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Electron-electron weak interaction in atoms and ions

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We discuss parity-nonconservation effects in two-electron atoms and ions, arising as the result of electron-electron neutral weak currents. Calculations are presented for the energy levels and transition probabilities for excited states of two-electron ions, and crossing of levels of opposite parity is observed in the region of charge $Z\sim 37$, which is the most suitable for observation of these effects. In the vicinity of this crossing the quantities characterizing the degree of parity nonconservation are of the order $\sim 10^{-8}$.

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The observation of neutral weak currents[1] has led to intensive searches for electron-nuclear weak interactions in atoms.[2–4] All methods of detection of weak interactions between an electron and a nucleus in an atom at low energies are based on the assumption of nonconservation of parity in neutral currents. The electron-electron weak interaction in atoms should have, apparently, the same order of magnitude as the electron-nuclear interaction. In observation of electron-electron weak interactions it is necessary to consider processes in which the electron-nuclear weak interaction is suppressed. Such processes are the emission of photons from atomic levels which have orbital angular momenta l greater than unity.[5]

Because of the centrifugal barrier, the wave functions of the electrons go to zero as r^l as $r\to 0$, where r is the distance from the center of the nucleus. The weak interaction, which does not conserve parity, has zero range and is proportional to $\mathbf{a}\cdot\mathbf{p}$, where $\mathbf{a}$ is some axial vector composed of the spins of the interacting particles and $\mathbf{p}$ is the momentum operator. The product of the initial and final wave functions of the electrons goes to zero as $r^{l_1+l_2}$, where l_1 and l_2 are the orbital angular momenta at the beginning and end of the process. The momentum $\mathbf{p}$ removes one power of r. Therefore after integration over the nuclear volume the matrix element of the weak-interaction potential turns out to be proportional to $(R/a)^{l_2+l_1-1}$, where R and a are the radii of the nucleus in the atom. For interaction with the nucleus the matrix elements between S and P states turn out to be nonvanishing as $R/a\to 0$. The matrix elements of the electron-electron interaction include integration over the entire volume of the atom and do not have parametric smallness, which depends on the magnitude of the orbital angular momenta. Therefore the matrix elements of the transitions $P\to P$, $D\to S$, and so forth for the electron-nuclear weak interaction are suppressed in comparison with $S\to P$ transitions by at least a factor $R/a\sim 10^5$, while the matrix elements of electron-electron transitions have their previous value.[1)] This statement remains valid even for large nuclear charges Z, where relativistic effects are important and it is necessary to use Dirac Coulomb functions.

The weak interaction U_W, which is invariant with respect to time reversal and which does not conserve spatial parity, can be constructed only from the product of vector and axial currents.[2–4] The electron-electron interaction contains only two independent relativistic invariants:

$$U_W = 2^{-1/2}Gg(\gamma_\mu)_1(\gamma_\mu\gamma_5)_2 + 2^{-1/2}Gh(\sigma_{\mu\nu}q_\nu)_1(\gamma_\mu\gamma_5)_2 + (1\rightleftarrows 2), \tag{1}$$

where $G=1.0\times 10^{-5}m_p^{-2}$, m_p is the proton mass, g and h are certain constants, and the subscripts 1 and 2 refer to the first and second interacting electrons. The identity of the electrons is taken into account by antisymmetrization of the wave functions of the initial and final states.

In the nonrelativistic approximation interaction (1) takes the form

$$U_W = 2^{-1/2}Gg(\sigma_1-\sigma_2)(\mathbf{v}_1^+-\mathbf{v}_2^+) + 2^{-1/2}G(g+h)[\sigma_1\times\sigma_2]\times(\mathbf{v}_1^--\mathbf{v}_2^-) + (1\rightleftarrows 2), \tag{2}$$

where $\mathbf{v}_i^+=(\mathbf{p}_i+\mathbf{p}_i')/2m$; σ_i, $\mathbf{p}_i$, and $\mathbf{p}_i'$ are the Pauli spin matrices and the initial and final momenta of the i-th electron. The second term of (1) contributes only to the second term of Eq. (2), while the first term of (1) contributes to both terms of Eq. (2). Therefore for simplicity we have carried out all calculations by setting $h=0$, $g=1$.

