ductivity is, apparently, produced only by solitons and moving walls.[9]

We express our gratitude to Yu. V. Kopaev for useful remarks.

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Translated by D. ter Haar

Electron-phonon interaction in one-dimensional systems in the adiabatic approximation

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(Submitted October 13, 1976)
Zh. Eksp. Teor. Fiz. 72, 1559–1567 (April 1977)

A one-dimensional Fermi gas interacting with lattice vibrations is considered in the adiabatic approximation. It is shown that the system should be a superconductor at small electron-phonon coupling constants g^2, in accord with the results of Fröhlich. [1] The critical velocity at which the superconducting state is stable with respect to scattering by impurities is found. The extreme case of large values of g^2 is investigated, and it is shown that in this case the problem reduces to the analysis of a system of almost noniteracting polarons. It is also shown that for the lattice Fermi gas the ground state corresponds to the Peierls state with a deformed lattice, and that the system is a dielectric.

PACS numbers: 74.20.–z

As is well known, the one-dimensional system of electrons interacting with a lattice, considered by Fröhlich in 1954,[1] was the first example of a model for which the dependence of the energy on the constant of the electron-phonon interaction bore a non-analytic character. It was later made clear, however, that the nature of the ground state of the electron-phonon system is completely different in the three-dimensional and one-dimensional cases. According to Ref. 1, in the one-dimensional case the system is in the so-called Peierls state, the characteristic feature of which is the presence of strongly excited lattice vibrations with wave number $q = 2k_F$, in whose periodic field the electrons move. Here the system of electrons and lattice displacements can move with the velocity v. On the basis of arguments similar to Landau's well-known argument, Fröhlich identified these states at $v \neq 0$ with the superconducting states. On the other hand, Allender, Bray, and Bardeen,[2] state that at $\rho = 1$ this system is a typical dielectric ($\rho = n/N$, where n is the number of electrons and N is the number of centers in the lattice).

The purpose of the present work is the study of the phase states of a one-dimensional electron-phonon system as a function of ρ in the adiabatic limit (we note that both the early work of Fröhlich[1]; and the work of Ref. 2 were carried out in just this approximation). We shall show that the analysis of Fröhlich is valid only at $g^2 \ll \rho \ll 1$. At these values of the parameters of the system, we investigate the stability of the superconducting states relative to scattering by impurities and calculate the critical velocity v_{cr}. It will be shown that in the presence of a strong electron-phonon interaction, $g^2 \gg \rho$ ($\rho \ll 1$), polaron states appear in the system. So far as the lattice Fermi gas is concerned, in this case the electron-phonon interaction forms a Peierls state with a distorted lattice and, as will be shown, the system is a dielectric.

 0038-5646/77/4504-0818$02.40

Thus, we consider a system characterized by the Hamiltonian

$$\hat{H}=\sum_{n\sigma}\beta_n(a_{n\sigma}^{+}a_{n+1\,\sigma}+a_{n+1\,\sigma}^{+}a_{n\sigma})+\frac{\varkappa}{2}\sum_{n}\xi_n^{2}+\frac{1}{2M}\sum_{n}p_n^{2}, \tag{1}$$

where $\xi_n=u_{n+1}-u_n$, u_n is the displacement of the n-th center from the equilibrium position, p_n is the momentum corresponding to this displacement, M is the nuclear mass, and $\varkappa$ is the elastic constant. At small displacements u_n, we have $\beta_n=\beta_0+\beta'\xi_n$.

Introducing the phonon creation and annihilation operators, we can write the Hamiltonian (1) in the form

$$\begin{aligned}\hat{H}&=2\beta_0\sum_{k\sigma}a_{k\sigma}^{+}a_{k\sigma}\cos k+\sum_{q}\omega_q\left(b_q^{+}b_q+\frac{1}{2}\right)\\&+\frac{2i\beta'}{N^{1/2}}\sum_{kq\sigma}[\sin(k+q)-\sin k]a_{k+q\,\sigma}^{+}a_{k\sigma}g(q)\,(b_q+b_{-q}^{+}),\\\omega_q&=\frac{2\varkappa}{M}\left|\sin\frac{q}{2}\right|,\quad g(q)=\frac{1}{(2M\omega_q)^{1/2}}.\end{aligned} \tag{2}$$

In going from (1) to (2), we assume that the lattice constant $a=1$ and $\hbar=1$.

