Loss of energy by an electron in a medium with periodic inhomogeneities

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A method is developed for calculation of the spectral density of energy loss by an ultrarelativistic particle in a periodically nonuniform medium, and calculations are carried out. It is shown that a periodic change in the absorbing properties of a medium substantially affects the spectral distribution of the energy loss.

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INTRODUCTION

Radiation arising in passage of fast charged particles through artificially created one-dimensional periodic structures presents interest as a result of the effect first noted by Ter-Mikaelyan.[1] This effect consists of the possibility of radiation of photons in such media by a uniformly moving charge in the region of the spectrum where Čerenkov radiation is clearly impossible. The radiation, called by Ter-Mikaelyan resonance radiation, has all the properties of Čerenkov radiation: threshold nature and directivity.

Radiation of rather energetic photons by ultrarelativistic electrons in a periodically inhomogeneous medium occurs essentially at small angles. Accordingly, it becomes important to take into account multiple scattering of the electrons.[2] Multiple scattering is the cause of bremsstrahlung by electrons. On the other hand, it leads to a change in the spectrum of resonance radiation. As a result of interference of the probability amplitudes, the multiple scattering of electrons can be treated by various methods, depending on what is convenient.[2]

Other important factors which require consideration in calculation of the radiation of electrons in periodic structures, in addition to multiple scattering, are the absorption and scattering of virtual photons in the radiation process, and also the energy loss by the electron, which leads to a change in the mean square scattering angle. Finally, as will be shown below, the resonance radiation effect can be produced not only by a change of the real part of the permittivity,[1] but also by a periodic change in the absorbing properties of the medium (the imaginary part of the permittivity).

In the work presented, in calculation of the energy loss by an electron in a periodically inhomogeneous one-dimensional structure we used the result obtained for homogeneous media[3,4] and an analogy, developed below, between radiation in a periodically inhomogeneous medium and in a homogeneous medium in the region of anomalous dispersion.

1. DERIVATION OF THE GENERAL EXPRESSION FOR ENERGY LOSS BY AN ELECTRON IN AN INHOMOGENEOUS MEDIUM

The energy loss by a relativistic electron in an inhomogeneous medium can be calculated from a general expression. This expression relates the probability W of radiation by an electron in an external field during the entire time of an interaction, summed over the final states of the electron with a definite energy, to the electron forward scattering amplitude T_{qq} calculated in second-order perturbation theory in the interaction of the electron with the radiation field. The result is as follows (see for example the book by Baĭer *et al.*,[5] page 98):

$$W=2\,\mathrm{Im}\,T_{qq}, \tag{1}$$

where

$$T_{qq}=ie^2\int\overline{\Psi}_q(x)\gamma^\mu S_F^{(e)}(x,x')\gamma^\nu D_{\mu\nu}^{(0)}(x-x')\Psi_q(x')\,d^4x\,d^4x',$$

$\Psi_q(x)=\psi_q(\mathbf{r})e^{-i\varepsilon_q t}$ is the wave function of the electron in the field corresponding to a state with energy ε_q and quantum numbers q, $S_F^{(e)}(x,x')$ is the Green's function of the electron in the external field, and $D_{\mu\nu}^{(0)}(x-x')$ is the photon Green's function.1)

The external field in the case of interest to us is the total field of the atoms of the material. In the case of random location of the atoms it is necessary in addition to average the probability W over the coordinates of the atoms of the medium, which we will designate below by angle brackets. It is easy also to take into account the interaction of the photons with the medium in the radiation process. For this it is sufficient in Eq. (1) to replace the Green's function of the free magnetic field $D_{\mu\nu}^{(0)}(x-x')$ by the photon Green's function in matter. The latter, taking into account the macroscopic lengths important in the problem (coherence lengths), can be taken in the long-wavelength approximation.[6] It is necessary here to have in mind that, in taking into account the interaction of the radiation photons with the medium, their removal from the intrinsic field of the electron can occur now not only as the result of scattering of the electron by the atoms of the medium (normal bremsstrahlung), but also without scattering of the electron. In the latter case the radiated photon interacts with the medium (Čerenkov radiation, direct ionization of atoms).