In coordinate space the interaction (1) takes the form

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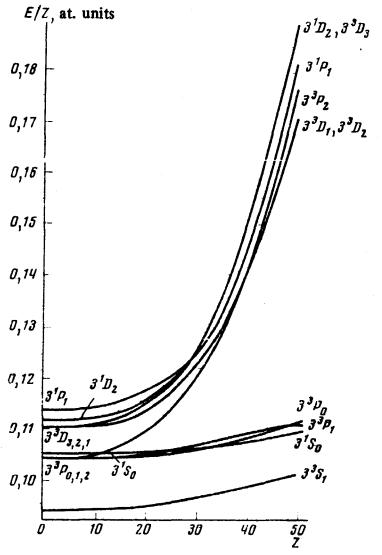

FIG. 1.

$$U_W = 2^{-1/2} G (\gamma_\mu)_1 (\gamma_\mu \gamma_5)_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) + (1 \rightleftarrows 2)$$
$$= 2^{-1/2} G \{(\gamma_5)_1 + (\gamma_5)_2 - \alpha_1 \Sigma_2 - \alpha_2 \Sigma_1\} \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (3)$$

where $\boldsymbol{\alpha}$ and Σ are the Dirac matrices.

The simplest variant of observation of weak electron-electron interaction effects is to use atoms and ions with two electrons. In Fig. 1 we have shown a plot of certain energy levels of two-electron ions as a function of the nuclear charge Z. This plot was obtained as the result of a completely relativistic calculation employing one-electron Dirac Coulomb functions and intermediate coupling. The scheme of such a calculation has been described in detail in a previous article.[6] The ordinate in Fig. 1 shows the energy of the levels in atomic units, divided by Z and measured from the value $\varepsilon_{1(1/2)} + \varepsilon_{3(1/2)}$, where ε_{nj} is the relativistic Coulomb energy of the electron in a state with principal quantum number n and total angular momentum j. The levels have been classified by taking the limit of the transition to small Z. Parity-nonconservation effects are maximal in the region of crossing of levels with different parities. The total angular momentum of the atom does not change when the weak interaction is turned on, and levels with different angular momenta cannot be mixed by the parity-nonconserving weak interaction. Therefore the only crossing which is of interest here is the crossing of the 3^3P_2 level with the level 3^3D_2 level in the region $Z = 37-39$ (ions of Rb, Sr, and Y). The uncertainty is due to the fact that we are neglecting corrections of second order in the Coulomb interaction of the electrons.[6]

The 3^3P_2 level has roughly ten times greater lifetime than the 3^3D_2 level. Therefore for reasons which we will discuss in detail below we shall choose the 3^3P_2 level as the object of study. The transitions to lower levels are determined by the sum of the two amplitudes shown graphically in Fig. 2. The graph of Fig. 2a corresponds to a direct transition, and the graph of Fig. 2b, to a transition through a weak admixture of the 3^3D_2 level. Interference of the two amplitudes leads to appearance of circular polarization of the radiation and asymmetry of the photon emission with respect to the initial ion spin $\mathbf{s}$.[2-5] We note that orientation of the spins always arises when multiply charged ions are obtained by the method of passing the beam through a foil whose position is determined by the pseudovector of the normal.[7]

The ratio of the amplitudes (Fig. 2b) and (Fig. 2a) determines the degree of circular polarization (or asymmetry coefficient) P, which is

$$P = (\Gamma_{DP}/\Gamma_{PP})^{1/2} \langle P | U_W | D \rangle / L,$$
$$L = E(3^3P_2) - E(3^3D_2), \quad (4)$$

where Γ_{PP} and Γ_{DP} are the widths for radiative transitions respectively from the 3^3P_2 and 3^3D_2 levels. The most advantageous situation is when $\Gamma_{DP} \gg \Gamma_{PP}$. This determines the choice of the lowest level 2^1P_1. In this case the first graph is the amplitude of the $M1$ transition, which is forbidden by spin for small values of Z. The second graph is the amplitude of the $E1$ transition, which is also forbidden by spin for small Z. The ratio of the amplitudes of these transitions is $(\Gamma_{PP}/\Gamma_{DP})^{1/2} \sim \alpha Z$.

