Theory of electron capture by attracting centers in photoexcited semiconductors

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The lifetime of the electrons under non-equilibrium conditions, following photoexcitation of the carriers, is calculated for the case when the energy relaxation is determined by the interaction with the acoustic phonons. It is shown that at large concentration of the capture centers the lifetime is practically independent of the concentration and coincides with the time of energy relaxation to the percolation level. The case of impurity photoexcitation in a compensated semiconductor is discussed in detail. The cross section for capture by a dipole is calculated. Special consideration is given to the case of pure materials at low temperatures, when the interaction of the carriers with the acoustic phonons is essentially inelastic.

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We have previously obtained[1] the cross section for electron recombination on an attracting center under the conditions that the conduction electrons have an equilibrium Maxwellian energy distribution. The analysis was carried out within the framework of the cascade capture model,[2] according to which the electron, emitting an acoustic phonon, is captured by highly excited levels of the impurity center. It turned out that the lifetime of the carriers is determined mainly by the capture of slow electrons with energy $E\sim ms^2$ (m is the effective mass and s is the speed of sound in the crystal). It follows therefore that the principal role in the capture is played by processes of emission and absorption of low-energy phonons. This result contradicts Lax's conclusion that the lifetime is determined by the capture of the thermal carriers, wherein phonons with thermal energy are emitted.[2] As shown in[1], the reason for the contradiction is that Lax[2] carried out an incorrect averaging of the transition probability over the electron orbit in the field of the center for electron collisions with acoustic phonons.

If there are N independent centers per unit volume, then the lifetime is determined by the formula

$$\tau=[N\langle\sigma v\rangle]^{-1}, \tag{1}$$

where v is the electron velocity, and σ is the effective cross section for its recombination on one center. The averaging is carried out over a Maxwellian distribution. Using the results of[1], we obtain the lifetime in the form

$$\tau=\left[\frac{16\sqrt{2}}{3\sqrt{\pi}}\left(\frac{e^2}{\varkappa}\right)^3 N\frac{m^{5/2}E_c^2}{\rho\hbar^4(kT)^{5/2}}\right]^{-1}. \tag{2}$$

Here $\varkappa$ is the dielectric constant of the crystal, E_c is the constant of the deformation potential, and ρ is the density of the crystal.

Formula (2) was derived under the assumption that the carriers are in thermal equilibrium with the lattice. The recombination process, however, always manifests itself under non-equilibrium conditions, for example, in photoexcitation of the carriers. The carriers do not have time to become thermalized, and then the recombination process itself will influence substantially the carrier distribution with respect to energy. Understandably, in this case formula (2) does not hold.

In this paper we consider the question of the energy distribution under stationary photoexcitation and calculate the lifetime of the carriers under these conditions. It is assumed that the electrons are produced at an energy ε_i, which for simplicity will be assumed to be lower than the energy of the optical phonon, but much higher than the thermal energy kT. The recombination process proceeds in this case in the following manner. Since capture of high-energy electrons, as shown in[1] have low probability, the electrons, emitting acoustic phonons, drop down to near-zero energies, and are then captured by high-excited levels of the charged centers and continue to lose energy via interaction with the acoustic lattice vibrations in the region of negative energies. The electron turns out to be practically captured by the center when its binding energy exceeds kT. Thus, the process of capture by a recombination center can be described as continuous diffusion in the space of total energy E at positive and negative values of the energy. At high positive energies, the influence of the centers on the energy diffusion can be neglected, and in the region of high negative energies the centers can be regarded as isolated. There exists, however, a region of intermediate energies near $E=0$, in which the presence of the centers cannot be neglected (since the frequency of the collisions with the acoustic phonons vanishes at zero kinetic energy of the electron), and at the same time it is necessary to take into account the superposition of fields of many centers. Actually this means that account must be taken of large-scale fluctuations of the potential. As a result of the fluctuations, there exists at all values of the total energy regions of space in which the kinetic energy is different from zero and emission of acoustic phonons is possible.

We denote the characteristic swing of the fluctuations of the potential by E_0. Then the results of the diffusion approximation considered in Sec. 1 can be formulated in the following manner. If $kT\gg E_0$, then the carriers manage to become thermalized, and formula (2) is valid for their lifetime. On the other hand, if $E_0\gg kT$ (but

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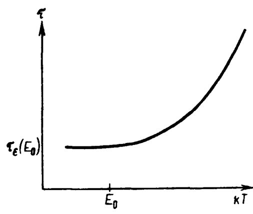

FIG. 1. Schematic form of the temperature dependence of the lifetime. At $kT \gg E_0$ the time is $\tau \sim T^{5/2}$, and $kT < E_0$ the time τ ceases to depend on the temperature.

