Andreev reflection and the electric resistance of superconductors in the intermediate state

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The electric resistance of superconductors in the intermediate state is calculated for various relations between the N and S layer thicknesses and the characteristic superconductor lengths (mean free path, energy relaxation length, and electric field decay length in the S region). It is shown that Andreev reflection at the S-N boundaries leads to electric field discontinuities at the boundaries, whereas the potential remains continuous.

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1. INTRODUCTION

In measurements of the resistance of a superconductor in the intermediate state, Landau[1] and then Pippard and coworkers[2] found that the resistance R of the system exceeds the resistance R_N of the N region, i.e., the S region has a finite resistance. The presence of an excess resistance $\delta R = R - R_N$ was connected by Pippard *et al.*[2] with a discontinuity in the electric potential φ at the S–N boundary, arising as a consequence of the Andreev reflection of the quasiparticles from the S–N boundary. They assumed the field $E = -\partial\varphi/\partial x$ to be equal to zero in the S region. It was subsequently made clear, however, both experimentally[3] and theoretically,[4] that an electric field exists in the S region and falls off exponentially with distances from the boundary.

The equation which describes the spatial change of the field E in a superconductor was derived phenomenologically by Rieger *et al.*,[4] who found that the characteristic field decay length is the correlation length $\xi(T)$. However, as shown by Gor'kov and Eliashberg,[5] this result is valid only for gapless superconductors with high concentration of impurities. In the case of ordinary superconductors with a gap, the field penetrates into the superconductor to a significantly large depth $l_b = (D\tau_b)^{1/2}$, where $D = vl/3$ is the diffusion coefficient, $\tau_b = \tau_e(T/\Delta)^{1/2}$ is the time in which equilibrium is established between the populations of the electron-like ($\xi = (p - p_0)v > 0$, p_0 and v are the Fermi momentum and velocity, respectively) and the hole-like ($\xi < 0$) branches of the spectrum, and τ_e is the energy relaxation time of the quasiparticles. This idea was advanced by Clarke and Tinkham,[6,7] who determined the time τ_b both theoretically and experimentally, and produced the asymmetry of branch populations by means of tunnel injection. The equation describing the spatial change of E in ordinary superconductors with a gap was derived for temperatures near T_c in the work of Schmid and Schön[8] for the case of dirty superconductors ($l \ll \xi/(T)$) and by us[9] for the case of pure superconductors ($l \gg \xi(t)$); see also Ref. 15.

The distribution of the electric field E (and thus the resistance) is calculated in the present paper for a superconductor in the intermediate state. The analysis will be carried out at various relations between the thickness of the layers of the S and N regions (L_S and L_N) and characteristic lengths l, l_e, l_b, where $l_e = (D\tau_e)^{1/2}$ is the energy relaxation length of the quasiparticles. In the case L_S, $L_N \gg l_e$, the problem is identical in its setup with that considered previously.[9] However, reflection of the quasiparticles at the S–N boundary, investigated by Andreev,[10] was not considered in Ref. 9. This can be done in the lowest approximation in Δ/T, since the fraction of quasiparticles reflected from the S–N boundary near T_c is small. Then the field E and the potential φ turned out to be continuous at the S–N boundary. Account of Andreev reflection in this case leads to a new effect—discontinuity of the electric field E at the S–N boundary and, consequently, a decrease in the resistance of the superconducting regions. This discontinuity decreases as one approaches the critical temperature, and increases with increase in the ratio l_e/l. It will also be shown that in the presence of a temperature gradient T'_x in the superconductor the reflection of quasiparticles from the boundary leads in first order in T'_x to perturbation of the gap near the boundary.

We note that a qualitative analysis of the effect of Andreev reflection on the coordinate dependence of field and potential was undertaken by Waldram.[11] However, as a consequence of the incorrect boundary conditions used by him, even his qualitative conclusions on the role of the reflection of quasiparticles turned out to be in error.

2. BOUNDARY CONDITIONS AT THE S–N BOUNDARY

We shall calculate the distribution of field and potential in a superconductor, as before,[9] with the help of the kinetic equation, which is applicable to the case of pure superconductors when the characteristic scale of change of all the quantities entering into the equation is large in comparison with $\xi(T)$.[12] On the S–N boundary, where the kinetic equation is inapplicable, we shall match its solutions together with the help of boundary conditions. The conditions of reflection of quasiparti-

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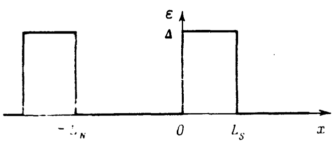

FIG. 1. Dependence of the energy gap on the coordinate.

