromagnetic field is comparatively weak. The rate of generation of the optical phonons with wave vector q is directly proportional to the intensity of the electromagnetic radiation S in the semiconductor, so long as it remains significantly smaller than some critical intensity $S_c(q)$, equal to

$$S_c(q) = \varepsilon^{1/2} m^2 c\Omega^4 / 8\pi e^2 q^2, \qquad (1)$$

where e and m are respectively the charge and the effective mass of the conduction electron, ε is the permittivity of the semiconductor at the frequency Ω. At $S > S_c$, the rate of generation of optical phonons, on the other hand, falls off with increase in the intensity. At $q = 10^6$ cm^{-1}, $m = 0, 1m_0$ (m_0 is the mass of the free electron), and $\Omega = 3 \times 10^{13}$ sec^{-1}, S_c amounts to about 10^7 W/cm^2. The number of phonons produced per unit time at $S \ll S_c$ is easily estimated, for example, by the formulas of Ref. 1. It is also important to know the interval of the wave vectors q of the generated optical phonons. In the resonance case the upper bound of this interval q_{max} is determined by the spread of the electron energy, and is consequently equal in order of magnitude to $(mT)^{1/2}/\hbar$. The lower bound of the interval that we studied, q_{min}, is described by the limits imposed by

electron collisions on the applicability of the theory developed here, namely, the quantity $\hbar q$ should be greater than the uncertainty of the electron quasimomentum $\hbar(m/T)^{1/2}\tau_e^{-1}$, where τ_e is the characteristic time of electron collisions. To be sure, the case $\Omega = \Omega_0$ encounters considerable experimental difficulties for the following two reasons.

First, it is difficult to choose an intense source of electromagnetic radiation with $\Omega = \Omega_0$. Second, in optically isotropic crystals, the frequency Ω_0 is the boundary of the region of opaqueness of the crystal. For this reason, we consider other cases, in which the frequency Ω is quite different from Ω_0.

2) The case $\Omega \gg \Omega_0$, from the viewpoint of the possibility of experiment, is much more realistic than the preceding one. The distribution function of the phonons has a maximum at $q = q_l \equiv [\,2m\hbar^{-1}(l\Omega - \Omega_0)]^{1/2}$. Since the quantity S_c is proportional to Ω^4, intensities of the order of S_c are hardly achievable at the present time. If $S \ll S_c$, then the height of the l-th peak is proportional to the l-th degree of the small parameter S/S_c; therefore the principal role is played by the process with $l = 1$. It must be noted that the nonequilibrium optical phonons in this case are not generated only in the process of multiphonon transition itself. The conduction electrons, thrown into a state with high energy, then return to the bottom of the conduction band, emitting in succession a series of optical phonons. If the frequency is a multiple of Ω_0 and the interaction with the optical phonons is appreciably greater than the interaction with the acoustic phonons, the distribution function of the electrons at the bottom of the conduction band can differ comparatively little from a Boltzmann function with temperature T.[3]

The intensity of the generated optical phonons is comparatively small. As has been mentioned, even at $l = 1$, it is proportional to the first power of the ratio S/S_c, i.e., at fixed intensity S, it is proportional to Ω^{-4}. Therefore, it is tempting to consider also in the opposite limiting case $\Omega \ll \Omega_0$.

3) in the case $\Omega \ll \Omega_0$, if $\exp(-\hbar\Omega_0/T) \ll 1$, there is a threshold in the intensity S, beginning with which the process of generation of optical phonons takes place. The position of the threshold depends on the phonon wave vector $\mathbf{q}$. It is easy to understand the cause of the threshold effect from the following discussion.