In addition to the $M1$ transition between the 3^3P_2 and 2^1P_1 states, the forbidden $E2$ transition and the allowed $M3$ transition also occur; their amplitudes do not interfere with the second graph of Fig. 2 but must be taken into account in calculation of the value of P. However, the amplitudes of these transitions, which are of the same order in the parameter αZ (α is the fine structure constant) as the amplitude of the $M1$ transition, are nevertheless numerically smaller for large values of Z. In fact the forbiddenness in the amplitudes of the $M1$ and $E2$ transitions is due to the difference of the spins of the initial and final states in the two cases. This forbiddenness is removed by the spin-orbit interaction, which allows transitions to the 2^1P_1 state through the intermediate state 2^3P_1. The admixture of the 2^3P_1 state to the 2^1P_1 state is of the order $(\alpha Z)^2 I/L_{31} \sim 0.1\alpha^2 Z$, where L_{31} is the distance between the 2^3P_1 and 2^1P_1 levels (according to the graph in our earlier article,[5] $L_{31} \approx 0.1Z^{-1}$). The amplitude of the $E2$ transition, which has the same enhancement I/L_{31} as for the $M1$ transition, is less than the $M1$-transition amplitude by a factor ω/I, where ω is the transition energy and I is the ionization energy.[8] In our case $\omega/I \sim 0.07$. The amplitude of the $M3$ transition does not have the enhancement I/L_{31} and consequently is smaller than the $M1$-transition amplitude by a factor L_{31}/I. Therefore in what follows we shall neglect the $M3$ and $E2$ transitions.

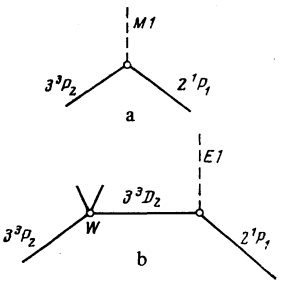

FIG. 2.

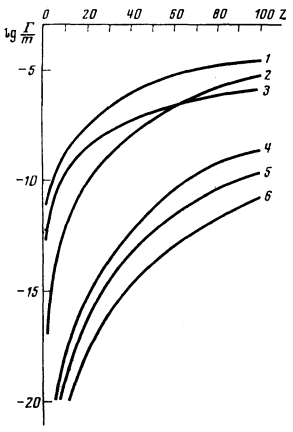

FIG. 3.

Thus, to obtain a large value of P it is necessary[4] that the admixed level have a larger partial transition value than the main level ($\Gamma_{DP} \gg \Gamma_{PP}$). This leads to difficulties of observing the main transition line. In most experiments excitation of close-lying levels occurs with identical filling numbers. In view of the Breit-Wigner distribution of the radiation intensity as a function of frequency, the radiation from the admixed level will occur also at the frequency of the main transition. In view of the high intensity of radiation from the admixed level this can lead to an effective reduction of the value of P. In this case it is necessary to strive for a low population of the admixed level. This can be achieved by choosing as the object of study a level with long lifetime; by delaying the beginning of the measurement with respect to the moment of excitation of the ions, this permits an emptying of the admixed level with an insignificant change in the population of the investigated level.[4] The lifetime of the $3\,^3P_2$ level is roughly ten times that of the $3\,^3D_2$ level, which determines our selection. Plots of the transition probability from the $3\,^3P_2$ and $3\,^3D_2$ levels to the low-lying levels are given in Fig. 3. These plots were calculated by us also by a completely relativistic method, as described in Ref. 9.