cles at the S–N boundary were made clear by Andreev.[10] We shall use the same model of the boundaries used in his work,[10] i.e., we shall assume that at equilibrium the gap changes jumpwise from zero in the N region to a value Δ in the S region (see Fig. 1). In the derivation of the boundary conditions, Andreev[10] copied the Bogolyubov–de Gennes equations and obtained from the condition of continuity of the solutions at the boundary solutions that corresponded to incident, reflected and transmitted waves. The transmission coefficient of quasiparticles with energy ε through the boundary ($w(\varepsilon)$) was then calculated, with the help of which a connection was established between the quasiparticle distribution functions n^N and n^S in the N and S regions;

$$\begin{gathered}n^S(v_g>0)=wn^N(v_g>0)+(1-w)n^S(v_g<0),\\ n^N(v_g<0)=wn^S(v_g<0)+(1-w)n^N(v_g>0),\\ w(\varepsilon)=2(\varepsilon^2-\Delta^2)^{1/2}/[\varepsilon+(\varepsilon^2-\Delta^2)^{1/2}]\\ \text{at}\ \ \varepsilon=|\xi^N|=((\xi^S)^2+\Delta^2)^{1/2}>0\end{gathered} \tag{1}$$

and

$$n^N(v_g<0)=n^N(v_g>0)\qquad \text{at}\quad \varepsilon<\Delta. \tag{2}$$

Here $v_g=v_x(\xi/\varepsilon)$ is the group velocity of the quasiparticles. The functions n^N and n^S enter into (1) and (2) at the same values of the energy ε and directions of the momentum $\mathbf{p}=m\mathbf{v}$ and, consequently, at different values of $\xi=(p-p_0)v$ in the N and S regions.

It is necessary for us to generalize the conditions (1) and (2) to the case of the presence of a field E and a current in the system. For this purpose, we must insert the scalar and vector potentials φ and $\mathbf{A}$ in the Hamiltonian in the Bogolyubov-de Gennes equations and, separating out the phase χ of the order parameter, introduce the gauge-invariant potentials

$$\Phi=e\varphi+\frac{1}{2}\frac{\partial\chi}{\partial t},\qquad \mathbf{p}_S=\frac{1}{2}\nabla\chi+\frac{e}{c}\mathbf{A}. \tag{3}$$

Here the equations, meaning therefore the transmission coefficient and the conditions (1) and (2), retain the same form, except that ξ must be replaced by

$$\tilde{\xi}=\xi+\Phi+{}^1/_2 mv_S^2,$$

and by n^N and n^S in (1) and (2), we must mean the distribution functions at the energies

$$\tilde{\varepsilon}=|\tilde{\xi}^N|=[(\tilde{\xi}^S)^2+\Delta^2]^{1/2}+\mathbf{p}\mathbf{v}_S.$$

We write down these conditions in the form

$$\begin{gathered}[n_\mp]=n_+^N-n_+^S=\gamma n_-^N\,\mathrm{sign}\,\mu\quad\text{at}\quad\tilde{\varepsilon}>\Delta,\\ n_-^N=n_-^S\theta(\tilde{\varepsilon}-\Delta),\end{gathered} \tag{4}$$

where $n_\pm^{N,S}$ are the even and odd parts (in $\tilde{\xi}^{N,S}$) of the distribution function in the N and S regions, $\mu=p_x/p_0$, and

$$\gamma=\frac{\varepsilon-|\tilde{\xi}^S|}{\tilde{\xi}^S}=\frac{2(1-w)}{w}\,\mathrm{sign}\,\tilde{\xi}^S.$$

We also derive the effective boundary conditions which we need in the case in which the thicknesses of the S and N layers exceed the mean free path. We write down the kinetic equation

$$\frac{\partial n}{\partial t}+\frac{\partial\xi}{\partial\mathbf{p}}\frac{\partial n}{\partial\mathbf{x}}-\frac{\partial\xi}{\partial\mathbf{x}}\frac{\partial n}{\partial\mathbf{p}}=I_{imp}(n)+I_{ph}(n), \tag{5}$$

where I_{imp} and I_{ph} are the collision integrals of the quasiparticles with the impurities and the phonons. At distances from the boundary that are large in comparison with the path length $l=v\tau$, in the case in which the frequency of the inelastic collisions with phonons is small ($\nu_e\ll\tau^{-1}$), the solution (5) in first order in the perturbation has the form[9]

$$n=n_0(\tilde{\varepsilon})+n_1,\quad n_1=a_0+\mu a_1, \tag{6}$$

where $n_0(\tilde{\varepsilon})$ is the Fermi distribution function and a_0 and a_1 satisfy the equations

$$a_1=-l\,\mathrm{sign}\,\xi\left[\frac{\partial a_0}{\partial x}-n_0'\varepsilon T_x'/T\right], \tag{7a}$$

$$\frac{|\xi|}{\varepsilon}D\frac{\partial^2 a_0}{\partial x^2}=-\bar{I}_{ph}(a_0),\qquad n_0'=\frac{\partial n_0}{\partial\varepsilon}=-\frac{1}{4T}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}, \tag{7b}$$

where $D=vl/3$, $\overline{I_{ph}}$ (a_0 is the linearized integral of the collision with the phonons, averaged over the angles). We have written out here the term proportional to the temperature gradient, aiming to analyze later the change in the gap Δ near the S–N boundary under the action of T_x'.

