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Tunnel effect in superconductors with nonequilbrium quasiparticle population under laser irradiation

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Small deviations of the volt-ampere characteristics of laser-irradiated Pb-PbO-Pb and Sn-SnO-Sn tunnel junctions from equilibrium are measured. The decrease of the junction conductivity due to nonequilibrium occupation of the quasi-particle states is investigated in detail for various temperatures and laser-radiation intensities. The tunnel current is calculated for a special model with a nonequilibrium excitation distribution function. The results of the calculation are in good agreement with the experiments. The role of thermal effects is analyzed.

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1. INTRODUCTION

A stationary energy distribution of the quasiparticles and phonons, which differs considerably from the equilibrium distribution, is established at sufficiently low temperatures ($kT \ll \Delta$) and under the influence of laser radiation in thin superconducting films.[1] The properties of the superconductor in such a nonequilibrium state are determined both by the total number of excess excitations and by the actual form of their energy distribution. However, experiments carried out to date have been limited principally to measurements of such integrated characteristics as the energy gap,[2] the dc resistance,[3] or the reflection coefficient at microwave frequencies.[4] Therefore, their results can be sufficiently well explained by various model-dependent distribution functions[5,6] under the condition that the concentration of the nonequilibrium excitations is small and that their total number remains unchanged.

A decrease in the conductivity of a laser-irradiated tunnel junction was first observed experimentally in Ref. 7 in a narrow range of voltages $eV \gtrsim 2\Delta$, the reason for this decrease is the occupation of part of the states over the energy gap by photo-excited nonequilibrium quasiparticles. The tunnel effect allows a direct measurement of the energy localization of the nonequilibrium occupation and, consequently, yields some information on the nonequilibrium contribution to the distribution function of the quasiparticles.

In the present we report here an experimental study of the behavior of the tunnel junctions of lead and tin irradiated by an He–Ne laser. The measurements were performed at various temperatures and radiation intensities. The calculation of the volt-ampere characteristics of the junctions was performed with the use of the nonequilibrium quasiparticle distribution function proposed by Vardanyan and Ivlev[8] and agrees well with the experimental results.

2. EXPERIMENTAL METHOD

The measurements were carried out on Pb–PbO–Ob and Sn–SnO–Sn tunnel junctions prepared by the usual techniques (condensation in a vacuum and oxidation in a glow discharge). The area of the junctions amounted to 0.15–0.06 mm^2, the resistance was 1–15 ohms. The thickness of the metallic films was within the range 1000–2000 Å. Crystalline quartz or sapphire was used as a substrate; in the initial states of the research, glass was also used. The sample was placed directly in liquid helium.[1)]

Radiation from an He–Ne laser of wavelength 1.15 μ was chopped at a rate of 420 Hz by a disc modulator (Fig. 1), guided into the cryostat by a light pipe, and focused with the help of a cone on the upper film of the sample in the form of a spot with a diameter of 2.2 mm. Part of the radiation was diverted to a calibrated photodetector to record the power level in the measurement process. The power was regulated by an iris diaphragm and by grid attenuators. The optical system was tuned against the red emission line as the laser resonator tuning was varied. To prevent heating and formation of a spatially inhomogeneous nonequilibrium state in the film,[3] relatively low levels of radiation intensity were used.

After amplification and synchronous detection, the

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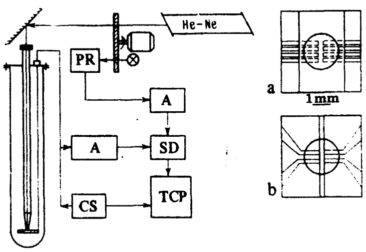

FIG. 1. Block diagram of measuring apparatus and configuration of tunnel junctions: A—amplifier, SD—synchronous detector, PR—photoresistor, TCP—two-coordinate potentiometer, CS—current scanning unit.