$E_0 \ll \varepsilon_i$), then the carrier energy distribution differs strongly from the Boltzmann distribution, and their lifetime is of the order of the energy relaxation time τ_ε of the electrons on the acoustic phonons, taken at the energy E_0,

$$\tau \approx \tau_\varepsilon(E_0) = \tau_0 \sqrt{ms^2/2E_0}, \tag{3}$$

where

$$\tau_0 = \pi\hbar^4\rho/2m^3 s E_c^2. \tag{4}$$

Usually we have $E_0 \approx e^2\varkappa^{-1}N^{1/3}$, so that in the latter case the lifetime does not depend on the temperature and is practically independent of the concentration N of the capture centers. The lifetimes as functions of the temperature and concentration, corresponding to the results above, are shown schematically in Figs. 1 and 2.

Formula (2) can be expressed in terms of the same time τ_0 in the following manner:

$$\tau = \frac{\sqrt{\pi}}{2}\frac{\tau_0}{\alpha}\left(\frac{2kT}{ms^2}\right)^{5/2}, \tag{5}$$

where we have introduced a parameter α proportional to the concentration of the capture centers

$$\alpha = \frac{4}{3}\pi\left(\frac{2e^2}{\varkappa ms^2}\right)^3 N. \tag{6}$$

Since $E_0 \sim \alpha(ms^2)^3$, it follows that the results described above can be represented in the following manner. So long as $kT > E_0$, the electron lifetime is longer than the time of their energy relaxation: $\tau = \tau_\varepsilon(E_0)(kT/E_0)^{5/2}$. On the other hand, if $kT < E_0$, the lifetime is determined by the time of energy diffusion to the level E_0.

Thus, the lifetime cannot be shorter than the energy relaxation time.[1)] Accordingly, the carrier energy distribution is a monotonically decreasing function even at a large capture-center concentration, when it differs substantially from the Boltzmann distribution (see Sec. 1).

The case when the swing of the fluctuation is $E_0 < ms^2$ calls for a special analysis. This situation is possible if pure semiconductors with large effective carrier mass. For example, for holes in silicon the energy ms^2 corresponds to 3 °K and $E_0 < ms^2$ at $N < 10^{13}$ cm^{-3}. In this case allowance for the fluctuations of the impurity potential cannot "save" the diffusion approach, so that at low energies the diffusion approximation cannot be used. However, even here, so long as $kT \gg ms^2$, formula (2) for the lifetime remains valid, since it is precisely kT which determines the scale of the variation of the distribution function in the total-energy space. On the other hand, if $ms^2 > kT$, then to find the distribution function and the lifetime it is necessary to solve an integral kinetic equation. This problem is considered in Sec. 2 of this paper. In the limit $ms^2 \gg kT$ (but $ms^2 \ll \varepsilon_i$), the following result is obtained:

$$\tau = 0.34\tau_0/\alpha. \tag{7}$$

1. DISTRIBUTION FUNCTION AND LIFETIME IN THE DIFFUSION APPROXIMATION

The change of the electron energy when they interact with acoustic phonons takes place in small batches, $\sim\sqrt{\varepsilon ms^2}$ (ε is the electron kinetic energy). When describing the energy relaxation process at $\varepsilon \gg ms^2$ we can therefore use the diffusion approximation for the collision integral

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \frac{1}{\rho(\varepsilon)}\frac{\partial}{\partial \varepsilon}j, \tag{8}$$

where j is the particle flux in energy space, taken with a minus sign,

$$j = \varepsilon\tau_\varepsilon^{-1}(\varepsilon)\rho(\varepsilon)\left[f + kT\frac{\partial f}{\partial \varepsilon}\right]. \tag{9}$$

Here $\tau_\varepsilon(\varepsilon) = \tau_0\sqrt{2\varepsilon/ms^2}$ is the energy relaxation time of electrons with energy ε, and the state density is

$$\rho(\varepsilon) = 8\sqrt{2}\pi(2\pi\hbar)^{-3}m^{3/2}\varepsilon^{1/2}.$$

The distribution function is normalized by the condition

$$\int \rho(\varepsilon) f\, d\varepsilon = n, \tag{10}$$

where n is the electron density.