Let Ω belong to the microwave band. For a wavelength of 1 cm, the ratio Ω_0/Ω is of the order of one hundred, i.e., the emission of the optical phonon is accompanied by absorption of a large number of electromagnetic quanta. In this situation, we shall describe the interaction of the electron with the electromagnetic field in classical terms (the corresponding criteria will be formulated in detail below). In an alternating electromagnetic field of frequency Ω the electron undergoes classical oscillatory motion and the amplitude of the alternating component of its quasimomentum is $\hbar\Delta p = eE_0/\Omega$. The location of the threshold is obtained if the quantity $\hbar\Delta\mathbf{p}$ reaches the minimum value of the electron quasimomentum, $\hbar p_{\min}(\mathbf{q})$, at which the conservation

laws permit the radiation of a phonon with a wave vector $\mathbf{q}$. We have

$$\frac{\hbar^2 p^2}{2m} = \frac{\hbar^2}{2m}(\mathbf{p}-\mathbf{q})^2 + \hbar\Omega_0,$$

whence

$$p = \left(\frac{q}{2} + \frac{m\Omega_0}{\hbar q}\right)\frac{1}{\cos\alpha},$$

where α is the angle between $\mathbf{p}$ and $\mathbf{q}$. This expression is a minimum at $\cos\alpha = 1$, when

$$p = \tfrac{1}{2}q + m\Omega_0/\hbar q.$$

Equating this value to Δp, we find the threshold field $E_0(q) = m\Omega\Omega_0/eq$ and the threshold intensity

$$S_{\text{th}}(q) = c\varepsilon^{1/2}m^2\Omega^2\Omega_q{}^2/8\pi e^2 q^2, \tag{2}$$

where $\Omega_g = \Omega_0 + \hbar q^2/2m$. With further increase in the intensity, the number of generated phonons falls off in comparison with the threshold value.

In order to deal with the lowest possible intensities, it is convenient to use the lowest microwave frequencies Ω compatible with the condition

$$\Omega\tau_e \gg 1$$

which is one of the basic criteria for the applicability of the theory developed here.[2]

The quantity $S_{\text{th}}(q)$ reaches a minimum value at $q = q_0 = (2m\Omega_0/\hbar)^{1/2}$:

$$S_{\text{th}}(q_0) = \hbar c\varepsilon^{1/2}m\Omega^2\Omega_0/4\pi e^2, \tag{3}$$

which then represents the absolute threshold in intensity for generation of optical phonons by the microwave field. At Ω of the order of 10^{11} sec^{-1}, this quantity turns out to be of the order of hundreds of watts per square centimeter.

In this situation, an important role can be played by the "heating" of the conduction electrons by the microwave field, and then all the quantitative results obtained below are valid if we take T to mean the electron temperature, which is generally not identical with the temperature T_0 of the crystal lattice. The electron-temperature approximation is useful if the characteristic time τ_{ee} of the electron-electron collisions, which is of the order of

$$\tau_{ee} = \varepsilon^2 m^{1/2}T^{3/2}/n_0 e^4 L, \tag{4}$$

where L is the Coulomb logarithm, is smaller than the relaxation time of electrons with respect to the optical phonons. At an electron concentration $n_0 = 10^{10}$ cm^{-1}, $\varepsilon = 10$ and $T = 100$ K, we have $\tau_{ee} \sim 10^{-12}$ sec. How can the generation of optical phonons be recorded? The most effective are apparently the optical methods, for example, Raman light scattering. If we use visible light, such a method allows us to record phonons with q smaller than or of the order of several units in 10^5 cm^{-1}.

Another optical method applies, in first degree, to

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optically anisotropic semiconductors. In such crystals, the optical phonons are generally not purely longitudinal or transverse, but mixed.[4] Being coupled with both the longitudinal and transverse electric fields, they should on the one hand interact with the conduction electrons, and on the other, they should emit at a frequency Ω_0 on approaching the boundary of the crystal. From this viewpoint, the greatest physical interest, in our opinion, attaches to the case $\Omega \ll \Omega_0$, which means that the crystal emits infrared light under action of microwave radiation.

