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Effect of dispersionless phonons on the kinetics of electrons in one-dimensional conductors

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A theory is constructed for the influence of dispersionless phonons on the diffusion and localization in one-dimensional systems. The conductivity of perfect one-dimensional crystals, is described by the Drude formula, as a result of exact cancellation of an extensive class of non-ladder diagrams. In imperfect crystals (impurities, structural disorder), the dispersionless phonons do not disturb the localization, and the localization length and form factor, as well as the dielectric constant, are preserved. The low-frequency asymptotic behavior of the conductivity, however, becomes different, taking the form $\omega^2\ln^3\omega$ instead of $\omega^2\ln^2\omega$. In addition, if the free path times for scattering by impurities and phonons (τ_i and τ_{ph}) satisfy the inequality $\tau_i \gg \tau_{ph}$, then the characteristic time that limits the region of the low-frequency asymptotic behavior is no longer the free-path value τ_i but the diffusion value τ_i^2/τ_{ph}. The results are valid in the region of relatively low temperatures $T \ll \omega_0$(ω_0 is the phonon frequency) and in the region of weak scattering $\tau_i T \gg 1$ and $\tau_{ph} T \gg 1$. The considered model is apparently of greatest importance from the point of view of the interaction of electrons with fully-symmetrical intramolecular phonons, for which the frequencies are quite high so that the upper bound on T is not very stringent. We conclude with a qualitative examination of the conductivity under certain other conditions.

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1. INTRODUCTION

The main singularity of the electron spectrum of one-dimensional disordered systems is that all the states are localized in them. [1] Therefore such systems have no static electric conductivity or diffusion. [2,3] These statements are valid for real systems only at zero temperature, when the electrons interact only with the static potential of the impurities.

At finite temperatures real phonons produce for the electrons a time-dependent potential that gives rise to transitions between the electron states with different energies. By the same token, the interaction with the phonons differs radically from the interaction with the impurities, which leads to localization of the electrons on account of the exact interference of waves scattered by different impurities. One should expect the change of the energy in scattering by phonons to disturb the interference and to lead to qualitatively new effects.

In our preceding paper[4] we considered the influence of weak electron-phonon interaction on the kinetics of electrons localized in an impurity field. The conditions were indicated under which the interaction with the phonons causes transitions between localized states of the electrons, disturb the localization partially, and by the same token give rise to a finite conductivity. In this case the electron motion consists of hops over distances on the order of the localization length in the impurity field, which follow each other in intervals on the order of the time of scattering by the phonons.

Gogolin *et al.* [5] have indicated the mechanisms through which the interaction with the phonons can intensify the localization of the electrons. These mechanisms in conjunction with the previously obtained results[4] have made it possible to propose an explanation for the experimental data on the temperature dependence of the electric conductivity and the dielectric constant of TCNQ salts with structural disorder.

Thus, the influence of the phonons on the electron kinetics in one-dimensional conductors is manifold—they can disrupt as well as enhance the localization. To determine the relative importance of the different types of electron-phonon interaction, it is therefore necessary

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to make use of qualitative concepts obtained from experience with the exact solution of model problems.

In the preceding paper[4] we considered phonons having a sufficiently large average frequency $\bar{\omega}$ and dispersion Δ. Namely, it was assumed that the criteria $\bar{\omega}\tau_i \gg 1$ and $\Delta\tau_i \gg 1$ were satisfied, where τ_i is the mean free path time relative to scattering by the impurities.

The last criterion is most strongly violated in the important case of dispersionless phonons, which is at present of particular interest from the point of view of interactions of electrons with intramolecular phonons. It is shown in the present paper that the case with $T \ll \omega_0$ admits of an exact solution. It turns out that localization is strictly preserved, and the localization length remains constant even if $\tau_{\text{ph}} \ll \tau_i$, where τ_{ph} is the mean free path time relative to scattering by phonons. The static conductivity σ is equal to zero at arbitrary ratio τ_i/τ_{ph}, while the static dielectric constant remains the same as at $\tau_{\text{ph}} = \infty$. When $\tau_i/\tau_{\text{ph}} \gg 1$, however, the conductivity σ begins to increase at an early stage: the characteristic frequency is $\omega \sim \tau_{\text{ph}}/\tau_i^2$, whereas at $\tau_i/\tau_{\text{ph}} \ll 1$ the characteristic frequency is $\omega \sim 1/\tau_i$. The sequence of the diagrams is much more numerous than the one corresponding to the kinetic equation; nonetheless, in the absence of impurities, the Drude formula is obtained when the sequence is summed for $\sigma(\omega)$.

2. SELECTION OF DIAGRAMS AND CALCULATION OF VERTICES

We consider a one-dimensional system of noninteracting electrons with a dispersion law $\varepsilon(p)$, situated in a random potential. We assume that the electrons interact with one branch of the dispersionless phonons with frequency ω_0. We confine ourselves below to the parameter range in which the Peierls transition does not set in. To determine the character of the localization, of the electric conductivity, and of the dielectric constant it is necessary to calculate the correlation functions of the density and current operators. A diagram technique suitable for this calculation was described in the preceding paper.[4]

If the phonon dispersion is large, then after absorption and emission of several phonons the electron energy practically never returns to the initial value. There are no interference effects in such an electron-phonon scattering. Therefore in the earlier calculation[4] of the polarization loop, all that was left in final analysis was an aggregate of phonon lines corresponding to the usual kinetic equation. For dispersionless phonons, as a result of consecutive absorption and emission of equal numbers of phonons, the electron energy returns to exactly the initial value. The result is an interference between the electronic states that correspond to multiple scattering by phonons. Therefore the class of essential phonon diagrams becomes much larger, approaching to a considerable extent the class of impurity diagrams. The phonon frequency ω_0 will be assumed, as in[4], to be high enough: $\omega_0\tau_i, \omega_0\tau_{\text{ph}} \gg 1$. At low temperatures $T \ll \omega_0$, the scattering of an electron by phonons proceeds via infrequent absorption and subsequent rapid emission of phonons. Consequently, in the presence of Fermi degeneracy the phonon absorption and emission processes couple the levels ε and $\varepsilon \pm \omega_0$, where ε lies near the Fermi energy ($\varepsilon \approx \varepsilon_F$). With the energy ε chosen in this manner, the attenuation of a Green's function with energy ε is small in comparison with the attenuation of Green's functions with energies $\varepsilon \pm \omega_0$ relative to the parameter $N \sim e^{-\omega_0/T} \ll 1$. The energy levels of significance for our problem, and the transitions between them, are shown and numbered in Fig. 1. The mean free path of an electron with $\varepsilon \approx \varepsilon_F$ is defined by the expressions

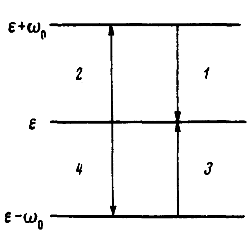

FIG. 1.