We assume that the distribution function depends only on the total energy. We multiply Eq. (8) by $\delta(E - \varepsilon - U(r))\rho(\varepsilon)\, V^{-1} d\varepsilon\, d^3\mathbf{r}$, where $U(r)$ is the potential energy of the electron in the field produced by the charge centers, and integrate over phase space.[3,1] The kinetic equation then takes the form

$$\bar{\rho}(E)\frac{\partial f(E)}{\partial t} = \frac{d}{dE}B(E)\left[f(E) + kT\frac{df(E)}{dE}\right]. \tag{11}$$

The coefficient $B(E)$ is proportional to the diffusion coefficient in energy space:

$$B(E) = V^{-1}\int \varepsilon\tau_\varepsilon^{-1}(\varepsilon)\rho(\varepsilon)\delta(E - \varepsilon - U(r))\, d\varepsilon\, d^3\mathbf{r}, \tag{12}$$

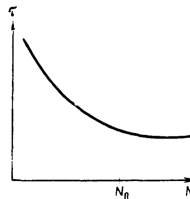

FIG. 2. Schematic form of the concentration dependence of the lifetime. At $N \ll N_0$ the time is $\tau \sim N^{-1}$, and at $N > N_0$ the time is $\tau \sim N^{-1/6}$, $N_0 \approx (\varkappa kT/e^2)^3$.

$\tilde{\rho}(E)$ is the density of states in the total-energy space, and the crystal volume is designated V.

Under stationary conditions, the right-hand side of (11) must be set equal to zero. We then obtain

$$B(E)\left[f(E)+kT\frac{df(E)}{dE}\right]=j, \tag{13}$$

where j is the numer of electrons excited by the light per unit time and per unit volume. We solve Eq. (13) in the region of energies E lower than the excitation energy ε_i, with the boundary condition

$$f(E)\to 0,\quad E\to -E_1;\quad E_1\gg kT. \tag{14}$$

In the final formulas we can replace E_1 by infinity. [3,1] The solution takes the form

$$f(E)=\frac{j}{kT}e^{-E/kT}\int_{-\infty}^{E}\frac{e^{E'/kT}}{B(E')}dE'. \tag{15}$$

Knowing $f(E)$, we can calculate the lifetime τ by the usual formula

$$\tau=n/j, \tag{16}$$

where n is the particle concentration, determined in terms of $f(E)$ from a formula similar to (10), but with $\rho(\varepsilon)$ replaced by $\tilde{\rho}(E)$. The coefficient $B(E)$ can be easily calculated in the region of large positive energies E, where the presence of the charged centers is insignificant (we can put $U(r)=0$ in (12)). We obtain

$$B(E)=E\tau_0^{-1}(E)\rho(E)=b(2E/ms^2)^2,\quad E>0, \tag{17}$$

where

$$b=(ms^2/2)\tau_0^{-1}\rho(ms^2/2). \tag{18}$$

In the region of large negative energies we can neglect the overlap of the charged centers and calculate $B(E)$ for one isolated center ($U(r)=-e^2\varkappa^{-1}r^{-1}$) and multiply by the number of centers. We obtain

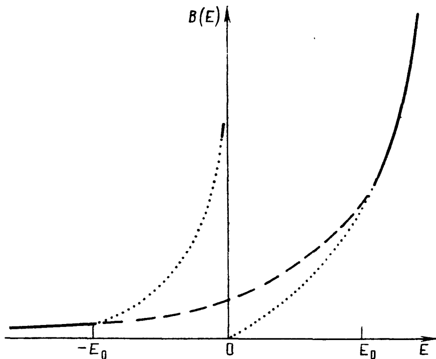

FIG. 3. Schematic form of energy dependence of the coefficient $B(E)$. E_0 is the characteristic swing of the large-scale fluctuations (see the text for an explanation).

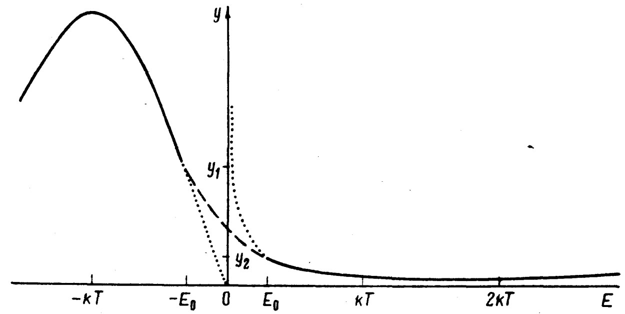

FIG. 4. Schematic form of the integrand $y=e^{E/kT}/B(E)$ (see formula (13), $E_0\ll kT$). The function reaches a maximum at $E=-kT$ and a minimum at $E=2kT$, with a ratio $y_{max}/y_{min}\sim(kT/E_0)^3$. The characteristic quantities are $y_1=y(-E_0)$, $y_2=y(+E_0)$, $y_1/y_2\sim 2E_0/ms^2$.

$$B(E)=bams^2/2|E|. \tag{19}$$

Formulas (17) and (19) are not valid near zero energy in an interval of width E_0, where E_0 is the characteristic swing of the large-scale fluctuations of the charged-center potential. The dependence of the coefficient $B(E)$ on the total energy E is shown schematically in Fig. 3. In this figure, the solid sections of the curve corrrespond to formulas (17) and (19), while the dotted sections are their continuations into the region $|E|<E_0$, where these formulas no longer hold. The section of the curve represented by the dashed line show schematically the behavior of $B(E)$ in the region of large-scale potential fluctuations.