It should also be noted that in all circumstances, the principal mechanism of the loss of the optical phonons is not the transition to the external electromagnetic field, but decay into acoustical phonons. Therefore, their nonequilibrium distribution function is larger the smaller the probability of this decay. From this point of view, there is special interest in the case considered by one of the authors,[5] when the frequency of the longitudinal optical phonons in the entire optical branch or in a significant part of it exceeds twice the frequency of the phonons of any other branch. Then the longitudinal optical phonon cannot decay into two phonons, and its lifetime can turn out to be anomalously large.

We proceed to the quantitative solution of the problem. We shall see below that, by virtue of the unusual mechanism of generation of the optical phonons, the frequency interval in which its distribution is different from zero at given q can be small in comparison, for example, with its natural width Γ. In this case, the state of the phonons is not conveniently described by specifying their usual distribution function. We shall therefore formulate the problem in terms of Keldysh diagram technique,[6] which allows us to introduce a generalized phonon distribution function.

In the Keldysh technique, the phonon Green's function D is the matrix

$$\hat{D} = \begin{pmatrix} -i\langle T\varphi(x)\varphi(x')\rangle & -i\langle \varphi(x')\varphi(x)\rangle \\ -i\langle \varphi(x)\varphi(x')\rangle & -i\langle \tilde{T}\varphi(x)\varphi(x')\rangle \end{pmatrix} = \begin{pmatrix} D^c & D^+ \\ D^- & \tilde{D}^c \end{pmatrix}. \tag{5}$$

Here T ($\tilde{T}$) is the symbol of chronological (antichronological) product, x is the set of variables $\mathbf{r}$ and t, and the angular brackets denote averaging over some initial density matrix, the explicit form of which does not play a role.[6] The rules of the diagram technique are essentially those of Feynman, with only the addition that each vertex is characterized by an index 1 or 2, which is the matrix index of the ends of the propagator that meet at this vertex. The indices 1 (2) correspond to the factor 1 (-1); summation is carried out over the matrix indices corresponding to each vertex.

The Hamiltonian of the system is the sum of the energy operators of the electrons in an external periodic field, which we assume to be linearly polarized, with a vector potential $\mathbf{A}(t)$, of the phonons (with account taken of the phonon-phonon interaction due to the anharmonicity), and of the electron-phonon interaction (see Ref. 7). Carrying out a canonical transformation,[8] we can transfer the entire dependence on the field to the electron-phonon interaction, as a result of which the ex-

pression for the electron-phonon vertex is of the form

$$C_q(t) = C_q \exp\left[-i\frac{e}{mc}\mathbf{q}\int_0^t \mathbf{A}(\tau)\,d\tau\right], \tag{6}$$

where

$$C_q^2 = \frac{2\pi\hbar\Omega_0 e^2}{q^2}(\varepsilon_\infty^{-1} - \varepsilon_0^{-1}) \equiv \frac{\pi\hbar(2\hbar\Omega_0)^{1/2}\alpha}{m^{1/2}q^2}. \tag{7}$$

Here ε_0 is the static permittivity, ε_∞ is the permittivity at the plateau of the dispersion curve, and α is the Fröhlich coupling constant.[7]

Since the external field depends explicitly on the time, the Green's function of the phonons not only depends on the time difference $t_1 - t_2$, but is also a periodic function of the combined variable $t = (t_1 + t_2)/2$ with period $2\pi/\Omega$. We shall assume that the damping of the phonons Γ is small:

$$\Gamma/\Omega_0 \ll 1. \tag{8}$$

It can be shown that all the harmonics of the Green's function, except the zeroth, are small quantities in terms of this parameter. Therefore, we shall calculate in what follows only the zeroth harmonic. In the stationary state, it depends only on the difference $x - x'$ and we can calculate its Fourier component $\hat{D}(k)$, where k denotes the set of variables ω and $\mathbf{q}$.

