Quantum oscillations of the superconducting transition temperature in metal-nonmetal systems

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The nature of the superconducting transition temperature (and conductivity) oscillations that arise in a metal-nonmetal film system as the thickness of the nonmetallic film is increased is considered. It is shown that the phenomenon is connected with the oscillations of the electron density of states in the metal caused by the oscillations of the boundary condition for the wave function at the metal-nonmetal film interface. The latter is connected with the nature of the motion and reflection of an electron in a semiconducting, or a semimetallic, film. The question of the period and amplitude of the T_c oscillations is analyzed in detail. The results obtained allow the explanation of all the distinctive features of the phenomenon, which was discovered by Mikheeva, Golyanov, *et al.* and Sixl.

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1. INTRODUCTION

Recently, reports have been published of the experimental discovery of the interesting phenomenon of oscillations in the superconducting transition temperature in metallic films with nonmetallic coatings as a function of the coating thickness. This effect was discovered in experiments performed by Mikheeva, Golyanov, *et al*,[1–3] who investigated three different metals (Mo, Tc, and V) with carbon coating, and, in a very sharp form, in the precision experiment by Sixl,[4] who carefully investigated a layered aluminum-silicon monoxide (SiO) system. It should, however, be noted that Naugle back in 1967[5] observed for the first time a maximum in the curve of T_c as a function of the thickness of a germanium coating on a thallium film. Recently a report was published of the discovery of a similar phenomenon in lead covered with a germanium or silicon film.[6]

The results of the experiments, chiefly the experiments,[2–4] allow us to formulate the general characteristics of the phenomenon, the main ones of which are the following:

1. The oscillations are observed only in those cases when semiconductors (or, possibly, semimetals) are used as the coating.

2. The locations of the extrema in the $T_c(t)$ curve (t is the coating thickness) do not depend on the thickness of the metallic film.

3. The amplitude of the oscillations increases with decreasing thickness, d, of the metallic film approximately as $1/d$.

4. Depending on the material of the coating, the oscillations can be close to being periodic, or, conversely, it can have a marked aperiodic character with a period that increases appreciably with thickness and, apparently, with a limited number of oscillations (see[4]).

5. If the metallic film is coated on both sides simultaneously, then the pattern of the T_c oscillations is very sensitive to the relation between the thicknesses of the two coatings (see[2,3]).

6. The longitudinal resistance oscillates simultaneously with the same period, but out of phase with T_c, and with a smaller amplitude.

The totality of all these characteristics demonstrates a very peculiar interlacement in the phenomenon of the properties of metallic and nonmetallic films, thereby predetermining the necessity in any analysis of a unified treatment for the system as a whole.

In the present paper we develop ideas about the behavior of the electron subsystem in such a layered structure that enable us to understand the nature of the resulting T_c oscillations and explain virtually all the above-enumerated characteristic features of the phenomenon. These ideas are based on the following. In an ideal metallic film of finite thickness d, the transverse motion of the electrons is quantized, and the energy density of states $g(\varepsilon)$ as a function of the energy undergoes oscillations. However, in a real situation, when the electron wavelength is of the order of the interatomic distance, these oscillations get primarily averaged out, owing to the nature of electron reflection from the surface and, of course, of electron scattering in the volume if the mean free path is less than the film thickness. The resulting averaged density of states, $\bar{g}(\varepsilon)$, differs from the density of states, $g_0(\varepsilon)$, in the bulk metal, and the higher the ratio of the surface area to the volume, i.e., the thinner the film, the more strongly pronounced the difference Δg. But the value of $\Delta g(\varepsilon)$ itself and even its sign depend on the boundary conditions at the surface (see, for example, Balian and Bloch's excellent paper[7]). The appearance of a second layer leads to a change in the boundary condition at the metal-nonmetal interface. In a semimetal or a semiconductor the wavelength of an electron with the same total energy ε turns out to be long compared to the interatomic distance, and the reflection of electrons from the boundary with vacuum has a coherent character. As a result, the quantum coupling between the boundaries is preserved in a thin nonmetallic film, and the logarithmic derivative of the wave function at the boundary with a metal depends on the thickness, t, of this film, and oscillates with increasing t. As a result, the boundary condition at the metallic boundary oscillates, giving rise to oscillations of the averaged density of electronic

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states in the metal. These oscillations of $\tilde{g}(\varepsilon)$, which are purely quantum in nature, explain, at least qualitatively, the entire observed oscillatory pattern of behavior of T_c in a superconducting metallic film, where there occurs a specific sharp increase, connected with the exponential dependence of T_c on the density of states, in the oscillation amplitude.

The oscillatory nature of the density of states in the metallic film as the thickness of the nonmetallic coating is varied, should lead to the oscillation of a number of other physical quantities, although, as a rule, on a smaller scale in comparison with T_c. This pertains, in particular, to the kinetic characteristics, such as the electrical conductivity.

In all the cases in which the oscillatory behavior was observed, the nonmetallic film was a semiconductor (a semimetal), whereas an ordinary dielectric film does not lead to such effects, which, in particular, was specifically noted by Sixl.[4] This circumstance is not accidental, for in a dielectric with a wide forbidden band, if the chemical potential of the metal lies within the limits of this band, the electron density falls off exponentially over a short distance into the dielectric. On account of this, as the thickness of the dielectric film increases, the logarithmic derivative of the electron wave function at the boundary with the metal does not undergo oscillations, which leads, naturally, to the absence of oscillations in the density of states $\tilde{g}(\varepsilon)$ in the metal. The effect can actually arise only at a metal-semimetal or a metal-semiconductor interface.

Having in mind to give a primarily qualitative description of the phenomenon, we shall use below the simplest models for a metal and a semiconductor (semimetal), assuming in all the cases intimate contact between the metal and the nonmetal, which, from all appearances, corresponds to the experimental situation.

In analyzing the critical temperature of the transition into the superconducting state, we shall restrict ourselves to the consideration of the effects connected with the variation of the electron density of states and the inhomogeneity of the gap (which effects seem to us to be the dominant effects in this problem), ignoring the possible variation of the phonon spectrum and of the effective electron-phonon coupling constant. The main attention here will be given to the T_c oscillations in the layered system under consideration, although we shall briefly discuss also the classical question of the sign of the variation of T_c with thickness in an isolated metallic film.