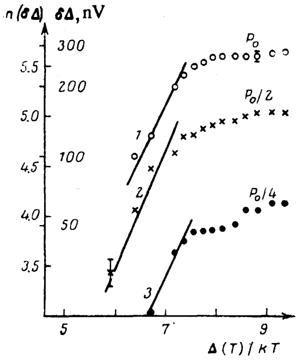

FIG. 2. Change in the energy gap with temperature for three radiation intensities ($P_0=65$ mW/cm^2).

alternating component of the voltage on the sample δV was recorded as a function of the current through the junction. The contribution to the tunnel current $\delta I(V)$ from the laser radiation is connected with the experimentally measured $\delta V(I)$ dependence by the relation

$$\delta I(V) = -\delta V(I) \Big/ \frac{dV}{dI}, \quad I = I(V), \tag{1}$$

where $I(V)$ is the volt-ampere characteristic of the tunnel junction and dV/dI is its first derivative. Both these characteristics were measured in the experiment and agreed well with values known from the literature.[9] The recording of $\delta V(I)$ was carried out at both polarities and the values of δV were defined as the arithmetic mean. For the change of the energy gap $\delta\Delta$ we took the value of δV at the level of one-half the jump in the quasiparticle current at the voltage $V=2\Delta/e$ (see Fig. 4 below).

The results for junctions of different configuration (see a and b in Fig. 1) were identical. The investigations were carried out chiefly on junctions of type b, which allowed the use of a four-probe circuit connection.

3. EXPERIMENTAL RESULTS

Effect of temperature and radiation power on the energy gap

It was first necessary to establish the fact that the observed characteristics were not due to ordinary heating of the tunnel junction by the radiation. For this purpose, we measured the dependences of the energy gap on the temperature and the radiation intensities which were reliably established for the photoexcitation processes both theoretically[5,8] and experimentally.[2] In correspondence with existing theories (see, in particular, Ref. 8) it is necessary at sufficiently low temperatures ($kT \ll \Delta$) to distinguish between two limiting regimes.

1. The weak injection regime, in which the number δN of nonequilibrium quasiparticles is much smaller than the number N_T of thermally excited equilibrium ones ($\delta N \ll N_T$). In this case, the temperature dependence of the gap change ($\delta\Delta=\Delta_0-\Delta$) is determined by the recombination time for quasiparticles ($\delta\Delta$ increases with decrease in temperature):

$$\delta\Delta(T, P) = \alpha P (\Delta/kT)^{1/2} e^{\Delta/kT}, \tag{2}$$

where α is a constant coefficient that determines the number of nonequilibrium quasiparticles, P is the radiation intensity and Δ_0 is the energy gap in the equilibrium state.

2. The strong injection regime, in which the reverse inequality holds, $N_T \ll \delta N$. The gap change depends very weakly on the temperature and is determined chiefly by the radiation intensity $\delta\Delta \sim \sqrt{P}$.

The experimental plots (Fig. 2) obtained by us for the tunnel junction Pb–PbO–Pb, show two distinct regions of change of $\delta\Delta$, corresponding to the two injection regimes. The measurements were carried out at three laser power levels. The quantity P_0 corresponds to a power level of 65 mW/cm^2. The boundary condition Δ/kT which separated one regime from the other, is equal to ≈ 7.7 for the upper curve and increases with decrease in power. The solid line is the result of calculation according to Eq. (2) in the case of αP equal to 0.058 (curve 1), 0.029 (curve 2), 0.012 (curve 3) nanovolts. The ratios of these quantities are in good agreement with the radiation-intensity ratios, which confirms the constancy of the coefficient α for all the plots.

Figure 3 shows plots of $\delta\Delta[(P/P_0)^{1/2}]$ for the tunnel junctions Pb–PbO–Pb (curve 1) and Sn–SnO–Sn (curve 2). The measurements were carried out at $T=1.65$ °K. A linear dependence of $\delta\Delta(\delta N \gg N_T)$ is observed for lead junctions up to $P=0.16\,P_0$. At lower power levels, a transition occurs to the regime of weak injection and the dependence becomes quadratic in $(P/P_0)^{1/2}$, i.e., the decrease in the energy gap is proportional to the radiation intensity. At a temperature of 1.65 °K, for a tin tunnel junction, $\delta\Delta \sim P/P_0$ ($P_0=23$ mW/cm^2) over the entire range of variation of the laser radiation intensity. This means that the inequality $\delta N \ll N_T$ holds for tin over the