We now use Furry's representation for the electron Green's function

$$S_F^{(e)}(x,x')=-i\sum_{n(\varepsilon_n>0)}\Psi_n(x)\overline{\Psi}_n(x')\theta(t-t'),$$

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where $\theta(t-t')$ is the Heaviside unitary function. For an inhomogeneous stationary medium[2)] the photon Green's function can be represented in the form

$$D_{\mu\nu}(x,x')=D_{\mu\nu}(\mathbf{r},\mathbf{r}',\tau)=(2\pi)^{-7}\int D_{\mu\nu}(\mathbf{k},\mathbf{l},\omega)e^{i(\omega\tau+\mathbf{k}\rho+\mathbf{l}\mathbf{r})}d^3k\,d^3l\,d\omega,$$

where $\tau=t'-t$, $\rho=\mathbf{r}'-\mathbf{r}$. Taking into account that $D_{\mu\nu}(\mathbf{k},\mathbf{l},\omega)$ is an even function of ω, we obtain for the probability of radiation of a photon with energy ω by an electron per unit time in an inhomogeneous medium the following expression:

$$\frac{d^2W}{d\omega\,dt}=\frac{e^2}{2^8\pi^7}\operatorname{Im}\int\int_0^\infty L^{\mu\nu}(\tau,t,\mathbf{k},\mathbf{l})D_{\mu\nu}(\mathbf{k},\mathbf{l},\omega)\,d\tau\,d^3k\,d^3l, \tag{2}$$

where

$$L^{\mu\nu}(\tau,t,\mathbf{k},\mathbf{l})=e^{-i\omega\tau}\sum_n (j^\mu(x)e^{-i(\mathbf{k}-\mathbf{l})\mathbf{r}})_{qn}(j^\nu(x')e^{i\mathbf{k}\mathbf{r}'})_{nq},$$
$$(j^\nu(x')e^{i\mathbf{k}\mathbf{r}'})_{nq}=\int\bar\Psi_n(x')\gamma^\nu\Psi_q(x')e^{i\mathbf{k}\mathbf{r}'}d^3r'. \tag{2'}$$

Within the accuracy of the notation, Eq. (2) coincides with the analogous result of Yakimets,[7] obtained by another method. It should be noted that in the general case, when the removal of the photon from the intrinsic field of the electron can occur by absorption of the photon, Eq. (2) is, generally speaking, the differential probability of energy loss at a frequency ω and not necessarily involving real radiation.

In the classical limit in which the quantization of the electron motion and the recoil on radiation are neglected, the matrix elements in Eq. (2) can be replaced by the Fourier components of the corresponding quantities, and the tensor $L^{\mu\nu}$ takes the form

$$L^{\mu\nu}_{(cl)}=v^\mu(t)v^\nu(t+\tau)e^{-i\omega\tau}\exp[-i(\mathbf{k}-\mathbf{l})\mathbf{r}(t)+i\mathbf{k}\mathbf{r}(t+\tau)], \tag{2''}$$

where $v^\mu(t)=(\mathbf{v}(t),1)$; $\mathbf{r}(t)$, $\mathbf{r}(t+\tau)$ are the locations of the electron in its trajectory at the moments of time t and t'; $\mathbf{v}(t)$ and $\mathbf{v}(t+\tau)$ are the velocities of the electron at the corresponding moments of time. The tensor $L^{\mu\nu}$ is expressed in terms of the trajectory and with inclusion of the quantized recoil in radiation by an ultrarelativistic electron.[5] Averaging over the coordinates of the scattering atoms reduces in the classical limit to averaging over all possible trajectories of the electron.

If we neglect the optical inhomogeneity of the medium, then the Green's function $D_{\mu\nu}(\mathbf{k},\mathbf{l},\omega)$ takes the form

$$D_{\mu\nu}(\mathbf{k},\mathbf{l},\omega)=(2\pi)^3D^{(h)}_{\mu\nu}(\mathbf{k},\omega)\delta(\mathbf{l}),$$

where $\delta(\mathbf{l})$ is the Dirac delta function. In this case the energy loss probability (2) is written in the form

$$\frac{d^2W^{(h)}}{d\omega\,dt}=\frac{e^2}{8\pi^4}\operatorname{Re}\int\int_0^\infty L^{\mu\nu}_{(h)}(\tau,t,\mathbf{k})\operatorname{Im}D^{(h)}_{\mu\nu}(\mathbf{k},\omega)\,d\tau\,d^3k,$$
$$L^{\mu\nu}_{(h)}(\tau,t,\mathbf{k})\equiv L^{\mu\nu}(\tau,t,\mathbf{k},0). \tag{3}$$