We consider first the case $E_0\ll kT$. A plot of the integrand of (15) is shown schematically for this case in Fig. 4. It is seen from the figure that at energies not exceeding several kT (we call this the first region) the integral in (15) is determined by negative energies and is practically independent of E. For values of the energy E from the second region (at $E\gg kT$), the main contribution to the integral (15) is made by the vicinity of the upper limit, where the integrand is exponentially large. Thus, the distribution function $f(E)$ defined by formula (15) has in the first region the Maxwellian form

$$f(E)\approx\frac{j}{kT}e^{-E/kT}\int_{-\infty}^{0}\frac{e^{E'/kT}}{B(E')}dE'. \tag{20}$$

In the calculation of the integral in (20) we can use formula (19) for $B(E)$. In the second region ($E\gg kT$) $f(E)$ takes the form

$$f(E)\approx j/B(E)\sim E^{-2}. \tag{21}$$

The coefficient $B(E)$ in (21) is determined by formula (17). The contribution made to the concentration by the first region is decisive, so that the lifetime, according to (16), is equal to

$$\tau=\frac{1}{kT}\int_0^{\infty}\tilde{\rho}(E)e^{-E/kT}dE\int_{-\infty}^{0}\frac{e^{E'/kT}}{B(E')}dE', \tag{22}$$

which agrees with formula (5). Thus, the result of[1]

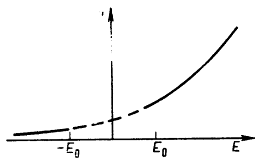

FIG. 5. Schematic form of the integrand $y = e^{E/kT}/B(E)$ (see formula (13), $E_0 \gg kT$).

is valid at $E_0 \ll kT$, which is natural, for in this case the photoelectrons have time to become thermalized. The contribution of the non-thermalized electrons (from the second region) to the concentration is small and constitutes a fraction $\sim \tau(kT)/\tau \sim (E_0/kT)^3$ of the total concentration.

When the temperature is lowered or when the capture-center concentration is increased, the characteristic swing of the large-scale fluctuations can exceed kT. We consider the limiting case $E_0 \gg kT$. Now the integrand in (15) has no more extrema, and its schematic form is shown in Fig. 5. The integrand in (15) is determined by the region near the upper limit, and the distribution function is $f(E) \approx j/B(E)$. At $E > E_0$ we have $f(E) \sim E^{-2}$. Thus, at large concentrations of the recombination centers or at low temperatures (i.e., when $E_0 > kT$) the distribution of the photoelectrons differs substantially from Maxwellian, since the electrons have no time to become thermalized. However, even in this case the distribution function decreases monotonically with increasing energy.

The lifetime at $E_0 \gg kT$ can be estimated by calculating the density of the electrons with energy $E > E_0$ and dividing it by the flux j. We then obtain formula (3), i.e., in this case the lifetime is of the order of the time of the energy relaxation of the electron to the energy level E_0.

It should be noted that electrons having an energy lower than the percolation level (which is $\sim E_0$) do not take part in the dc conductivity.

2. DISTRIBUTION FUNCTION AND LIFETIME IN PURE CRYSTALS

In very pure crystals the characteristic swing of the fluctuations of the impurity potential E_0 is small and may turn out to be the minimum kinetic energy ms^2 at which the electrons are still capable of emitting acoustic phonons. The energy region $\sim ms^2$ then assumes a major role in the determination of the distribution function. In this region, the interaction of the electrons with the acoustic phonons is essentially inelastic, and the distribution function must be obtained by solving an integral kinetic equation. However, if $kT \gg ms^2$, then an analysis similar to that of Sec. 1 for the case $kT \gg E_0$ shows that the result of[1] is valid for the lifetime (formula (5) of the present paper). In this section we consider the inverse limiting case $kT \ll ms^2$.

The kinetic equation in the total-energy space takes the form

$$f(E)\int_{-\infty}^{+\infty} w(E,E')\,dE' - \int_{-\infty}^{+\infty} f(E')\,w(E',E)\,dE' = j\delta(E-\varepsilon_i). \tag{23}$$

The right-hand side describes here generation of photoelectrons with energy ε_i. We consider first the case $T = 0$. Then the only significant processes are those with emission of acoustic phonons, and the electron can only lose energy. Consequently, the probability $w(E', E) = 0$ at $E' < E$.