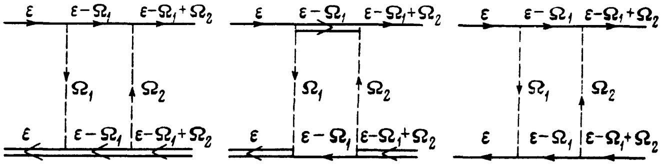

FIG. 2. Diagrams with "hot" lines.

$$\frac{1}{l^{\pm}(\varepsilon)} = \frac{1}{l_2^{\pm}(\varepsilon)} + \frac{1}{l_4^{\pm}(\varepsilon)}, \tag{1}$$

$$\frac{1}{l_2^{\pm}(\varepsilon)} = \frac{1}{v(\varepsilon)} \int \frac{d^3\mathbf{q}}{(2\pi)^2} |c_{\mathbf{q}}|^2 \frac{1-n(\varepsilon+\omega_0)}{1-n(\varepsilon)} \frac{N(\omega_0)}{v(\varepsilon+\omega_0)} \delta(q_x + p(\varepsilon) \mp p(\varepsilon+\omega_0)),$$

$$\frac{1}{l_4^{\pm}(\varepsilon)} = \frac{1}{v(\varepsilon)} \int \frac{d^3\mathbf{q}}{(2\pi)^2} |c_{\mathbf{q}}|^2 \frac{1-n(\varepsilon-\omega_0)}{1-n(\varepsilon)} \frac{N(\omega_0)+1}{v(\varepsilon-\omega_0)} \delta(q_x + p(\varepsilon) \mp p(\varepsilon-\omega_0)).$$

This is the same length which entered in the linearized kinetic equation (see[4], Sec. 7). The lengths l_2 and l_4 correspond respectively to production of a hot electron with energy $\varepsilon + \omega_0$ and a hot hole with energy $\varepsilon - \omega_0$. The processes of phonon emission by hot carriers correspond to the lengths

$$l_1^{\pm} \approx l^{\pm}(\varepsilon+\omega_0), \quad l_3^{\pm} \approx l^{\pm}(\varepsilon-\omega_0). \tag{2}$$

If $\omega_0 \ll \varepsilon_F$, then $l_1^{\pm} \approx l_3^{\pm}$, $l_2^{\pm} \approx l_4^{\pm}$.

The minus and plus indices correspond to scattering with and without rotation of the momentum, respectively. As indicated above, $l_2, l_4 \gg l_1, l_3$.

The correlation functions are represented by a diagram in the form of an electron loop consisting of two Green's functions. Typical examples of the phonon inserts in the loop are shown in Fig. 2. Each diagram contains two phonon lines corresponding to successive absorption and emission of a phonon, so that the electron Green's functions contained between them are rapidly damped. The integral over the distance between the phonon lines converges over the "hot" lengths l_h

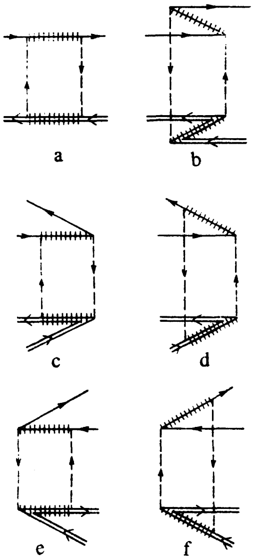

FIG. 3. Two-phonon skelton vertices. The hot lines are cross-hatched.

$\sim l_1, l_3$, whereas the integral over the position of a pair of phonon lines converges over the "cold" length l. After passage of a pair of phonon lines, the energy of the electron line returns to its initial value and in this respect the pair of phonon lines is analogous to an impurity line. We shall henceforth not separate the diagram segments containing hot lines, and calculate initially integrals that converge over the hot lengths; this will yield expressions for the effective vertices that join only cold lines.

Assuming l_h to be small in comparison with l, we shall nevertheless regard l_h as long enough in comparison with the electron wavelengths $\lambda(\varepsilon)$, $\lambda(\varepsilon \pm \omega_0)$. The contribution of diagrams of the type of Fig. 2c, in which the rapidly oscillating factors do not cancel out, is small relative to the parameter $\varepsilon\lambda/\omega_0 l_h \ll 1$, and such diagrams will therefore be omitted.

The contribution from the diagrams of Fig. 2b vanishes exactly. The segment of this diagram, containing the left-hand phonon line, is identical with the diagram of Fig. 6b of[4], taken at coinciding energies; but it is precisely in this case, as shown in[4], that the contribution of this diagram vanishes.

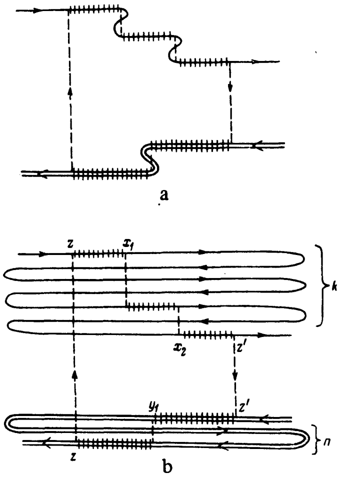

FIG. 4. Complication introduced in the skeleton diagram by drawing out the cold lines from the self-energy parts: k—number of "tongues" in the upper part, n—in the lower.

When integrating diagram Fig. 2a with respect to the frequencies Ω_1 and Ω_2, the main contribution proportional to N results from the term with $\Omega_1 = \Omega_2$. It is by this that we mean that absorption and subsequent rapid emission of phonons predominate in the scattering, as a result of which the electron returns to a state with the same energy as before. The principal skeleton vertices corresponding to such processes are shown in Fig. 3. The contribution from diagrams with $\Omega_1 = -\Omega_2$, corresponding to a change of $2\omega_0$ in the electron energy, is of higher order in N, and will henceforth be omitted.

Naturally, the self-energy diagrams for the hot lines involve states with $\varepsilon \approx \varepsilon_F$, corresponding to the return of the electron to the initial energy (see Fig. 4a). Therefore the diagrams of Fig. 3 are made more complicated because of these cold electron lines are drawn out over distances on the order of l beyond the limits of the section bounded by the pairs of the initial phonon lines, followed by their attachment to other cold lines via any one of the diagrams of Fig. 3. By way of illustration, Fig. 4a shows three self-energy parts; when the cold lines contained in them are drawn out, the result is the diagram of Fig. 4b. It is clear that the diagrams must be made more complicated in two ways: first, by increasing the number of self-energy parts from which the cold lines are drawn out, and second, by increasing the number of tongues on each of the drawn-out cold lines.

In what follows, we shall draw out the cold lines only from the self-energy diagrams of Fig. 5a (a′). The analogous contribution from the diagrams of Fig. 5b (b′) contains a factor $(T\tau_{\rm ph})^{-1}$, which we assume to be small (see the Appendix for details).

The next task is to obtain an expression corresponding to the diagram of Fig. 6. This diagram has been introduced to designate assemblies of diagrams of the type of Fig. 4, in which integration is carried out over all the internal variables and summation over all the variants of the rearrangements of the internal phonon lines at a fixed number of external ends. The diagram of Fig. 6 does not change the number of lines in the vertical section, and corresponds therefore to the skeleton diagrams of Figs. 3a and 3b. We present detailed calculations for the contribution made by the skeleton diagram 3a, after which we write out the expressions corresponding to all other skeleton diagrams in Fig. 3.