The probability of radiation for the transition $3\,^3P_2 \to 2\,^1P_1$ has the form

$$\Gamma_{PP}^{W}=\Gamma_{PP}(1+P\mathbf{ns}),$$

where Γ_{PP} is the transition probability in the absence of the weak interaction, P is defined by Eq. (4), $\mathbf{n}$ is the direction of photon emission, $\mathbf{s}$ is the photon spin, expressed in the well known way in terms of the polarization vector, $\mathbf{s}=i(\mathbf{e}\times\mathbf{e}^*)$, or the initial spin of the ion. The weak-interaction matrix element, which determines P in accordance with Eq. (4), is calculated in the Appendix. For the strontium ion Sr the distance between the $3\,^3P_2$ and $3\,^3D_2$ levels is minimal and is given by $L=6.5\times10^{-3}m\alpha^2$. The value of the parity-nonconservation coefficient (4) for strontium is $P=1.8\times10^{-6}$. We have also calculated values of P for smaller Z values. In the region far from the point of crossing of the $3\,^3P_2$ and $3\,^3D_2$ levels, the main contribution to the second plot of Fig. 2b is from the intermediate level $3\,^1D_2$ with the allowed $E1$ transition $3\,^1D_2 \to 2\,^1P_1$. In this case the ratio of probabilities in Eq. (4) is $(\Gamma_{PP}/\Gamma_{DP})^{1/2} \sim (\alpha Z)^3$ and as a result the value P increases for small Z values to $P=6.0\times10^{-8}$ at $Z=2$ (the helium atom).

A plot of P as a function of the nuclear charge Z is given in Fig. 4. In the region of small Z, however, strong restrictions arise on the external electric fields, which as the result of Stark broadening of the lines lead to a reduction of P. In addition, the effect of collisions, which limits the density of the buffer gas,[4] turns out to be important. For large Z these limitations do not arise.

Let us estimate the measurement time necessary for observation of the effect. To determine the degree of parity nonconservation P in a transition with partial probability B it is necessary to record $N \sim B^{-1}P^{-2}$ events. If we use an ion beam with density ρ traveling with velocity v in a volume V with a cross section S and length l, the number of decays N of the level in time t in this volume is

$$N=\rho v S t[1-\exp(-l/v\tau_P)],$$

where τ_P is the total lifetime of the $3\,^3P_2$ level. As a result we obtain the following expression for the time of observation t:

$$t=\tau/V\rho BP^2,$$

where

$$\tau=\frac{l}{v}\left[1-\exp\left(-\frac{l}{v\tau_P}\right)\right]^{-1}=\begin{cases} l/v & (\tau_P \ll l/v), \\ \tau_P & (\tau_P \gg l/v). \end{cases}$$

In the case of experiments with ion beams[10] we have $v \sim 10^6$ cm/sec, $\rho \sim 10^5$ cm^{-3}, and for our transition we find $\tau \sim l/v$. If we assume $B \sim 10^{-5}$ this gives $t \sim (10^7/V\,[\text{cm}^3])$ sec. Hence it follows that to obtain an acceptable observation time $t \sim 10^3$ sec it is necessary to use a volume $V \sim 10^4$ cm^3.

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APPENDIX

We have calculated the matrix elements of the operator U_W on the wave functions $\psi_{JMjj'll'}$ constructed according to jj coupling from the one-electron Dirac functions ψ_{njlm}. To take into account the relativistic mixing

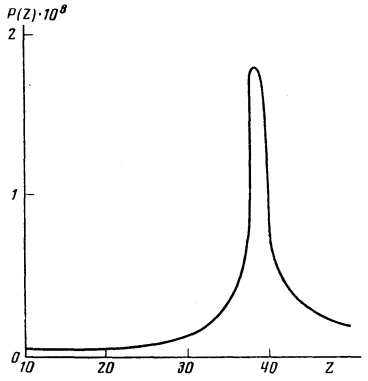

FIG. 4.