2. THE ELECTRON DENSITY OF STATES IN A METALLIC FILM

Let us consider an ideal metallic film of thickness d, and let the boundary conditions for the electron wave function at both surfaces have the form

$$\psi|_{x=-d}=0, \quad (d\psi/dx+\varkappa(\varepsilon)\psi)_{x=0}=0. \tag{2.1}$$

Let us, for simplicity, assume the square-law dispersion for the electrons in the unbounded bulk of the metal. The longitudinal and transverse motions in the film are then separable, and in the second boundary condition we shall assume that $\varkappa$ depends only on the energy, ε, of the transverse motion. The self-consistent representation for the electron spectrum in the plate with allowance for (2.1) has the form

$$E_n(k_\parallel)=\hbar^2k_\parallel^2/2m_1+\varepsilon_n(\varkappa(\varepsilon_n)). \tag{2.2}$$

The values of ε_n are then determined from the solution of the dispersion relation

$$\operatorname{ctg} k_xd=-\varkappa(\varepsilon)/k_x, \quad \varepsilon=\hbar^2k_x^2/2m_1. \tag{2.3}$$

The density of electron states in the film per unit volume is equal to

$$g(E)=\frac{1}{d}\sum_n\int\frac{d^2k_\parallel}{(2\pi)^2}\delta(E-E_n(k_\parallel)).$$

After the trivial integration, we obtain

$$g(E)=\frac{m_1}{2\pi\hbar^2d}\sum_{n=0}^{\infty}\theta(E-\varepsilon_n), \tag{2.4}$$

where $\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for $x<0$. We shall assume that $\varkappa(E)$ changes relatively slightly when the energy is changed by a value of the order of the level spacing $\Delta\varepsilon$. Let us use the Poisson transformation. Then

$$\begin{gathered}\sum_{n=0}^{\infty}\theta(E-\varepsilon_n)=\int_{-1/2}^{\infty}dy\,\theta\left(\frac{kd}{\pi}-y-\xi(y)\right)\\+2\operatorname{Re}\sum_{m=1}^{\infty}\int_{-1/2}^{\infty}dy\,\theta\left(\frac{kd}{\pi}-y-\xi(y)\right)e^{i2\pi my},\\\xi(y)=\frac{1}{2}+\frac{1}{\pi}\operatorname{arctg}\frac{\varkappa(y)}{k_x(y)}, \quad k=\left(\frac{2m_1}{\hbar^2}E\right)^{1/2}.\end{gathered} \tag{2.5}$$

Here we have used the fact that, according to (2.3),

$$k_{xn}d=\pi\left(n+\frac{1}{2}\right)+\operatorname{arctg}\frac{\varkappa(k_{xn})}{k_{xn}}.$$

After the integration in (2.5) for the density of states, we find directly that

$$\begin{gathered}g(E)=\tilde{g}(E)+g_\sim(E),\\\tilde{g}(E)=g_0(E)-\frac{m_1}{2\pi^2\hbar^2d}\operatorname{arctg}\frac{\varkappa(E)}{k}\equiv g_0(E)+\Delta g,\end{gathered} \tag{2.6}$$

where $g_0(E)$ is the density of states in the bulk metal. The oscillatory part of the density of states is equal to

$$g_\sim(E)=1/2-\{kd/\pi+\xi\},$$

where $\{\cdots\}$ denotes the fractional part modulo 1.

The actual character of electron reflection from the boundary, as well as of scattering in the volume, of the film leads to the disappearance of the oscillating part $g_\sim(E)$, and the actual density of states in the metallic film of thickness d is determined by the function $\tilde{g}(E)$, (2.6). Notice that $\tilde{g}(E)$ depends on the boundary condition defined for the maximum value, $\varepsilon=E$, of the energy of the transverse motion.

If the wave function vanishes at the second boundary as well ($\varkappa=\infty$), then for the change in the averaged density of states we arrive at the well-known result

$$\Delta g=-m_1/4\pi\hbar^2 d \qquad (2.7)$$

(see, for example,[7]). This decrease in the density of states corresponds precisely to the case when the metallic film has a free boundary. If the Neumann boundary condition ($\varkappa=0$) is realized at the second boundary, then $\Delta g=0$, and the density of states corresponding to the bulk metal is reestablished. For $\varkappa<0$ the correction to the density becomes positive. It should be noted that the result (2.6), if we assume that $\varkappa$ does not depend on E, corresponds to the results of Balian and Bloch's analysis.[7]

Thus the averaged density of states in a metallic film varies with the boundary conditions, the characteristic scale of these variations being determined by the quantity (2.7). It should be noted that this scale may be significantly larger in a real metal, because of the special role of small electron clusters. Indeed, on account of the nondependence (or weak dependence) of Δg on energy, each small cluster will make a contribution similar to (2.7), whereas the correction to $g_0(E)$ will be small. Physically, this is connected with the fact that the local variation of the density of states near a boundary occurs in a layer of thickness proportional to the particle wavelength. As a result, the electrons of a small cluster feel the distorting role of the boundary in a much greater volume than electrons of the main group with the same energy, and this compensates for their small number. Below we shall write the general expression for Δg in the form

$$\Delta g(E)=-\zeta\frac{1}{d}\operatorname{arctg}\frac{\varkappa(E)}{k}, \qquad (2.8)$$

remembering only that in the general case the effective constant ζ can be greater than the constant figuring in (2.6).

A change in the density of states entails a change in the chemical potential of the electrons. Under the assumption of constancy of the number of electrons in a unit volume of the metal, we have immediately

$$\Delta\mu=-\frac{1}{g_0(\mu_0)}\int_0^{\mu_0}\Delta g(E)\,dE. \qquad (2.9)$$

The change in the density of states at the Fermi surface then turns out to be equal to

$$\bar{g}(\mu)-g_0(\mu_0)=\Delta g(\mu_0)-\frac{\partial \ln g_0(\mu_0)}{\partial\mu_0}\int_0^{\mu_0}\Delta g(E)\,dE. \qquad (2.10)$$

3. THE METAL-NONMETAL BOUNDARY, OSCILLATIONS OF THE ELECTRON DENSITY OF STATES

A. The metal-semimetal boundary. Let us consider the metal-semimetal layered structure, assuming intimate contact between the media. Let, for definiteness, the chemical potential of the metal be higher than the chemical potential of the semimetal, and let us assume, for simplicity, that the electronic spectrum of the latter has a square-law dispersion with effective mass m_2. The flow of electrons from the metal into the semimetal creates near the boundary an inhomogeneous potential distribution that is described in the self-consistent approximation by the equation

$$\begin{gathered}\frac{d^2\Phi_s(x)}{dx^2}=-\gamma_s^2\{(\mu-V_s^0-\Phi_s(x))^{3/2}-(\mu_s^0-V_s^0)^{3/2}\},\\ \Phi_s=e\varphi_s,\quad \gamma_s^2=\frac{4e^2(2m_s)^{3/2}}{3\pi\hbar^3\varepsilon_s}\quad(\varepsilon_1=1).\end{gathered} \qquad (3.1)$$

Here the index s assumes the value 1 for the metal and 2 for the semimetal; μ_s^0 and V_s^0 are the chemical potential and the bottom of the conduction band for isolated films; μ is the total chemical potential of the sandwich, presupposing in the general case a self-consistent definition; ε_2 denotes the part of the permittivity not connected with the free carriers.