We note that $\hat{H}$ commutes with $\hat{P}$, where $\hat{P}$ is the quasimomentum operator and is of the form:

$$\hat{P}=\sum_{k\sigma}ka_{k\sigma}^{+}a_{k\sigma}+\sum_{q}qb_q^{+}b_q. \tag{3}$$

Therefore the eigenstates of $\hat{H}$ are characterized by definite values of the quasimomentum.

In the adiabatic approximation, i.e., at $\omega_q=0$ or $M\to\infty$, we can find the exact eigenvalues and eigenfunctions of the system. In fact, to find the energy of the ground state, it is necessary to minimize the total energy with respect to the displacements u_n. It is easy to see that

$$\xi_n=-\frac{\beta'}{\varkappa}\sum_{\sigma}\langle a_{n\sigma}^{+}a_{n+1\,\sigma}+a_{n+1\,\sigma}^{+}a_{n\sigma}\rangle, \tag{4}$$

where the average should be calculated in terms of the eigenfunction of the ground state. Substituting (4) in (1), we obtain a Hamiltonian of the type of the Hartree Hamiltonian, which can be solved exactly if we assume that the numbers ξ_n form the following sequence:

$$\xi_n=\{\ldots\xi_1,\ \xi_2,\ \ldots,\ \xi_p,\ \xi_1,\ \xi_2,\ \ldots,\ \xi_p,\ \ldots\}, \tag{5}$$

$\rho=2/p$ is a rational number. There are p self-consistent solutions of the quantities ξ_n of the type (5), which can be obtained by cyclic permutation of the numbers $\xi_1,\xi_2,\ldots,\xi_p$. The wave functions of the system are linear combinations of p functions with the ξ_n, determined from (5):

$$\Phi_n=\sum_{k=0}^{p-1}\exp\left(-\frac{2\pi i}{p}nk\right)\psi_k,\quad n=0,\ldots,p-1, \tag{6}$$

and possess the correct symmetry relative to the translation group, while the eigenvalue P_n of the quasimomentum in the states (6) are equal to

$$P_n=\exp\{2i\pi n/p\}. \tag{7}$$

In particular, for $p=2$ (i.e., $\rho=1$) the solution of the type (5) means that the distances between the centers alternate and two choices of (5) correspond to the two possible configurations of the lattice with $\xi_{n1}=(-1)^n u$ and $\xi_{n2}=(-1)^{n+1}u$ (u is a parameter determined by minimization of the total energy[3]). A gap appears at $k=k_F$ in the spectrum of the energy of the electron states ($k_F=\pi\rho/2$), and the magnitude of the gap, for example, at $\rho=1$, is equal to $4|\beta'|u$.

We now calculate the mean value of the current operator $\hat{I}$ in the states (6). The operator $\hat{I}$ has the form

$$\hat{I}=ie\sum_{n\sigma}(a_{n+1\,\sigma}^{+}a_{n\sigma}-a_{n\sigma}^{+}a_{n+1\,\sigma})=2e\sum_{k\sigma}a_{k\sigma}^{+}a_{k\sigma}\sin k. \tag{8}$$

In the calculation of $I_n=\langle\Phi_n\hat{I}\Phi_n\rangle$ we have in mind the fact that in the adiabatic approximation each of the ψ_k entering into (6) is represented in the form $\psi_k=u_k\chi_k$, where u_k and χ_k are the electron and vibrational wave functions, respectively; χ_k describes the vibrations of the nuclei near the new equilibrium position corresponding to the sets (5). Here $\langle\psi_k\hat{I}\psi_k\rangle=0$ by virtue of the fact that all the states below the gap are filled and $\langle u_k\hat{I}u_k\rangle=0$, while $\langle\psi_k\hat{I}\psi_{k'}\rangle=0$ at $k\neq k'$, since we have for the overlap integral of the oscillator functions corresponding to lattices with non-identical displacements $\langle\chi_k\chi_{k'}\rangle\to0$ as $N\to\infty$ (for example, in the case $\rho=1$ we have $\langle\chi_1\chi_2\rangle=\exp[-N(M\varkappa)^{1/2}u^2/2]$). As a result, $I_n=0$ for all n and the system is a dielectric.