In a uniform medium, according to Eqs. (3) and (2''), the differential probability of energy loss (3) does not depend on time, strictly speaking, only when the electron is moving in a straight line: $\mathbf{v}(t+\tau)=\mathbf{v}(t)\equiv\mathbf{v}$. For ultrarelativistic particles the deviation from straight-line motion is small. Therefore after averaging over the trajectories the tensor $\langle L^{\mu\nu}_{(h)}\rangle$ can be assumed not to depend explicitly on the time and, consequently, to introduce a time-independent differential probability[3)] $d^2\langle W^{(h)}\rangle/d\omega\,dt$.

In an optically inhomogeneous medium, as can easily be seen from Eqs. (2) and (2''), the differential probability (2) depends, generally speaking, on time even in the case of straight-line motion of the electron. There are, however, at least two types of optically inhomogeneous media in which this dependence disappears. Thus, in a medium with random inhomogeneities, discussed by Kalashnikov and Ryazanov,[9] the photon Green's function $D_{\mu\nu}(\mathbf{k},\mathbf{l},\omega)$ averaged over the inhomogeneities contains a dependence on $\mathbf{l}$ in the form $\delta(\mathbf{l})$. Another case, which is discussed in detail in the present work, corresponds to a medium with periodically located inhomogeneities.

Suppose that the medium is optically inhomogeneous with a period a along a direction $\mathbf{n}$. Using the translational symmetry of the photon Green's function $D_{\mu\nu}(\mathbf{r},\mathbf{r}',\tau)=D_{\mu\nu}(\mathbf{r}+\mathbf{n}a,\mathbf{r}'+\mathbf{n}a,\tau)$, it can be represented in the form

$$D_{\mu\nu}(\mathbf{r},\mathbf{r}',\tau)=\sum_{m=-\infty}^{\infty}D^{(m)}_{\mu\nu}(\mathbf{r}-\mathbf{r}',\tau)e^{i\mathbf{K}\mathbf{r}m}, \tag{4}$$

where $\mathbf{K}$ is the base vector of the reciprocal lattice ($e^{i\mathbf{K}\mathbf{n}a}=1$). In particular, the probability of energy loss in a medium with periodic inhomogeneities by a rectilinearly moving electron is obtained by substitution in Eq. (2) of expression (2''), and the Fourier component of the Green's function (4) and has the form

$$\frac{d^2W}{d\omega\,dt}=-\frac{e^2}{(2\pi)^3}\int v^\mu v^\nu\delta(\omega-\mathbf{k}\mathbf{v})\sum_{m=-\infty}^{\infty}\operatorname{Im}[D^{(m)}_{\mu\nu}(\mathbf{k},\omega)e^{i\mathbf{K}\mathbf{v}mt}]d^3k, \tag{5}$$

where

$$D^{(m)}_{\mu\nu}(\mathbf{k},\omega)=\int D^{(m)}_{\mu\nu}(\rho,\tau)e^{-i(\omega\tau+\mathbf{k}\rho)}d^3\rho\,d\tau.$$

For simplicity we shall assume that the electron is moving along the direction $\mathbf{n}$. In an infinite periodic medium the probability (5) has the meaning of an average over the time of flight $t_a=a/v$ between neighboring inhomogeneities. After such averaging the contribution to the energy loss probability (5) is only from terms with $m=0$. This statement obviously remains valid also in the case of "almost rectilinear" trajectories, which occur when relativistic electrons are scattered by the atoms of the medium. Thus, the probability of energy loss (radiation) by a relativistic electron at frequency ω per unit time, averaged over the coordinates of the scattering atoms (trajectories) and over the time of flight between neighboring optical inhomogeneities, for a periodically nonuniform medium has the form

$$\frac{d^2W}{d\omega\,dt}=\frac{e^2}{8\pi^4}\operatorname{Re}\int\langle L^{\mu\nu}(\tau,\mathbf{k},0)\rangle\operatorname{Im}D^{(0)}_{\mu\nu}(\mathbf{k},\omega)\,d^3k. \tag{6}$$

