Bremsstrahlung spectra of 1-GeV electrons and positrons in crystals of silicon, germanium, and niobium

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Bremsstrahlung spectra of 1-GeV electrons and positrons have been measured. It is found that for crystals of silicon, niobium, and germanium, when the condition of axial channeling is satisfied, there is a substantial difference in the measured bremsstrahlung spectra of electrons and positrons. The experimental values for positrons are less than those calculated in the Born approximation, while those for electrons are greater than the Born values. If the condition of planar channeling is satisfied, the difference between theory and experiment for silicon and germanium crystals is observed only for positrons; for electrons agreement of theory and experiment is observed over the entire energy region except for the region of the coherent peak for germanium and niobium crystals.

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It has been shown theoretically by Ter-Mikaelyan[1] and Überall[2] and later experimentally by several authors[3–5] that the bremsstrahlung spectrum of electrons for a crystal differs substantially from the spectrum for an amorphous target. The periodicity of the crystal lattice leads to appearance in the bremsstrahlung spectrum of peaks whose intensity and location depend on the initial-particle energy and the crystal orientation relative to the particle-beam direction.

The bremsstrahlung spectra of electrons in a crystal measured experimentally in Refs. 3–5 are in good agreement with the theory of coherent bremsstrahlung in the Born approximation.[6] This theory does not depend on the sign of the charge of the particle, and consequently should also describe the bremsstrahlung of positrons in a crystal.

However, in a study of the interaction of 1-GeV positrons with a silicon crystal[7] we observed a channeling effect which appeared as a decrease in the multiple-scattering angle and the intensity of bremsstrahlung. The channeling effect has also been observed[8–10] for positrons and electrons of low and intermediate energies

With increasing energy of the electrons and positrons their motion in a periodic potential can be described by means of the theory of channeling for heavy charged particles, based on classical mechanics.[11] If the direction of the particle beam coincides with the direction of a crystallographic axis, it follows from classical channeling theory that the maximal density of positrons is observed near the channel axis and that of electrons near the string of atoms. A similar redistribution of the initial flux of particles in motion through a crystal is observed when their initial direction is parallel to one of the crystallographic planes.

The radiation of high-energy photons $\omega \sim E_0$, where E_0 is the initial energy of the particle, is a quantum effect. The spatial redistribution of the electron and positron flux means from the quantum-mechanical point of view that the wave function of these particles in the crystal differs from a plane wave: Near the lattice sites the positron wave function dies out exponentially and the electron wave function has a maximum.[12] In the present work we have investigated the effect of axial and planar channeling on the bremsstrahlung spectra of electrons and positrons in crystals of silicon, germanium, and niobium.

The experimental studies were carried out in the 2-GeV linear accelerator at our institute. An electron or positron beam was accelerated to an energy $E_0 = 1$ GeV and directed onto a crystalline target mounted in a goniometer. After radiation of photons of energy ω the secondary electrons or positrons with energy $E_0 - \omega$ were separated by a spectrometer and recorded by an ionization chamber. The initial divergence of the electron or positron beam did not exceed 2×10^{-4} rad. The bremsstrahlung spectra were measured in the photon-energy range $0.2 < X < 0.8$, where X is the photon energy expressed in units of the initial energy of the electron or positron, $X = \omega/E_0$. The experiment was performed with crystals of silicon (240 μ), germanium (165 μ), and niobium (150 μ). The technique and the experimental arrangement have been described in more detail in a previous article.[13]

Bremsstrahlung spectra of electrons and positrons were measured for the cases in which a) the axial-channeling condition was satisfied (the particle direction coincides with the direction of the [110] crystallographic axis for silicon and of the [111] axis for germanium and niobium; b) the planar-channeling condition was satisfied (orientation angles $\theta = 1.6\times10^{-2}$ rad, $\Psi = 1.0\times10^{-2}$ rad, $\Phi = 1.75\times10^{-2}$ rad, where θ is the angle between the direction of the particle beam and the [110] crystallographic axis in the (001) plane of the silicon crystal, Ψ and Φ are the angles between the particle-beam direction and the [111] axis in the $(\bar{1}10)$ plane for germanium and niobium crystals, respectively).