As is not difficult to see, a similar consideration is valid at all rational values of ρ and, it would seem, one should expect that the one-dimensional electron-phonon system is always a dielectric in the adiabatic approximation. However, one can cite the following qualitative arguments that indicate the inapplicability of the considerations given above at $\rho\ll1$. We first consider the case $\rho=1$. The energy of the ground state as a function of ξ_n has the form shown in Fig. 1. (Although ξ_n takes on discrete values, E is shown as a continuous curve in the figure for the sake of clarity). The minimum E corresponds to the values ξ_{n1} and ξ_{n2}. A similar situation occurs for the other ρ. The corresponding plots of $E(\xi_1,\ldots,\xi_p)$ will have a more complicated form, but there will always exist a barrier for transition of the lattice from one configuration to the other. However, we note that we have neglected, up to now, the kinetic energy of the nuclei. Account of the vibrations of the atoms about the new equilibrium positions can alter the picture drawn above. In fact, if $\hbar\omega$ (the vibrational quantum) becomes much greater than the height of the barrier, the different configurations of the lattice will be indistinguishable and the lattice itself can be represented as a continuous distribution of nuclei (a string). Since the barrier height and the corresponding deformation energy is proportional to $\varepsilon_F\exp(-1/g^2)$ (ε_F is the Fermi energy), the inequality $\hbar\omega\gg\varepsilon_F\exp(-1/g^2)$ will be satisfied at $k_F\ll1$ (and, consequently, $\rho\ll1$). A rigorous criterion for the transition to the continuous distribution require the finding of values of ξ_n that min-

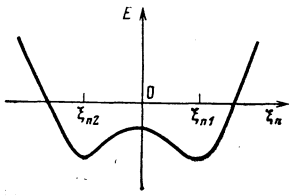

FIG. 1. Dependence of E on ξ_n at $\rho = 1$.

imize the total energy for $\rho \ll 1$, and this is very difficult to obtain.

We now proceed to consider a Fermi gas which interacts with the vibrations of a string. We shall consider the string vibration semiclassically here, i.e., we shall describe the displacements of points of the string by means of the function $\varphi(x,t)$ (here $\xi_n \to \partial\varphi/\partial x$). In addition, in view of the fact that $\rho \ll 1$, the energy of an electron $\varepsilon(k)$ with momentum k is equal to $k^2/2m$, where $m = 1/2|\beta_0|$. It is also convenient to transform from the second-quantization representation to the coordinate representation. This transition is accomplished by introduction of a wave function having the form

$$\psi = \sum_{x_1,\ldots,x_N} \Phi(x_1,\ldots,x_N;t)\, a^+_{x_1\sigma_1}\ldots a^+_{x_N\sigma_N}|0\rangle,$$

where $a^+_{x\sigma}$ is the creation operator for an electron with spin σ at the point x. We represent $\Phi(x_1,\ldots,x_N;t)$ in the form

$$\Phi(x_1,\ldots,x_N;t) = u_1(x_1,t)\,u_2(x_2,t)\ldots u_N(x_N,t),$$

where the $u_j(x,t)$ are orthonormalized. With account of this, the Lagrangian of the considered system L is written down in the form

$$\begin{aligned} L = i\sum_i \int \left\{u_i^*(x,t)\frac{\partial}{\partial t}u_i(x,t) - u_i(x,t)\frac{\partial}{\partial t}u_i^*(x,t)\right\}dx \\ + \frac{M}{2}\int\left(\frac{\partial\varphi}{\partial t}\right)^2 dx + \frac{1}{2m}\int u_i^*(x,t)\frac{\partial^2}{\partial x^2}u_i(x,t)\,dx \\ - \frac{\varkappa}{2}\int\left(\frac{\partial\varphi}{\partial x}\right)^2 dx + 2\beta'\sum_i\int\frac{\partial\varphi}{\partial x}|u_i(x,t)|^2\,dx. \end{aligned} \tag{9}$$