The boundary conditions have the following form (Fig. 1):

$$\frac{d\Phi_2(t)}{dx}=\frac{d\Phi_1(-d)}{dx}=0, \qquad (3.2a)$$

$$\Phi_2(0)=\Phi_1(0),\quad \left.\frac{\partial\Phi_2}{\partial x}\right|_{x=0}=\left.\frac{\partial\Phi_1}{\partial x}\right|_{x=0}, \qquad (3.2b)$$

where t and d are the thicknesses of the semimetallic and metallic films. In (3.1), (3.2) we have virtually ignored the possible variation of the parameters in a narrow region near the boundary. It is easy to check that allowance for the latter leaves all the qualitative results obtained below unchanged.

In the metallic film, $|\Phi_1|\ll\mu-V_1^0$, and Eq. (3.1) becomes linearized:

$$\begin{gathered}\frac{d^2\Phi_1}{dx^2}(x)=-\eta_1^2\left[\frac{2}{3}\frac{\mu^{3/2}-(\mu_1^0)^{3/2}}{\mu^{1/2}}-\Phi_1(x)\right],\\ \eta_s^2={}^3/_2\gamma_s^2(\mu-V_s^0)^{1/2}.\end{gathered} \qquad (3.3)$$

Here and below we assume $V_1^0=0$ ($\mu_1^0=\varepsilon_F^0$).

Setting $\Phi_1(-d)=0$ and assuming that, since $\eta_1\sim 1/a_1$ (a_1 is the interatomic distance in the metal), $d\eta_1\gg 1$, and we have with exponential accuracy the relation

$$\mu=\mu_1^0. \qquad (3.4)$$

In the second layer Eq. (3.1) cannot, in general, be linearized. But we can find directly its general solution, which, with allowance for (3.2), has the following form:

$$\begin{gathered}\int_{\Phi_2(t)}^{\Phi_2(x)} dz\{{}^2/_5[(\mu-V_2^0-z)^{5/2}-(\mu-V_2^0-\Phi_2(t))^{5/2}]\\ +(\mu_2^0-V_2^0)^{3/2}(z-\Phi_2(t))\}^{-1/2}=2^{1/2}\gamma_2(x-t).\end{gathered} \qquad (3.5)$$

The expression (3.5) with allowance for the boundary conditions (3.2) determines the dependence $\Phi_2(x)$ in the semimetal layer. For $\eta_2 t\gg 1$ the function $\Phi_2(t)$ reaches a limit not depending on t and equal to

$$\Phi_2(t)=\mu_1^0-\mu_2^0. \qquad (3.6)$$

The characteristic—for the semimetal—inequality

$\eta_2 a_2 \ll 1$ predetermines a smooth behavior for the potential in the second layer.

By considering the solution of the one-electron Schrödinger equation in the self-consistent field $V_2^0+\Phi_2(x)$, we can find the wave function, $\psi_2(x)$, satisfying the boundary condition

$$\psi_2(t)=0. \tag{3.7}$$

It is significant that the small magnitude of the Fermi momentum in the semimetal allows us to assume a purely specular character for the reflection from the boundary with the vacuum. Therefore, in a thin semimetallic layer, when the scattering in the volume plays a very slight role, the value of the wave function ψ_2 at the boundary with the metal will depend on the thickness t, the coupling between the boundaries being preserved precisely at the wave-function level.

The separation of the longitudinal and transverse motions in the film leads to a situation in which the logarithmic derivative at the boundary with the metal

$$\varkappa=-\left(\frac{1}{\psi_2}\frac{d\psi_2}{dx}\right)_{x=0} \tag{3.8}$$

depends only on the transverse energy. If the transverse energy is equal to the total energy, i.e., if the longitudinal momentum is equal to zero, then the expression (3.8) determines precisely the quantity $\varkappa(E)$, which figures in the expression, (2.6), for the averaged density of states in the metallic film.

If the chemical potentials in the metal and the semimetal are nearly equal, and the flow of charge can be neglected, then we have for $\varkappa(E)$ the simple relation

$$\varkappa(E)=k_2\operatorname{ctg}k_2t,\quad k_2=(2m_2(E-V_2^0))^{1/2}/\hbar. \tag{3.9}$$

In the general case, when there is a flow of charge and, by the same token, distortion of the bottom of the conduction band occurs, we find, using the quasi-classical approximation,

$$\varkappa(E)\approx k_2(0)\operatorname{ctg}\chi(E,t),\quad \chi=\int_0^t k_2(x,E)\,dx, \tag{3.10}$$

where

$$k_2(x,E)=[2m_2(E-\Phi_2(x)-V_2^0)]^{1/2}/\hbar. \tag{3.11}$$

In the presence at the boundary of a transition layer with continuously varying parameters, the value of the quasi-classical phase can be directly refined. (We neglect some distortion, determined by the function $\Phi_1(x)$ and extending over a distance of the order of atomic distances, of the bottom of the conduction band in the metal.)

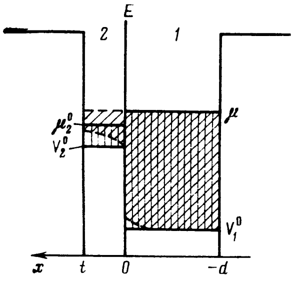

FIG. 1.

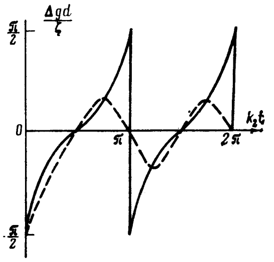

FIG. 2.

As the thickness t is varied, the value of $\varkappa(E)$ (given by (3.10) and (3.9)) for fixed E oscillates, the oscillations being, in the general case, aperiodic in character. For $\mu_1^0>\mu_2^0$, the period increases with the thickness, which is connected with the continuous decrease of the local quasi-classical value of the momentum (3.11) with increasing x (see Fig. 1) from its maximum value at $x=0$ to the minimum value corresponding to the limit (3.6). For thicknesses $t>1/\eta_2$, the oscillations will already have a periodic character, so long as the coherent coupling between the boundaries is preserved. If we substitute (3.10) or (3.9) into the relation (2.6), then we can easily verify that the density of states $\tilde{g}(E)$ oscillates with increasing coating thickness, the positions of the extrema being dependent only on the parameters of the semimetal. The oscillation amplitude, on the other hand, depends only on the properties of the metallic film, and is inversely proportional to its thickness d. It is interesting that the initial increase in the film thickness is accompanied by the growth of $\tilde{g}(E)$.