The internal phonon lines can be inserted in the two ways shown in Fig. 7. Carrying out integration over all x_i and y_i and over $z' - z$ (see Fig. 4) and recognizing that the internal phonon lines are arranged in sequence along

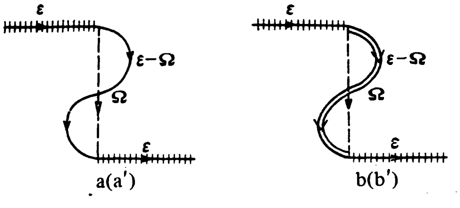

FIG. 5. Self-energy parts. The dashes correspond to permutation of single and double lines.

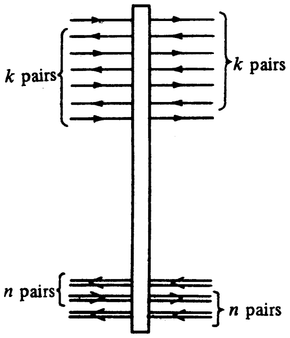

FIG. 6. Effective phonon vertex produced after integration along the hot lines.

the coordinate, we obtain for a diagram with r internal phonon lines in the upper part and s lines in the lower part the expression

$$\frac{1}{l_2^+l_1^+}\int_0^\infty d(z'-z)\frac{1}{r!s!}\left[-\left(\frac{1}{l_1^+}+\frac{1}{l_1^-}\right)(z'-z)\right]^{r+s}$$
$$\times\exp\left\{-\left(\frac{1}{l_1^+}+\frac{1}{l_1^-}\right)(z'-z)\right\}=\frac{l_1^-}{l_2^+(l_1^++l_1^-)}(-1)^{r+s}C_{r+s}^r. \tag{3}$$

The expression $1/l_1^+ + 1/l_1^-$ is the result of summation over phonon lines of type a and b (or a′ and b′) of Fig. 7. Only absorption of a phonon with production of a hot electron is taken into account in (3). The processes of absorption with production of a hot Fermi hole correspond to the substitutions $l_1 \to l_3$ and $l_2 \to l_4$.

We introduce for the calculated contribution the notation

$$\frac{l_1^-}{l_2^+(l_1^++l_1^-)}\Gamma_{kn}. \tag{4}$$

The quantity Γ_{kn} is the result of summation over all the arrangements the internal phonon lines

$$\Gamma_{kn}=\sum_{r,s}(-1)^{r+s}C_{r+s}^{r}C_{k-1}^{r-1}C_{n-1}^{s-1},\quad k\geqslant1,\quad n\geqslant1,$$
$$\Gamma_{00}=1,\quad \Gamma_{k0}=\sum_{r}(-1)^{r}C_{k-1}^{r-1},\quad k\geqslant1,\quad \Gamma_{0n}=\sum_{s}(-1)^{s}C_{n-1}^{s-1},\quad n\geqslant1. \tag{5}$$

The expression for Γ_{kn} simplifies if one uses the identity

$$C_{r+s}^{r}=\frac{1}{2\pi i}\int\frac{dz}{z^{r+1}(1-z)^{s+1}} \tag{6}$$

where the integration is over a small circle around $z=0$.

Substituting (6) in (5), we obtain after simple transformations

$$\Gamma_{kn}=(-1)^{k-n}C_2^{k-n+1}=2\delta_{k,n}-\delta_{k,n+1}-\delta_{k,n-1},\quad k\geqslant1,\quad n\geqslant1,$$
$$\Gamma_{00}=1,\ \Gamma_{01}=\Gamma_{10}=-1,\ \Gamma_{k0}=0,\ k\geqslant2,\ \Gamma_{0n}=0,\ n\geqslant2. \tag{7}$$

Expressions (7) correspond to the skeleton diagram of Fig. 3a. The remaining skeleton diagrams of Fig. 3 correspond to analogous expressions

$$\frac{l_1^-}{l_2^+(l_1^++l_1^-)}\Gamma_{kn}\ (a),\quad \frac{l_1^+}{l_2^-(l_1^++l_1^-)}\Gamma_{kn}\ (b),$$
$$\frac{l_1^+}{l_2^+(l_1^++l_1^-)}\Gamma_{kn}e^{2i\omega z/v}\ (c),\quad \frac{l_1^-}{l_2^-(l_1^++l_1^-)}\Gamma_{kn}e^{2i\omega z/v}\ (d) \tag{8}$$
$$\frac{l_1^+}{l_2^+(l_1^++l_1^-)}\Gamma_{kn}e^{-2i\omega z/v}\ (e),\quad \frac{l_1^-}{l_2^-(l_1^++l_1^-)}\Gamma_{kn}e^{-2i\omega z/v}\ (f).$$

The presence of the lengths l^+ and l^- in these formulas is due to the contribution of the diagrams of the type of Figs. 7a and 7b.

Processes with hot holes correspond to the substitutions $l_1 \to l_3$, $l_2 \to l_4$.

We thus arrive at a diagram technique with effective vertices of the type shown in Fig. 6, corresponding to expressions (8). This technique is similar to that constructed by Berezinskiĭ[3] for impurities, but differs in that the vertices connect not pairs but entire bundles of electron lines.

3. ELECTRIC CONDUCTIVITY OF A PERFECT CRYSTAL

To calculate the correlation functions, we use a procedure developed earlier.[3,4] Each diagram for the polarization loop is divided into three parts that lie respectively to the left, in between, and to the right of the outer vertices x' and x. The number of pairs of single and double lines passing through the vertical sections of the diagram at the points x and x' will be designated m and m'. We then obtain for the right and central parts the equations

$$-\frac{d\tilde{R}_m}{dx}=-\frac{2m\tilde{R}_m}{L}+\frac{1}{l}\Gamma_m(\tilde{R}_{m+1}e^{-2i\omega x/v}+\tilde{R}_{m-1}e^{2i\omega x/v})$$
$$+2\left(\frac{1}{L}-\frac{1}{l}\right)\Gamma_m\tilde{R}_m, \tag{9}$$

$$\frac{dZ_{m'm}}{dx}=\frac{i\omega}{v}Z_{m'm}-\frac{1}{L}(2m+1)Z_{m'm}+\frac{1}{l}(\Gamma_mZ_{m'm-1}e^{-2i\omega x/v}$$
$$+\Gamma_{m+1}Z_{m'm+1}e^{2i\omega x/v})+\left(\frac{1}{L}-\frac{1}{l}\right)(\Gamma_m+\Gamma_{m+1})Z_{m'm}, \tag{10}$$

where

$$\Gamma_m=\sum_{k,n=0}^{m}(m-k)(m-n)\Gamma_{kn}. \tag{11}$$

$$\frac{1}{l}=\frac{1}{l_1^++l_1^-}\left(\frac{l_1^+}{l_2^+}+\frac{l_1^-}{l_2^-}\right)+\frac{1}{l_3^++l_3^-}\left(\frac{l_3^+}{l_4^+}+\frac{l_3^-}{l_4^-}\right), \tag{12}$$

$$\frac{1}{L}=\frac{1}{l_2^+}+\frac{1}{l_2^-}+\frac{1}{l_4^+}+\frac{1}{l_4^-}. \tag{13}$$

Substituting (7) in (11) we obtain $\Gamma_m=m$. Taking this into account, we see that the length L drops out of Eqs. (9) and (10).

