localized in the domains; the role of amplifying resonator is again played by the plate. In the third case, the displacements will be detected at all frequencies for the nonresonant component and in the mechanical resonance range of 1 to 15 odd modes for the resonance component. The nonresonant component is excited at the plate edges by uniform precession. In the resonance range, displacements excited by uniform precession at the plate edges are partially compensated by displacements excited by uniform precession at the walls. On destruction of the domain structure by an external biasing field, the compensation is removed, and the range of the detectable resonance modes extends to 59 MHz. In both cases the plate is an amplifying resonator.

## 5. CONCLUSION

From the estimates given, it is evident that excitation of acoustical oscillations by Bloch walls in a multidomain structure is quite effective. These oscillations can be detected experimentally, and investigation of them is useful in the study of multidomain structures. With decrease of the frequency of the rf field, the effectiveness of the excitation of sound at mechanical resonances of the specimen increases. These "low-frequency oscillations" may be responsible for an effect in the hundreds-of-kilohertz range in hematite.[12] The effectiveness of the sound excitation depends strongly on the orientation of the rf field, in accordance with the results of[14]; furthermore, the acoustical signal in a multidomain structure is not determined solely by the intensity of the exciting field and the properties of the specimen as an acoustic resonator. It is necessary to treat a multidomain medium as a single magnetoacoustical resonator.

A similar picture of sound excitation is possible in a domain structure of stripe domains, and also in ferroelectric materials. In the latter case, electric domains must be considered instead of magnetic.

In conclusion, the authors thank É. A. Kaner for his interest in the research.

1) This result differs from that obtained in a paper of Nedlin and Shapiro[7], which appeared while our paper was being prepared for printing. There, in the case $H_0=0$, a wall can excite only sound waves with polarization perpendicular to the magnetization in the domains. The difference is apparently due to their use of a more simplified phenomenological model.


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# Transient processes in the region where the NMR and FMR overlap

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Transient processes in a nuclear system (spin echo, decay of free induction, etc.) are investigated in the region where the NMR and FMR frequencies overlap. The singularities of the relaxation of a nuclear system via the electron system (NER) is analyzed in detail. It is shown that at small values of the nuclear magnetization $\mu$ the character of the transient processes remains unchanged in the region where the NMR and FMR frequencies overlap, but the gain of the high-frequency field and of the nuclear signal increase strongly. At large $\mu$, the transient processes in the nuclear system are determined by the NER mechanism.

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The question of the influence of the overlap of the NMR and FMR frequencies on the NMR signal observed by intermittent methods was considered earlier[1] where electron-nuclear magnetic resonance (ENMR) was predicted, a phenomenon consisting of multiple amplification of the NMR signal by the FMR signal. This result was subsequently observed in experiment.[2] Theoretically, quasistationary transient processes in a nuclear system were investigated under the conditions of overlap of NMR and FMR.[3] It is of interest to examine the sin-

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gularities that may appear in the character of the nonstationary transient processes investigated by pulsed experimental methods when the NMR and FMR frequencies overlap.

The amplification of the spin-echo amplitude in the region where the NMR and FMR frequencies overlap has already been observed, [4] but no direct calculations of the pulsed characteristics of a nuclear signal have been performed as yet. In the present article we investigate theoretically the main features of nuclear transient processes, such as the decay of the free induction and spin echo, in the region where the FMR and NMR frequencies overlap.

## CHARACTER OF MOTION OF NUCLEAR MAGNETIZATION

As in the preceding papers, [1,3,5] we start from the experimental fact that most ferromagnets are characterized by a large electron relaxation parameter $\Gamma_e$: as a rule, $\Gamma_e$ is of the order of the NMR frequency $\omega_n$, and greatly exceeds all other characteristic parameters of the motion of the nuclear system. The motion of the electron magnetization M and of the nuclear magnetization in such a situation is described by Eqs. (2.1) of[3], and for an ellipsoid of arbitrary shape, using the notation[3], we have

$$\omega_1=\gamma_e[H_1+(N_x-N_z)M],\quad \omega_2=\gamma_e[H_2+(N_y-N_z)M].$$

where $H_1=H_2=H+H_k$, for a sample magnetized along the anisotropy axis and $H_1=H-H_k$, $H_2=H$ for a sample magnetized perpendicular to the anisotropy axis. We put henceforth $\omega_1=\omega_2=\omega_e$, for simplicity, where $\omega_e$ is the FMR frequency.