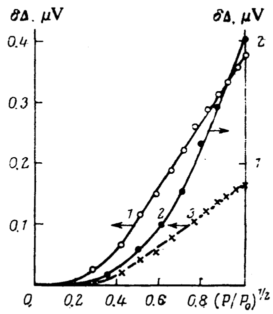

FIG. 3. Dependence of the change in the energy gap $\delta\Delta$ and depth of the minimum δV_m on the radiation intensity (1—$\delta\Delta$, 3—δV_m for Pb-PbO-Pb; 2—$\delta\Delta$ for Sn-SnO-Sn), $T=1.65$ K.

entire range of change of the power level. We recall that the conditions for the satisfaction of this inequality are determined by the power level, the temperature, the energy gap, and the recombination time of the quasiparticles.

The good agreement of the obtained experimental results with the theory[8] that takes into account only the photoexcitation of the quasiparticles can serve as a sufficiently convincing proof of the absence of direct heating of the tunnel junction by the laser radiation at the power levels selected in the temperature range studied. As our special studies have shown, the heating actually takes place for the tunnel junction on a glass substrate of thickness 0.1 mm at temperatures above the λ point of liquid helium. In order to eliminate the effect of photoexcitation on the change in the gap, in the study of heating we applied the laser radiation to the side of the substrate opposite to the junction, a side covered by an opaque layer of silver paste of thickness ~0.1 mm, or by a film of lead with thickness ~1 μ. In these experiments, ln δΔ decreased monotonically with increasing Δ/kT, in contrast to Fig. 2, and δΔ tended toward zero as $T \to T_\lambda$.

Pb-PbO-Pb junctions

The family of plots of the additional voltage δV that arises at the Pb–PbO–Pb tunnel junction under the action of laser radiation, vs the current I through the junction, is shown in Fig. 4. The maximum intensity ~100 mW/cm² corresponded to the upper curve. For succeeding curves, the intensity decreases by a constant amount $\delta P = 8.3$ mW/cm² in correspondence with plot 1 of Fig. 3, where the values of δN at $I \cong 1$ mA are shown (see the vertical dashed line for the current 1 mA in Fig. 4). The current through the junction serves as the independent variable specified in the experiment. The voltage at the junction could be determined from the volt-ampere characteristic, shown by the dotted lines in Fig. 4, by projecting with its help any point of the curve $\delta V(I)$ onto the V axis.

As is seen from Fig. 4, the jump of the single-particle current with increase in the radiation intensity is increasingly smoothed, i.e., the gap singularity in the density of states is smeared out over the energy. The depth of the minimum δV_m at $V = 3.1$ mV increases with increase in the radiation intensity (curve 3 in Fig. 3), while the course of this dependence is similar to that of $\delta\Delta(P)$. The temperature dependence of the depth of the minimum is discussed below. In correspondence with

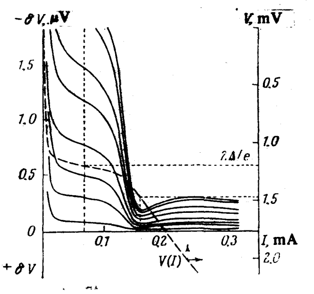

FIG. 5. Dependence of the additional voltage δV on the current through the tunnel junction Sn-SnO-Sn; $T = 1.65$ K.

Eq. (1), the contribution to the tunnel current is positive at voltages below $2\Delta/e$, and changes sign at voltages above $2\Delta/e$.

In addition to the negative contribution to the current, which is localized directly behind the gap, two smeared minima are observed in the region of the phonon singularities of lead at $eV = 2\Delta + \hbar\omega_{TA} = 7.5$ meV and $eV = 2\Delta + \hbar\omega_{LA} = 11.6$ meV (see Fig. 2 in Ref. 7). The location of the minima on the V axis corresponds to maxima in the density of states of transverse TA and longitudinal LA acoustic phonons of lead. The depth of the minima is practically independent of the temperature.