If we neglect the influence of the potential of the centers on the phonon emission process, then we can obtain for $w(E', E)$ the expression

$$w(E',E) = \frac{b}{4(ms^2)^4}(E'-E)^2, \quad E<E'<(\sqrt{E}+\sqrt{2ms^2})^2 \tag{24}$$

in the region $E > ms^2/2$ and $w(E', E) = 0$ at other values of E'. Changing over to the dimensionless variables $x = 2E/ms^2$ and denoting by $f_1(x)$ the distribution function of $x > 1$, we obtain the kinetic equation in the form

$$f_1(x)(\sqrt{x}-1)^3 - \frac{3}{64}\int_x^{(\sqrt{x}+2)^2}(x'-x)^2 f_1(x')\,dx' = 0. \tag{25}$$

At $x \gg 1$ we can use the Fokker-Planck approximation and obtain for $f_1(x)$

$$f_1(x) = j/bx^2. \tag{26}$$

The constant in (26) is determined from the condition that the particle flux in energy space (see formula (13) with $T = 0$) is equal to j.

An investigation of (25) near $x = 1$ shows that $f_1(x)$ behaves here in the following manner:

$$f_1(x) = cj/b(x-1)^{3+\nu}, \tag{27}$$

where c is a numerical coefficient and ν satisfies the equation $4\nu(\nu+1)(\nu+2) = 3$ and is approximately equal to $\nu \approx 0.26$. The divergence of the distribution function near $E = ms^2/2$ (i.e., $x = 1$) is due to the fact that the probability of emission of acoustic phonons tends to zero when the kinetic energy approaches $ms^2/2$. The large-scale fluctuations of the potential of the impurities smooth out this singularity, so that expression (27) is valid only at $(x-1) > 2E_0/ms^2$.

We proceed now to the energy region $E < ms^2/2$. In this region, emission of acoustic phonons is possible only in the presence of impurities. Therefore the departure term is determined entirely by the impurity centers

$$w(E,E') = \frac{4}{3}\pi r_m^3 N\frac{b(E'-E)^2}{4(ms^2)^4}, \quad E'<E<\frac{ms^2}{2}, \tag{28}$$

where r_m is how close an electron of energy E must come to the impurity center in order to become capable of emitting an acoustic phonon and go over into a state with energy E':

$$r_m = e^2\varkappa^{-1}\{ms^2/2 + (E'-E)^2/8ms^2 - (E'+E)/2\}^{-1}. \tag{29}$$

We write out the kinetic equation in the region $0 < E < ms^2/2$ in the dimensionless variables x, denoting by $f_2(x)$ the distribution function in the interval $0 < x < 1$:

$$\alpha\left\{-f_2(x)\int_{-\infty}^{x}K(x,x')\,dx'+\int_{x}^{1}f_2(x')K(x',x)\,dx'\right\}+\int_{(\sqrt{x}-2)^2}^{(\sqrt{x}+2)^2}(x'-x)^2 f_1(x')\,dx'=0, \tag{30}$$
$$K(x,x')=(x'-x)^2[1-(x'+x)/2+(x'-x)^2/16]^{-3}.$$

In this equation, the first term in the curly brackets takes into account the electrons that go over into a bound state on centers, the second term takes into account the arrival of electrons from the region $x<x'<1$, which is possible only in the presence of impurity centers. The last term in (30) describes the arrival at the level $0<x<1$ as a result of transitions of electrons from the region $x'>1$. The function $f_1(x)$ which enters in the last term is a solution of Eq. (25). Equation (30) can be formally obtained if the integral kinetic equation for electrons emitting acoustic phonons is multiplied by $\delta(E-\varepsilon-U(r))\,V^{-1}\rho(\varepsilon)\,d\varepsilon\,d^3\mathbf{r}$ and then integrated over phase space, neglecting the overlap of the potentials of the individual capture centers. The parameter α is then assumed to be small and all that is left in the equation are the terms of zeroth order in α (it must be borne in mind that $f_2(x)\sim\alpha^{-1}$).

The electron density is represented in the form $n=n_1+n_2$, where

$$n_1=\frac{ms^2}{2}\rho\left(\frac{ms^2}{2}\right)\int_1^{x_1}\sqrt{x}f_1(x)\,dx,\quad n_2=\frac{ms^2}{2}\rho\left(\frac{ms^2}{2}\right)\int_0^1\sqrt{x}f_2(x)\,dx. \tag{31}$$

Let us show that $n_1\ll n_2$. The main contribution to the density n_1 is made by the region near $x=1$, in which $f_1(x)$ is given by formula (27) with $x-1>2E_0/ms^2$. Taking this into account, and also recognizing that $f_1(x)/f_2(x)\sim\alpha$, we obtain $n_1/n_2\approx\alpha(2E_0/ms^2)^{-2-\nu}$. Inasmuch as usually $E_0\approx e^2\varkappa^{-1}N^{1/3}$, we obtain $n_1/n_2\approx\alpha^{(1-\nu)/3}\approx\alpha^{0.25}$. Thus, the lifetime is $\tau\approx n_2/j$.