Equation (6) for the probability differs from the corresponding expression (3) for the case of an optically homogeneous medium only in the fact that $D^{(0)}_{\mu\nu}(\mathbf{k}, \omega)$ now represents the Fourier component of that part of the total Green's function of the photon in the inhomogeneous medium (4) which depends only on the difference of coordinate ρ. This analogy is the principal result which will be used subsequently in the calculation. As will be shown below, with certain restrictions, the function $D^{(0)}_{\mu\nu}(\mathbf{k}, \omega)$, which determines the differential probability of energy loss by the electron (6), can be represented in the form of a linear superposition of Green's functions of homogeneous media. Thus, almost all of the results for the spectral and angular distributions of the energy loss, obtained previously for homogeneous media,[3,4] can be transferred directly to the case of a medium with periodic inhomogeneities.

2. THE PHOTON GREEN'S FUNCTION IN A PERIODIC MEDIUM

In the approximation of macroscopic electrodynamics and in the gauge with a scalar potential equal to zero, the photon Green's function in an inhomogeneous medium satisfies the equation[6]

$$[\varepsilon(\omega, \mathbf{r})\omega^2\delta_{il}-\mathrm{rot}_{im}\mathrm{rot}_{ml}]D_{lk}(\mathbf{r}, \mathbf{r}', \omega)=4\pi\delta_{ik}\delta(\mathbf{r}-\mathbf{r}').$$

We take the coordinate axis $x_1 \equiv x$ along the direction of optical inhomogeneity. Since the medium is assumed uniform in the remaining directions, it is convenient to go over to the Fourier components

$$D_{ik}(x, x', \mathbf{q}, \omega)$$
$$=\int \exp[-iq_2(x_2-x_2')-iq_3(x_3-x_3')]D_{ik}(\mathbf{r}, \mathbf{r}', \omega)\,d(x_2-x_2')\,d(x_3-x_3')$$

and to direct the axes x_2 and x_2' along the vector $\mathbf{q}$. Then for the nonzero Fourier components we obtain the equations

$$L_1\left(x, \frac{d}{dx}\right)D_{33}=4\pi\delta(x-x'), \quad L_2\left(x, \frac{d}{dx}\right)D_{22}=4\pi\delta(x-x'),$$
$$L_2\left(x, \frac{d}{dx}\right)D_{21}=4\pi i\frac{d}{dx}\frac{\delta(x-x')}{Q^2(x)}, \tag{7}$$
$$D_{12}=\frac{iq}{Q^2(x)}\frac{dD_{22}}{dx}, \quad D_{11}=Q^{-2}(x)\left[iq\frac{dD_{21}}{dx}+4\pi\delta(x-x')\right],$$

where

$$L_1\left(x, \frac{d}{dx}\right)=\frac{d^2}{dx^2}+Q^2(x),$$
$$L_2\left(x, \frac{d}{dx}\right)=\varepsilon(\omega, x)+q^2\frac{d}{dx}\left(\frac{1}{Q^2}\frac{d}{dx}\right)+\frac{d}{dx^2},$$
$$Q^2(x)=\varepsilon(\omega, x)\omega^2-q^2.$$

Below we will be interested in the radiation of rather energetic photons, for which the complex dielectric permittivity $\varepsilon(\omega, x)$ is close to unity and the angles of radiation are small ($\theta_{eff} \ll 1$). A periodic variation of $\varepsilon(\omega, x)$ along the x axis is achieved in practice by varying the density of the material. Therefore ε can be represented in the form $\varepsilon(\omega, x)=\varepsilon_0(\omega)+\varepsilon_1(\omega, x)$, where $|\varepsilon_0-1| \ll 1$, and the periodic part of the dielectric permittivity $\varepsilon_1(\omega, x)$ is significantly smaller than unity and, consequently, smaller than the constant part $\varepsilon_0(\omega)$. In addition, for practical purposes it is sufficient to find the solution of the system of Eqs. (7) in the WKB approximation, requiring satisfaction of the inequalities

$$\left|\frac{1}{2}Q^{-1/2}\frac{d^2}{dx^2}Q^{-1/2}\right| \ll \left|\frac{1}{2}\frac{d}{dx}\frac{1}{Q}\right| \ll 1, \tag{8}$$

$$\frac{1}{2}\left|\int_{x'}^{x} Q^{-1/2}\frac{d^2}{dx^2}Q^{-1/2}\,dx\right| \ll 1. \tag{9}$$