The equilibrium part of the solution $n_0(\varepsilon)$ satisfies the conditions (4). We should require that the correction n_1 also satisfy the boundary conditions. Since n_1 is proportional to the perturbation, and we solve a problem that is linear in the perturbation, it suffices to require that n_1 satisfy the condition (4), in which $\tilde{\xi}$ and $\tilde{\varepsilon}$ should be replaced by ξ and ε. The increment n_1 in the form (6) does not satisfy the boundary condition at $\varepsilon>\Delta$. This means that at $|x|\lesssim l$ we can no longer limit ourselves to the first two terms in the expansion of nL in Legendre polynomials. It is necessary to seek n_1 in the form

$$n_1=a_0+\mu a_1+f(x),$$

where the function $f(x)$, the expansion of which contains everything with the exception of the first Legendre polynomial, tends to zero at $|x|\gg l$. We need to obtain the matching condition for functions a_0 and a_1 that do not vanish at $|x|\gg l$. Since $f(x)$ falls off at distances

$$l\ll l_e=(D\tau_e)^{1/2},$$

its behavior is determined only by the collisions with the impurities; therefore, we omit the collision integral with phonons in the equation for $f(x)$. The boundary conditions for a_0 and a_1 follow from the equations for $f(x)$ (see the Appendix).

$$a_{1+}{}^{N}-a_{1+}{}^{S}=[a_{1+}]={}^{3}/{}_{2}\gamma a_{0-},\ [a_{0-}]=0,\ \varepsilon>\Delta, \tag{8}$$

$$[a_{0+}]={}^{3}/{}_{4}\gamma a_{1-},\ [a_{1-}]=0,\ \varepsilon>\Delta, \tag{9}$$

where the plus and minus signs denote the even and odd (in ξ) parts of the functions. The boundary conditions (8) and (9) allow us to find the form of the distribution functions and, consequently, the $E(x)$ dependence at distances from the boundary exceeding the path length l.

3. THICK LAYERS

In this section we calculate the electrical conductivity of the system for the case in which the thicknesses of the layers are large in comparison with the path length; $L_{S,N}\gg l$. The continuity of the potential Φ at the boundary at any ratio of the thickness of the layers to the path length follows immediately from the continuity of the odd part of a_0 in ξ and from the expression which follows from the condition of neutrality[9]

$$\Phi=\int\limits_{-\infty}^{+\infty}d\xi\frac{\xi}{\varepsilon}a_0, \tag{10}$$

which is valid for any temperature. To find E, we must solve Eq. (7b) with the matching conditions (8) and (9). We first consider the case of a temperature close to critical ($\Delta\ll T$).

In lowest approximation in Δ/T, continuity of the distribution function on the S–N boundary follows from (8) and (9), and the problem reduces to one previously solved.[9] In this approximation, the electric field created in the N region by the flowing current is continuous at the S–N boundary, and the solution of Eq. (7b) is of the form

$$a_0=-n_0'\Phi(x)\operatorname{sign}\xi. \tag{11}$$

In the S region, Φ satisfies the equation[9]

$$\frac{\partial^2\Phi}{\partial x^2}-l_b^{-2}\Phi=0, \tag{12}$$

where $l_b=l_e(T/\Delta)^{1/2}$. Allowance for the finiteness of Δ leads to the result that the solution should differ from (11) in the energy range $\varepsilon\ll T$, where the parameter γ in the boundary conditions is not small.

At $\varepsilon\ll T$, the collision integral with phonons $\bar{I}_{ph}(a_0)$ can be simplified (for the expression for $\bar{I}_{ph}$, see Ref. 12, for example). We substitute expression (11) in that part of $\bar{I}_{ph}(a_0)$ in which a_0 is contained under the integral sign. This is valid with accuracy to small corrections, since the essential contribution to the integral is made by the region $\varepsilon\ll T$. Calculating the integrals entering into $\bar{I}_{ph}(a_0)$, we obtain

$$\bar{I}_{ph}(a_0)=\tau_e^{-1}\left[-n_0'\frac{\xi}{\varepsilon}\Phi(x)-a_0\right],$$

where

$$\tau_e^{-1}=7\pi\zeta(3)\zeta_{ph}T^3/\Theta_D^2$$

is the energy-relaxation frequency. In the same way, the equation reduces at $\varepsilon\ll T$ to the differential equation

$$\frac{|\xi|}{\varepsilon}D\frac{\partial^2a_0}{\partial x^2}-a_0=n_0'\frac{\xi}{\varepsilon}\Phi. \tag{13}$$

At $\varepsilon\ll T$, this equation is exact, and at $\varepsilon\lesssim T$ it gives the qualitatively correct result.