The correlation functions $\mathscr{X}^0(\omega, k)$ and $\mathscr{X}^1(\omega, k)$ of the densities and of the currents are expressed in the following manner in terms of $R_m(x)$ and $Z_{m'm}(x', x)$[3,4]:

$$\mathscr{X}^a(\omega,k)=4\int\frac{d\varepsilon}{2\pi}n(\varepsilon)(1-n(\varepsilon))\frac{1}{v(\varepsilon)}X^a(\varepsilon,\omega,k),\quad a=0,1, \tag{14}$$

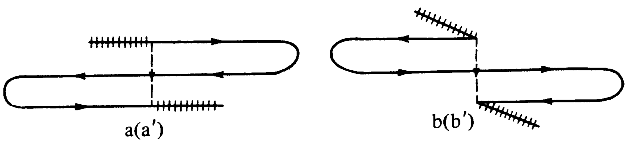

FIG. 7. Inclusion of cold lines with the aid of rotating and nonrotating vertices. The diagram a(a′) corresponds to the factor $(-l^+(\varepsilon \pm \omega_0))^{-1}$, and the diagram b(b′) to the factor $(-l^-(\varepsilon \pm \omega_0))^{-1}$.

$$X^a(\varepsilon,\omega,k)=\frac{2l}{v(\varepsilon)}\left[\frac{v(\varepsilon)}{2}\right]^{2a}\sum_{m=0}P_m{}^a(\omega)\{Q_m{}^a(\omega,k)+Q_m{}^a(\omega,-k)\},\quad(15)$$

where

$$\tilde{R}_m(x)=R_me^{2im\omega x/v},\quad P_m{}^0={}^1/_2(R_m+R_{m-1}),\quad P_m{}^1=R_m-R_{m+1},\quad(16)$$

$$Q_m{}^a(\omega,k)=\frac{1}{l}\sum_{m'=0}^{\infty}\int_{x'}^{\infty}dx\,e^{ik(x'-x)}e^{-2im'x'\omega/v}Z_{m'm}(x',x)\,e^{2imx\omega/v}P_{m'}{}^a.\quad(17)$$

The equations for R_m and $Q^a_m(\omega,k)$ are

$$i\tilde{\nu}R_m+R_{m-1}+R_{m+1}-2R_m=0.\quad R_0=1,\quad(18)$$

$$i\tilde{\nu}(m+{}^1/_2)Q_m{}^a+(m+1)(Q^a_{m+1}-Q_m{}^a)-m(Q_m{}^a-Q^a_{m-1})-i\tilde{\varkappa}Q_m{}^a+P_m{}^a=0.\quad(19)$$

The dimensionless frequency and momentum are equivalent to (we put henceforth $\tau_{\text{ph}}=\tau$)

$$\tilde{\nu}=2l\omega/v=2\omega\tau,\quad\tilde{\varkappa}=kl.\quad(20)$$

Equations (18) and (19) are solved by making the exponential substitution

$$R_m=e^{-\gamma m},\quad Q_m{}^a=Q_0{}^ae^{-\gamma m},\quad(21)$$

$$4\operatorname{sh}^2\frac{\gamma}{2}=-i\tilde{\nu},\quad Q_0{}^1=\frac{1-e^{-\gamma}}{\operatorname{sh}\gamma+i\tilde{\varkappa}},\quad Q_0{}^0=\frac{1}{2}\frac{1+e^{-\gamma}}{\operatorname{sh}\gamma+i\tilde{\varkappa}}.\quad(22)$$

From this we get

$$X^0(\varepsilon,\omega,k)=\left(-i\omega+\frac{k^2}{1-i\omega\tau(\varepsilon)/2}\frac{l^2(\varepsilon)}{2\tau(\varepsilon)}\right)^{-1},\quad(23)$$

$$X^1(\varepsilon,\omega,k)=\frac{1}{2}\tau(\varepsilon)\left(1-i\omega\tau(\varepsilon)/2+\frac{k^2}{-i\omega}\frac{l^2(\varepsilon)}{2\tau(\varepsilon)}\right)^{-1}.\quad(24)$$

We note that at low frequencies $\tilde{\nu}\ll 1$ the main contribution to the sums is made by the terms with $m\sim\gamma^{-1}\sim\tilde{\nu}^{-1/2}$.

In the region $\omega\tau\ll 1$, the correlator (23) has a diffusion form with a diffusion coefficient $D(\varepsilon)=l^2(\varepsilon)/2\tau(\varepsilon)$. For the conductivity in a homogeneous field ($k=0$) we obtain the Drude formula

$$\sigma(\omega)=\frac{2}{\pi}e^2\int d\varepsilon\,v(\varepsilon)\left(-\frac{\partial n}{\partial\varepsilon}\right)\frac{\tau(\varepsilon)/2}{1-i\omega\tau(\varepsilon)/2}.\quad(25)$$

The quantity $\tau(\varepsilon)/2$ is the usual transport relaxation time.

Expressions (23)–(25) coincide with the results of the solution of the general one-dimensional kinetic equation. Moreover, even Eqs. (18) and (19) are already identical to those obtained by summing the ladder diagrams corresponding to the kinetic equation. We emphasize, however, that Eqs. (18) and (19) were obtained by summation of a rather broad class of diagrams, including diagrams of the type of Fig. 4. The main difference between the one-dimensional and three-dimensional cases is that in the former the ladder diagrams dominated relative to the parameter $\varepsilon\tau\gg 1$, while here the analogous result is the consequence of exact cancellation of more complicated diagrams, each having the same order of magnitude as the ladder diagrams.

It is seen that there is no localization of the electrons in the case of electron interaction with dispersionless phonons, and the conductivity at zero frequency remains finite and is determined by the usual Drude formula. It is of interest to emphasize in this connection that in the three-dimensional case elastic two-phonon scattering is practically equivalent to impurity scattering.[6] The qualitative difference between the results for these two types of scattering in the one-dimensional case is connected with the introduced limitation on the hot path length $l_h\gg\lambda\varepsilon/\omega_0$, which is sufficient to eliminate definite interference diagrams.

4. CONDUCTIVITY OF CRYSTAL WITH IMPURITIES

We consider now the joint influence of impurities (or of structural disorder) and of dispersionless phonons on the localization and electricconductivity. The problem can be solved at $l_h\ll l_i^-$. In this case the inclusion of the impurity lines in the internal part of the diagram of Fig. 4 (i.e., in the interval z, z') results in small corrections of the order of $l_h/l_i^-\ll 1$, which can be neglected. In this approximation, the problem reduces to simultaneous summation of the contribution of the impurity lines and of all the phonon vertices considered above (for example, of the type of Fig. 6). The equations for R_m and $Q^a_m(\omega,k)$ include additively terms that correspond to both types of interaction

$$i\tilde{\nu}R_m+(1+\beta m)(R_{m+1}+R_{m-1}-2R_m)=0,\quad(26)$$

$$i\tilde{\nu}(m+{}^1/_2)Q_m{}^a+(m+1)(Q^a_{m+1}-Q_m{}^a)-m(Q_m{}^a-Q^a_{m-1})$$
$$+\beta(m+1)^2(Q^a_{m+1}-Q_m{}^a)-\beta m^2(Q_m{}^a-Q^a_{m-1})-i\tilde{\varkappa}Q_m{}^a+P_m{}^a=0,\quad(27)$$

where

$$\beta=l/l_i^-=\tau/\tau_i,\quad\tau_i=l_i^-/v.$$

Interaction with impurities corresponds in Eq. (26) to the term linear in m, and in Eq. (27) to terms quadratic in m and $m=L$. In the region of low frequencies, the large m are always important: in the case of interaction with impurities $m\sim(\omega\tau_i)^{-1}\gg 1$,[3] and in the case of interaction with phonons $m\sim(\omega\tau)^{-1/2}\gg 1$ (see Sec. 3). It follows therefore that at low frequencies the terms due to the interaction with impurities predominate in (26) and (27). This means that in the limit as $\omega\to 0$ we can neglect the interaction with the phonons at any ratio of τ_i and τ, so that at large times the correlators retain the same form as in the absence of phonons. Thus, the asymptotic form of the density correlator, meaning also the localization length, is determined exclusively by the interaction with the impurities, and the static conductivity is equal to zero.