Using the condition of smallness of the angles of radiation ($q \ll |\varepsilon^{1/2}(\omega, x)\omega|$) and the closeness of $|\varepsilon(\omega, x)|$ to unity, we can show that the first of the inequalities (8) is equivalent to the condition of smallness of the wavelength of the radiation in comparison with the period of the medium ($a\omega \gg 1$), which is always satisfied for artificially produced periodic media and for x-ray frequencies. The second of the inequalities (8) and inequality (9) are equivalent to the less rigid condition $a\omega \gg |\varepsilon_1|$. The restrictions enumerated permit the components of the Green's function in Eq. (7) to be expressed in terms of D_{33} as follows:

$$D_{22}=\frac{Q_0^2}{\varepsilon_0\omega^2}D_{33}, \quad D_{21}=-\frac{iq}{\varepsilon_0\omega^2}\frac{dD_{33}}{dx'}$$
$$D_{12}=\frac{iq}{\varepsilon_0\omega^2}\frac{dD_{33}}{dx}, \quad D_{11}=\frac{q^2}{\varepsilon_0\omega^2}D_{33}. \tag{10}$$
$$Q_0^2=\varepsilon_0\omega^2-q^2.$$

Since we are interested in that part of the Green's function which depends only on the difference in coordinates (see Eq. (6)), we can assume, in accordance with Eq. (10), that $D_{21}=D_{12}$. Then the relations (10) formally coincide with the corresponding relations for a uniform medium.

The solution of the first of Eqs. (7) with inclusion of the restrictions which we have introduced has the form

$$D_{33}(x, x', \mathbf{q}, \omega)=-\frac{2\pi i}{Q_0}\exp(iQ_0|x-x'|)\exp\left[\frac{i\omega}{2}\mathrm{sign}(x-x')\int_{x'}^{x}\varepsilon_1(\omega, x)\,dx\right]. \tag{11}$$

The last factor in Eq. (11) is a periodic function of x and x' and can be represented in the form of a double Fourier series. Then the part of $D^{(0)}_{33}$ which depends only on the difference $x-x'$ takes the form

$$D^{(0)}_{33}(x-x', \mathbf{q}, \omega)=-\frac{2\pi i}{Q_0}\exp(iQ_0|x-x'|)\sum_{r=-\infty}^{\infty} P_r\exp(iKr|x-x'|), \tag{12}$$

where

$$P_r=G_rH_r, \quad K=2\pi/a,$$
$$G_r=\frac{1}{a}\int_0^a \exp\left[\frac{i\omega}{2}\int_0^x \varepsilon_1(\omega, x)\,dx\right]e^{-iKrx}\,dx,$$
$$H_r=\frac{1}{a}\int_0^a \exp\left[-\frac{i\omega}{2}\int_0^{x'} \varepsilon_1(\omega, x)\,dx\right]e^{iKrx'}\,dx'.$$

In the particular case where $\varepsilon_1(\omega, x)$ varies according to a law $\varepsilon_1(\omega, x)=\Delta\cos Kx$, the weighting factors P_r are expressed in terms of the Bessel functions $J_r(\alpha)$: $P_r=J_r^2(\alpha)$, where $\alpha=\omega a\Delta/4\pi$ (a is the period of the medium).

Taking into account the closeness of $|\varepsilon_0(\omega)|$ to unity, we obtain for the Fourier component of the Green's function $D^{(0)}_{33}(\mathbf{k}, \omega)$ the following expression[4]:

$$D_{ss}^{(0)}(\mathbf{k},\omega)=4\pi\sum_{r=r_{min}}^{r_{max}}\frac{P_r}{\mathbf{k}^2-\varepsilon_{eff}\omega^2},\tag{13}$$

where we have introduced the designation $\varepsilon_{eff}=\varepsilon_0(\omega)+4\pi r/a\omega$. In the transition from Eq. (12) to Eq. (13) it is assumed that we can neglect terms in Eq. (12) with $r<r_{min}$ and $r>r_{max}$, where r_{min} and r_{max} are chosen from the condition of closeness of the effective dielectric permittivity to unity for the corresponding values of the index r:

$$|\varepsilon_{eff}(r_{min},\omega)-1|\ll 1,\quad |\varepsilon_{eff}(r_{max},\omega)-1|\ll 1.$$

It is always possible to neglect these terms if the radiation of particles in a periodic medium occurs mainly at small angles (see below).