We now consider the structure shown in Fig. 1. As is known, a phase difference that increases linearly with time arises between two superconductors separated by a normal region. We shall therefore assume that the quantity $\dot{p}=\partial p_S/\partial t$, determined by Eq. (3), is different from zero in the N layer. We note that a sufficiently thin layer is considered in the present work ($L_{S,N}\gg\xi(0)$), so that we do not have to take the Josephson effect into account (see Ref. 13, for example). The electric field in the N region is equal to

$$eE=\dot{p}_S-\partial\Phi/\partial x, \tag{14}$$

where $\dot{p}_S$ does not depend on the time. In the S region

$$eE=-\partial\Phi/\partial x. \tag{15}$$

In correspondence with what has been said, the expression for a_1 in the N region is of the form

$$a_1=-l\left[\operatorname{sign}\xi\frac{\partial a_0}{\partial x}+n_0'\dot{p}_S\right]. \tag{16}$$

We find a_0 from (13):

$$a_0{}^S(x)=n_0'\left\{-\Phi(x)\frac{\xi}{\varepsilon}+C^S\operatorname{sh}\left[\frac{x-L_S/2}{l_e}\left(\frac{\varepsilon}{|\xi|}\right)^{1/2}\right]\right\},$$
$$a_0{}^N(x)=n_0'\{-\Phi(x)\operatorname{sign}\xi+C^N\operatorname{sh}[(x+L_N/2)/l_e]\}. \tag{17}$$

The constants $C^{N,S}$ are determined from the matching conditions (8) and (9) and the relation (16). We need the expression for $a_0^S(0)$ at $\varepsilon\gg\Delta$; it is of the form

$$a_0{}^S(0)=-n_0'\operatorname{sign}\xi\frac{E^N-E^S+\Phi(0)\alpha}{\alpha+{}^3/{}_2\gamma(l_e/l)},$$
$$\alpha=\operatorname{cth}\frac{L_N}{2l_e}+\operatorname{cth}\frac{L_S}{2l_e}. \tag{18}$$

From the expressions (7a), (10) and (14), (15) for the field difference at the S–N boundary, we have

$$E^N-E^S=2l^{-1}\int\limits_0^\Delta a_1{}^N(0)\,d\varepsilon+3l^{-1}\int\limits_\Delta^\infty\gamma a_0{}^S(0)\,d\varepsilon. \tag{19}$$

The fundamental contribution is made here by the second term. Substituting $a_0^S(0)$ from (18) in (19), we find

$$E^N-E^S=E^S\alpha\left(\frac{T}{\Delta}\right)^{1/2}\operatorname{th}\left(\frac{L_s}{2l_b}\right)\begin{cases}\pi\varepsilon_0/4T, & \varepsilon_0\ll T\\ 3\varepsilon_0{}^2/\pi^2T^2, & \varepsilon_0\gg T\end{cases}, \tag{20}$$

where $\varepsilon_0=\Delta(3l_e/4\alpha l)^{1/2}\gg\Delta$ is the characteristic energy in the integrand in Eq. (19). In obtaining (20), we have used the form of $\Phi(x)$ in the S region:

$$\Phi^S(x)=E^S l_b \operatorname{ch}^{-1}(L_S/2l_b)\operatorname{sh}[(x-L_S/2)/l^b]. \tag{21}$$

We note that at $\varepsilon_0 \ll T$, the formula (20) is exact and at $\varepsilon_0 \gg T$ it has only a qualitative character, since the model collision integral in (13) is used in this case. However, if $L_{S,N} \ll l_e$, then the formula (20) is valid at any ε_0, since in this case the integral of inelastic collisions is unimportant. Moreover, at $L_{S,N} \ll l_e$, we can obtain an expression for the discontinuity of the fields that is valid at any temperature:

$$E^N=E^S J^{-1}\left(1-\frac{L_S}{L_N}J\right), \quad J=-\int_\Delta^\infty \frac{2n_0'\,d\varepsilon}{1+L_S/L_N+{}^3/_4\gamma(L_S/l)}. \tag{22}$$

At $\Delta \ll T$, the formula (20) follows from (22) at $L_{S,N} \ll l_e$ and at $\Delta \gg T$, we obtain

$$E^S=E^N\frac{4(2\pi)^{1/2}}{3}\frac{l}{L_S}\left(\frac{T}{\Delta}\right)^{1/2}e^{-\Delta/T}.$$

What is measured in the experiment is the effective electrical conductivity σ^*, is defined by the equation

$$j=\sigma^*\bar{E}=\sigma^*(L_S+L_N)^{-1}\int_0^{L_S+L_N}E(x)\,dx=\sigma^*\frac{E^N L_N+2l_b E^S \operatorname{th}(L_S/2l_b)}{L_S+L_N}. \tag{23}$$

Taking it into account that $j=\sigma E^N$ in the N layer, we find

$$\frac{\sigma^*}{\sigma}=1+\frac{E^N L_S-2l_b E^S \operatorname{th}(L_S/2l_b)}{E^N L_N+2l_b E^S \operatorname{th}(L_S/2l_b)}. \tag{24}$$

Formulas (20), (22), and (24) connect the measured quantity σ^* with the characteristics of the system $L_{S,N}$, l, l_e, and so on.