The dependence of τ on the parameters of the problem can be separated in explicit form by introducing the new functions $\varphi_1=(b/j)f_1$ and $\varphi_2=(\alpha b/j)f_2$, which are satisfied by the following equation that is obtained from (25) and (30):

$$\begin{aligned}&\varphi_1(x)(\sqrt{x}-1)^3-\frac{3}{64}\int_x^{(\sqrt{x}+2)^2}(x'-x)^2\varphi_1(x')\,dx'=0,\\&-\varphi_2(x)\int_{-\infty}^{x}K(x,x')\,dx'+\int_x^1\varphi_2(x')K(x',x)\,dx'+\int_{(\sqrt{x}-2)^2}^{(\sqrt{x}+2)^2}(x'-x)^2\varphi_1(x')\,dx'=0\end{aligned} \tag{32}$$

with the boundary condition $\varphi_1(x)=x^{-2}$, $x\gg1$. For the lifetime τ we obtain

$$\tau=A\frac{ms^2}{2}\rho\left(\frac{ms^2}{2}\right)\alpha^{-1}b^{-1}, \tag{33}$$

where A is a numerical coefficient equal to

$$A=\int_0^1\sqrt{x}\,\varphi_2(x)\,dx. \tag{34}$$

An analysis of the equation for $\varphi_2(x)$ in (32) shows that this distribution function has an integrable singularity $\varphi_2=-0.48c/(1-x)^{0.26}\ln(1-x)$ as $x\to1$ and is finite in the entire remaining region all the way to $x=0$. The system (32) was solved with a computer, and it turned out that $c=4.7$ and $A=0.34$. Substituting in (33) the obtained value of A, as well as α and b (formulas (6) and (18)), we can rewrite the formula for the lifetime in the form (7). The criterion for the applicability of this result (derived under the assumption $T=0$) can be obtained by adding to (30) a term corresponding to the departure of electrons to higher energies as a result of phonon absorption. By obtaining the corresponding correction to the function $f_2(x)$ and its contribution to the density n_2, we find that formula (7) is valid if in addition to the condition $\alpha\ll1$ the following inequality is satisfied

$$\alpha\gg\gamma^{3.7}/(\ln\gamma)^2,$$

where $\gamma=2kT/ms^2$. On the other hand, if the opposite inequality takes place (even though $\gamma\ll1$ as before), then there is time for a Maxwellian distribution to establish itself, and the capture time can be calculated by counting the number of transitions per unit time from the region of positive energies into the region of negative energies (since $kT\ll ms^2$, the ejection of the electrons from the capture centers can be neglected). Using for the effective cross section of this transition formula (32) of[1], under the condition that the sticking function is $p=1$, we obtain for this case

$$\tau=15\sqrt{\pi}\frac{\tau_0}{\alpha}\left(\frac{2kT}{ms^2}\right)^{1/2}. \tag{35}$$

3. ELECTRON LIFETIME IN A COMPENSATED SEMICONDUCTOR

As shown above, the electron lifetime depends on the ratio of the characteristic swing of the x fluctuations of the impurity potential E_0 to the thermal energy kT (or ms^2). In turn, E_0 depends on the temperature and on the character of the doping. In addition, an essential role can be played by the photoexcitation method (impurity or interband) and its intensity. In particular, the concentration of the capture centers N can depend on the intensity of the light, and when interband excitation takes place, cases are possible when the captures of the electrons and holes are not independent processes. Understandably, each individual case calls for a special analysis. By way of example of the application of the developed theory we consider one concrete situation.

Assume that we have a compensated n-type semiconductor. We denote the donor concentration by N_D and the electron concentration by N_A. We assume the degree of compensation to be small ($k_0=N_A/N_D\ll1$). In this case all the acceptors are negatively charged, and the number of positively charged donors is equal to the number of acceptors. (We recall that our entire analysis pertains to the case when the temperature T is much lower than the ionization energy of the donors, and the doping is weak, so that the overlap of the wave functions of the donors is insignificant.)

We assume that the photoexcitation of the electrons proceeds via impurity absorption from neutral donors. The intensity of the light is assumed to be so small that the concentration and the disposition of the charged donors and acceptors remain in equilibrium and under

conditions of stationary illumination. Let us see how the capture time varies in this case with decreasing tem temperature. Let at first the temperature be relatively high, so that

$$kT>e^2\varkappa^{-1}N_D^{1/3}. \qquad (36)$$

In this case the negatively charged acceptors are screened by the positively charged donors, which can be regarded as a gas of holes. In order of magnitude, E_0 is equal in this case to $E_0=(e^2/\varkappa r_D\sqrt{N_A r_D^3}$, where the Debye radius is $r_D=(kT\varkappa/4\pi e^2 N_A)^{1/2}$. It is easy to verify that when the condition (36) is satisfied we have $kT>E_0$, and consequently the results (2) and (5), where the concentration of the capture centers is the acceptor concentration ($N=N_A$) and $\tau\sim T^{5/2}$, is valid.