Under conditions when the interaction with the phonons does not influence the static characteristics, it is of particular interest to investigate the frequency dependences. We consider two limiting cases.

a. Strong scattering by phonons ($\tau\ll\tau_i$)

The low-frequency region is determined here by the inequality $\tilde{\nu}\ll\beta^2\ll 1$, i.e., $\omega\tau_i^2/\tau\ll 1$.

Analogously,[3] in the low-frequency limit we can go from difference equations to differential equations. In this approximation, Eq. (26) can be solved exactly:

$$R(p)=\frac{(p-i\nu\alpha)^{1/2}K_1[2(p-i\nu\alpha)^{1/2}]}{(-i\nu\alpha)^{1/2}K_1[2(-i\nu\alpha)^{1/2}]},\quad(28)$$

and the equation for Q^1 takes the form

$$-pQ^1+\frac{d}{dp}\left(p(p-i\nu\alpha)\frac{dQ^1}{dp}\right)+P^1(p)=0,\quad P^1(p)=i\nu\frac{dR}{dp}, \tag{29}$$

where

$$p=-i\nu m,\ \ \nu=\tilde{\nu}/\beta=2\omega\tau_i,\ \ \alpha=1/\beta=\tau_i/\tau, \tag{30}$$

and K_1 is a Bessel function. Equation (29) was written out for $k=0$. The conductivity $\sigma(\omega)$ is expressed in terms of the correlator

$$X^1(\varepsilon,\omega)=\frac{v(\varepsilon)l_i^-(\varepsilon)}{-i\nu}\int_0^\infty P^1(p)Q^1(p)\,dp=v(\varepsilon)l_i^-(\varepsilon)\int_0^\infty\frac{dQ_1}{dp}(R(p)-1)\,dp. \tag{31}$$

In contrast to the impurity problem proper,[3] the differential equation for Q^1 is in this case accurate enough to calculate the principal term in $\operatorname{Re}\sigma(\omega)$. The reason is that the contribution of the terms with small m in the sum (15) is small in terms of the parameter $\beta\ll 1$. We shall solve Eq. (29) by expansion in terms of $|\nu\alpha|\ll 1$. This yields in the lowest order the answer for the impurity problem proper: $X^1(\varepsilon,\omega)=v(\varepsilon)l_i^-(\varepsilon)2\zeta(3)(-i\nu)$. Therefore $\operatorname{Im}\sigma(\omega)$ has at low frequencies the same form as in the absence of phonons.[3,4]

The interaction with the phonons comes into play in the next order in $\nu\alpha$. The quantity $\operatorname{Re}\sigma(\omega)$ is now of the order of $\alpha\nu^2$, i.e., it increases in comparison with its value $\sim\nu^2$ in the absence of phonons.

To calculate $\operatorname{Re}\sigma(\omega)$, we break up the integral (31) into two: over the region $0\leqslant p\leqslant p_0$ and over the region $p_0\leqslant p<\infty$, where $|\nu\alpha|\ll p_0\ll|\nu\alpha|^{1/2}\ll 1$. In the region $p\ll|\nu\alpha|^{1/2}$ we can neglect the first term of (29). As a result we obtain

$$\frac{dQ^1}{dp}=-i\nu\frac{R(p)-1}{p(p-i\nu\alpha)}. \tag{32}$$

The integration constant is determined from the condition that $Q^1(p)$ be regular as $p\to 0$. Formula (32) yields at $p=p_0$ the boundary condition for the derivative dQ^1/dp obtained for the region $p\gg|\nu\alpha|$.

Expanding $R(p)$ in terms of $|\nu\alpha|/p_0\ll 1$, we obtain from (32)

$$\left.\frac{dQ^1}{dp}\right|_{p=p_0}=(-i\nu)\left[\frac{\ln p_0+2C-1}{p_0}+\frac{i\nu\alpha}{p_0^2}(\ln(-i\nu\alpha)+2C-2)\right], \tag{33}$$

where C is the Euler constant.

In the region $p\gg|\nu\alpha|$, Eq. (29) can be solved by expanding in terms of $|\nu\alpha|\ll 1$. We seek Q^1 in the form $Q^1=Q_0^1+Q_1^1$, where (cf.[3])

$$Q_0^1=\frac{-i\nu}{p}[p^{1/2}K_1(2p^{1/2})(\ln p+2C)+K_0(2p^{1/2})]. \tag{34}$$

The equation for Q_1^1 is

$$-pQ_1^1+\frac{d}{dp}\left(p^2\frac{dQ_1^1}{dp}\right)-i\nu\alpha\frac{d}{dp}\left(p\frac{dQ_0^1}{dp}\right)+P_1^1(p)=0, \tag{35}$$

$$P_1^1(p)=i\nu\frac{d}{dp}(R(p)-2p^{1/2}K_1(2p^{1/2})). \tag{36}$$

The substitution $2p^{1/2}=z$ transforms (36) into an inhomogeneous Bessel equation for the function $zQ_1^1(z)$. Its solutions, which satisfies the condition that it decrease at infinity, is of the form

$$Q_1^1(z)=\frac{I_1(z)}{z}\int_z^\infty\xi\,d\xi\,K_1(\xi)f(\xi)+\frac{K_1(z)}{z}\int_0^z\xi\,d\xi\,I_1(\xi)f(\xi)+C_1\frac{K_1(z)}{z}, \tag{37}$$

$$f(z)=\nu^2\alpha\frac{16}{z^2}\left\{K_1-\frac{d}{dz}\left[z\frac{d}{dz}\left(\frac{K_1}{z}\left(\ln\frac{z}{2}+C\right)+\frac{K_0}{z^2}\right)\right]\right\}$$
$$+\nu^2\alpha(\ln(-i\nu\alpha)+2C-1)\frac{8}{z}K_0. \tag{38}$$