In addition, we need the transformed phonon Green's function

$$\hat{D}' = U\hat{D}U^{-1} = \begin{pmatrix} 0 & D^a \\ D^r & D^d \end{pmatrix}, \tag{9}$$

where the unitary matrix $U = (1 - i\sigma_y)\sqrt{2}$, σ_y is the second Pauli matrix. Here $D^r(D^a)$ is the retarded (advanced) Green's function; in place of the function $D^d(k)$, it is convenient to introduce the generalized phonon function $N(k)$[3]:

$$D^d(k) = [2N(k) + 1][D^r(k) - D^a(k)]. \tag{10}$$

The various Green's functions satisfy the following relations, which are useful in specific calculations:

$$\begin{aligned} D^d &= D^c + \tilde{D}^c = D^+ + D^-, \\ D^a &= D^c - D^- = D^+ - \tilde{D}^c, \\ D^r &= D^c - D^+ = D^- - \tilde{D}^c. \end{aligned} \tag{11}$$

Using these relations and the definition (10), we obtain the following expressions for the functions $D^\pm(k)$ in terms of the generalized distribution function:

$$\begin{aligned} D^+(k) &= N(k)[D^r(k) - D^a(k)], \\ D^-(k) &= [N(k) + 1][D^r(k) - D^a(k)]. \end{aligned} \tag{12}$$

We wish to derive the kinetic equation for the function $N(k)$. It is contained in the Dyson equation for the function $\hat{D}(k)$, in which the polarization operator $\hat{\Pi}(k)$ enters. It constitutes a matrix of the form

$$\hat{\Pi}(k) = \begin{pmatrix} \Pi^c & \Pi^+ \\ \Pi^- & \tilde{\Pi}^c \end{pmatrix}; \quad \hat{\Pi}' = U\Pi U^{-1} = \begin{pmatrix} \Pi^d & \Pi^r \\ \Pi^a & 0 \end{pmatrix}, \tag{13}$$

which has the following properties:

$$\hat{\Pi}^*(k) = -\sigma_x \hat{\Pi}(-k)\sigma_x, \quad \hat{\Pi}'^*(k) = -\sigma_x \hat{\Pi}'(-k)\sigma_x,$$

$$\hat{\Pi}(-k) = \Pi^T(k), \quad \hat{\Pi}'(-k) = \hat{\Pi}'^T(k), \qquad (14)$$

where the asterisk denotes the complex conjugate and the superscript T denotes the transposed matrix. Using these properties, we break up $\Pi^{r,a}$ into a sum of real and imaginary parts.

$$\Pi^{r,a} = \Lambda \mp i\Gamma, \qquad (15)$$

where Λ and Γ are real functions satisfying the relations

$$\Lambda(k) = \Lambda(-k), \quad \Gamma(k) = -\Gamma(-k). \qquad (16)$$

By means of the above relations we can verify that the function is real and has the property

$$N(-k) = -[N(k)+1]. \qquad (17)$$

Further, limiting ourselves to the lowest approximation in the electron-phonon and phonon-phonon interactions, we shall neglect the small phonon frequency shift due to the quantity $\Lambda(k)$. By virtue of the smallness of the damping coefficient $\Gamma(k)$ in comparison with the frequency Ω_0, it is sufficient to know its value at $\omega = \Omega_0$ (on the mass shell). We set $\Gamma_q = \Gamma(q, \Omega_0)$.

The generalized kinetic equation is the equation for the function $D^+(t, q, \omega)$. In the spatially homogeneous case, it is of the form

$$i\frac{\partial D^+}{\partial t} = \Pi^+(k)D^-(k) - \Pi^-(k)D^+(k). \qquad (18)$$