With the help of Eq. (40) (see the Appendix), we can clarify the behavior of the field and the potential at distances less than or of the order of the path distance from the boundary. The function $\bar{f}_-(x)$, obtained with the aid of the Fourier transform of the principal (second) term in the right hand side of Eq. (4), is negative, i.e., the Andreev reflection leads to a decrease in the difference of the electron-like and hole-like perturbations at distances less than or of the order of l from the boundary. It turns out, moreover, that as $x \to 0$, the function $\bar{f}_-(x)$ and, in accord with (10), $\Phi(x)$ also, behaves as $x\ln x + O(1) + O(x)$; consequently, the field $E=-e^{-1}\Phi_x'$ diverges logarithmically near the boundary. However, if we take it into account that the energy gap at the S–N boundary does changes not discontinuously but smoothly over the correlation length, the divergence is removed. This can be verified by replacing $\delta(x)$ in (37) by a smooth function that differs from zero at $|x| \lesssim \xi(T)$. As a result, the electric field has an addition with a maximum in the N region and a minimum in the S region, the value of which is of the order of

$$E^S\left(\frac{\Delta}{T}\frac{l_e}{l}\right)^{1/2}\ln\frac{l}{\xi(T)}.$$

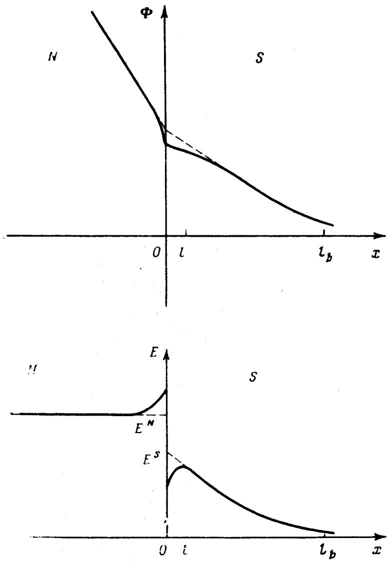

FIG. 2. Coordinate dependence of the electric field and of the potential near the boundary.

At distances $|x|>l$ the contributions to the field and the potential due to $\bar{f}_-(x)$ fall off exponentially. This means that they do not make a contribution to the measured difference in the potentials. The variation of the electric field and the potential with the coordinate are shown in Fig. 2 for the case $L_{N,S} \gg l_b$.

We note that, in spite of the fact that the electric field existing in the interior of the S region (at distances $\sim l_b$) produces a quasiparticle current, there is no total current in the interior of the superconductor. The component of the current of quasiparticles in the direction perpendicular to the boundary is cancelled by the current of pairs in the interior of the superconductor

$$j_x=\sigma E+eN_S v_{Sx}=0$$

as a consequence of the Meissner effect, since the total current distribution is described by the same equation as in the absence of an electric field, i.e., the total current in the S region flows in the Meissner layer along the S–N boundary.

4. THIN LAYERS

In this case, the solution of Eq. (31) (on the left side of which there should be added a term $n_0'\dot{p}_S v\mu$) is sought in the form (32). For the nonequilibrium antisymmetrical (in the angles) contribution to the distribution function η we have:

$$\eta=-\mu l\left[\operatorname{sign}\xi\frac{\partial\psi}{\partial x}+\dot{p}_S n_0'\right]. \tag{25}$$

For the contribution ψ that is symmetric over the angles, we obtain the following from Eq. (33), in which only the first term on the left side should remain,

$$\psi(x)=C_1x+C_2. \tag{26}$$

In addition, we must again substitute the solutions (26) and (25) in the boundary conditions (3), taking into account the antisymmetry of $\psi(x)$ relative to the median of the S and N layers. We can neglect the component with γ in the right side of (3), since this leads to contributions of the type $L_{S,N}/l$. Acting in this fashion, we find that

$$\psi^{S,N}(x)=\pm\frac{p_S n_0' L_{N,S}}{L_N+L_S}\left(x\mp\frac{L_{S,N}}{2}\right)\operatorname{sign}\xi, \tag{27}$$

where the plus sign refers to the S region and the minus sign to the N region.

It is not difficult to verify with the help of Eqs. (27) and (25) and also Eq. (9), in which a_0 should be replaced by ψ, that the current in the N region is equal to $j=\sigma E^N$ and the average field $\bar{E}$ is equal to $\bar{E}=(E^S L_S+E^N L_N)\times(L_S+L_N)^{-1}$, since the field $E^{S,N}$ in each layer does not depend on the coordinates. If we determine next the field $E^{S,N}$ by means of (4), (15) and (9), we find, finally, that

$$\frac{\sigma^*}{\sigma}=1+\frac{L_S}{L_N}\operatorname{th}\frac{\Delta}{2T}. \tag{28}$$

This expression is valid at all temperatures.