Sn-SnO-Sn junctions

Figure 5 shows a series of $\delta V(I)$ plots for the Sn–SnO–Sn tunnel junction, measured at different laser radiation intensities. The upper curve corresponds to the maximum radiation intensity 23 mW/cm². The values of the intensity for the remaining curves can be determined from curve 2 of Fig. 3. Each $\delta V(I)$ curve on Fig. 5 corresponds to an appropriate point on curve 2 of Fig. 3. As is seen from the figure, the contribution to the tunnel current from the radiation, is positive everywhere (see Eq. (1)). However, some dip is observed at $V \geq 2\Delta/e = 1.45$ mV at relatively high intensity levels.

The changes in the volt-ampere characteristics in the region of voltages $V \gg 2\Delta/e$ are shown in Fig. 6. In the voltage interval 5.2 mV < V < 20.4 mV, δV becomes positive and increases with increase in the intensity. The range of energies in which $\delta V > 0$ is identical with the region of high density of states of the phonon spectrum of tin,[9] shifted in energy by the value 2Δ. Outwardly, the dependence $\delta V(I)$ plot recalls remotely the dependence of the phonon density of states on the energy with a smoothed fine structure. As has already been

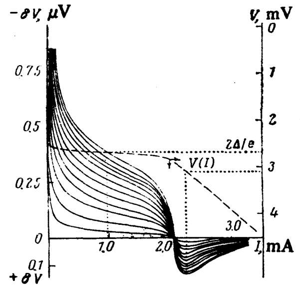

FIG. 4. Dependence of the additional voltage δV on the current through the tunnel junction Pb-PbO-Pb. The radiation intensity is maximum for the upper curve; $T = 1.7$ K.

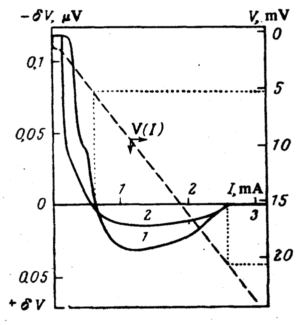

FIG. 6. Plots of $\delta V(I)$ and $V(I)$ for the junction Sn-SnO-Sn (1—$P = 23$ mW/cm²; 2—$P = 9.2$ mW/cm²) $T = 1.65$ K.

noted in the previous section, similar singularities have been observed by us in lead (see Fig. 2 in Ref. 7).

4. DISCUSSION OF EXPERIMENTAL RESULTS

Theoretical calculation of the tunnel current in superconductors with a nonequilibrium distribution function

The decrease in the conductivity of tunnel junctions in a narrow range of voltages past the gap, which arises following irradiation by a laser, can be explained by the nonequilibrium occupation of the quasiparticle states (the "blocked" states in the film irradiated by photoexcited quasiparticles[7]). For the calculation of this effect, we make use of the expression for the current through the tunnel junction of two superconductors[10]:

$$I(V)=\frac{1}{eR_N}\int_{-\infty}^{\infty} n_1(\varepsilon)n_2(\varepsilon+eV)[f_1(\varepsilon)-f_2(\varepsilon+eV)]d\varepsilon, \quad (3)$$

where $n_{1,2}$ is the density of states, $f_{1,2}$ is the quasiparticle distribution function, and R_N is the tunnel resistance of the junction in the normal state. We define the contribution to the tunnel current arising in laser irradiation as the difference

$$\delta I(V,\Delta,T,P)=I-I_0, \quad (4)$$

where I_0 is the current through the junction without the irradiation, and δI is a function of the voltage V at the junction, of the energy gap, of the temperature, and of the radiation intensity.

In the general case, calculation of the current I through a junction irradiated by the laser must be carried out with the use of the nonequilibrium values $n(\varepsilon)$, $f(\varepsilon)$ and Δ. Taking into account the complexity of such a problem, the necessity of calling on certain model representations is evident. For subsequent calculations, we employ the following model. Radiation leads to a nonequilibrium distribution of the quasiparticles only in one irradiated film of the tunnel junction. The distribution function for this electrode has the form of the usual Fermi function with the nonequilibrium increment[8] in the form

$$\delta f(|\varepsilon|)=\left(\frac{2\Delta}{\pi kT}\right)^{1/2}\frac{\delta\Delta}{2\Delta}\exp\left[-\frac{|\varepsilon|-\Delta}{kT}\right]. \quad (5)$$