The following pole approximation should be used, at the accuracy assumed by us, for the functions $D^\pm(k)$ which enter explicitly in the right-hand side:

$$D^+(k) = -2\pi i N(k)\Delta(\omega - \Omega_0, \Gamma_q),$$

$$D^-(k) = -2\pi i [N(k)+1]\Delta(\omega - \Omega_0, \Gamma_q),$$

$$\Delta(\omega, \Gamma) = \Gamma/\pi(\omega^2 + \Gamma^2). \qquad (19)$$

Substituting (19) in (18), we find for the rate of generation of the phonons a general expression that can be represented in the form of a sum of two terms (describing the induced and the spontaneous transitions, respectively):

$$\frac{\partial N(k)}{\partial t} = -2\Gamma(k)N(k) + G(k), \qquad (20)$$

where

$$2\Gamma(k) = i[\Pi^+(k) - \Pi^-(k)], \quad G(k) = -i\Pi^+(k). \qquad (21)$$

The polarization operator $\Pi(k)$, in lowest approximation, is the sum of the electron and phonon terms. To calculate the first, we use the zeroth approximation for the electron Green's functions. Finally, we obtain the electron contribution to the phonon damping[4]

$$\Gamma^e = \frac{2\pi}{\hbar^2}|C_q|^2 \sum_{l=-\infty}^{+\infty} J_l^2(a_0 q)\int \frac{d^3p}{(2\pi)^3}\delta\left(\frac{\varepsilon_p}{\hbar} + \Omega_0 - l\Omega - \frac{\varepsilon_{p+q}}{\hbar}\right)(F_p - F_{p+q}),$$
$$(22)$$

where F_p is the distribution function for electrons with quasimomentum $\hbar p$, $a_0 = e\,E_0/m\Omega^2$ is the amplitude of

the oscillations of the electrons in an alternating field of amplitude E_0, and $J_l(z)$ is a Bessel function of order l. If F_p is the Boltzmann function, then

$$\Gamma_q^e = \frac{2\pi}{\hbar^2}|C_q|^2 \sum_{l=-\infty}^{+\infty} J_l^2(a_0 q)P(q, \Omega_0 - l\Omega), \qquad (23)$$

where

$$P(q, \omega) = \frac{n_0}{2\pi^{1/2} q v_T} e^{\hbar\omega/2T}(1 - e^{-\hbar\omega/T})\exp\left\{-\left(\frac{m\omega^2}{2q^2 T} + \frac{\hbar^2 q^2}{8mT}\right)\right\},$$
$$v_T = (2T/m)^{1/2}.$$

The phonon contribution to the damping Γ_q^{ph} was measured experimentally in Refs. 12, 13 and was computed in Ref. 9.

The rate of spontaneous generation of phonons—the second term on the right side of (20)—is

$$G^e(k) = \frac{2}{\hbar^2}|C_q|^2 \sum_{l=-\infty}^{+\infty} J_l^2(a_0 q)\int \frac{d^3p}{(2\pi)^2} F_{p+q}(1 - F_p)$$

$$\times \delta\left(\frac{\varepsilon_p}{\hbar} + \omega - l\Omega - \frac{\varepsilon_{p+q}}{\hbar}\right) \qquad (24)$$

When $F_p \ll 1$ is the Boltzmann function, this expression goes over into

$$G^e(k) = 4\pi^{1/2}\frac{(\hbar\Omega_0)^{1/2}}{\hbar T^{1/2}}\alpha\frac{n_0}{q^3}\sum_{l=-\infty}^{+\infty} J_l^2(a_0 q)\exp\left[-\frac{m}{2q^2 T}\left(\omega - l\Omega + \frac{\hbar q^2}{2m}\right)^2\right].$$
$$(25)$$

In the stationary case of interest to us, the time derivative on the right side of (20) is equal to zero, and at $\omega > 0$ we get

$$N(k) = G(k)/2\Gamma_q, \quad G = G^e + G^{ph}, \quad \Gamma = \Gamma^e + \Gamma^{ph}.$$

As is clear from Eq. (25), the generalized distribution function $N(k)$ can be a rather sharp function of frequency ω. In particular, its frequency width $\Delta\omega$ can turn out to be small in comparison with the quantity Γ_q. Properly speaking, it is this which brings about the necessity of a description in terms of the generalized function $N(k)$ rather than the ordinary distribution function N_q. If the frequency width is large in comparison with Γ_q, then we can introduce the ordinary distribution function[9]

$$N_q = \int d\omega\, N(k)\Delta(\omega - \Omega_0, \Gamma_q).$$

In other words, at $\Delta\omega \gg \Gamma_q$, the expression for N_q is obtained if we set $\omega = \Omega_0$ in $N(k)$. At equilibrium, when the external electromagnetic field is absent and the distribution function of the electrons and the acoustic phonons are respectively the Fermi and Planck functions, we likewise obtain for N_q, as we should, the Planck function, equal to $\exp(-\hbar\Omega_0/T)$ at $T \ll \hbar\Omega_0$.