In the investigation of transient processes in a nuclear system, in the region where the NMR and FMR frequencies coincide, we shall use the following approximation. Let the resonant high-frequency (HF) field $h$ acting on the system be turned off (on) at the instant $t=0$. We consider times $t\gg\Gamma_e^{-1}$, in which the natural oscillations of the electronic magnetization M attenuate. Then the solution of the equation for M can be written approximately in the form

$$M_\pm=\chi_e(h_\pm-A\mu_\pm), \tag{1}$$

where

$$\chi_e=\frac{\gamma_e M}{\omega_e-\omega_n+i\Gamma_e}=\chi_e'-i\chi_e'' \tag{2}$$

is the complex electric susceptibility, $M_\pm=M_x\pm iM_y$, $\mu_\pm=\mu_x\pm i\mu_y$, $h_\pm=h_x\pm ih_y$.

In contrast to[6,7] and other studies in which the motion of the nuclear magnetization was investigated far from the overlap region, we take into account here the imaginary part of the electronic susceptibility, i.e., account is taken of the fact that M lags the effective field acting on it by an angle $\varphi_e(\tan\varphi_e=\chi_e''/\chi_e')$. Using (1), we obtain the nonlinear equations of motion of the nuclear magnetization, which are valid in the region where the NMR and FMR frequencies overlap. At $h=0$, in a coordinate frame rotating at a frequency $\omega_n$, these equations take the form

$$\begin{gathered}\dot{\mu}_\pm+(\mp i\gamma_n A^2\chi_e'\mu_z+\gamma_n A^2\chi_e''\mu_z+\Gamma_n)\mu_\pm=0,\\ \dot{\mu}_z=\gamma_n A^2\chi_e''\mu_-\mu_+-\Gamma_1(\mu_z+\mu),\end{gathered} \tag{3}$$

where $\Gamma_n=T_2^{-1}$, $\Gamma_1=T_1^{-1}$, while $T_2$ and $T_1$ are the times of the transverse and longitudinal nuclear relaxation, respectively.

When the system (3) is linearized in the normal ($\mu_z=-\mu$) or inverted ($\mu_z=\mu$) state, the first equation leads to expressions for the dynamic frequency shift $\Delta\omega_n'$ and the nuclear-like oscillation damping coefficient:

$$\begin{gathered}\Delta\omega_n'=\mp D=\mp\gamma_n A^2\chi_e'\mu,\quad \omega_n''=\Gamma_n\pm\Gamma_\varkappa,\\ \Gamma_\varkappa=\gamma_n A^2\chi_e''\mu.\end{gathered} \tag{4}$$

Here $D$ is the parameter of the dynamic frequency shift, $\Gamma_\varkappa$ is the parameter of the nuclear-electron relaxation (NER), where NER is taken to mean the damping in the nuclear system due to relaxation via the electron system. We note that the maximum dynamic frequency shift, which is reached at the point $\omega_e(H)=\omega_n+\Gamma_e$, and the maximum increase of the damping coefficient, which is reached at $\omega_e=\omega_n$, are determined, accurate to a factor $\frac{1}{2}$, by one and the same expression $\varkappa=\omega_q^2/4\Gamma_e$, where $\omega_q=(4\gamma_e\gamma_n A^2 M\mu)^{1/2}$ is the parameter of the dynamic electron-nuclear interaction.

In all probability, the principal changes in the character of the transient processes in the nuclear system, when NMR and FMR frequencies overlap, should be connected with the abrupt enhancement of the NER (at $\omega_e\approx\omega_n$ we have $D\ll\Gamma_\varkappa$). On the other hand, if the nonlinear terms describing the NER and the dynamic frequency shift in (3) can be neglected, then we obtain the usual Bloch equation. These equations (with allowance for the inhomogeneous broadening $\Gamma_3$ of the NMR) describe[8] numerous pulsed phenomena, such as the relaxation of the free induction, spin echo, etc. Thus, at a sufficiently low value of $\mu$, when the contribution of the NER to the nuclear relaxation can be neglected, pulsed phenomena in the region where the NMR and FMR overlap do not differ in their character from pulsed phenomena far from the overlap region. The only singularity, as follows from (1), is that the gains of the HF field and of the nuclear signal are now determined by the expression $\eta=A|\chi_e|$ and they increase strongly as $\omega_e\to\omega_n$ because of the increase of the electronic susceptibility. It appears that this situation took place in the experiment of[4], where an abrupt enhancement of the nuclear spin-echo signal was observed for the first time at $\omega_e\to\omega_n$. It is clear from the foregoing that it is more convenient to observe weak nuclear signals when the NMR and FMR frequencies overlap.