The quantity $\delta\Delta/2\Delta$ determines the relative number of excess excitations, which depends on the radiation intensity, and can be measured directly. The nonequilibrium phonons that are generated through energy relaxation of the quasiparticles and can lead to a nonequilibrium distribution of the quasiparticles also in the other film of the junction are not taken into account in the calculation. We also neglect the dependence of δf on the coordinates, since the film thicknesses are much less than the diffusion path length of the quasiparticles.[2,3] The changes in the gap, measured experimentally, are sufficiently small ($\delta\Delta/\Delta_0\approx 10^{-3}$); therefore we can use the equilibrium value Δ_0 in the calculations. In particular, for lead, this value, measured as the voltage of the midpoint of the jump in the single-particle current, amounts to $2\Delta_0=2.85$ meV at $T=1.65$ K. For the density of states, we assume the ordinary expression of BCS theory. In the non-irradiated film $n(\varepsilon)$ and $f(\varepsilon)$ are taken at their equilibrium values.

After substitution of the corresponding expressions in (3) and a number of simple transformations, the increment of the current will be of the form

$$\delta I(V,T,P)=\frac{\delta\Delta(T,P)}{2\Delta eR_N}\left(\frac{2\Delta}{\pi kT}\right)^{1/2}[\theta(eV-2\Delta)J_1+J_2], \quad (6)$$

where $\theta(eV-2\Delta)$ is the Heaviside step function,

$$J_1=\frac{1}{2}\int_{-1}^{1}\frac{[(eV-2\Delta)^2x^2-(eV)^2]\exp[(2\Delta-eV)/2kT(1+x)^{-1}]}{(1-x)^{1/2}[(eV+2\Delta)^2-(eV-2\Delta)^2x^2]^{1/2}}dx, \quad (7)$$

$$J_2=(kT)^{1/2}\left[1-\exp\left(-\frac{eV}{kT}\right)\right]\int_0^{\infty}\frac{(kTx+eV+\Delta)(kTx+\Delta)x^{-1/2}e^{-x}}{\{(kTx+2\Delta)[(kTx-eV+\Delta)^2-\Delta^2]\}^{1/2}}dx. \quad (8)$$

The integration of Eqs. (7) and (8) has been performed numerically on a high-speed computer with the use of the Gaussian quadrature formulas.[11] J_1 is negative at $eV\geqslant 2\Delta$ and has a sharp minimum near 2Δ with a minimum value of $\pi\Delta/2$. With increase in the voltage, $J_1\to 0$; $J_2>0$ over the entire range of displacements and also approaches zero at V greater than 2Δ. Thus, J_1 and J_2 enter into the expression for the contribution to the tunnel current with different signs; therefore the sign of δI is determined by the absolute values $|J_1|$ and $|J_2|$, which depend on the temperature and the value of the energy gap. There will be no negative increment to the current whenever $|J_1|<|J_2|$ over the entire energy range. This occurs at some temperature $T>T^*$, where T^* is equal to 5.2 and 2.3 K, respectively, for tunnels of lead and tin.

Comparison of theory with experiment

It follows from the experimental results (see Figs. 4 and 5) that the model used describes best the situation in Pb–PbO–Pb tunnel junctions; therefore, we give a quantitative comparison with theory for these junctions. Figure 7 gives (solid line) the calculated $\delta I(V)$ dependence, while the circles represent values taken from the experimental curves $\delta V(I)$ with the use of Eq. (1). Both dependences are in good agreement in shape and scale of change of δI.[2)] However, there are some differences. We shall analyze the reason for them.

The stepwise decrease in δI obtained in the theory at $eV=2\Delta$ is due to the choice of the form of the function of the density of states. As is known, a finite slope of