We are interested in those cases in which the function $N(k)$, as a consequence of the significant intensity of the radiation, is much larger than its equilibrium value and, in particular, does not contain an exponential small factor, i.e., the case in which the electron generation germ G^e is sufficiently large. We shall not take into consideration the phonon term G^{ph}, which describes the

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reverse process of sticking of the acoustic phonons with formation of an optical phonon since there are usually a number of other channels which more effectively ensure the destruction of these acoustic phonons. (In those comparatively rare cases in which this is not so, the distribution function $N(k)$ will be determined not by the rate of decay of the optical phonons but by the rate of destruction of the acoustic phonons via these other channels.)

So far as the damping coefficient Γ is concerned, we shall first discuss the case in which the electron concentration n_0 or the electron-phonon coupling constant is so small that the phonon contribution predominates in the damping. We can assume Γ_q^{ph} to be a constant weakly dependent on q, since the vector q is much smaller than the reciprocal of the lattice constant. We present the quantitative results which follow from the general formula (25) in the different limiting cases.

Let $\Delta\Omega_l \equiv \Omega_0 - l\Omega > 0$. The argument of the exponential in the l-th term of the sum turns out to be smaller than or of the order of unity if $\hbar\Delta\Omega_l/T \lesssim 1$, and in this case the values of q lie in the range $q_l^2/q_T \lesssim q \lesssim q_T$, where

$$q_T = \hbar^{-1}(2mT)^{1/2}, \quad q_l = (2m|\Delta\Omega_l|/\hbar)^{1/2}. \tag{26}$$

We now consider $\Delta\Omega_l < 0$. At $\hbar|\Delta\Omega_l|/T \lesssim 1$ the previous interval (26) of allowed values of q is obtained. At $\hbar|\Delta\Omega_l|/T \gg 1$, the condition under which the exponent is less than or of the order of unity gives an interval of width $\Delta q \sim q_T$ near $q = q_l$. Thus, if $\hbar|\Omega_0 - \Omega| \gg T$, the phonon distribution function has no longwave tail and it forms a system of narrow peaks near $q = q_l$.

In order to estimate the order of the quantity $N(k)$, we must estimate the argument of the Bessel function. As before, we shall be interested in such q at which the argument of the exponential in (25) is less than or of the order of unity. As an example, we consider the first resonance, when $\hbar|\Omega_0 - \Omega|/T \ll 1$ and the fundamental contribution to the sum (25) is made by the terms with $l = 1$. If $S \ll S_c$, the argument of the Bessel function is much less than unity and we can use the experience $J_1(z) = z/2$. If we assume for the subsequent estimates $n_0 = 10^{16}$ cm^{-3}, $\Gamma_q^{ph} = 5\times10^{10}$ sec^{-1}, and $\alpha \sim 0.1$, then we obtain from (25) at $\omega = \Omega_0$ and $q = 10^5$ cm^{-1} that $N(k) = 1$ at $S = 7\times10^5$ W/cm^2.