In Fig. 2 we show the nature of the dependence of $\Delta g(E)$, (2.6), on the coating thickness.

Let us emphasize one important circumstance. Since $k_2 a_2 \ll 1$, the possible inhomogeneity of the semimetallic film on the atomic scale has a relatively slight effect on the value of $\varkappa(E)$ and thereby on the oscillations of the electron density in the metal. If we average Δg over some finite thickness range, Δt, with allowance for the relation $k_2/k_1 \ll 1$, then we obtain a dependence of the type shown in the same figure by the dashed curve. The initial growth of Δg with the thickness then turns out to be distinct. Notice in conclusion that the qualitatively found picture remains absolutely the same in the case of the inverse relation between the chemical potentials, i.e., when $\mu_1^0<\mu_2^0$.

B. *The metal-semiconductor boundary.* Considering the metal-semiconductor layered structure, we encounter different cases depending on the relative positions of the chemical potentials in the two media and on the impurity concentration in the semiconductor.

1. Let the chemical potential of the metal lie above the bottom, V_2^0, of the conduction band of the semiconductor, and let the temperature be close to zero, so that there are virtually no intrinsic carriers in the semi-

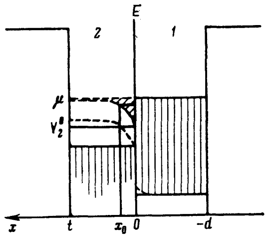

FIG. 3.

conductor. The electron liquid will then flow from the metal into the semiconductor, penetrating to a depth of the order of $1/\eta_2$ (see (3.3)), which in a semiconductor with a narrow forbidden band is quite large and may be tens of angstroms. Such a flow will be accompanied by a rise of the bottom of the conduction band, and can be found from the solution to the same Eqs. (3.5) and (3.3) if we set $\mu_2^0 - V_2^0 = 0$. Now for $E < \mu$ there arises a limiting penetration depth $x_0(E)$ (Fig. 3) determined by the relation

$$\Phi_2(x_0) = E - V_2^0. \qquad (3.12)$$

In the quasi-classical approximation the logarithmic derivative at the semiconductor-metal boundary is determined by the expression (3.10), where, for the determination of the phase χ, the upper integration limit is equal to $\min\{t, x_0(E)\}$. For a semiconductor-film thickness $t > x_0(E)$, to the argument of the cotangent in (3.10) is added a phase δ_2 connected with the subbarrier tail of the wave function:

$$\chi(E,t) = \int_0^{x_0(E)} k_2(x)\,dx + \delta_2(E,t) \equiv \chi_0 + \delta_2. \qquad (3.13)$$

The quantity δ_2 reaches its limiting value close to $\pi/4$ with the growth of $t - x_0(E)$. The value of the phase $\chi_0(E, t)$ depends on the dimensionless parameter (see (3.3), (3.11))

$$k_2(0,E)\frac{1}{\eta_2} \approx \left(\frac{E - V_2^0}{E_0}\right)^{1/4}, \quad E_0 = \frac{1}{2}\frac{m_2 e^4}{\hbar^2 \varepsilon_2^2}. \qquad (3.14)$$

For large thicknesses the numerical integration of (3.5) leads to a limiting value for the phase (the electron density drops to zero: see Fig. 3) equal to

$$\chi(\mu, t) \approx 5.5 k_2(0, \mu)/\eta_2. \qquad (3.15)$$

It follows from (3.15) and (3.14) that, as t is increased, a large change in the argument of the cotangent in (3.10) is attained in semiconductors with a narrow forbidden band, which are characterized by a small value of the Bohr energy E_0. For such semiconductors, the substitution of (3.10) into (2.6) leads to the appearance, as the semiconductor-film thickness is increased, of some limited number of aperiodic oscillations in the electron density of states $\tilde{g}(E)$, followed by the attainment of some constant value. In this case, because of the drop in density, the period sharply increases with thickness, and, in particular, the function may be so drawn out that it reaches a constant value. The oscillation amplitude is, as before, inversely proportional to the thickness of the metallic film. In Fig. 4, we show a characteristic form of such a dependence as obtained in the direct evaluation of (3.5), (3.10), and (2.6) with the semiconductor parameters $m_2/m_0 = 0.2$, $\varepsilon_2 = 4$, and $\mu - V_2^0 = 0.2$ eV.

If the chemical potential of the metal lies close to the bottom of the conduction band of the semiconductor, then the interaction of the carriers in the semiconductor with the polarization induced in the metal (i.e., with the "image" forces) becomes important. This interaction leads to an appreciable depression of the bottom of the band in the region immediately adjoining the boundary with the metal:

$$V_2(x) = V_2^0 - e^2/4\varepsilon_2 x. \qquad (3.16)$$

Actually, the function $V_2(x)$, (3.16), should figure in (3.5) and (3.11) instead of V_2^0, which, obviously, is responsible for the growth of $k_2(x, E)$, as well as the growth of the phase $\chi(E, t)$. For $\mu_1^0 - V_2^0 \approx 0$ the t dependence of χ in a pure semiconductor is, in general, determined virtually by only the flowing in of electrons as a result of the lowering of the bottom of the band, (3.11).

If we neglect the quantity $\Phi_2(x)$, then in this case we immediately have

$$\chi(\mu, t) \approx (2t/r_B)^{1/2}, \qquad (3.17)$$

where

$$r_B = \hbar^2 \varepsilon_2 / m_2 e^2 \qquad (3.18)$$

is the Bohr radius in the semiconductor.

In the case when the chemical potential of the metal falls within the forbidden band of the semiconductor ($\mu_1^0 < V_2^0$), the limiting penetration depth $x_0(E)$ is determined from the condition

$$V_2(x_0) = E$$

and, correspondingly,

$$\chi_0(\mu) \approx {}^1/_4\pi[E_0/(V_2^0 - \mu)]^{1/2}. \qquad (3.19)$$

The electron density flowing into the semiconductor actually screens off the field produced by the image forces. It is not difficult to show that the characteristic screening radius is close to r_B. (For thicknesses t or x_0 greater than r_B, it is already necessary to take into account the increasing potential $\Phi_2(x)$.) Therefore, as t increases, the phase χ attains a limiting value,

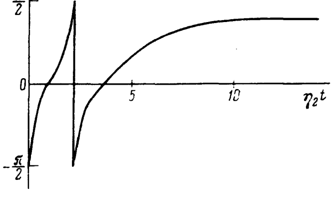

FIG. 4.

which is characterized by the expression (3.17) with $t \sim r_B$.