The corresponding equations of motion for $u_i(x,t)$ and $\varphi(x,t)$ are

$$\begin{aligned} \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} &= \frac{\partial^2\varphi}{\partial x^2} - \frac{2\beta'}{\varkappa}\sum_i\frac{\partial}{\partial x}|u_i|^2, \\ i\frac{\partial u_i}{\partial t} &= -\frac{1}{2m}\frac{\partial^2 u_i}{\partial x^2} - 2\beta'\frac{\partial\varphi}{\partial x}u_i, \end{aligned} \tag{10}$$

$c = (\varkappa/M)^{1/2}$ is the sound velocity. In view of the linearity of the first of Eqs. (10), $\varphi(x,t)$ can be written in the form

$$\varphi(x,t) = \sum_i \varphi_i(x,t),$$

where $\varphi_i(x,t)$ satisfies the equation

$$\frac{1}{c^2}\frac{\partial^2\varphi_i}{\partial t^2} = \frac{\partial^2\varphi_i}{\partial x^2} - \frac{2\beta'}{\varkappa}\frac{\partial}{\partial x}|u_i|^2.$$

We note the following property of the solutions of the set (10). If $\varphi_i(x,t) = \varphi_i(x)$, then the functions $u_i(x,t)$ satisfy the system of equations

$$i\frac{\partial u_i}{\partial t} = -\frac{1}{2m}\frac{\partial^2 u_i}{\partial x^2} - 4\lambda^2 u_i\left(\sum_j |u_j|^2\right), \tag{11}$$

where $\lambda^2 = \beta'^2/\varkappa$. It is easy to see that the Galilean transformation $x' = x + vt$ leads to the replacement of λ^2 in (11) by $\lambda'^2 = \lambda^2(1 - v^2/c^2)^{-1}$, i.e., if the system moves with the velocity v, a renormalization of the quantity λ^2 takes place.

There is a basis for assuming that the set (11) can be solved exactly. This follows from the fact that this system is completely integrable and we apply the method of the inverse scattering problem for its solution. However, we limit ourselves to the investigations of limiting small and large values of the electron-phonon interaction constant g^2 ($g^2 = 2\lambda^2 m$).

The stationary states of electrons in the field of forces due to interaction with the lattice vibrations are determined from the set of equations

$$E_i\psi_i = -\frac{1}{2m}\frac{\partial^2\psi_i}{\partial x^2} - 4\lambda^2\psi_i\left(\sum_j |\psi_j|^2\right), \tag{12}$$

where $\psi_i(x) = u_i(x,t)\exp\{iE_i t\}$. We first consider the limit of small g^2. 1) Let $k_F \gg g^2$. The solution of (12) in this case can be obtained in self-consistent fashion. We represent Eq. (12) in the form

$$E_i\psi_i = -\frac{1}{2m}\frac{\partial^2\psi_i}{\partial x^2} + V(x)\psi_i. \tag{13}$$

Following Fröhlich,[1] we shall assume $V(x)$ to be a periodic potential with period $2k_F$. We represent $V(x)$ in the form

$$V(x) = -4\lambda^2\sum_j |\psi_j|^2 = a_0 + \sum_{n=1}^{\infty} a_n\cos 2k_F n x, \tag{14}$$

where a_0 and a_n are coefficients subject to determination. Since the term a_0 in the expansion (14) leads only to a shift of all the energy levels by a constant amount, we can set it equal to zero. We note that all a_n with $n \geqslant 2$ were neglected in Ref. 1.

We shall seek a solution of (13) by means of perturbation theory. (At $V(x) = 0$, the functions ψ_i are plane waves, the states with $|k| < k_F$ are being occupied and those with $|k| > k_F$ free.) In the application of perturbation theory it must be taken into account that the states with $k \lesssim k_F$ and $k \gtrsim -k_F$ are almost degenerate relative to states with $k' = k - 2k_F$ and $k' = k + 2k_F$, respectively. Lifting of the degeneracy leads to the functions

$$\psi_k^{(1)}(x) = \frac{1}{N^{1/2}}\left[\cos\frac{\beta}{2}\exp\{ikx\} + \sin\frac{\beta}{2}\exp\{i(k-2k_F)x\}\right], \tag{15a}$$