The scattering of the primary and secondary electrons or positrons in the crystal and the bremsstrahlung process lead to an increase in the exit angles of the secondary electrons or positrons. Therefore in comparison with a theoretical spectrum integrated over all emission angles of photons and secondary electrons (positrons),

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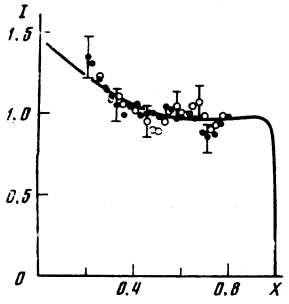

FIG. 1. Bremsstrahlung spectra of electrons and positrons in an amorphous aluminum target: ●—electrons, ○—positrons, curve—theory.

the experimental results must be corrected for the finite angular acceptance of the spectrometer. Corrections were calculated by the Monte Carlo method with allowance for the beam size and its initial divergence. To determine the total error due both to the measurement of the intensity of the secondary electrons (positrons) and to the calculation of the corrections, we carried out control measurements of the bremsstrahlung spectra of electrons and positrons for an amorphous aluminum target of thickness 500 μ. The results of these measurements (with inclusion of the corrections) are shown in Fig. 1. The ordinate is the normalized bremsstrahlung intensity and the abscissa is the photon energy in units of the initial electron or positron energy. Here and below the normalizations were made to the intensity of photons with energy $X = 0.5$ in the bremsstrahlung of electrons (positrons) in an amorphous aluminum target 500 μ thick. From comparison of the measured electron and positron bremsstrahlung spectra with a Born approximation calculation it follows that the total error does not exceed 10%.

Figures 2–4 show the bremsstrahlung spectra of electrons and positrons when the axial-channeling condition is satisfied for crystals of silicon, germanium, and niobium. For comparison we have shown in the same figures the theoretical bremsstrahlung spectra in the Born approximation. The calculations utilized a screened Dirac-Slater potential.[14] Multiple scattering of the initial particles was taken into account by analogy with Ref. 6. The mean square multiple-scattering angles of the electrons and positrons for various orientations of the crystal were determined experimentally and are given in Table I. The value of the mean square multiple-scattering angle depends on the crystal orientation and on the sign of the charge of the initial particle. Therefore in calculations carried out for different crystal orientations and initial particles it is necessary to take into account the different values of the mean square multiple-scattering angles. The theoretical values of the mean square multiple-scattering angles were obtained from the theory of Molière[15] for an amorphous target and consequently they do not depend on the target orientation. The dashed curve in Figs. 2–4 shows the change in the bremsstrahlung spectrum as the result of multiple scattering of electrons in the crystal.

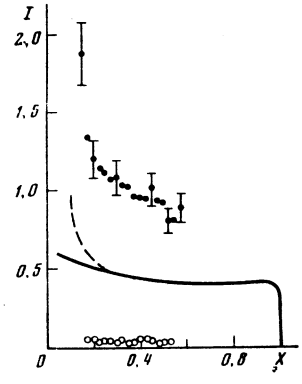

FIG. 2. Bremsstrahlung spectra of electrons and positrons in a silicon crystal, $\theta = 0$, ●—electrons, ○—positrons, solid curve, theory; the dashed curve is the theory with inclusion of multiple scattering of electrons.

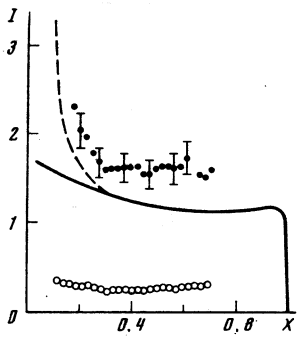

FIG. 3. Bremsstrahlung spectra of electrons and positrons in a germanium crystal, $\Psi = 0$. (The designations are the same as in Fig. 2.)

From comparison of the experimental results with the theory it follows that axial channeling of the particles has a substantial effect on the bremsstrahlung intensity both for electrons and positrons. In the measured range of photon energies the bremsstrahlung intensity of electrons in a silicon crystal is about twice that given by the theory, while for positrons a decrease in bremsstrahlung intensity by about a factor of ten is observed. For germanium the difference is somewhat smaller, and only for niobium do we observe relatively good agreement with the theory for electrons; for positrons the difference between theory and experiment is preserved.