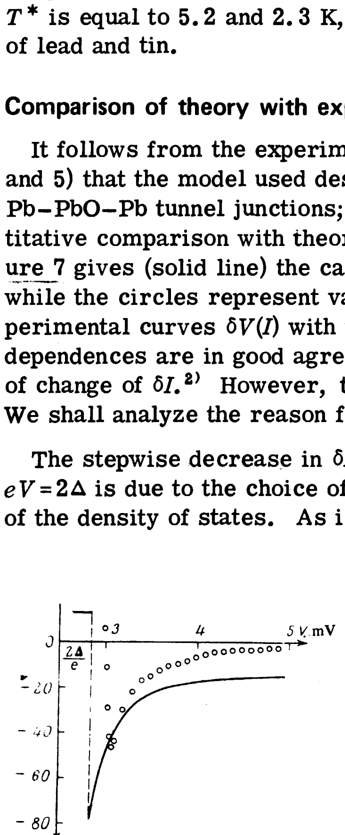

FIG. 7. Decrease in the tunnel current of the junction Pb-PbO-Pb in the case of radiation by the laser (continuous curve—calculated for the model); $T=1.65$ K.

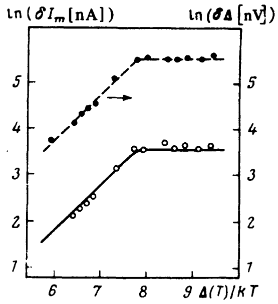

FIG. 8. Temperature dependence of the contribution to the current upon radiation of the junction Pb-PbO-Pb (continuous curve—calculated for the model) and $\delta\Delta$ (dashed).

the jump of the single-particle current is observed experimentally at the threshold voltage.[12] Account of the smearing out of the singularity of the density of states near the threshold should lead to a smoothing of the shape of the minimum of δI on the calculated curve. Another discrepancy between the calculated and experimental values of δI is the more rapid decrease of $\delta I(V)$ at large V, obtained experimentally. It can be due to the creation of nonequilibrium quasiparticles in the nonirradiated film of the tunnel junction, resulting from the penetration into it of recombination phonons from the irradiated film.[7] The presence of nonequilibrium quasiparticles in both films of the junction is equivalent to a certain effective heating of the junction as a whole and should lead to a positive increment to the tunnel current (increase in the amplitude of J_2 in Eq. (6)). The value of the temperature T^* turned out to be lower, experimentally, for the junctions Sn–SnO–Sn for the same reason. In special experiments on the study of the effect of heating, described above, $\delta V(I)$ was always positive, similar to the plots shown in Fig. 5, but without the dip in the region $eV \gtrsim 2\Delta$.

The effect of the laser radiation intensity on the increment to the tunnel current is determined, according to (6), by the $\delta\Delta(P)$ dependence and is well supported by the experimental data (Fig. 3, curves 1 and 3).

The depth of the minimum of δI as a function of the temperature is determined essentially by the $\delta\Delta(T)$ dependence and is demonstrated in Fig. 8, where the calculated (solid curves) and experimental (points) values are shown. For comparison, the dependence of $\ln(\delta\Delta)$ on Δ/kT (dashed) is also shown in the figure.

The source of the phonon singularities obtained in the present research for tin junctions, just as in Ref. 7 for lead junctions, is apparently the smoothing of the singularities of the density of states of the quasiparticles as a function of the energy, a consequence of which is a corresponding smoothing of the volt-ampere characteristics.

5. CONCLUSION

The investigation of the tunnel effect in superconductors with nonequilibrium occupation of quasiparticle states allows us in principle to establish completely the nonequilibrium increment to the distribution function. Interest centers here on the determination not only of its principal part, localized directly over the gap, but also on the exponentially small tail, which extends into the range of energies of the order of the characteristic phonon frequencies. Whereas the principal part of the nonequilibrium increment of the distribution function carries information principally on the recombination time of quasiparticles with formation of a Cooper pair, the exponentially small contribution in the region of high energies reflects the energy dependence of the time of electron scattering, i.e., it is directly connected with the imaginary part of the self-energy of the electron in the phonon field. However, extraction of this information from the tunnel characteristics of the nonequilibrium superconductors in the given setup of experiments is not possible. The reason for this lies not only in the smallness of the corresponding increments of the tunnel current, but also in the significant departures from the theoretical model, which are observed upon increase of the radiation intensity. As experiment confirms, the density of states undergoes appreciable changes upon increase in the radiation intensity. Singularities in the density of states, due to the energy gap and the phonon characteristics, are smoothed out, similar to what occurs under the action of other strong mechanisms of pair breaking, for example a current state with a high current density, the effect of a strong magnetic field on superconductors with impurities, the effect of magnetic impurities, etc. As is seen from the present work, this effect appreciably exceeds the expected change in the tunnel current due to the nonequilibrium character of the distribution function. This conclusion follows also from the results of our previous research.[7]