If now we return to the expressions (3.10), (3.13), and (2.6), then we can infer from the obtained results that the inevitable "semimetallization" of the semiconductor near the boundary with the metal leads, even when $\mu_1^0 - V_2^0 \lesssim 0$, to the growth of $\bar{g}(\mu)$ with the thickness t, it being possible for the density of states to vary through a complete amplitude characteristic of the given d and even pass through a maximum before $\bar{g}(\mu)$ reaches a constant value.

Upon further decrease of the chemical potential of the metal right down to the value corresponding to the middle of the forbidden band of the semiconductor, the maximum value, (3.19), of the phase becomes small. If μ_1^0 lies below the middle of the forbidden band, the entire picture turns out to be inverted, the electron flow now being from the semiconductor into the metal. We shall not expressly dwell on this situation.

2. The above-performed analysis pertains, strictly speaking, to pure semiconductors. In the real situation the semiconducting film will, in many cases, contain an appreciable impurity concentration. Semiconductors with a narrow forbidden band then turn out again to be especially distinct. Indeed, the large value of the Bohr radius (3.18) in this case leads to a situation in which such semiconductors turn out to be already "highly doped" at relatively low impurity concentrations. The semiconducting film will then be virtually a semimetal at $T=0$, and the sandwich system will behave, as the thickness of the coating is varied, just as described in Subsec. A, exhibiting oscillations in the electron density of states. The fact that the transition into the semimetallic state occurs at a sufficiently low impurity concentration allows us to assume the latter to be so low that electron scattering in the volume does not play an important role and does not lead to the destruction of the coherent coupling between the boundaries with the vacuum and the metal.

If the impurity concentration is not sufficient for the transition into the semimetallic state to occur in the semiconductor with a narrow forbidden band near the boundary with the metal, then the effect of impurity autoionization begins to play an important role (see[8]). The point here is that, owing to the polarization of the metal and, accordingly, the "image" forces in a layer of thickness $\sim r_B$, the transition of the electrons from the impurity levels (having a binding energy $\sim E_0$) into the continuous spectrum turns out to be energetically advantageous. This leads to the formation of a region of free carriers near the boundary with the metal, and the logarithmic derivative (3.10) can now have a phase χ close to (3.17), even if μ_1^0 lies in the middle of the forbidden band.

Notice that even in the case of a low impurity concentration, a quite narrow transition layer at the boundary with the metal will quite probably possess semimetallic properties.

3. If the chemical potential of the metal has a value falling somewhere in the middle of the forbidden band of the semiconductor, then in the absence of impurities in the latter the electron density exponentially falls off from the boundary with the metal into the semiconductor. Although in the case of a narrow forbidden band this decrease may be comparatively prolonged (see, for example,[9]), the logarithmic derivative $\varkappa(E)$, (3.8), does not oscillate with the thickness of the semiconducting film. This can easily be verified if, for the determination of the electron wave function in the semiconductor, we use, for example, the two-band model (see, for example,[10]), and determine $\varkappa(E)$ in its explicit form, assuming the boundary condition (3.7). Then averaging over thicknesses of the order of the interatomic distance with a view to eliminating the oscillations connected with the modulating Bloch function, we find that $\varkappa$ varies monotonically with t within finite limits equivalent to the maximum value of the phase in (3.10), i.e., equivalent to $\pi/4$ (see the comment connected with the phase δ_2 in (3.13)). In the case of a wide forbidden band this value is even smaller. Thus, in the situation under consideration the electron density of states in the metal will not oscillate with increasing coating thickness.

4. QUANTUM OSCILLATIONS OF THE SUPERCONDUCTING TRANSITION TEMPERATURE

Let us consider the superconducting properties of the metal-nonmetal sandwich system, assuming, for simplicity, that the interaction responsible for the pairing of the electrons is different from zero only in the metallic film. In accordance with the results of the preceding section, the role of the nonmetallic film in this case should be manifested through a change in the boundary conditions for the electrons in the metal.

Let us begin with the consideration of the ideal metallic film without a coating, and let us limit ourselves to the case of a weak electron-phonon coupling. In the framework of the standard temperature technique,[11] the determination of T_c reduces to the solution of the equation for the gap

$$\begin{gathered}\Delta(x)=|\lambda| \int dx_1 K_T(x, x_1)\Delta(x_1),\\ K_T(x, x_1)=T\sum_{|\omega_n|<\omega_D}\int\frac{d^2k_\parallel}{(2\pi)^2}G_{-\omega_n}(xx_1;k_\parallel)G_{\omega_n}(x_1x;k_\parallel).\end{gathered} \tag{4.1}$$

Here the G_{ω_n} are Green functions in the mixed representation. Using for their determination the normalized eigenfunctions $\varphi_l(x)$ corresponding to the Hamiltonian of the transverse electron motion in the film and satisfying the corresponding boundary conditions, we have (see for example,[12])

$$K_T(x, x_1)=T\sum_{|\omega_n|<\omega_D}\sum_{lm}\int\frac{d^2k_\parallel}{(2\pi)^2}\frac{\varphi_l(x)\varphi_m(x)\varphi_l(x_1)\varphi_m(x_1)}{(-i\omega_n+\xi_l(k_\parallel))(i\omega_n+\xi_m(k_\parallel))} \tag{4.2}$$

where

$$\xi_l(k_\parallel)=\hbar^2k_\parallel^2/2m+\varepsilon_l-\mu.$$

Let us represent the solution to Eq. (4.2) in the form

$$\Delta(x)=\Delta_0+\Delta_1(x), \quad \int_0^d \Delta_1(x)\,dx=0.$$

Carrying out the iterative procedure and integrating both sides of Eq. (4.1) with respect to x, we find after simple calculations

$$|\lambda| g(\mu)\ln(2\gamma\omega_D/\pi T_c)=1-\beta, \tag{4.3}$$

$$\beta=\Big(\sum\theta(\mu-E_l)\Big)^{-2}\sum_{lm}\theta(\mu-E_l)\theta(\mu-E_m)\Big[d\int_0^d dx\,\psi_l^2(x)\psi_m^2(x)-1\Big]. \tag{4.4}$$

Here $g(\mu)$ is the density of states determined by the expression (2.4).