For crystal orientation angles $\theta \sim 10^{-2}$ relative to the particle-beam direction the bremsstrahlung spectrum

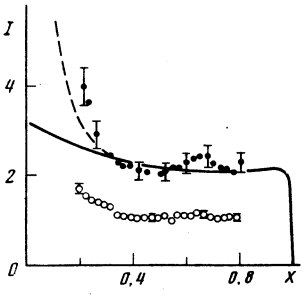

FIG. 4. Bremsstrahlung spectra of electrons and positrons in a niobium crystal, $\Phi = 0$. (The designations are the same as in Fig. 2.)

TABLE I. Mean square multiple-scattering angles of electrons and positrons for various crystal orientations.

Crystal	Thickness, μ	Orientation angle, 10^{-2} rad	Mean square multiple-scattering angle of electrons, 10^{-3} rad	Mean square multiple-scattering angle of positrons, 10^{-3} rad	Theoretical mean square multiple-scattering angle, 10^{-3} rad
Silicon	240	0	1.2	0.14	0.67
		1.6	0.65	0.48	
Germanium	165	0	1.45	0.6	1.2
		1.0	1.35	1.0	
Niobium	150	0	1.7	1.2	1.7
		1.75	1.5	1.5	

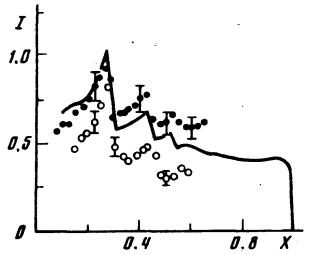

FIG. 5. Bremsstrahlung spectra of electrons and positrons in a silicon crystal, $\theta = 1.6 \times 10^{-2}$ rad, ●—electrons, ○—positrons, curve—theory with inclusion of electron multiple scattering.

is appreciably influenced by coherent effects, as a result of which the intensity of photons of a definite energy is increased. Figures 5–7 show the measured spectra of coherent bremsstrahlung of electrons and positrons in crystals of silicon, germanium, and niobium. The theoretical spectra of coherent radiation in the Born approximation with inclusion of scattering of the initial particles are shown by the solid curves in the figures. The calculations utilized the mean square multiple-scattering angles for electrons. For the case when the planar-channeling condition is satisfied a difference between theory and experiment over the entire energy region is observed only for positrons in crystals of silicon and germanium. For electrons (in those same crystals) satisfactory agreement between theory and experiment is observed except for the region of the first coherent peak for the germanium crystal. For niobium the experimental results are in good agreement with theory, both for electrons and positrons (except for the region of the coherent peak).

As follows from the experimental data, the effect of planar channeling of positrons on the bremsstrahlung spectra is less noticeable than the effect of axial channeling. This is due to the fact that the critical angle of planar channeling is less than the critical angle for axial channeling[16] by a factor of about $2Z^{1/6}$, where Z is the atomic number of the target. Therefore, when we take into account the initial beam divergence, fewer particles are captured into the planar-channeling regime than in the case of axial channeling.

Since the critical angle of planar channeling of 1-GeV positrons is $\Psi_{cr} \sim 10^{-4}$ rad, the channeling process is substantially affected by imperfections in the crystal structure which do not affect the coherent bremsstrahlung. Therefore, the absence of planar channeling of positrons in the niobium crystal can be explained by the presence in the crystal of regions disoriented relative to each other by angles $\sim 10^{-4}$ rad. From the agreement of the experimental results (for crystals of silicon, germanium, and niobium) with the theory for electrons it follows that no spatial redistribution of the electron flux occurs as the result of their motion parallel to a plane.

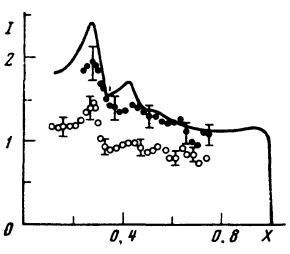

FIG. 6. Bremsstrahlung spectra of electrons and positrons in germanium crystal, $\Psi = 1.0 \times 10^{-2}$ rad. (The designations are the same as in Fig. 5).