5. THE PERTURBATION OF THE ENERGY GAP NEAR THE S–N BOUNDARY

By studying the passage of current through the S and N layers, we have seen that the equilibrium-distribution-function increment symmetric in the angles is asymmetric in ξ[Eqs. (18) and (27)], i.e., perturbation of the gap does not occur in the linear case considered. A different situation arises in the presence of a temperature gradient T_x' in the system. In this case, because of the Andreev reflection of the quasiparticles, the gap $\Delta(T)$ is perturbed at distances $\sim l_e$ from the boundary.

We limit our consideration to thick layers $L_{S,N}\gg l_e$ and assume that the temperature is near critical. Eq. (13) has then a solution that satisfies the boundary conditions (8) and (9) in the S region, in the form

$$a_0^S=\frac{\Phi_0}{4T}\frac{\xi}{\varepsilon}\exp\left(-\frac{x}{l_b}\right)+\left\{\frac{\Phi_0}{4T}\frac{\operatorname{sign}\xi-(\xi/\varepsilon)(1+{}^3/_2|\gamma|l_e/l)}{1+(\varepsilon/|\xi|)^{1/2}+{}^3/_2|\gamma|l_e/l}+\frac{3lT_x'|\gamma|\varepsilon}{16T^2[1+(\varepsilon/|\xi|)^{1/2}]}\right\}\exp\left[-\frac{x}{l_b}\left(\frac{\varepsilon}{|\xi|}\right)^{1/2}\right],\quad \varepsilon\ll T. \tag{29}$$

In the case $T_x'\neq 0$, the symmetric (in the angles) increment (29) to the equilibrium function contains a part that is even in ξ and, consequently, leads to a perturbation of the gap in first order in the temperature gradient. In obtaining the expression (29), we have taken it into account that the temperature gradient is the same in the N and S layers, to within terms of order $(\Delta/T)^2$. Moreover, since the correction (29) to the distribution function leads to a change proportional to $(\Delta/T)^2$ in the mean energy of the quasiparticles and the heat flux, while the coefficient of thermal conductivity in the N and S regions also differs by an amount $\sim(\Delta/T)^2$, it follows from the condition of constancy of the heat flux

$$q=-\varkappa T_x'$$

that

$$d^2T/dx^2=0$$

to within terms $\sim(\Delta/T)^2$. For this reason, we have omitted the term with $\partial^2T/\partial x^2$ from (7b) and (13).

It follows from (29) that the change in the thermoelectric field on the S–N boundary is also determined by Eq. (20), where the field in the N region is now

$$E^N=\frac{\beta}{\sigma}T_x'.$$

Here β/σ is the differential thermoelectric power in the normal state.

In order to calculate the increase in the energy gap, we make use of the self-consistency condition, which is applicable also in the nonequilibrium case[12,14]

$$1=\lambda\int_0^{\omega_D}\frac{1-2n_0-2a_0}{\varepsilon}d\xi. \tag{30}$$

The expression for the increase in the gap at $\Delta\ll T$ follows from (30):

$$\delta\Delta=-\frac{8T^2}{C\Delta}\int_\Delta^\infty\frac{a_{0+}}{\varepsilon}d\xi,\qquad C=\int_0^\infty\frac{dx}{x^2}\left(\frac{\operatorname{th}x}{x}-\operatorname{ch}^{-2}x\right)\approx 0.79.$$

Then, substituting the even part of a_0 from (29), we obtain

$$\delta\Delta\approx-1.9lT_x'J_1(x/l_e),$$

$$J_1(z)=\int_0^\infty dy[(1+y^{-2})^{1/4}-1]\exp[-z(1+y^{-2})^{1/4}].$$

6. CONCLUSION

At the boundary of superconducting and normal phases, there is a jump in the electric field if flow of current or heat is produced in the transverse direction and the potential changes jumpwise on going through the boundary. Thus, for example, in the case of sufficiently thick layers ($L_{S,N}\gg l$) at $(T_c-T)/T_c\ll(l/l_c)^2$, in accord with (20), the jump in the field is small and the contribution of the S region to the total resistance or thermal emf of a superconductor in the intermediate state is the same as in the normal state (if $L_S\ll l_b$) or is equal to the resistance of the normal layer of thickness $2l_b=2l_e(T/\Delta)^{1/2}$ (if $L_S\gg l_b$). If $(T_c-T)/T_c\gtrsim(l/l_e)^2$, which, by virtue of the condition $l/l_e\ll 1$, is possible even at temperatures near critical, the field in the S region becomes much less than the field in the N region. At low temperatures ($\Delta>T$) the electric field in the superconductor is proportional to $e^{-\Delta/T}$ (see (22) and (28)) and, correspondingly, the contribution of the S region to the resistance and the thermal emf of the superconductor in the intermediate state become exponentially small.

The energy gap in the superconductor is not perturbed by passage of current in first order in the current (to within small terms $\sim T/\xi_F$). The presence of a temperature gradient leads to the perturbation of the gap at distances of the order of the energy relaxation length. The increase or decrease of the gap (depending on the direction of the thermal flux) is in this case a consequence of the Andreev reflection of excitations at the S–N boundary.