The non-integrable divergences in $f(z)$ cancel each other at small z. Differentiating and expanding (37) in the small quantity $p_0=z_0^2/4$, we obtain after comparison with (33)

$$C_1=-4\nu^2\alpha(\ln(-i\nu\alpha)+2C-2). \tag{39}$$

Substituting (37) in (31), we obtain the contribution made to the integral by the upper region:

$$\int_{p_0}^\infty dp\frac{dQ^1}{dp}(R(p)-1)=-i\nu 2\zeta(3)(1+i\nu\alpha\ln(-i\nu\alpha))$$
$$+i\nu p_0[\ln^2 p_0+4\ln p_0(C-1)+4C^2-8C+5]$$
$$+\nu^2\alpha\left[\frac{1}{3}\ln^3 p_0+\ln^2 p_0-(2C-3)(2C-1)\ln p_0\right]$$
$$-\nu^2\alpha\ln(-i\nu\alpha)[\ln^2 p_0+2(2C-1)\ln p_0+4C^2-4C+2]+\text{const}\,\nu^2\alpha. \tag{40}$$

Using (32), we obtain the contribution from the lower region

$$\int_0^{p_0} dp\frac{dQ^1}{dp}(R(p)-1)=-i\nu p_0[\ln^2 p_0+4\ln p_0(C-1)+4C^2-8C+5]$$
$$+\nu^2\alpha\left[-\frac{2}{3}\ln^3(-i\nu\alpha)+(3-4C)\ln^2(-i\nu\alpha)+(8C-4C^2-5)\ln(-i\nu\alpha)\right]$$
$$+\nu^2\alpha\left[-\frac{1}{3}\ln^3 p_0+\ln^2 p_0\ln(-i\nu\alpha)-\ln^2 p_0\right.$$
$$\left.+2(2C-1)\ln p_0\ln(-i\nu\alpha)+(2C-1)(2C-3)\ln p_0\right]+\text{const}\,\nu^2\alpha.$$

Adding (40) in (41), we obtain ultimately

$$\sigma(\varepsilon,\omega)=\frac{2}{\pi}e^2l_i^-\left\{-4\zeta(3)i\omega\tau_i+4(\omega\tau_i)^2\frac{\tau_i}{\tau}\left[-\frac{2}{3}\ln^3\left(-2i\omega\frac{\tau_i^2}{\tau}\right)\right.\right.$$
$$\left.\left.+(3-4C)\ln^2\left(-2i\omega\frac{\tau_i^2}{\tau}\right)+(12C-8C^2-7+2\zeta(3))\ln\left(-2i\omega\frac{\tau_i^2}{\tau}\right)+\text{const}\right]\right\}. \tag{42}$$

Here $\sigma(\varepsilon,\omega)$ is the contribution made to the conductivity by electrons with energy ε. To obtain the total conductivity $\sigma(\omega)$, it must be integrated with respect to ε with a weight $-\partial n/\partial\varepsilon$. According to (1), (12), and (20), the phonon time τ decreases like $\exp\{-|\varepsilon-\varepsilon_F|/T\}$ with increasing distance from the Fermi level. In the calculation of $\operatorname{Re}\sigma$, this factor cancels out the exponential contained in $\partial n/\partial\varepsilon$, and as a result it turns out that the actual integration region includes the entire interval between $\varepsilon_F-\omega_0/2$ and $\varepsilon_F+\omega_0/2$ (in contrast to the usual situation, cf. in particular, (25), when the integration region has a width on the order of T).

In connection with the establishment of the integration limits, it should be noted that the lengths $l(\varepsilon\pm\omega_0)$ play the role of the hot lengths and satisfy the condition $l(\varepsilon$

$\pm \omega_0) < l(\varepsilon)$ only at $\varepsilon_F - \omega_0/2 < \varepsilon < \varepsilon_F + \omega_0/2$. At $\varepsilon > \varepsilon_F + \omega_0/2$ the length $l(\varepsilon)$ is the hot one, and $l(\varepsilon - \omega_0)$ the cold one, and analogously for the region $\varepsilon < \varepsilon_F - \omega_0/2$. The electric conductivity is determined in all cases by the cold lengths. Their dependence on the energy is such that cancellation of the exponential factors takes place only in the interval $(\varepsilon_F - \omega_0/2,\ \varepsilon_F + \omega_0/2)$. Outside this interval, the integrand decreases like $\exp\{-2|\varepsilon - \varepsilon_F|/T\}$ and the relative contribution of the corresponding regions is small in terms of the parameter $T/\omega_0 \ll 1$. It should be noted that our derivation of (42) is not utterly invalid at the boundaries of the indicated interval, namely in regions of width of the order of T near $\varepsilon_F \pm \omega_0/2$, since the hot and cold lengths become comparable there.

The integration of (2) over the principal interval yields in the principal logarithmic approximation

$$\operatorname{Re}\sigma(\omega) = \int\limits_{\varepsilon_F-\omega_0/2}^{\varepsilon_F+\omega_0/2} d\varepsilon \left(-\frac{\partial n}{\partial \varepsilon}\right) \operatorname{Re}\sigma(\varepsilon, \omega) \approx \frac{\omega_0}{4T} \operatorname{Re}\sigma(\varepsilon_F, \omega).$$

This formula is valid at low frequencies, when $\ln[\tau(\varepsilon_F)/\omega\tau_i^2] \gg \omega_0/2T$.

All the foregoing complications disappear in the case of a nondegenerate (Boltzmann) electron gas. According to (1), $\tau(\varepsilon)$ is constant in this case; as a result, the actual integration region has a width $\sim T$, and for an explicit calculation of the integral of (42) it is necessary to know only the function $\tau_i(\varepsilon)$. The formula for the conductivity is in this case valid in the entire region $\tau/\omega\tau_i^2 \gg 1$.

The low-frequency dielectric constant $\varepsilon = 4\pi\sigma(\omega \to 0)/(-i\omega)$ is determined by the first term of (42). It retains exactly the same value[3,4] as in the absence of phonons; this fact has already been noted above. At the same time, the absorption (i.e., $\operatorname{Re}\sigma$) changes significantly in comparison with the value $\operatorname{Re}\sigma = 8\pi^{-1} e^2 l_i^-(\omega\tau_i)^2 \ln^2(\omega\tau_i)$ in the absence of phonons.[3] First, the highest-order term has the structure $\omega^2 \ln^3(\omega\tau_i^2/\tau)$, as against $\omega^2 \ln^2(\omega\tau_i)$ in the absence of phonons. Second, the entire expansion is in terms of $\omega\tau_i^2/\tau$ rather than $\omega\tau_i$. The characteristic time τ_i corresponds to ballistic motion of the electron over a length l_i^-, while τ_i^2/τ corresponds to diffuse motion over the same length, with a diffusion coefficient $D \sim v^2\tau$ determined exclusively by phonon scattering. Since, however, the localization is fully preserved, this diffusion has a pure quantum character.

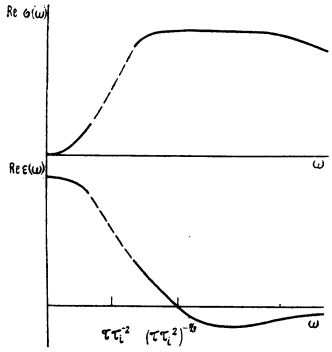

FIG. 8. Frequency dependence of the conductivity and of the dielectric constant.