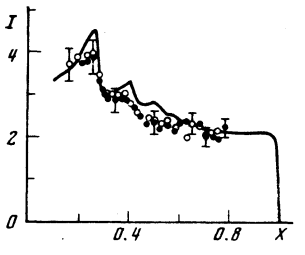

FIG. 7. Bremsstrahlung spectra of electrons and positrons in a crystal of niobium, $\Phi = 1.75 \times 10^{-2}$ rad (the designations are the same as in Fig. 5).

Planar channeling leads to a decrease in the bremsstrahlung of positrons in comparison with that of electrons, but this decrease is different for the coherent and incoherent parts of the radiation; the incoherent part of the radiation decreases more than the coherent part. This circumstance may lead to an increase in the polarization of the coherent bremsstrahlung of positrons in comparison with that of electrons.

A certain discrepancy between theory and experiment for electrons in the vicinity of the coherent peak can be explained by the inapplicability of the Born approximation in this region for the germanium and niobium crystals. The applicability of the Born approximation for a single atom is determined by the condition

$$Ze^2/\hbar v \ll 1, \tag{1}$$

where $\hbar$ is Planck's constant, v is the particle velocity, and e is the electronic charge. For a crystal, where the coherent effects are maximal (the interaction occurs not with one but with several atoms), the condition of applicability of the Born approximation is

$$NZe^2/\hbar v \ll 1, \tag{2}$$

where N can be defined as $N = R/\theta d$, R is the screening radius, θ is the crystal-orientation angle, and d is the distance between atoms along the string.[17,18] The values of N for the germanium and niobium crystals are respectively 3.5 and 3. When we take it into account that as the result of thermal vibrations not all N atoms interact coherently, the real value is $N_{real} < N$, but nevertheless can be sufficient for Eq. (2) to be violated. For the germanium crystal the difference between theory and experiment in the vicinity of the coherent peak is greater than for the niobium crystal. This may be due to the fact that the quantity $N_{real}Z$ is greater for germanium than for niobium, since the amplitude of thermal vibrations for niobium is 1.6 times that for germanium.

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Spectrum of hydrogen plasma at the series limit

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A theory of the spectral intensity distribution near the series limit is proposed. It is based on the inclusion of subbarrier ionization of atoms from excited Stark sublevels in the statistical microfield of the plasma. It is found that spectral lines disappear when the ionization probability exceeds the radiative transition probability by two or three orders of magnitude. The transmission of the potential barrier is then still much less than unity and this means that perturbation theory can be used to calculate the line emission, and the sum of the oscillator strengths remains constant during the transformation of lines into the continuum. The latter results can be used to calculate the photocapture spectrum under the disappearing lines. The experimental results are compared with the predictions of simplified models of the spectrum near the series limit.

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Experiment shows (see[1–4] and elsewhere) that the spectral lines of hydrogen (and of other gases) become increasingly broad, overlap, and gradually merge into the continuum as the series limit is approached. The transition region, i.e., the region between the line and the continuous spectra, expands with increasing plasma density, and eventually covers the entire series. There is no satisfactory theory of this type of spectrum at present.[5–7] On the other hand, such a theory would be useful, firstly, because this spectral region may play an important role in radiative heat transfer and thus determine the conditions for the production and maintenance of plasma. Secondly, plasma emission, including emission at the series limit, is an important source of information in contactless diagnostics. Another important application is the development of a primary intensity standard based on the electric arc in hydrogen.[8] Finally, comparison of theoretical predictions with precision measurements would yield further information about the interaction between plasma particles.

The field ionization of excited atoms in the electric field of the charged plasma particles must be taken into account when the radiation at the series limit is calculated.[9] The external field (the plasma microfield) is then comparable with the internal atomic field experienced by an optical electron. This throws some doubt[7] on the validity of standard perturbation theory in this case. It is shown below that such reservations are unjustified, at least for the spectral lines. Insofar as the continuum is concerned, on the other hand, it is possible to construct a satisfactory model based on the constancy of the sum of oscillator strengths.

In this paper, which is a continuation of[10,11], a method is reported for taking into account the effect of the statistical microfield of plasma on spectral-line profiles,

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