$$g_k^{(1)}(x) = \frac{1}{N^{1/2}}\left[-\sin\frac{\beta}{2}\exp\{ikx\} + \cos\frac{\beta}{2}\exp\{i(k-2k_F)x\}\right], \tag{15b}$$

where $\tan\beta = a_1/2k_F(k - k_F)$. The energies corresponding to $\psi_k^{(1)}$ and $g_k^{(1)}$ are

$$E_{1,2}(k)=\tfrac{1}{2}[\varepsilon(k)+\varepsilon(k-2k_F)]$$
$$\mp\{[\varepsilon(k)-\varepsilon(k-2k_F)]^2+a_1^2\}^{1/2}, \quad (16)$$

where the minus sign before the radical corresponds to (15a) and the plus sign to (15b). We note that (15) and (16) correspond to $k>0$. The corresponding expressions for $k<0$ have a similar form. As follows from (16), the quantity a_1 is equal to the gap in the spectrum of the single particle excitations.

In addition, applying the second order (in $V(x)$) perturbation theory, we obtain the following expression for $\psi_k(x)$

$$\psi_k(x)=\psi_k^{(1)}(x)+\frac{e^{ikx}}{2N^{1/2}}\Big\{\sum_{n=2}^{\infty}\Big(a_n\cos\frac{\beta}{2}+a_{n-1}\sin\frac{\beta}{2}\Big)\times\frac{\exp(-2ik_Fnx)}{\varepsilon(k)-\varepsilon(k-2k_Fn)}$$
$$+\sum_{n=1}^{\infty}\frac{(a_n\cos{}^1/_2\beta+a_{n+1}\sin{}^1/_2\beta)\exp(2ik_Fnx)}{\varepsilon(k)-\varepsilon(k+2k_Fn)}\Big\}, \quad (17)$$
$$0<k<k_F.$$

It is easy to find the coefficients a_n from the self-consistency conditions with the use of (17) and (14):

$$a_1=16\varepsilon_F\exp(-\pi k_F/2g^2), \quad a_2=-a_1^2/8\varepsilon_F,$$
$$a_{2n+1}\sim\alpha^n a_1, \quad a_{2n}\sim\alpha^{n-1}a_2; \quad \alpha=\frac{g^2}{k_F}\exp\Big(-\frac{\pi k_F}{2g^2}\Big). \quad (18)$$

As is seen from (18), $|a_2/a_1|=2\exp(-\pi k_F/2g^2)\ll 1$, and the approximation used in Ref. 1 corresponds to neglect of the exponentially small terms. However, we note that allowance for these terms gives, in accord with (18), a value for the gap that is twice as large as that obtained in Ref. 1.

We now proceed to the study of the current states of the system at $k_F\gg g^2$. Since the total momentum of the system is conserved at $k_F\ll 1$ (the quasimomentum operator (3) is the momentum operator at $k_F\ll 1$), a current appears when the system moves with velocity v. Fröhlich[1] drew the conclusion that such current states are superconducting. The argument for this in Ref. 1 is based on the fact that the existence of a gap in the spectrum of single-particle excitations excludes the possibility of elastic scattering if v is sufficiently small. Along with this, the critical v_{cr} itself was not given in Ref. 1.

We shall assess the possibility of identifying the states mentioned above with superconducting states by investigating their stability relative to scattering by impurities. In the presence of impurities, the system (11) takes the form

$$i\frac{\partial u_k(x,t)}{\partial t}=-\frac{1}{2m}\frac{\partial^2 u_k(x,t)}{\partial x^2}$$
$$-4\lambda'^2u_k\Big(\sum_{k'}|u_{k'}(x,t)|^2\Big)+\tilde{V}(x-vt)u_k, \quad (19)$$

where $\tilde{V}(x)$ is the impurity potential (it is assumed that the impurities are randomly distributed and $\tilde{V}(x)\ll\lambda^2$). The set (19) is written down in a system of coordinates moving relative to the impurities with velocity v.