With increasing NER, the character of the transition processes should change. We consider the limiting case, when the NER time $\Gamma_\varkappa^{-1}$ is the shortest relaxation time in the nuclear system (usually the shortest time is the time of dephasing of the nuclear spins as a result of the inhomogeneity of the hyperfine field $\Gamma_3^{-1}$). This situation is not too exotic at low temperatures. In fact, if far from the overlap region we have $D > \Gamma_3$, i.e., if at $\omega_n \ll \omega_e$ a dynamic frequency shift is observed with assurance in the ferromagnet (and such experiments were performed in[9]), then at $\omega_e \approx \omega_n$ the quantity $\Gamma_\varkappa^{-1}$ should be the shortest relaxation time, inasmuch as $\max D = \max \Gamma_\varkappa/2$. In this case the relaxation processes are determined primarily by the NER mechanism. This mechanism was first considered in[6], but was not investigated in detail, inasmuch as far from the overlap region it makes no substantial contribution to the nuclear relaxation.

To study the NER, we turn to Eqs. (3) and put $\Gamma_1 = \Gamma_n = 0$. We note immediately the main feature of NER: if this mechanism predominates, then the relaxation of the nuclear magnetization proceeds with conservation of the modulus $|\mu|$.

Changing over in the resultant equations to spherical coordinates ($\mu_\pm = \mu e^{\pm i\varphi} \sin\theta$, $\mu_z = -\mu \cos\theta$), we see that the variables $\theta$ and $\varphi$ separate, and we have

$$\operatorname{tg}\frac{\theta(t)}{2} = e^{-\Gamma_\varkappa t} \operatorname{tg}\frac{\theta_0}{2}, \quad \dot{\varphi} = D\cos\theta(t), \tag{5}$$

where $\theta_0$ is the initial angle between $\mu$ and the equilibrium position.

Recognizing that $\dot{\theta} = -\Gamma_\varkappa \sin\theta$, the expressions for the longitudinal ($\mu_z$) and transverse ($\mu_\perp = |\mu_+|$) components of $\mu$ can be written in the form

$$\dot{\mu}_z = -(\Gamma_\varkappa/\mu)\mu_\perp^2; \quad \dot{\mu}_\perp = (\Gamma_\varkappa/\mu)\mu_z\mu_\perp. \tag{6}$$

We see therefore that in the NER process the components $\mu_z$ and $\mu_+$ are not independent. Naturally, these expressions differ quite substantially from the ordinary Bloch expressions ($\dot{\mu}_z = -\Gamma_1(\mu_z + \mu)$, $\dot{\mu}_\perp = -\Gamma_n \mu_\perp$).

The experimentally observed transverse component of the electronic magnetization is given by

$$M_\perp = \eta\mu_\perp = \eta\mu \sin\theta. \tag{7}$$

Curve 1 in Fig. 1 shows the envelope of the free-induction signal at $\theta_0 \leqslant \pi/2$. At $\theta_0 > \pi/2$ the decay of the free induction should give way to a burst of radiation (curve 2). It must be borne in mind, however, that at $\theta_0 > \pi/2$, and especially at $\theta_0$ close to $\pi$, a possibility exists[10] of spontaneous buildup of nuclear-like spin waves (this question will be discussed later on). We emphasize once more that the NER mechanism leads only to a change in the angle between **M** and $\mu$, but does not change the value of $|\mu|$. The NER, for example, cannot take the nuclear magnetization out of the saturation state ($\mu_\perp = 0$, $-\mu < \mu_z < 0$). We note also that at $\Gamma_\varkappa > \Gamma_3$ it is impossible to observe nuclear spin echo, since the nuclear magnetization relaxes within a time shorter than the time of the reversible dephasing of the nuclear spins.

We note that all the foregoing results can be easily extended to include the case of strong asymmetry ($\omega_2 \gg \omega_1$), which is typical of the experiments in[2,4]. For this situation, as can be easily shown, we have $\eta = 2A|\chi_e|$, and $\omega_q = (\gamma_e A\mu\omega_2)^{1/2}$, where

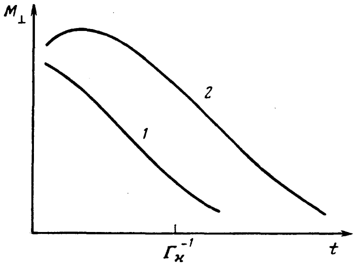


FIG. 1. Plot of $M_\perp$ against $t$: curve 1 corresponds to $\theta_0 < \pi/2$, and curve 2 to $\theta_0 > \pi/2$.

$$2\chi_e = \gamma_e M\omega_2/[(\omega_e^2 - \omega_n^2) + 2i\Gamma_e\omega_n]. \tag{8}$$

Expressions (4) for $\Gamma_\varkappa$ and $D$ remain in force when (8) is taken into account.