Thus, only the region of energy localized directly above the gap remains accessible for experimental investigation. The present work has been devoted to a detailed investigation of the principal part of the nonequilibrium increment of the distribution function (Eq. (5)), localized in this region of energies. We shall subsequently attempt to improve the agreement of theory with experiment by taking into account phenomenologically the smearing of the density of states near the threshold by introduction of a modified density of states $n'(\varepsilon)$, equal to[13]

$$n'(\varepsilon)=\int n(\varepsilon')\Delta(\varepsilon'-\varepsilon)\,d\varepsilon', \tag{9}$$

where the function of smearing of the gap $\Delta(\varepsilon-\varepsilon')$ appears under the integral. This function has a finite width. The shape of this function and the magnitude of the smearing are chosen by comparing the experimental volt-ampere characteristics with the calculated ones. In addition, we can attempt to take into account the nonequilibrium heating up of the lattice by the recombination phonons, or set up experiments which eliminate the excitation of quasiparticles by the recombination phonons in the non-irradiated film, for example, by irradiating an Sn–Pb junction from the tin side. We propose to carry out these investigations in the near future.

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Thermodynamics of an impurity uniaxial ferromagnet below the Curie point

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A four-dimensional Ising model and an easy-axis ferromagnet with dipole interaction ($d = 3$), containing randomly distributed pinned impurities, are considered. The renormalization-group methods yields exact equations of states for both systems. The temperature and field dependences of the susceptibility, the magnetization, and the heat capacity are obtained below T_c and in an external magnetic field.

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In systems having randomly disposed pinned impurities, scattering by impurities leads to an additional interaction of the critical fluctuations of the order parameter, their sign being that of attraction. Despite the appearance of the new type of interaction, a second-order phase transition takes place in such systems as before, and scale invariance is present with critical exponents that differ from the critical exponents of the "pure" system.[1-3] There is no doubt that one of the most interesting is the case of impurity systems with single-component order parameter, examples of which are Ising's 4-dimensional impurity model with short-range exchange forces, or a three-dimensional impurity easy-axis ferromagnet (ferroelectric) with dipole interaction.[3-5] The interest in these systems is due to several factors. First, the renormalization-group equations for them can be solved exactly, so that the singularities of the thermodynamic quantities at the phase-transition point can be determined exactly; second, as shown in[5], these singularities (above T_c in a zero external field) contain besides powers of t and $\ln t$ (here $(t = T_c)/T_c$) also the unusual factor $\exp\{-(D|\ln t|)^{1/2}\}$, where D is a certain number (see below). Finally, the conclusions of the theory can be verified experimentally on impurity easy-axis ferromagnets (ferroelectrics), e.g., $LiTbF_4$,[6] with nonmagnetic atoms as impurities. In this paper we obtain by the renormalization-group method an exact equation of state for impurity systems, with a single-component order parameter, and the temperature and field dependences of the thermodynamic quantities below T_c and in an external field.

To make the exposition clearer, we describe first the general procedure for the analysis of impurity systems (the effective-Hamiltonian method) and the necessary relations obtained for the region above T_c in[5]; this is followed by a derivation of the equation of state in the spirit of the known paper of Larkin and Khmel'nitskiĭ[7] and an examination of its consequences. In the conclusion we discuss the possibility of an experimental verification.

The Hamiltonian of Ising's 4-dimensional impurity model is

$$H\{\Phi(x), \psi(x)\} = \frac{1}{2}\int d^4x \left\{r_0\Phi^2 + (\nabla\Phi)^2 + \frac{v_0}{12}\Phi^4 + 2\Phi^2\psi\right\}, \qquad (1)$$

where $\psi(\mathbf{x}) \sim n(\mathbf{x}) - \langle n(\mathbf{x})\rangle$ is a random variable *describing* the local fluctuations of the temperature in the average-field approximation, $n(\mathbf{x})$ is the impurity density, and r_0 is a linear function of the temperature.

For a given impurity configuration, the free energy is