We shall seek the solution of (19) by perturbation theory, limiting ourselves to first order in $\tilde{V}(x)$. Let $u_k(x,t)=u_k^{(0)}(x,t)+\chi_k(x,t)$, where $u_k^{(0)}$ is the solution of (19) at $\tilde{V}(x)=0$. Then χ_k is found from the linearized set of equations:

$$i\frac{\partial\chi_k}{\partial t}=-\frac{1}{2m}\frac{\partial^2\chi_k}{\partial x^2}-4\lambda'^2u_k^{(0)}\sum_{k'}(\chi_{k'}u_{k'}^{(0)}+u_{k'}^{(0)}\chi_{k'}^*)$$
$$-4\lambda'^2\chi_k\sum_{k'}|u_{k'}^{(0)}|^2+\tilde{V}(x-vt)u_k^{(0)}. \quad (20)$$

We seek the functions χ_k in the form

$$\chi_k=\sum_{k'}a_{k'k}(t)\tilde{\psi}_{k'}(x,t)+\sum_p b_{pk}\tilde{g}_p(x,t), \quad (21)$$

where

$$\tilde{\psi}_k(x,t)=\exp\{-iE_1(k)t\}\psi_k(x), \quad \tilde{g}_p(x,t)=\exp\{-iE_2(p)t\}g_p(x).$$

As is seen from (17), with accuracy up to exponentially small terms, $\psi_k(x)$ and $g_p(x)$ are equal respectively to $\psi_k^{(1)}$ (15a) and $g_p^{(1)}(x)$ (15b). ($E_1(k)$ and $E_2(k)$ are defined in accord with (16).)

Since the total wave function of the system is a determinant constructed of the functions $u_k(x,t)$ with $|k|<k_F$, it follows that the functions χ_k are completely defined by the coefficients $b_{pk}(t)$. For the determination of these coefficients, we have the set of equations

$$i\frac{\partial b_{wk}}{\partial t}=-4\lambda'^2\sum_{k'p}b_{pk'}\int\tilde{g}_w^*\tilde{\psi}_k\tilde{\psi}_{k'}^*\tilde{g}_p dx$$
$$-4\lambda'^2\sum_{k'p}b_{pk'}^*\int\tilde{g}_w^*(x)\tilde{\psi}_k(x)\tilde{\psi}_{k'}(x)\tilde{g}_p^*(x)dx$$
$$+\int\tilde{g}_w^*\tilde{V}(x-vt)\tilde{\psi}_k dx. \quad (22)$$

We shall seek the solution of (22) in the form

$$b_{wk}(t)=c_{wk}(t)\exp\{-i(E_1(k)-E_2(w))t\}. \quad (23)$$

The coefficients $c_{wk}(t)$ satisfy the equations

$$i\frac{\partial c_{wk}}{\partial t}+(E_1(k)-E_2(w))c_{wk}$$
$$=-4\lambda'^2\sum_{k'p}c_{pk'}\int g_w^*\psi_k\psi_{k'}^*g_p dx$$
$$-4\lambda'^2\sum_{k'p}c_{pk'}^*\int g_w^*\psi_k\psi_{k'}g_p^*dx+\int g_w^*\tilde{V}(x-vt)\psi_k dx. \quad (24)$$

We first consider the homogeneous set of equations corresponding to (24). Its solution is naturally sought in the form

$$c_{k+q,k}(t)=B_{k+q,k}e^{-i\tilde{\omega}(q)t}, \quad w=k+q. \quad (25)$$

Since scattering of electrons with momenta $|k|\approx k_F$ is the most significant from the viewpoint of stability of the current states (the transferred momentum here is approximately equal to zero or $2k_F$), we shall consider the solution of (24) in the form (25) for k and q satisfying the conditions

$$\{|q|,\ |k-k_F|\}\ll k_F\exp(-\pi k_F/2g^2). \quad (26)$$

The solution of the homogeneous system corresponding to (24) can be obtained by a method similar to the method of solution of the set of equations of the time-depen-

dent Hartree-Fock approximation (see, for example, Ref. 4). Solving this system, we obtain the following expression for $\tilde{\varepsilon}(q)$:

$$\tilde{\varepsilon}(q)=a_1 \operatorname{sign} q, \tag{27}$$

where a_1 is the gap in the spectrum of Fermi excitations of the system, defined as in (18). We shall seek a solution of the inhomogeneous set of equations (24) in the form

$$c_{k+q,\,k}(t)=B_{k+q,\,k}(t)\,e^{-i\tilde{\varepsilon}(q)t}. \tag{28}$$

As is easy to see, the condition that $|B_{k+q,\,k}|^2$ (with k and q satisfying (26)) does not increase linearly with time, and that therefore the system be stable relative to scattering by impurities, is of the form

$$2k_F|v|<a_1. \tag{29}$$

At $v<v_{cr}$, where $v_{cr}=a_1/2k_F$, the system is found in the superconducting state.[1)]