## ELECTRON-NUCLEAR SPIN WAVES

As already mentioned, if the nuclear magnetization is rotated through an angle $\theta_0 > \pi/2$, then the relaxation in the nuclear system can become inhomogeneous because of the spontaneous growth of electron-nuclear spin waves. Such a possibility is particularly significant at $\theta_0$ close to $\pi$, when the initial amplitude of the homogeneous deviation is small. To analyze this situation, we write down the dispersion law of the electron-nuclear spin waves in a ferromagnet with allowance for the relaxation. From the coupled system of equations of motion for **M** and $\mu$ and the equations of magnetostatics we obtain the following dispersion equation:

$$\begin{gathered}(\omega^2 + 2i\Gamma_{e\mathbf{k}}\omega - \omega_{e\mathbf{k}}^2)(\omega^2 + 2i\Gamma_n\omega - \omega_n^2) - d_\mathbf{k}^4 = 0,\\ d_\mathbf{k}^4 = 2\gamma_e A\mu\omega_n(\omega^2 + \sigma\omega_n),\\ \sigma = \gamma_e[(H_1 + H_2)/2 - N_z M + \alpha M\mathbf{k}^2 + 2\pi M \sin^2\theta_\mathbf{k}].\end{gathered} \tag{9}$$

Here $\omega_{e\mathbf{k}}$ and $\Gamma_{e\mathbf{k}}$ are the frequency and damping parameter of the electron spin wave, expressions for which are given, for example, in[11]; $\alpha$ is the exchange-interaction constant.

If the parameter $\omega_{q\mathbf{k}} = [2\gamma_e A\mu(\omega_n + \sigma)]^{1/2}$ of the dynamic interaction between the electron and nuclear spin waves is much less than $\Gamma_{e\mathbf{k}}$, then, accurate to terms $\sim\mu$, we obtain

$$\begin{gathered}\tilde{\omega}_{e\mathbf{k}} = \omega_{e\mathbf{k}}\left(1 + d_{e\mathbf{k}}^4 \frac{\omega_{e\mathbf{k}}^2 - \omega_n^2}{2\omega_{e\mathbf{k}}^2 B_\mathbf{k}^4}\right) + i\Gamma_{e\mathbf{k}}\left(1 - \frac{d_{e\mathbf{k}}^4}{B_\mathbf{k}^4}\right),\\ \tilde{\omega}_{n\mathbf{k}} = \omega_n\left(1 + d_{n\mathbf{k}}^4 \frac{\omega_n^2 - \omega_{e\mathbf{k}}^2}{2\omega_n^2 B_\mathbf{k}^4}\right) + i\left(\Gamma_n + \Gamma_{e\mathbf{k}}\frac{d_{n\mathbf{k}}^4}{B_\mathbf{k}^4}\right),\\ B_\mathbf{k}^4 = (\omega_{e\mathbf{k}}^2 - \omega_n^2)^2 + 4\Gamma_{e\mathbf{k}}^2\omega_n^2.\end{gathered} \tag{10}$$

Here $\tilde{\omega}_{e\mathbf{k}}$ is the complex frequency of the electron-like spin wave, $\tilde{\omega}_{n\mathbf{k}}$ is the complex frequency of the nuclear-like spin wave, $d_{e\mathbf{k}} = d_\mathbf{k}(\omega = \omega_{e\mathbf{k}})$, $d_{n\mathbf{k}} = d_\mathbf{k}(\omega = \omega_n)$.

When the nuclear magnetization is changed into the inverted state $\theta_0 \approx \pi$, the expressions (10) remain in force, subject to the substitution $\mu \to -\mu$.

The damping coefficient of the nuclear-like spin wave is determined in this case by the expression

$$\omega_{n\mathbf{k}}'' = \Gamma_n - \Gamma_{\varkappa\mathbf{k}}, \quad \Gamma_{\varkappa\mathbf{k}} = \Gamma_{e\mathbf{k}} d_{n\mathbf{k}}^4/B_\mathbf{k}^4. \tag{11}$$

It is seen therefore that at $\Gamma_{\varkappa\mathbf{k}} > \Gamma_n$ the nuclear-like spin wave does not attenuate but increases. It is obvious that if in this case $\Gamma_{\varkappa k} > \Gamma_\varkappa$, then the homogeneous transient process connected with the NER will be impossible because of the decay of the nuclear magnetization into nuclear-like spin waves.


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