In the region of high frequencies $\omega\tau_i^2/\tau \gg 1$, Eqs. (26) and (27) are solved directly by expanding in powers of β. In the lowest order in $\beta m \sim \beta|\tilde{\nu}|^{-1/2} \ll 1$ we have

$$R_m = e^{-\gamma m}\left\{1 + \beta\frac{m}{2}\left[m\frac{e^\gamma - 1}{e^\gamma + 1} + \frac{e^{2\gamma}+1}{(e^\gamma+1)^2}\right]\right\}, \tag{43}$$

$$Q_m{}^1 = e^{-\gamma m}\left\{Q_0{}^1 + \beta\left[m^2\frac{e^\gamma-1}{(e^\gamma+1)^2} + m\frac{3e^{2\gamma}-10e^\gamma-3}{3(e^\gamma+1)^3} - \frac{2e^\gamma(9e^{2\gamma}+3e^\gamma+4)}{3(e^\gamma+1)^4(e^\gamma-1)}\right]\right\}. \tag{44}$$

From this we obtain ultimately

$$\sigma(\omega) = \frac{2}{\pi} e^2 l \frac{1}{1 - i\omega\tau/2}\left[1 - \frac{\beta(1 + i\,\mathrm{sign}\,\omega)}{2|\omega\tau|^{1/2}} \frac{1 - i\omega\tau}{(1 - i\omega\tau/2)^{3/2}}\right]. \tag{45}$$

It follows from (45) that the impurities decrease $\operatorname{Re}\sigma$ and $\operatorname{Im}\sigma$ simultaneously; the expansion is now in terms of $|\nu\alpha|^{-1} \ll 1$. The imaginary part of $\sigma(\omega)$, meaning also the dielectric constant $\operatorname{Re}\varepsilon(\omega)$, goes through zero and reverses sign even in the region where (45) is valid. Thus, the dielectric constant acquires the characteristic dielectric sign already in that frequency region where the conductivity is still well described by the Drude formula. The frequency dependences of $\operatorname{Re}\sigma(\omega)$ and $\operatorname{Re}\varepsilon(\omega)$ are shown schematically in Fig. 8. The expansions (45) and (42) are continuously joined together at $|\nu\alpha| \sim \omega\tau_i^2/\tau \sim 1$.

It follows from the results that in the case of strong interaction with phonons the form of the correlators is determined by different mechanisms at different times. At $t < \tau$ the correlators correspond to free particles, at $\tau < t < \tau_i^2/\tau$ the decisive role is played by scattering by phonons, which leads to ordinary diffusion, while at $t > \tau_i^2/\tau$ the mechanisms of scattering by phonons and by the impurities act jointly and in the limit as $t \to \infty$ the asymptotic values of the correlators are determined only by impurity scattering.

b. Weak scattering by phonons ($\tau \gg \tau_i$)

In this case the accuracy of the differential equations of the type (29) for R and Q is insufficient for the determination of the low-frequency $\operatorname{Re}\sigma$. Therefore, in analogy with[3], we use the exact solution of (26)

$$R_m = \int\limits_0^\infty ds \left(\frac{s}{s+1}\right)^{m+\alpha} e^{i\nu s} \Big/ \int\limits_0^\infty ds \left(\frac{s}{s+1}\right)^{\alpha} e^{i\nu s}. \tag{46}$$

Substituting (46) in (15), we obtain

$$X^1(\varepsilon, \omega) = v(\varepsilon) l_i^-(\varepsilon) \int\limits_0^\infty ds \left(\frac{s}{s+1}\right)^{\alpha} e^{i\nu s} B(s) \Big/ \int\limits_0^\infty ds \left(\frac{s}{s+1}\right)^{\alpha} e^{i\nu s}, \tag{47}$$

where

$$B(s) = \frac{1}{s+1} \sum_{m=0}^{\infty} Q_m{}^1 \left(\frac{s}{s+1}\right)^m \tag{48}$$

satisfies the equation

$$\frac{d}{ds}\left(s(s+1)\frac{dB}{ds}\right) + \alpha\frac{dB}{ds} + i\nu\left[s\frac{d}{ds}((s+1)B) + \frac{1}{2}B\right] + \frac{P^1(s)}{s+1} = 0. \tag{49}$$

Here

$$P^1(s)=\int_0^\infty dt\,\frac{s+1}{t+s+1}\left(\frac{t}{t+1}\right)^\alpha e^{i\nu t}\Big/\int_0^\infty dt\left(\frac{t}{t+1}\right)^\alpha e^{i\nu t}. \tag{50}$$

Equation (49) must be solved accurate to terms linear in $\alpha \ll 1$; this is carried out in the same manner as the calculation of Re σ in Berezinskiĭ's paper.[3] It is necessary next to calculate in the same approximation the conductivity in accordance with (47).

We present here only the highest-order logarithmic term in the conductivity increment $\delta\sigma$ due to the interaction with the phonons

$$\delta\sigma(\varepsilon,\omega)=-\frac{16}{3\pi}e^2 l_i\frac{\tau_i}{\tau}(\omega\tau_i)^2\ln^3(-2i\omega\tau_i)+\ldots \tag{51}$$

It is seen that the highest-order logarithmic term in (51) coincides with the analogous term in (42).

5. CONCLUSION

We have investigated above the influence of low-frequency dispersionless phonons on the localization and electric conductivity. The obtained solutions are valid when the following criteria are satisfied:

$$\omega_0 \gg T,\quad T\tau_i \gg 1,\quad T\tau \gg 1,\quad l_h \ll l_i,\quad l_h \gg \lambda\varepsilon_F/\omega_0,\ \lambda.$$

The principal results consist in the following:

1. In a perfect crystal, dispersionless phonons ensure conductivity described by the Drude formula.

2. In an imperfect crystal (impurities, structural disorder), dispersionless phonons do not disturb the localization, whose form factor of and length l_i^- are strictly preserved, as is also the static dielectric constant.

The very fact that the localization is preserved can be easily interpreted from qualitative considerations, since it is impossible to ensure energy conservation for dispersionless phonons in hops between localized states. It is much more difficult to interpret the constancy of the length l_i^-.

Phonons with small dispersion Δ should upset the localization relatively little, to the extent that the parameter $\Delta\tau_i \ll 1$ is small. Since the energy change in the two-phonon process is of the order of Δ, we can assume, in analogy with the $\sigma \sim \omega^2$ law, that the electric conductivity is proportional to $(\Delta\tau_i)^2$.

We shall discuss in this connection certain results of an analysis of the conductivity of TCNQ salts with asymmetrical cations Qn and Adz.[5] Since the values obtained for the energy difference of the localized states of order τ_i^{-1} were ~350 °K, which is large in comparison with the Debye frequency $\omega_D \sim 70$ K, it was assumed that the delocalization is caused in the main by the interaction with the fully symmetrical intramolecular phonons. The constant g of the intramolecular electron-phonon interaction at the site n can be naturally introduced in the form $H_{\text{int}}=(g\omega_0\varepsilon_F)^{1/2}a_n+a_n(b_n^+ + b_n)$. From the value of τ_{ph} obtained by reducing the experimental data[5] it follows that $g \sim 0.03$–0.04. This is unexpectedly low; we note for comparison that in molecular crystals of the aromatic series (such as anthracene) the intramolecular exciton-phonon interaction is usually noticeably larger and reaches $g \sim 1$ for a number of vibrations. It can be assumed that the obrained small effective value of g is connected with the smallness of the parameter $\Delta\tau_i$. It is also possible that an important role is played in delocalization by two-phonon processes with participation of fully symmetrical intramolecular and acoustic phonons. The former ensures a sufficiently large average frequency, and the latter makes it possible to reconcile the initial and final energies in the hops.