2. We now consider the other limiting case, $k_F \ll g^2$. We first solve the problem of the interaction of a single electron with the field of lattice (string) vibrations. The set (12) in this case reduces to the following nonlinear equations

$$E\psi(x)=-\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2}-4\lambda^2\psi(x)|\psi(x)|^2. \tag{30}$$

The solution of (30), bounded at infinity, is of the form

$$\psi(x)=(g^2/2)^{1/2}\operatorname{ch}^{-1}(2g^2x). \tag{31}$$

The wave equation $\psi(x)$ describes an electron surrounded by a cloud of phonons. We call such a quasiparticle a soliton (or polaron). As is seen from (31), the dimensions of the region of localization of the soliton is $\sim 1/g^2$. If there are n electrons with density $\rho=n/N$, then, since $\rho \sim k_F \ll g^2$, the mean spacing between particles will be much greater than the dimension of the soliton, and we can speak of the system of almost non-interacting solitons. Here the solitons form a "crystal lattice," the constant of which is equal to $2/\rho$ and the wave function of the m-th soliton is of the form

$$\psi_m(x)=(g^2)^{1/2}\operatorname{ch}^{-1}(2g^2(x-x_0m)), \quad x_0=2/\rho. \tag{32}$$

The wave equation (32) differs from (31) because the soliton (32) represents two electrons with opposite spins, surrounded by a cloud of phonons (there are altogether $n/2$ solitons) i.e., (32) satisfies the equation

$$E\psi(x)=-\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2}-8\lambda^2\psi(x)|\psi(x)|^2. \tag{33}$$

The collision energy of two solitons (32) can be estimated from the overlap of the wave functions of solitons localized at a distance R from one another ($R \gg 1/g^2$):

$$V(R)=\int_{-\infty}^{\infty}\psi_R(x)\psi_0(x)\,dx=4g^2Re^{-2g^2R}. \tag{34}$$

On the other hand, if at $R \sim 2/\rho$ this energy is much less than the adiabatic energy of the solitons, their lattice will be disrupted. In order to obtain the corresponding criteria, we calculate the width $4\beta_H$ of the polaron band.

According to Holstein,[5] we have

$$\beta_H=\beta_0 e^{-S}, \quad S=\pi\beta'^2\sqrt{M}/2\varkappa^{1/2}. \tag{35}$$

Thus, in first order in the nonadiabaticity, the system of solitons can be described by the Hamiltonian

$$\hat{H}_{sol}=\beta_H\sum_m(b_m^+b_{m+1}+b_{m+1}^+b_m)+\sum_{m,m'}V(R_m,R_{m'})\,b_m^+b_mb_{m'}^+b_{m'}, \tag{36}$$

where b_m^+ and b_m are the Pauli operators of the solitons. In this case, the criterion for the transformation of the soliton lattice into a soliton liquid takes the form

$$V(2/\rho)\leqslant 2\beta_H. \tag{37}$$

Equating the exponents of (24) and (35) (g^2R and $S \gg 1$), we can write this criterion in the form

$$\rho^{-1}=\pi|\beta_0|\sqrt{M}/4\varkappa. \tag{38}$$

There is a basis for expecting that this system is superconducting in the liquid phase. This is suggested by the fact that the correlation function $G(R)=\langle b_R^+b_0\rangle$ falls off with distance according to a power law, and the integral

$$\int G(R)\,dR$$

diverges.[6] This argument has, however, only a qualitative character. In order to speak of this with assurance, we must investigate the effect of the impurities.