Using Eq. (10), we obtain the following expression for the transverse part of the Green's function $D_{ik}^{(0)}(\mathbf{k},\omega)$, which in accordance with Eq. (6) determines the differential probability of energy loss in a periodically inhomogeneous medium[5]:

$$D_{ik}^{(0)}(\mathbf{k},\omega)=\sum_{r=r_{min}}^{r_{max}}P_rD_{ik}^{(h)}(r,\mathbf{k},\omega),\tag{14}$$

where

$$D_{ik}^{(h)}(r,\mathbf{k},\omega)=\frac{4\pi}{k^2-\varepsilon_{eff}(r,\omega)\omega^2}\left(\delta_{ik}-\frac{k_ik_k}{k^2}\right)$$

is a quantity which formally coincides with the transverse part of the Green's function in a homogeneous medium with a dielectric permittivity $\varepsilon_{eff}(r,\omega)=\varepsilon_0(\omega)+4\pi r/a\omega$.

The expressions (14) determine the dispersion of photons with wavelength significantly less than the period of the medium a which are propagated at small angles to the direction of periodic change of the optical properties of the medium.

3. SPECTRAL DISTRIBUTION OF ENERGY LOSS BY AN ELECTRON

Relations (3), (6), and (14) show that for purely real weighting factors P_r the spectral distribution of the energy loss in a periodically inhomogeneous medium is directly expressed in the form of a linear superposition of the corresponding quantities for uniform media.[6] This relation is preserved also in the general case of complex P_r, since the spectral distribution of the energy loss (6) is determined in fact by the residues at the points where $k^2=\varepsilon_{eff}(r,\omega)\omega^2$,[8] and near these points the imaginary part of $D_{ik}^{(h)}(r,\mathbf{k},\omega)$ coincides with the real part (see Eq. (14)). Thus, the spectral density of energy loss $I(\omega)$ by an electron per unit length of path in a periodically inhomogeneous medium can be represented in the form

$$I(\omega)=\sum_{r=r_{min}}^{r_{max}}(\operatorname{Im}P_r+\operatorname{Re}P_r)I^{(h)}(r,\omega),\tag{15}$$

where $I^{(h)}(r,\omega)$ is the corresponding quantity for a homogeneous medium with the effective permittivity as defined in (13).

The quantity $I^{(h)}(r,\omega)$ for a homogeneous absorbing medium with inclusion of multiple scattering and electron energy loss has been calculated by Bazylev *et al.*[3] In the simpler case in which we can neglect the effect of energy loss on the mean square multiple-scattering angle, the result has the following form[3]:

$$I^{(h)}(r,\omega)=\frac{2e^2}{\pi}(q\omega)^{1/2}F(s_r)+\frac{e^2\omega}{\pi}(1-\varepsilon'_{eff}+E^{-2})\left[\operatorname{arctg}\frac{1-\varepsilon_{eff}+E^{-2}}{\varepsilon''_{eff}}-\frac{\pi}{2}\right]+\frac{e^2\omega}{2\pi}\varepsilon''_{eff}\ln\frac{1}{\omega^4[(1-\varepsilon'_{eff}+E^{-2})^2+(\varepsilon''_{eff})^2]}.\tag{16}$$

Here

$$F(s_r)=4\operatorname{Im}\left\{s_r\left[\psi\left(\frac{\beta}{2}\right)+\frac{1}{\beta}-\ln\frac{\beta}{2}\right]\right\}-8\pi\theta(\varepsilon'_{eff}-1-E^{-2}-\varepsilon''_{eff})\operatorname{Re}\frac{s_r}{e^{i\pi\mu}-1},$$

$$s_r=\frac{1-\varepsilon_{eff}+E^{-2}}{8}\left(\frac{\omega}{q}\right)^{1/2},\quad \mu=2(1+i)s_r,$$

θ is the unit step function, $\operatorname{sign}x=x/|x|$, $\psi(x)=d(\ln\Gamma(x))/dx$ is the logarithmic derivative of the Γ function,

$$\varepsilon'_{eff}=\varepsilon_0'(\omega)+4\pi r/a\omega,\quad \varepsilon''_{eff}=\varepsilon_0''(\omega)$$

are the real and imaginary parts of the effective dielectric permittivity: $\beta=\mu\operatorname{sign}(1-\varepsilon_{eff}+E^{-2}+\varepsilon''_{eff})$, $4q$ is the mean square multiple-scattering angle per unit length of a medium with the average density over the period.[7]