We now consider the case $\Omega_0/\Omega \gg 1$, where Ω is the microwave frequency. The value of the function $N(k)$ in this case depends essentially on the parameter

$$\gamma = m\Omega^2/2q^2 T. \tag{27}$$

Accordingly, we distinguish between two limiting cases.

a) The parameter $\gamma \gg 1$. It is most convenient to analyze the results with the help of Eq. (25). In this case, only one term in the sum turns out to be important, with l equal to the nearest (larger or smaller) integer value of the ratio Ω_0/Ω if, in addition, the following inequality is satisfied:

$$(\Omega_0 - l\Omega)^2 \lesssim q^2 T/m$$

(it is assumed that the radiofrequency $\Omega \ll T/\hbar$). The maximum yield of radiated phonons is obtained at the intensities S_{th} (2).

b) At $\gamma \ll 1$, many terms in the sum (25) are important and, in order to study the expression (25), we first transform it. We represent the sum over l in (25) in the following form

$$A = \frac{1}{\pi^{1/2}}\int_{-\infty}^{+\infty} dy \exp\left[-y^2 - 2i\left(\frac{m}{2T}\right)^{1/2}\frac{\Omega_q}{q}y\right]\sum_{l=-\infty}^{+\infty} J_l^2(z)e^{2i\gamma^{1/2}y l}, \tag{28}$$

where $\Omega_q = \Omega_0 + \hbar q^2/2m$ and $z = \mathbf{a}_0\cdot\mathbf{q}$, and we make use of the Neumann expansion theorem for Bessel functions, which gives

$$\sum_{l=-\infty}^{+\infty} J_l^2(z)e^{il\varphi} = J_0\left(2z\sin\frac{\varphi}{2}\right).$$

As a result, (28) takes the form

$$A = \frac{1}{\pi^{1/2}}\int_{-\infty}^{+\infty} dy \exp\left[-y^2 - 2i\left(\frac{m}{2T}\right)^{1/2}\frac{\Omega_q}{q}y\right]J_0[2z\sin(\gamma^{1/2}y)].$$

In this integral, the region $y \lesssim 1$ is important and therefore at $\gamma \ll 1$ we can set $\sin(\gamma^{1/2}y) = \gamma^{1/2}y$. Furthermore, making use of the integral representation of the Bessel function J_0, we obtain[5]

$$G^e = 4\pi^{1/2}\frac{(\hbar\Omega_0)^{1/2}}{\hbar T^{1/2}}\alpha\frac{n_0}{q^3}\int_{-\pi/2}^{\pi/2} d\psi \exp\left\{-\frac{m}{2q^2T}(\Omega_q - \Omega|z|\sin\psi)^2\right\}. \tag{29}$$

At high temperatures, when $T \gg m\Omega_q/q^2$, we immediately obtain

$$G^e = 4\pi^{1/2}\frac{(\hbar\Omega_0)^{1/2}}{\hbar T^{1/2}}\alpha\frac{n_0}{q^3}. \tag{30}$$

Since

$$\sum_{l=-\infty}^{+\infty} J_l^2(z) = 1,$$

the quantity (30) represents the maximum possible rate of phonon generation via the mechanism considered by use (see (25)).

We now consider the opposite case $T \ll m\Omega_q/q^2$. If $\Omega_q - \Omega|z| \gg q(T/m)^{1/2}$, then the expression (29) is exponentially small.

If the field intensity is so close to threshold that $|\Omega_q - \Omega|z|| \ll q(T/m)^{1/2}$, then the main contribution to the integral is made by values of ψ close to $\pi/2$, and we obtain

$$G^e = (2\pi)^{1/2}\Gamma(1/4)\frac{(\hbar\Omega_0)^{1/2}}{\hbar T^{1/2}}\alpha\frac{n_0}{q^3}\left(\frac{2q^2T}{m\Omega_q^2}\right)^{1/4}. \tag{31}$$

At high microwave intensities, when $\Omega|z|/\Omega_q \gg 1$, the principal contribution to the integral is made by ψ close to zero. Setting $\sin\psi = \psi$ and integrating, we obtain

$$G^e = 2^{3/4}\pi^{1/2}\frac{(\hbar\Omega_0)^{3/4}\alpha n_0}{\hbar\Omega m^{1/4}|z|q^2}. \tag{32}$$