In conclusion, we note the following circumstance. The electron-electron interaction is neglected in the Hamiltonian (1). Yet it is known that this interaction is not small in real highly conducting quasi-one-dimensional systems, such as TTF-TCNQ crystals. Furthermore, it plays a dominant role in the formation of the electron structure of the ground state of these crystals—the ground state of such systems corresponds to a Wigner crystal with incomplete charge transfer from the donor molecules (D) to the acceptor molecules (A).[7] Excitations in such a Wigner crystal on the other hand, correspond to the creation of charged defects D^+A^-. However, since the concentration of these excitations is small (at any rate at low temperatures), the gas of these charged defects is similar in many respects to a weakly interacting gas of spinless Fermi particles and can be described by a Hamiltonian of the type of (1).

[1)] By virtue of the fact that the effect of the impurities is taken into account here only in first-order perturbation theory, the condition (29) is apparently necessary but not sufficient.

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Translated by R. T. Beyer

Two-frequency microwave resonance in the hyperfine structure of the ground state of alkali metals

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(Submitted October 20, 1976)
Zh. Eksp. Teor. Fiz. 72, 1568–1574 (April 1977)

The problem of two-particle resonance in a three-level system is considered. A stationary solution is obtained for the coherence of two levels coupled by alternating fields with the third; this solution is valid for arbitrary field intensities and frequencies. Two-frequency resonance was investigated experimentally in the hyperfine structure of the ground state of potassium under spectrally selected optical pumping. Modulation of the pump light registered transversely to the magnetic field through a circular analyzer was observed at a frequency equal to the difference between the two microwave fields that cause transitions between the level $F = 1$, $m_F = 1$ and the levels $F = 2$, $m_F = 1, 2$.

PACS numbers: 32.80.Bx

1. INTRODUCTION

It is known that in magnetic-resonance techniques one uses two types of observable quantities, proportional to the imaginary and real components of the magnetic susceptibility. The former are connected with the populations of the magnetic sublevels and are responsible for energy absorption. The latter are connected with coherences, i.e., are described by the off-diagonal elements of the density matrix. Examples of these quantities are the oscillating (transverse) components of the magnetization. The populations and the coherences behave essentially differently at resonance. This is manifest by a different dependence of these quantities on the amplitude and the frequency of the alternating field and, frequently more importantly, in a difference in the dynamics. The registration of the quantities connected with the coherence is more convenient because the corresponding signals vary in time. Under stationary conditions they are expressed in the general case by a series in harmonics of the alternating fields that initiate the resonance. This makes it possible to register the resonance without resorting to a scanning technique, the use of which makes the dynamics of the observation worse and difficult in the case of very narrow resonance. In contrast to the populations, the time of establishment of which is limited by the relaxation constant, the phases of the coherence oscillations react without delay to changes in the system energy.

The advantages of observing the coherence can be realized in the region of relatively low frequencies—in the NMR technique[1] and in experiments on optical orientation of atoms in weak and medium fields.[2,3] Among the applications are the so called M_x-self-generating magnetometers with optical orientation, in which use is made of Larmor polarized-light precession due to the coherence of the atomic magnetic sublevels. The extension of this magnetic-resonance registration method to the region of higher frequencies encounters great technical difficulties. When optical means are used to monitor the coherence, the limitations are imposed primarily by the inertia of the photoreceivers. These limitations could be overcome in demonstration experiments,[4,5] in which microwave modulation of the light passing through potassium and rubidium vapor was observed under conditions of magnetic resonance in the hyperfine structure.

In this article we demonstrate the possibility of converting the microwave oscillations of coherence into radio-frequency oscillations by producing two-frequency microwave resonance on adjacent sublevels of the magnetic splitting of the hyperfine-structure states. The foregoing is illustrated in Fig. 1, which shows the energy structure of the ground state of an alkali-metal atom with nuclear spin 3/2 in a magnetic field. Assume that there exists a process that leads to predominant population of, say, the upper hyperfine level of $F = 1$. (This is a natural result of thermal relaxation, but the obtained population difference is very small and in practice one uses the forcing process of optimal pumping, see below). Superposition of two microwave fields of suitable polarization and frequency gives rise to transitions between one sublevel of the state $F = 1$ and two sublevels of $F = 2$, as is shown, for example, by the arrows of Fig. 1. Under these conditions, coherence sets in between the upper pair of levels at a frequency equal to the difference frequency of the two microwave fields.

 0038-5646/78/4504-0823$02.40