For a transparent medium ($\varepsilon''(\omega,x)\equiv 0$) which changes its properties according to a cosine law $\varepsilon_1'(\omega,x)=\Delta\cos Kx$, Eqs. (15) and (16) coincide with the result of Ter-Mikaelyan.[2]

In the general case we can separate for each discrete index r three dimensionless parameters:

$$p_1=(q/\omega)^{1/2},\quad p_2=\varepsilon'_{eff}-1-E^{-2},\quad p_3=\varepsilon''_{eff}.$$

If $p_1\gg|p_2|,p_3$, then Eq. (16) is dominated by the first term, which represents in this limit the intensity of bremsstrahlung in the r-th harmonic. For $p_2\gg p_1,p_3$ the energy loss is determined mainly by the second term (resonance radiation). If $p_3\gg p_1,|p_2|$, the energy loss is determined by the third term, which represents (for an appropriate $\varepsilon_0''(\omega)$) the contribution to ionization loss from the r-th harmonic. In intermediate cases when at least two parameters are comparable in magnitude, this simple interpretation of Eq. (16) becomes impossible (see Refs. 2 and 3). Thus, only in the case $p_3\ll p_1,|p_2|$ when the coherence length of the radiation is significantly less than the photon-absorption length can we assume that the entire energy loss is determined by the radiation of the relativistic electron.

In the opposite case ($p_3\gg p_1,|p_2|$), if we are interested, for example, in the radiation emerging from a periodic absorbing medium of finite size, further calculation is necessary.

The following result is important. According to Eqs. (15) and (16) the spectral density of the energy loss is determined not only by the periodic variation of $\varepsilon_1'(\omega, x)$,[1] but to an equal degree also by the periodic variation of the absorbing properties of the medium $\varepsilon_1''(\omega, x)$. Thus, for example, in the case of a cosine-function medium $\varepsilon_1(\omega, x) = \Delta\cos Kx$ inclusion of periodic variation of $\varepsilon_1''(\omega, x)$ substantially affects the value of the weighting factors $P_r = J_r^2(\omega a\Delta/4\pi)$ in Eq. (15), particularly in the frequency region where $\mathrm{Im}\,\Delta \gtrsim \mathrm{Re}\,\Delta$.

It follows from the results obtained that a periodic inhomogeneity of the medium not only affects the bremsstrahlung spectrum and leads to appearance of resonance radiation.[2] The energy loss to direct ionization of the atoms in the medium and formation of electron-positron pairs, and also the radiation from the recoil Compton electrons, whose spectral density is determined by the third term in Eq. (16),[3,8] depend substantially on the periodic variation of the dielectric properties of the medium.

The calculation which we have presented of the electromagnetic processes in a periodic medium may be useful not only in construction of relativistic-particle detectors (see for example Ref. 2). An excess of ε'_{eff} over unity permits in principle use of radiation in a periodic medium as an intense source in the x-ray region, in analogy with Čerenkov radiation in homogeneous media in the region of atomic frequencies.[10] In this case a broader possibility exists than in homogeneous media, since the creation of an appropriate periodic medium is limited only by technical problems.

[1)]We use the system of units in which $\hbar = m = c = 1$, and the metric tensor has the form $g_{\mu\nu} = 0$ $(\mu \neq \nu)$, $g_{11} = g_{22} = g_{33} = -g_{44} = 1$.

[2)]The approach described can easily be generalized to the case of a nonstationary medium.

[3)]A dependence on time remains only if we introduce phenomenologically a time-dependent mean square scattering angle.[8]

[4)]Small deviations from strict periodicity can be taken into account by introduction into Eq. (13) of a factor of the Debye-Waller type.[2b]

[5)]The effect of periodicity on radiation of longitudinal modes is not considered in this work.

[6)]It is assumed that the deviation of the density of the medium from the average does not have a substantial effect on the motion of the electron.

[7)]The formula for $F(s_r)$ in the article of Bazylev *et al.*[3] is given with an incorrect sign in front of the θ function.

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