As is seen from the obtained formulas, the effective electric conductivity σ^* of the superconductor in the intermediate state depends on many parameters ($L_{S,N}$, l, l_e, l_b, $\Delta(T)$). By changing the magnetic field and the temperature, we can vary the relation between the thickness of the layers $L_{S,N}$ and the characteristic lengths of the system l, l_e, l_b and obtain important information on the parameters of the superconductors from measurements of σ^*.

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APPENDIX

We introduce the boundary conditions (8) and (9). We linearize (5) relative to perturbation of the distribution function n_1. Omitting I_{ph}, we obtain

$$\mu l \operatorname{sign}\xi \frac{\partial n_1}{\partial x} = -(n_1 - \bar{n}_1), \tag{31}$$

where the bar indicates averaging over the angles and the scattering by impurities is assumed to be isotropic. Since $a_0 + \mu a_1$ satisfies Eq. (31), it follows that $f(\mu, x)$ also satisfies this equation. We divide $f(\mu, x)$ into parts symmetric and antisymmetric with respect to the angles:

$$f = \psi(x) + \eta(x). \tag{32}$$

From (31) we find equations for η and φ:

$$\eta = -\mu l \frac{\partial \psi}{\partial x} \operatorname{sign}\xi, \qquad l^2\mu^2 \frac{\partial^2 \psi}{\partial x^2} - \psi = -\bar{\psi}, \tag{33}$$

with boundary conditions that follow from (4):

$$[\eta_-(0)] \equiv \eta^N(0) - \eta^S(0) = 0, \tag{34}$$

$$[\psi_+(0)] = [a_{0+}(0)] + \gamma\{|\mu| a_{1-} + \eta_-(0) \operatorname{sign}\mu\}, \tag{35}$$

$$[\eta_+(0)] = \mu[a_{1+}(0)] + \gamma\{a_{0-}(0) + \psi_-(0)\} \operatorname{sign}\mu, \tag{36}$$

where $\gamma = \xi^{-1}(\varepsilon - |\xi|)$.

We first consider the equations for the parts of ξ odd in ψ, i.e., for ψ_-. Using (33) and (36), we write down the equation for ψ_-:

$$\mu^2 l^2 \frac{\partial^2 \psi}{\partial x^2} - \psi_-(x) = -\bar{\psi}_-(x) + \delta(x)\{\mu^2[a_{1+}] + \gamma|\mu|(a_{0-}(0) + \psi_-(0))\} \operatorname{sign}\xi. \tag{37}$$

We now carry out a Fourier transformation in the dimensionless coordinate x/l, and express $\psi_-(0)$ in terms of the integral of the Fourier components $\psi_-(k)$ and $\psi_-(k) = \bar{f}_-(k)$ with respect to k. Then, eliminating $\psi_-(0)$ and averaging over μ, we obtain the integral equation

$$f_-(k) = -\frac{|\gamma|}{2\pi(2+|\gamma|)D(k)} \int_{-\infty}^{\infty} dk_1 \frac{\bar{f}_-(k_1)}{(k_1^2 - k^2)} \ln\frac{1+k_1^2}{1+k^2} + \frac{2\operatorname{sign}\xi}{2+|\gamma|}\frac{1}{k^2}[a_{1+}] - \frac{|\gamma|}{2+|\gamma|}\frac{\ln(1+k^2)}{k^2 D(k)} a_{0-}(0), \tag{38}$$

where $D(k) = 1 - k^{-1}\tan^{-1}k$. We can obtain conditions on $[a_{1+}(0)]$ and $a_{0-}(0)$ from the requirement $\bar{f}_-(k)$ be finite as $k \to 0$:

$$-\frac{2}{3}[a_{1+}(0)] + |\gamma| a_{0-}(0) + \frac{|\gamma|}{2\pi}\int_{-\infty}^{\infty} dk \frac{\ln(1+k^2)}{k^2}\bar{f}_-(k) = 0. \tag{39}$$

Eliminating $[a_{1+}]$ from (39) and (38), we finally have

$$\bar{f}_-(k) = -\frac{|\gamma|}{2\pi(2+|\gamma|)}\int_{-\infty}^{\infty} dk_1 \bar{f}_-(k_1)\left\{\frac{\ln[(1+k_1^2)/(1+k^2)]}{(k_1^2-k^2)D(k)} - \frac{3\ln(1+k_1^2)}{k^2 k_1^2}\right\} - \frac{|\gamma| a_{0-}(0)}{2+|\gamma|}\left\{\frac{\ln(1+k^2)}{D(k)} - 3\right\}\frac{1}{k^2}. \tag{40}$$