The correction β, which appears because of the inhomogeneity of the gap, is of the order of a/d. Thus, in the case of zero boundary conditions

$$\beta=\frac{1}{2}\Big[\sum_l\theta(\mu-E_l)\Big]^{-1}. \tag{4.5}$$

In the expressions (4.3)–(4.5) figure the metallic film's true chemical potential, which differs from the value μ_0 in the bulk metal by the quantity (2.9).

Let the metallic film now have a nonmetallic film on it. The change arising then in the boundary condition will be felt only by the electrons with a transverse energy, E_l, lying in a narrow interval of the order of $\mu - V_2^0$ near μ (V_2^0 is the bottom of the conduction band in the semiconductor or semimetal). This pertains only to a small number of levels, as compared to their total number in the metallic film. Therefore, β, (4.4), just as $\Delta\mu$, (2.9), varies relatively little with the thickness of the nonmetallic film, and the oscillations in these quantities are very feebly marked. At the same time the quantity Δg, which is of the same order of smallness as β and $\Delta\mu$, varies (oscillates) with the thickness, t, of the nonmetallic film with an amplitude of the order of its magnitude. Hence it follows from the relation (4.3) that the superconducting transition temperature oscillates with t, the oscillations being correlated precisely with the oscillations of the density of states at the Fermi surface. Determining T_c from (4.3), and going over to the smoothed-out electron density of states, $\tilde{g}(\mu)$, in the metallic film (see Sec. 2), we find for the transition temperature corresponding to the nonmetallic-film thickness t the expression

$$\ln\frac{T_c(t)}{T_c(0)}=\frac{\delta g(t)}{g_0(\mu)}\frac{1}{|\lambda| g_0(\mu)},\quad \delta g(t)=\Delta g(t)-\Delta g(0). \tag{4.6}$$

The transition temperature, $T_c(0)$, in the metallic film without a coating is determined by the relation (see (2.10))

$$\ln\frac{T_c(0)}{T_0}=\Big[\beta+\Delta g(\mu_0)+\frac{\partial g_0(\mu_0)}{\partial\mu_0}\Delta\mu\Big]\frac{1}{|\lambda| g_0(\mu_0)}, \tag{4.7}$$

where T_0 is the transition temperature in the bulk metal.

Thus, the variation of the transition temperature with the thickness of the nonmetallic film is proportional to $\delta g(t)$, and the entire pattern, obtained in the preceding section, of density-of-states oscillations can be wholly referred to the quantity $\delta T_c = T_c(t) - T_c(0)$. In this case, however, it is very important that there occur a sharp amplification, connected with the large factor $1/|\lambda| g_0 \times (\mu_0)$, of the oscillation amplitude. The locations of the extrema remain unchanged.

The circumstance that the density of states at the Fermi surface figures in all the formulas is connected, in fact, with the usual assumption that $g(E)$ varies little in an energy interval of the order of ω_D near the chemical potential. In the case of a sandwich system the dependence of Δg in (2.6) on energy through the logarithmic derivative of the wave function at the boundary, $\varkappa(E)$, (3.8), then leads to the condition imposed on the variation of the phase χ (see (3.10), (3.13)):

$$(\partial\chi(E,t)/\partial E)_{E=\mu}\,\omega_D\ll\pi. \tag{4.8}$$

When this inequality is violated, it is necessary, in deriving the expression for T_c, to take into account the energy dependence of Δg, and this leads to a decrease in the amplitude of the transition temperature oscillations.

In the case of a semiconducting film the inequality (4.8) does not impose rigid conditions on the parameters. For a semimetallic coating $\partial\chi/\partial E \sim t$, and for a sufficiently large thickness t the condition (4.8) begins to be violated. A consequence of this is the decrease of the oscillation amplitude with increasing t.

Let us now briefly discuss the question of the influence of the finite thickness on the transition temperature, $T_c(0)$, in a thin film without a coating, a problem which has been widely discussed in recent years (see, for example,[12–15]). According to (4.7), the variation of T_c in a thin film is determined simultaneously by three causes: the variation of the density of states, the inhomogeneity of the gap along the thickness, and the shift in the chemical potential. For zero boundary conditions the contributions from the variation of the density of states, (2.7), and the inhomogeneity of the gap along the thickness, (4.5), cancel each other out. As a result, the variation of the transition temperature is determined by only the shift, (2.9), in the chemical potential, which leads to the growth of T_c with decreasing thickness. This result was first obtained by Shapoval[12] and later by Ovchinnikov.[15] However, the above-noted cancellation has a relatively fortuitous character and is absent, for example, in the cases of other boundary conditions. In a real metal, Δg and β in the general case cease altogether to correlate with each other. This manifests itself especially clearly in metals with a complex electronic structure in the presence of relatively small electron clusters. As has already been noted, a small cluster makes the same contribution to Δg as the main group of carriers, whereas its contributions to $g_0(\mu)$ and β are small. On the other hand, the contribution of a small cluster to the change in the chemical potential yields (see (2.9))

$$\frac{\partial g_0(\mu_0)}{\partial\mu_0}\Delta\mu=-\frac{\partial\ln g_0(\mu_0)}{\partial\mu_0}\mu_0\Delta g\approx-\frac{1}{2}\Delta g,$$

i.e., effectively decreases the contribution of the small cluster to the total value of Δg by roughly a factor of two. But the sign of the contribution remains unchanged, and for zero boundary conditions each small cluster leads to the lowering of T_c. Thus, in a polyvalent metal, depending on the electronic structure, a decrease in the film thickness can lead to either an increase or a decrease in T_c.

In the above-presented discussion, we abstracted ourselves from other possible influences of the finiteness of the film thickness on T_c, specifically from the role of the fluctuations of the order parameter and the electromagnetic field, as well as from the role of the variation of the phonon spectrum (see, from example, [14,15]), which falls outside the limits of the present paper. Notice, however, that, from all appearances, their effect is weaker than the effect of the variation of the density of states, which was directly demonstrated in respect of fluctuations by Ovchinnikov. [15]

5. OSCILLATIONS OF THE ELECTRICAL CONDUCTIVITY

Let us consider the longitudinal conductivity, $\sigma_\parallel$, of a film with a nonmetallic coating, and show that $\sigma_\parallel$ oscillations appear under the same conditions under which the Δg oscillations arise. It is significant that the effect can be completely elucidated within the framework of the quasi-classical Boltzmann equation alone (in contrast to, for example, the Shubnikov-de Haas effect, which requires for its description allowance for the off-diagonal elements of the density matrix). Restricting ourselves at low temperatures to taking account of electron scattering by impurities only, we have the well-known simplest form of the solution to this equation.