When the temperature is lowered, the inequality (36) no longer holds and the screening becomes nonlinear. The equilibrium distribution of the charged centers was considered in detail in this case in[4], where the expression obtained for the rms potential of the large-scale fluctuations is

$$E_0=0.26k_0^{-1/12}(e^2/\varkappa)N_A^{1/3}=0.26k_0^{1/4}(e^2/\varkappa)N_D^{1/3}. \qquad (37)$$

It turns out here that the charged donors are constituents of 1- and 2-complexes. (A 1-complex is a dipole made up of a charged donors and an acceptor, which are separated from each other by a distance $\sim N_D^{-1/2}$, while 2-complexes are formations consisting of two positively charged donors located at a distance $\sim N_D^{-1/3}$ from the negatively charged acceptor.) It is shown in[4] that at $T=0$ the concentration of the 2-complexes is 1.5% and the concentration of the 1-complexes is 97% of the concentration of the acceptors (the remaining 1.5% is made up of solitary acceptors—0-complexes).

At finite temperatures, a fraction of the positively charged donors breaks away from the acceptors and is not contained in the 1- and 2-complexes. The behavior of the chemical potential as a function of the temperature was considered in[5], where formulas were derived with which to calculate the concentrations N_1 of the 1-complexes, N_2 of the 2-complexes, and N of the solitary positively-charged donors. The electron neutrality condition yields $N_2+\tilde{N}=N_0$, where N_0 is the concentration of the 0-complexes.

An important factor in the cross section for the capture by a complex is the value of the potential of the complex at the distance at which the potential energy of the electron is $\sim kT$. At

$$kT<e^2\varkappa^{-1}N_D^{1/3} \qquad (38)$$

this distance is much larger than $N_D^{-1/3}$, so that the 1-complex should be regarded as a point dipole and the 2-complex as a singly-charged Coulomb center.

The lifetime of the electrons is determined by two independent processes: by capture by the positively-charged Coulomb centers (2-complexes) and solitary charged donors (time τ_c) and by capture by dipoles—1-complexes (time τ_d)

$$\tau^{-1}=\tau_c^{-1}+\tau_d^{-1}. \qquad (39)$$

τ_c is satisfied by formula (2) or (5), if N is taken to mean the total number of positively charged Coulomb centers, which in this case is equal to N_0. The capture time τ_d is calculated in the appendix (see A.3)). Formula (39) can be written in the form

$$\tau=\frac{\sqrt{\pi}}{2}\frac{\tau_0}{\alpha_A}\left(\frac{2kT}{ms^2}\right)^{5/2}\frac{N_A}{N_0}\left\{1+0.5\frac{N_1}{N_0}\frac{kT}{\varepsilon_D}\right\}^{-1}, \qquad (40)$$

where α_A is determined by formula (6), in which N is replaced by the acceptor concentration N_A, and $\varepsilon_D=[(4\pi/3)N_D]^{1/3}e^2\varkappa^{-1}$. When the temperature becomes lower than ε_D, the decrease of the lifetime with decreasing temperature slows down mainly as a result of the decrease in the effective number of the Coulomb capture centers, since an ever-increasing number of donors becomes bound into dipoles. With further decrease of the temperature, the dipole concentration N_1 increases and their role in the capture processes can be decisive (the second term in the curly brackets of (40) can become larger than unity). Figure 6 shows the temperature dependence of the relative contribution of the dipoles to the recombination—$\tau_c/\tau_d=0.5(N_1 kT/N_0\varepsilon_D)$ for two values of the degree of compensation. We use here the formulas of[5]. When the temperature drops below the level E_0 (formula (37)) the capture time ceases to depend on the temperature and its order of magnitude becomes $\tau_\varepsilon(E_0)$. We note that this pertains only to the capture time determined by measuring the dc conductivity, in which only photoelectrons with energy above the percolation level take part. When the capture time is determined by measuring the high-frequency photoconductivity or by some method in which the time of arrival to the ground state at the donor is significant, the results for $kT<E_0$ can be different.

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APPENDIX: EFFECTIVE CROSS SECTION FOR CAPTURE BY DIPOLE

In the calculation of the lifetime for the capture by a dipole we use formula (22), in which the coefficient

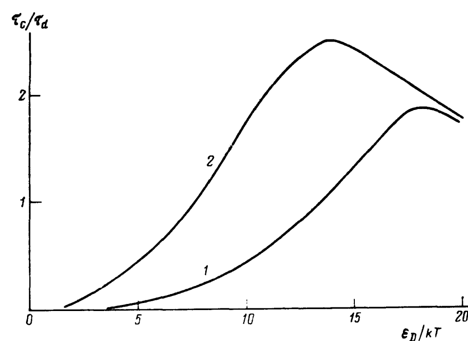

FIG. 6. Temperature dependence of the relative contribution of the dipoles to the recombination. 1—$k_0=0.005$, 2—$k_0=0.08$.