The Hamiltonian of the interaction with the non-fully-symmetrical (NFS) intramolecular phonons $\frac{1}{2}\Delta_\omega a_n^+ a_n(b_n^+ + b_n)^2$ contains a term $\Delta_\omega a_n^+ a_n b_n^+ b_n$ corresponding to elastic scattering of electrons by phonons; in analogy with the impurities, it ensures localization. The terms with b_n^{+2} and b_n^2, which describe the inelastic processes, are relatively small at low temperature $N(\omega_0) \ll 1$. According to the proof presented above, they do not disturb the localization produced by the elastic scattering component, provided that the dispersion of the phonons can be neglected. Therefore the static conductivity of such a model system is equal to zero even in the absence of impurities.

This result is valid in the lowest order in the electron-phonon coupling parameter $g' = \Delta_\omega/\varepsilon_F$. In the next higher orders it is necessary to take into account the multiple elastic scattering of the electrons by the NFS phonons, which are themselves moved over the crystal by successive inelastic processes (of the type b_n^{+2} and b_n^2). Since the displacement of the phonons means a change of their potential with time, these processes should lead to a finite electric conductivity (which, however, should be small to the extent that g' is small).

For impurities, in contrast to NFS phonons, there are no grounds for assuming that allowance for the deviation from the Born approximation disturbs the localization; we therefore have, as before, $\sigma(\omega = 0) = 0$. However, the low-frequency asymptotic value of $\sigma(\omega)$ may change. Indeed, when the electron is localized in a bounded region it inevitably experiences successive collisions with each impurity at time intervals $\sim \tau_i c l_i$, where c is the impurity concentration. Such repeated collisions do not occur in the Born approximation. One can therefore expect the function $\sigma(\omega)$ to change its form at frequencies $\omega \lessdot (c l_i \tau_i)^{-1}$.

We have shown in the preceding paper[4] that phonons upset the electron localization if two criteria are simultaneously satisfied: $\bar{\omega}\tau_i$, $\bar{\omega}\tau_{\text{ph}} \gg 1$ and $\Delta\tau_i$, $\Delta\tau_{\text{ph}} \gg 1$. By rigorously solving the equations we have demonstrated above the extent to which the second criterion is essential: strong violation of this criterion leads to a radical change in the results. We shall discuss here qualitatively the consequences ensuing from violation of the first criterion. We have already noted[5] that at $\bar{\omega}\tau_i \ll 1$ an adiabatic situation arises, namely, the potential produced by the phonons can be regarded as quasistatic. This random potential, when added up to the impurity potential, enhances the localization. As a result, the mean free path, and consequently also the localization length, decrease with increasing T. An adiabatic regime can arise also in a perfect crystal, but only at high temperatures and in the case of a sufficiently strong electron-phonon coupling. Indeed, if the frequency of the electron in the instantaneous field of the phonons τ_{ph}^{-1} greatly exceeds $\overline{\omega}$ (i.e., $\overline{\omega}\tau_{\text{ph}} \ll 1$ or $\overline{\omega} \ll gT$), then the adiabaticity criterion is satisfied. A short-term localization is then produced wherein the electron remains in the region $\sim v_F\tau_{\text{ph}}$ during the time that the phonon potential is stationary. For phonons with strong dispersion (i.e., $\Delta \sim \overline{\omega}$), the potential is destroyed within a time $\sim \overline{\omega}^{-1}$. This enables us to estimate the diffusion coefficient[1)] $D \sim (v_F\tau_{\text{ph}})^2\overline{\omega} \sim D_{Dr}\overline{\omega}\tau_{\text{ph}}$. Here $D_{Dr} \sim v_F^2\tau_{\text{ph}}$ is the ordinary diffusion coefficient. We see that in the adiabatic region $D \ll D_{Dr}$, with $D \sim T^{-2}$; at $\overline{\omega}\tau_{\text{ph}} \gg 1$, the ordinary diffusion is preserved with $D \sim D_{Dr}$. For dispersionless phonons, the potential is exactly reconstructed within a time $2\pi/\omega_0$. Therefore the diffusion can proceed only as a result of the non-adiabaticity, and it is then natural to expect an even lower value of D.

The authors are grateful to L. P. Gor'kov for a very useful discussion.

APPENDIX

The summation of the self-energy parts of Fig. 5 leads to the following expression for the Green's function:

$$G^{+}(\varepsilon|x-x') = -\frac{i}{v(\varepsilon)}\exp\left\{ip(\varepsilon)|x-x'| + \frac{1}{v(\varepsilon)}|x-x'|(\Sigma^{+}+\Sigma^{-})\right\}, \tag{A.1}$$

where

$$\Sigma^{+} = \int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\int_{-\infty}^{\infty} dx\, e^{-ip(\varepsilon)x}G_0^{+}(\varepsilon-\Omega|x)\,[\,(1-n(\varepsilon-\Omega))D_{21}(\Omega,x) + n(\varepsilon-\Omega)D_{12}(\Omega,x) + n(\varepsilon-\Omega)(D_{21}(\Omega,x)-D_{11}(\Omega,x))\,] \tag{A.2}$$

and

$$\Sigma^{-} = \int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\int_{-\infty}^{\infty} dx\, e^{-ip(\varepsilon)x}G_0^{-}(\varepsilon-\Omega|x)\,n(\varepsilon-\Omega)\,(D_{11}(\Omega,x)-D_{12}(\Omega,x)) \tag{A.3}$$

correspond respectively to the diagrams of Figs. 5a and 5b. The entire real part of $\Sigma^{+}+\Sigma^{-}$, which is equal to $-\frac{1}{2}v(1/l^{+}+1/l^{-})$ and yields the damping of G^{+}, is the result of the first two terms in (A.2). The term Σ^{-} and the last term in Σ^{+} are different from zero because of the residues at the poles $n(\varepsilon-\Omega)$. It can be shown, however, that their sum is always imaginary. It therefore contributes only to the renormalization of the spectrum and drops out completely from the correlations functions (just as the imaginary part of the first two terms in (A.2)—see[4]). This sum was left out in[4]; it follows from the foregoing that this has no influence whatever on the results obtained there.

On going to more complicated diagrams of the type of Fig. 4b, the expressions analogous to (A.3) and to the last term enter in (A.2) separately, but are different from zero, as before, only as a result of the residues at the poles $n(\varepsilon-\Omega)$. At $T \lesssim \varepsilon_F$ the contribution from these residues leads to a strong damping of the Green's functions, of the type $e^{-T|x|/v}$, where x is the length of the "tongues" and therefore the corresponding terms can be left out. Accordingly, it is not necessary to draw out the cold lines from the diagrams of the type of Fig. 5b (b′).

[1)] L. P. Gor'kov has kindly informed us that a similar estimate for D was obtained in an unpublished paper of J. Hertz and M. H. Cohen. We wish to emphasize the decisive significance of the adiabatic criterion for the validity of this result.

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