Carrying out a similar procedure with the function ψ_+, we can obtain expressions similar to (39) and (40) for $f_+(k)$, $[a_{0+}(0)]$ and a_{1-}:

$$-\frac{4}{3}[a_{0+}(0)] + |\gamma| a_{1-}(0) + \frac{|\gamma|}{\pi}\int_{-\infty}^{\infty} dk\left\{1 - \frac{\ln(1+k^2)}{k^2}\right\}\bar{f}_+(k) = 0, \tag{41}$$

$$\bar{f}_+(k) = \frac{|\gamma|}{2\pi(2+|\gamma|)}\int_{-\infty}^{\infty} dk_1 \frac{\bar{f}_+(k_1)}{k^2}\left\{\frac{k^2\ln(1+k_1^2) - k_1^2\ln(1+k^2)}{(k^2-k_1^2)D(k)} - 3\left(1 - \frac{\ln(1+k_1^2)}{k_1^2}\right)\right\} + \frac{\gamma}{2+|\gamma|}\frac{a_{1-}(0)}{k^2}\left\{\frac{k^2 - \ln(1+k^2)}{k^2 D(k)} - \frac{3}{2}\right\}. \tag{42}$$

We must determine $\bar{f}_-(k)$ from (41) and substitute in (39). The solution (40) cannot be found in the general case but, fortunately, it turns out that this is not necessary since, as is seen from (18), we are interested in the values of a_0 and a_1 at small $\gamma \sim l/l_e$, i.e., at $\varepsilon \sim \Delta(l_e/l)^{1/2} \gg \Delta$. In this case, the solution can be found by iteration, assuming the integral term to be small (we note that even for values of γ that are not small, the solution of Eqs. (40) and (42) can also be found by iteration, assuming the integral term to be small, since its smallness is assured by the numerical coefficient $1/2\pi$). On the basis of what was stated above, we find in the principal approximation in l/l_e the first of the conditions (8) from (39).

We find the condition (9) for $[a_{0+}]$ in similar fashion from Eqs. (41) and (42), to within a few percent. The conditions

$$[a_{0-}] = [a_{1-}] = 0$$

immediately follow from the second condition (4) and from the relation

$$3\overline{\mu n_1(0,\mu)} = a_1(0), \quad 3\overline{\mu^2 n_1(0,\mu)} = a_0(0),$$

which follows from Eq. (31) under the condition $\overline{\mu f(x)} = 0$.

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Inverted hot-electron states and negative conductivity in semiconductors

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The feasibility of obtaining inverted hot-electron distribution functions and a negative differential conductivity (NDC) in semiconductors at high frequencies and in strong electric and magnetic fields is considered for the case when a strong scattering mechanism is turned on only above a certain critical electron energy $\epsilon=\epsilon_0$. In pure n-GaAs in a moderate electric field at lattice temperatures $T \ll \hbar\omega_0$, the electron energy ϵ_0 can be identified with the optical phonon energy $\hbar\omega_0$. Numerical calculations of the electron distribution function by the Monte Carlo method show that in this case it should be possible to obtain in crossed electric and magnetic fields a maser type NDC at frequencies $\omega \gtrsim 10^{12}$ Hz near the cyclotron resonance. In a strong electric field, when ϵ_0 is the intervalley transition energy in n-GaAs, the NDC should appear at frequencies of the order of the reciprocal free-flight time of a light-valley electron in the electric field. The conditions for the appearance of NDC at frequencies determined by the time of flight of the electrons in a field E between $\epsilon=0$ and $\epsilon=\epsilon_0=\hbar\omega_0 \gg T$ in n-InSb in a magnetic field $\mathbf{H} \parallel \mathbf{E}$ are also considered for the case when the electrons occupy one lower Landau level.

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1. INTRODUCTION

Negative conductivity should be possible in inverted distributions of charged particle systems with a region of energies ε for which the population of the high-energy states is greater than the population of the low-energy states. For "hot" electrons, however, inverted distributions are not possible as a rule. In fact, under typical conditions elastic (or quasielastic) scattering predominates for hot electrons over all other processes. Hence, the distribution function is almost isotropic, and is of the Druyvesteyn type with a maximum at $\varepsilon=0$ for an arbitrary energy dependence of the scattering intensity (see, e.g., [1,2,3]). Under these conditions, inverted hot-electron distributions and the resulting negative conductivity can arise, apparently, only when the electron creation and annihilation (capture and recombination) are substantial[1)] (cf. [4]). In addition, as shown by Rabinovich, [6] there is no inverted hot-electron distribution also when elastic scattering predominates, while the electron-to-lattice energy transfer proceeds via inelastic processes.

In pure semiconductors, however, conditions exist under which strong scattering sets in (is "turned on") only at electron energies above a certain threshold ε_0, and at $\varepsilon<\varepsilon_0$ the scattering is small and the electrons move almost freely under the influence of the electromagnetic fields. Under these conditions the hot-electron distribution can be strongly anisotropic and inverted due to electron recoil or accumulation at $\varepsilon=\varepsilon_0$. The energy ε_0 can be the optical phonon energy $\hbar\omega_0$ (for $\hbar\omega_0 \gg T$), the energy ε_h at which the intervalley transfer begins to take place (as for example in n-GaAs), and others. Such conditions are also favorable to the formation of a bulk negative differential conductivity (NDC) of hot electrons at microwave frequencies. The microwave NDC can be of two types.

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