To find the t dependence of the conductivity, we can use the same procedure used in Sec. 2, i.e., determine $\sigma_\parallel$ in the ideal metallic film with the boundary conditions (2.1) for the electron wave function and then separate out the nonoscillatory—for fixed $\varkappa(E)$—part corresponding to the actual averaged picture.

With allowance for (2.2) the basic expression for $\sigma_\parallel$ has the form

$$\sigma_\parallel=\frac{e^2\hbar^2}{dm_1{}^2}\sum_n\int\frac{d^2k_\parallel}{(2\pi)^2}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\tau k_\parallel^2, \tag{5.1}$$

where f_0 is the equilibrium electron distribution function and τ is the relaxation time.

After integrating, and assuming, for simplicity, that τ depends only on the energy, we find

$$\sigma_\parallel=\frac{e^2\tau(\mu)}{\pi d\hbar^2}\sum_{n=0}^{\infty}(\mu-\varepsilon_n)\theta(\mu-\varepsilon_n). \tag{5.2}$$

To determine the sum figuring in (5.2), we use the Poisson transformation, retaining the notation used in (2.5):

$$\sum_{n=0}^{\infty}(\mu-\varepsilon_n)\theta(\mu-\varepsilon_n)=\frac{\hbar^2}{2m_1}\left(\frac{\pi}{d}\right)^2\left\{\int\limits_{-1/2}^{k_Fd/\pi-\xi}dy\left[\left(\frac{k_Fd}{\pi}\right)^2-(y+\xi)^2\right]\right.$$
$$\left.+2\,\mathrm{Re}\sum_{m=1}^{\infty}\int\limits_{-1/2}^{k_Fd/\pi-\xi}dy\left[\left(\frac{k_Fd}{\pi}\right)^2-(y+\xi)^2\right]e^{2\pi imy}\right\}, \tag{5.3}$$

Assuming, as before, that $k_Fd\gg 1$, and retaining only the leading—in this parameter—terms, we have for the first integral in (5.3)

$$\frac{2}{3}\left(\frac{k_Fd}{\pi}\right)^3-\left(\frac{k_Fd}{\pi}\right)^2\frac{1}{\pi}\operatorname{arctg}\frac{\varkappa(\mu)}{k_F}. \tag{5.4}$$

The sum over m leads to an expression containing both oscillating and nonoscillating terms, but all of the them are at least k_Fd times smaller than the second term in (5.4). Taking this into consideration, we find for the smoothed-out value of the longitudinal electrical conductivity the expression

$$\bar{\sigma}_\parallel=\sigma_0\left[1-\frac{3}{2}\frac{1}{k_Fd}\operatorname{arctg}\frac{\varkappa(\mu)}{k_F}\right]=\sigma_0\left[1+\frac{3}{2}\frac{\Delta g}{g_0(\mu)}\right], \tag{5.5}$$

$$\sigma_0=\frac{1}{3\pi^2}\frac{e^2k_F{}^3\tau(\mu)}{m_1}. \tag{5.6}$$

It is interesting that if $\varkappa$ and, consequently, Δg do not depend on energy, e.g., in the case of a boundary with vacuum, then allowance for the variation of the Fermi energy in the film (see (2.9)) in the expression (5.6) leads to the cancellation of the second term in the brackets in (5.5), and the longitudinal electrical conductivity remains equal to the electrical conductivity of the bulk metal. (This result naturally remains valid until the finite thickness begins to affect the electron mean free path in the film.) But as has already been noted in the preceding section, the Δg oscillations arising when the thickness, t, of the semimetallic or semiconducting film in the sandwich system is varied virtually does not lead to the variation of μ. As a result, the oscillations of the electron density of states in the metal with t will cause the longitudinal conductivity to oscillate in phase with the T_c oscillations (out of phase with the electrical resistance oscillations). The relative $\bar{\sigma}_\parallel$-oscillation amplitude will be less than in the T_c case, since the specific—for superconductivity—enhancement of the dependence on the variation of the density of states is absent.

Let us now elucidate the question of the influence of temperature on the quantum oscillations of the conductivity. In going from (5.1) to (5.2), we replaced $-\partial f_0/\partial\varepsilon$ by a delta function, which led to the temperature independent expression (5.4). In fact, we can do this at a finite temperature only under the condition that Δg, as a function of E, oscillates in energy interval $\delta E\gg T$. At low temperatures this inequality is always fulfilled. However, it may begin to be violated as the temperature increases, and the thicker the coating, the earlier this will happen. This will lead to the decrease of the oscillation amplitude with temperature. It is worth noting that, because of the large value of δE, especially for the small thicknesses corresponding to the initial period, the slowly decaying quantum oscillations of $\sigma_\parallel$ can, in principle, exist in a wide temperature range.

6. THE THREE-LAYERED SYSTEM

We have thus far considered only the two-layered, metal-nonmetal system. However, of considerable interest is the system where nonmetallic films cover the metallic film on both sides. Now the boundary conditions for the electrons in the metal at both boundaries vary with the variation of the deposited-film thickness.

Extending the results of Sec. 2 to this case, we have for the smoothed-out electron density of states, (2.6), in the metal (cf. (2.8))

$$\Delta g=-\frac{\zeta}{d}\left[\operatorname{arctg}\frac{\varkappa_1(E)}{k}+\operatorname{arctg}\frac{\varkappa_2(E)}{k}-\frac{\pi}{2}\right], \qquad (6.1)$$

where the $\varkappa_{1,2}$ are the values of the logarithmic derivative (3.8) at the two boundaries, which should be independently determined for each of the nonmetallic layers. It follows from (6.1) that the variation of the density in the metal depends simultaneously on the size effect in both nonmetallic films. The phase χ in (3.10) and (3.13) then has in these films uncorrelated values. As a result, there arises a distinctive interference between the two nonmetallic layers, even though the electron mean free path in the metallic film may be small compared to d. The variation of Δg is determined now by the values of the thicknesses, t_1 and t_2, of the two nonmetallic films. The nature of the T_c and $\sigma_{\parallel}$ oscillations, when t_1 and t_2 are varied in the three-layered system, will be determined by the results of the preceding sections if for Δg we insert the expression (6.1).

7. COMPARISON WITH EXPERIMENT. CONCLUDING REMARKS

The obtained results enable us to explain the whole set of available experimental facts connected with the observation of the superconducting transition temperature (and resistance) oscillations in metal-nonmetal layered systems.