$B(E)$ is determined by formula (12). The potential $U(r)$ is given in this case by

$$U(r)=-\frac{e^2}{\varkappa r}+\frac{e^2}{\varkappa(r^2+2rd\cos\vartheta+d^2)^{1/2}}, \tag{A1}$$

where d is the length of the dipole and ϑ is the angle between the radius vector $\mathbf{r}$ and the dipole axis. Calculation for $E<0$ and $|E|\ll e^2/\varkappa d$ yields

$$B(E)=2\pi(\ln 4-1)Nbd(2e^2/\varkappa ms^2)^2, \tag{A2}$$

where N is the dipole concentration and b is defined by formula (18). In this energy region, B does not depend on the energy. With the aid of (22) we obtain for the lifetime

$$\tau=\frac{\sqrt{\pi}}{3(\ln 4-1)}\frac{\tau_0}{\alpha}\left(\frac{2e^2}{\varkappa ms^2d}\right)\left(\frac{2kT}{ms^2}\right)^{3/2}, \tag{A3}$$

where τ_0 and α are defined by formulas (4) and (6), in which N is the dipole concentration. Accordingly we have for the effective cross section for capture by a dipole

$$\sigma=[\tau N\langle v\rangle]^{-1}=2^5(2\ln 8-3)\sigma_1\left(\frac{ms^2}{2kT}\right)^3\left(\frac{\varkappa kTd}{e^2}\right)\approx\sigma_c\left(\frac{\varkappa kTd}{2e^2}\right), \tag{A.4}$$

where

$$\sigma_1=\frac{\pi}{6}\left(\frac{e^2}{\varkappa ms^2}\right)^3\frac{1}{\tau_0 s},\quad \sigma_c=2^6\sigma_1\left(\frac{ms^2}{2kT}\right). \tag{A5}$$

For the dipole length d we can use the average distance to the nearest donor, $d=0.55N^{-1/3}$.

[1)]Indeed, assume that the electron has an energy $E\gg ms^2$. We compare the probabilities of two processes: 1) the electron in the field of the center emits an acoustic phonon of high energy and is immediately captured by the center; 2) the electron "diffuses" from the energy E to the energy E_0. The electron lifetime τ_1 in the first process is

$$\tau_1^{-1}=3\alpha(ms^2/\sqrt{EE_0})(kT/E+U)(ms^2/E+U)^2\tau_\varepsilon^{-1}(E_0).$$

This expression was obtained from formula (21) of [1]. Here U is the binding energy, and the capture takes place when $U\gtrsim kT$. Since $\alpha\sim(E_0/ms^2)^3$, and $E>E_0$, we see that the first process is less probable than the second ($\tau_1>\tau_\varepsilon(E_0)$).

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Resonance of a substrate surface polariton with a longitudinal phonon of a thin lithium-fluoride film

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We have investigated the spectra and the angular dependences of attenuated total internal reflection (ATIR) of surface polaritons of sapphire, rutile, and yttrium iron garnets with lithium-fluoride films 100–800 Å thick. The dispersion curves obtained at fixed incidence angles and at fixed frequencies are compared with the calculation. The thickness dependence obtained for the splitting in the ATIR spectrum agrees with the theoretical predictions.

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The extensive use of thin films in optics and electronics calls for knowledge of their properties, which in many cases differ from the properties of the bulk material from which these films are made. To investigate thin films deposited on a crystal it is possible to use, besides the traditional spectroscopic methods, also the surface polaritons of the crystal.[1] The dependence of the frequency ω of a surface polariton on the wave vector k_x is given for the interface between two isotropic media with dielectric constants $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ by the formula

$$k_x=\frac{\omega}{c}\left(\frac{\varepsilon_1(\omega)\varepsilon_2(\omega)}{\varepsilon_1(\omega)+\varepsilon_2(\omega)}\right)^{1/2}. \tag{1}$$

Surface polaritons can exist at frequencies for which the dielectric constants of the media in contact have opposite signs. The field of a surface polariton is concentrated mainly near the surface, and decreases exponentially with increasing distance from the interface, so that surface polaritons are very sensitive to the properties of films on this interface. Consequently, the spectroscopy of surface polaritons can be a valuable source of information on thin films on crystal surfaces. If the frequency of the transverse or longitudinal oscillation of the dielectric film falls in the region of the existence of the substrate surface polariton, then a gap should be observed at this frequency in the dispersion curve of the

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