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It is important to note that a rather sharp directivity diagram of generated phonons arises near the threshold. To be precise, sufficiently close to threshold, when $|\Omega_q - |z|\Omega| \ll q(T/m)^{1/2}$, the range of angles θ between the wave vectors of the generated phonons and the direction of the electric field is

$$\theta \ll 2q(T/m\Omega_q^2)^{1/2}. \tag{33}$$

Up to now, we have assumed that the electron damping of the optical phonons is small in comparison with the damping due to phonon anharmonicity. At sufficiently high concentrations n_0 and values of the coupling constant α, the contribution of the electrons to the damping can in principle be more important. The expression (23) for the quantity Γ^e, which describes this contribution, will not be investigated in detail here, since it is not clear to us at the present time whether it is possible to satisfy in the experiment the condition $\Gamma^e \gtrsim \Gamma^{ph}$ simultaneously with the other criteria formulated above for the applicability of our theory.

We wish, however, to indicate that, under certain relations between the frequencies Ω and Ω_0, the quantity Γ^e can be negative. The reason for this consists in the following.

The number of induced transitions with absorption of an optical phonon and emission of l electromagnetic quanta is proportional to the difference in the probabilities of the corresponding direct and reverse transitions, i.e., to the difference $F(\varepsilon_p) - F(\varepsilon_p + \hbar\Omega_0 - \hbar l\Omega)$ (see (23)). For the equilibrium distribution function $F(\varepsilon_p)$, this difference is positive if $\Omega_0 > l\Omega$. If $\Omega_0 < l\Omega$, then the corresponding term of the sum (23) turns out to be negative. This means that, because of the energy entering from the external field, the number of reverse transitions with emission of an optical phonon exceeds the number of direct transitions. If such negative terms make the dominant contribution to the sum (23), then an unusual instability of the system of optical phonons develops in the presence of an intense external electromagnetic field.

In conclusion, we discuss the question of the value of the electron temperature reached under our conditions. If scattering from acoustic phonons predominates, then we have at $eE_0 \gg \Omega(mT_0)^{1/2}$ [14]:

$$T = \frac{\pi^3}{32}\frac{e^4 E_0^4}{\Omega^4 m^3 s^2}$$

where s is the sound velocity. If scattering from optical phonons predominates, then a comparatively complicated integral equation is obtained for T. Its numerical solution gives the value $\exp(-\hbar\Omega_0/T) \sim 0.2$ in the case when the amplitude of the electric field is equal to the threshold value $E_0(q_0)$, so that a rather sharp threshold is obtained for generation of optical phonons.

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1) We neglect the small dispersion of the optical phonons.
2) If the carrier density is sufficiently large, so that the frequency Ω turns out to be less than the plasma frequency,

then the permittivity of the semiconductor is negative and the electromagnetic field falls off exponentially on going from the surface of the semiconductor to the interior. The attenuation length can, nevertheless turn out to be comparatively large and then the generation of optical phonons can be observed in the near-surface layer.
3) The Keldysh diagram technique for the study of nonequilibrium phonons in solids was used by Levinson.[9] His definition of the function $N(k)$ differs somewhat from ours.
4) Sums of this type, which describe the probabilities of multiphoton transitons, were encountered in the papers of Épshteĭn.[10,11]
5) The transition from summation over l to an integral of the form (29) means a transition from quantum theory to classical theory. The corresponding result could obviously be obtained also from the kinetic equation directly. On the other hand, the transition from the sum to the integral can be made not only when the electrons have a Boltzmann distribution, but also in the general case.[14] This gives the following general formula for the rate of generation of phonons:

$$G^e(k) = \frac{2}{\pi\hbar^3}|C_q|^2\frac{m}{q}\int_{-\pi/2}^{\pi/2}d\psi\,\Phi\left(\frac{eE_0q}{\hbar q\Omega}\sin\psi - \frac{q}{2} - \frac{m\Omega_0}{\hbar q}\right),$$

where

$$\Phi(p_z) = \sum_{p_x p_y}F(p),$$

the p_z axis is parallel to q.

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