As is apparent from the above-presented analysis, the oscillations of T_c or the nonmonotonic behavior of this quantity with the thickness of the nonmetallic film can arise in the case when this film is a semimetal or a semiconductor, and then primarily when the semiconductor has a comparatively narrow forbidden band and thereby an appreciable Bohr radius, (3.18), and a low first-Bohr-level energy, (3.14). On the other hand, in the case of a dielectric with a wide forbidden band the T_c oscillations should, as a rule, be absent. This assertion is apparently in complete correlation with the experimental results.[1–6] In particular, Sixl,[4] who discovered the T_c oscillations in aluminum covered with a semiconducting SiO film, verified subsequently that other semiconductors with a narrow forbidden band also give a similar effect. The results of the investigations[5,6] pertain to the deposition on a metal of semiconducting Ge or Si films. Not quite clear is the state in which the carbon film used to produce the metal-nonmetal system in[1–3] was. It could have been both semiconducting and semimetallic.

Since the considered oscillation mechanism is connected with the variation of the boundary condition for the electrons in the metal, a boundary condition which arises as a result of the size effect in the nonmetallic film, the locations of the extremum points on the $T_c(t)$ curve turned out in all the variants of the metal-nonmetal boundary to be independent of the thickness of the metallic film. This result was experimentally established in a clear-cut form in[1–4]. As has been shown in Sec. 3, if the nonmetallic film is a semiconductor, then the period of the oscillations of the electron density of states in the metal and, consequently, of T_c (see (4.6)) should increase sharply with the thickness t, and the number of oscillations should be severely limited. Such a picture was first observed by Sixl,[4] and the shape of his curves is qualitatively quite close to that of the theoretical curve shown in Fig. 4. It should be noted that, in accordance with the results of Sec. 3, Δg and T_c should increase during the initial growth of the thickness of the semiconducting film in the case of an ideal boundary. The subsequent variation in the general case depends on the relation between the parameters of the metal and the semiconductor. In particular, the situation is possible when the finite scale of the inhomogeneous electron density in the semiconductor does not allow the oscillations to develop and the transition temperature T_c can only pass through a maximum and then approach a limiting value with increasing t, or even simply approach a liniting value. This was, in fact, experimentally observed in Naugle's[5] and Strongin's[14] early investigations. To the ideal boundary will, to all appearances, correspond a thin transition layer with a high density of free carriers. In this layer should occur a rapid growth of the phase (see (3.10), (3.11), and (3.13)), and T_c will attain a maximum over a very small thickness t. (This assertion is of qualitative nature, since the formulas of Sec. 3 may not, strictly speaking, be applicable to such thicknesses.) This was, in fact, observed in all the investigations.[1–6] If the coating on the metal is a semimetal, then, as follows from the above-obtained results, the initial variation of the period with t should, on the contrary, change into oscillations (in principle, damped oscillations) with a constant period. It would be interesting to verify experimentally the appearance of such a pattern by measuring T_c specifically in a metal-semimetal system.

The amplitude of the T_c oscillations is proportional to the scale of the variation of Δg, (2.6), as $\varkappa(E)$ in the boundary condition (2.1) passes through the entire range of possible values. But $\Delta g \sim 1/d$, and therefore the T_c-oscillation amplitude should be inversely proportional to the thickness of the metallic film.

The decrease of the amplitude of the T_c oscillations with thickness was first observed in experiments performed by Mikheeva's group.[1–3]. Sixl[4] established in a very clear-cut form that the nature of the dependence corresponds precisely to $\delta T_c \sim 1/d$ (see also[6]). It should be noted that the absolute magnitude of the T_c variation with t can itself easily be quantitatively accounted for if the enhancement effect, which obtains precisely for T_c (as compared to Δg—see Sec. 4), is taken into account. In the case of[4] it is sufficient for this purpose to use (4.6) and the simplest expression for Δg, (2.6), which corresponds to a single spherical Fermi surface. It should be pointed out that the scale of the T_c oscillations can be quite considerable even in the case of an appreciable metallic-film thickness, and is moreover amplified owing to the presence of small electron clusters.

As was shown in Sec. 5, the conductivity oscillations are much more feebly marked than the T_c oscillations.

Nevertheless, they were experimentally observed by Mikheeva *et al.*[3] The theoretically predicted nontrivial—on the fact of it—result that the resistance and T_c should undergo antiphased oscillations is in complete agreement with the dependence found in[2]. Among the results obtained in[2,3], of special interest is the observation of a critical sensitivity of the oscillatory pattern to the simultaneous covering of the metal with nonmetallic films on both sides. The ideas developed in the preceding section allow us to understand that this effect is connected with the phase difference in the boundary conditions that is introduced by the independent size effects in the two nonmetallic films.

Let us note, in conclusion, that the remarks made in Sec. 4 about the variation of T_c in an isolated metallic film enable us to understand the possible nature of the different—in sign—dependences of T_c on d in different metals.

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New measurements of the viscosity of water behind a shock wave front

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A new method is put forward for measuring the viscosity of shock-compressed dielectrics, based on magnetoelectric recording of the velocity of cylindrical conductors behind the shock front. The viscosity of water at pressures between 30 and 80 kbar was determined. The measured viscosity was found to be greater by five orders of magnitude than the viscosity of water under normal conditions.

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Viscosity under high dynamic pressures was determined in[1–3] from the violation of similarity in the attenuation of perturbations on a shock wave front. This method was used to determine the viscosity of water for pressures in the range 80–250 kbar and the results were found to lie in the range between 1.5×10^4 and 3×10^4 P. Other values of the viscosity of water, lower by a factor of a million, are reported in[4]. The latter are based on measurements of the electrical conductivity of shock-compressed water electrolytes. In this paper, we describe a new and more direct method of measuring the viscosity of dielectrics behind a shock wave front. The method has been used to determine the viscosity of water at pressures in the range 30–80 kbar.

The method is based on recording the acceleration of "heavy" cylindrical bodies by the flow of a lighter material behind a shock wave front. Plane waves were produced by detonating a charge, 100 mm in diameter, in a layer of water, 30 mm deep. The bodies to be accelerated were in the form of copper and tungsten wires, 0.3–0.5 mm in diameter. They were placed in the central cross section of the layer, parallel to the plane of the shock wave front. The velocity of the wires dragged by the flow of compressed water was recorded by a magnetoelectric method.[5,6] To do this, the entire assembly was placed in a constant uniform magnetic field of 350 Oe, and the emf induced in the wire as it cut the magnetic lines of force was recorded by an oscillograph. In analogous experiments using a 0.1-mm aluminum foil instead of the wire, measurements were made of the mass velocity $u(t)$ of water in the central cross section of the layer. A change in the length of the wires from 10 to 20 mm did not lead to a change in the recorded ac-