of the $3\,^3P_2$ and $3\,^3D_2$ states we have used wave functions of a more general form

$$\psi_{JM}(12)=\sum_{jj'}\sum_{ll'}a_J(jj'll')\psi_{JMjj'll'}, \tag{A.1}$$

where the mixing coefficient a_j are determined from the condition of minimum energy. The matrix element of most general form for the operator U_W on the functions $\psi_{JMjj'll'}$ appears as follows:

$$\begin{aligned}
&\langle J'M'j_2j_2'l_2l_2'|U_W|JMj_1j_1'l_1l_1'\rangle\\
&=-\frac{i}{\sqrt{2}}G(1-\mathscr{P}_{22'})(1+\mathscr{P}_1\mathscr{P}_{2'}-\mathscr{P}_1Q_2+Q_2\mathscr{P}_{2'}+\mathscr{P}_1\mathscr{P}_{2'}-\mathscr{P}_1\mathscr{P}_{1'}\\
&\quad-\mathscr{P}_1\mathscr{P}_{1'}Q_2Q_{2'})R(g_1g_1'j_2g_2')K(j_1j_2j_1'j_2'J|l_1\bar{l}_2l_1'l_2')\delta_{JJ'}\delta_{MM'},\\
&R(g_1g_1'j_2g_2')=\int_0^\infty g_{n_1\kappa_1l_1}(r)\,g_{n_1'\kappa_1'l_1'}(r)\,f_{n_2\kappa_2l_2}(r)\,g_{n_2'\kappa_2'l_2'}(r)\,r^2\,dr,\\
&\times K(j_1j_2j_1'j_2'J|l_1\bar{l}_2l_1'l_2')=(-1)^{l_1+l_2+l_1'+l_2'}\Pi_{j_1j_2j_1'j_2'}\Pi_{l_1l_2l_1'l_2'}\sum_l(2l+1)\\
&\cdot\begin{pmatrix}l_1&l_2&l\\0&0&0\end{pmatrix}\begin{pmatrix}l_1'&l_2'&l\\0&0&0\end{pmatrix}\begin{Bmatrix}l_1&l_2&l\\j_2&j_1&1/2\end{Bmatrix}\begin{Bmatrix}l_1'&l_2'&l\\j_2'&j_1'&1/2\end{Bmatrix}\begin{Bmatrix}j_1&j_2&l\\j_2'&j_1'&J\end{Bmatrix}\\
&\quad+\begin{Bmatrix}\bar{l}_2'&l_2&j_2'&j_2&1/2\\1/2&l&J&1&\bar{l}_1'\\l_1&j_1'&j_1&1/2&1/2\end{Bmatrix}.
\end{aligned}\tag{A.2}$$

Here we have designated by g and f respectively the upper and lower components of the Dirac bispinors; the operators $\mathscr{P}_1$, $\mathscr{P}_{1'}$, ... denote the substitution $g_1\to f_1$, $g_1'\to f_1'$, and so forth in the integral $R(g_1g_1'f_2g_2')$ with simultaneous replacement of l_1, l_1', ... by $\bar{l}_1\equiv 2j_1-l_1$, $\bar{l}_1'\equiv 2j_1'-l_1'$, etc. in the expression for K. The operator Q_2 signifies the inverse substitution $f_2\to g_2$, $\bar{l}_2\to l_2$. The symbol $\mathscr{P}_{22'}$ represents the operator of the permutation $n_2j_2l_2\rightleftarrows n_2'j_2'l_2'$. In addition, we have used standard designations for the $3j$, $6j$, and $15j$ symbols. Finally,

$$\Pi_{l_1l_2l_1'l_2'}\equiv[(2l_1+1)(2l_2+1)(2l_1'+1)(2l_2'+1)]^{1/2}.$$

Equation (A. 2) can be used to calculate the matrix element of the electron-electron weak interaction between two arbitrary states of two-electron ions with arbitrary nuclear charge Z.

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