Effects impeding observation of weak neutral interaction between a muon and a nucleus in light mesic atoms

D. P. Grechukhin and A. A. Soldatov

I. V. Kurchatov Atomic Energy Institute
(Submitted July 7, 1976)
Zh. Eksp. Teor. Fiz. 72, 390–404 (February 1977)

An estimate is given of the intensity of the satellite lines of the $2s^{1/2}\rightarrow 1s^{1/2}$ radiative transition in a mesic atom due to configurational interaction between the meson and the shell electrons of the mesic atom. It is shown that the intensity of these satellite lines is by approximately six orders of magnitude greater than the intensity of the radiation in a $2s^{1/2}\rightarrow 1s^{1/2}$ meson transition. The necessity to exclude the satellite lines in an experiment aiming to observe nonconservation of parity requires an energy resolution for the quantum detector of $\Delta\hbar\omega/\hbar\omega \leq 10^{-4}$. The effect of the configuration interaction between a meson and the electrons of the conduction band in metals does not allow us to utilize metal targets for stopping the mesons in such an experiment. In the case of formation of a mesic atom in a gaseous medium the Stark effect of the electric field of the dipole induced in the collision of an atom of the medium with the mesic atom ($Z \geq 2$) stripped of electrons, and the Stark effect of the intra-atomic field in the case of a collision of a μH atom with an atom of the medium, as shown by the estimates that have been made, give essential upper limits on the density of the medium respectively of the order of $\leq 10^{14}$ at/cm^3 and $\leq 10^{11}$ at/cm^3.

PACS numbers: 36.10.Gv

§1. INTRODUCTION

1. The weak neutral interaction between a muon and a nucleus leads to mixing of the states of mesic atoms of opposite parities (for example, $|2s\tfrac{1}{2}\rangle$ and $|2p\tfrac{1}{2}\rangle$). In the case of a mesic atom polarized beforehand this mixing manifests itself, in particular, in the angular distribution $W(\theta)=1+\alpha\cos\theta$ of quanta emitted when the meson makes a transition. But if the mesic atom was not polarized, then the effect linear in the interaction occurs only in the circular polarization of the quanta radiated by the mesic atom. These effects were estimated in a number of papers[1–7] for different variants of the weak neutral interaction between a meson and the nucleus for the case of mixing of the $|2s\tfrac{1}{2}\rangle$ and $|2p\tfrac{1}{2}\rangle$ states of the mesic atom, with the calculation in one of these papers[2] being carried out for the region $3 \leqslant Z \leqslant 82$, while the other papers considered the region of the light ($1 \leqslant Z \leqslant 10$) mesic atoms. The magnitude of the effect in the case of a radiative transition to the $|1s\tfrac{1}{2}\rangle$-orbit is determined by the amplitude $\delta(2s\tfrac{1}{2};2p\tfrac{1}{2})$ of the admixture of the $|2p\tfrac{1}{2}\rangle$ state to the "initial" $|2s\tfrac{1}{2}\rangle$ state of the mesic atom and by the ratio of the probabilities of $E1$ and $M1$ radiative transitions, i.e., by the factor

$$\frac{[W(E1;2p\rightarrow 1s)\,W(M1;2s\rightarrow 1s)]^{1/2}}{W(M1;2s\rightarrow 1s)+|\delta(2s^1/_2;2p^1/_2)|^2 W(E1;2p\rightarrow 1s)}\operatorname{Im}\delta(2s^1/_2 2p^1/_2). \quad (1)$$

In the case of light mesic atoms ($1 \leqslant Z \leqslant 10$) we have for the probabilities of the radiative $M1$ and $E1$ transitions (cf.,[1,2])

$$W(E1;\ 2p\rightarrow 1s)\approx 1.29\cdot 10^{11}Z^4\ [\mathrm{sec}^{-1}],$$
$$W(M1;\ 2s\rightarrow 1s)\approx 5.16\cdot 10^{-4}Z^{10}\ [\mathrm{sec}^{-1}], \quad (2)$$

other data required for the calculation of spectra of light mesic atoms are given in Table I. For heavy mesic atoms the probabilities $W(M1)$ and $W(E1)$ have been calculated in[2].

The competitive ratio $W(E1)/W(M1)$ falls off rapidly with increasing nuclear charge Z, while $|\delta(2s\tfrac{1}{2}2p\tfrac{1}{2})|$ is of the order of $\sim 10^{-7}$ in the range $6 \leqslant Z \leqslant 82$ and has a somewhat greater value in specific cases $1 \leqslant Z \leqslant 5$ (cf.,[1–5]). Therefore light nuclei can be regarded as the most convenient systems for designing an experiment to observe the effects of the weak neutral interaction between a meson and a nucleus.

2. It should be noted that the estimates made in[1–7] refer only to an isolated mesic ion totally lacking an electron shell. This circumstance is sufficiently firmly emphasized only in[1]. Under actual experimental conditions the capture of a meson into an orbit occurs in targets with finite atomic and electron (metals) densities. Therefore processes are possible of partial or total neutralization of a mesic ion involving the filling of an electron shell; a collision of a mesic ion with atoms of the medium, the capture of a mesic atom into a molecule; the interaction of a mesic atom with conduction electrons in the case of metallic targets, etc. It is necessary to make an estimate of the extent to which these possible processes are dangerous, i.e., the extent to which they can distort that ideal picture of the effect of a neutral weak interaction which is obtained in the case of an isolated mesic ion.[1–7]

TABLE I. Data for the spectrum of states of light mesic atoms.

μA_Z	$\varepsilon(2p^1/_2)-\varepsilon(2s^1/_2)$, eV	$W(E1)$ $(2p\rightarrow 1s)$, sec^{-1}	$W(M1;$ $2s\rightarrow 1s)$, sec^{-1}	$W_{\gamma\gamma}(2s\rightarrow 1s)$, sec^{-1}	$\frac{W(E1;\ 2p\rightarrow 1s)}{W(M1;\ 2s\rightarrow 1s)}$	Reference	$\hbar\omega$ $(2s\rightarrow 1s)$, keV
μH_1^1	0.2	$1.29\cdot 10^{11}$	$4.6\cdot 10^{-4}$	$1.5\cdot 10^{3}$	$2.8\cdot 10^{14}$	[1]	1.90
μHe_2^4	1.4	$2.06\cdot 10^{12}$	$5.1\cdot 10^{-1}$	$1.1\cdot 10^{5}$	$4.05\cdot 10^{12}$	»	8.21
μLi_3^6	1.1	$1.03\cdot 10^{13}$	$3.0\cdot 10$	$1.2\cdot 10^{6}$	$3.44\cdot 10^{11}$	»	18.60
μLi_3^7	1.5	$1.03\cdot 10^{13}$	$3.0\cdot 10$	$1.2\cdot 10^{6}$	$3.44\cdot 10^{11}$	»	18.60
μBe_4^9	−2.2	$3.3\cdot 10^{13}$	$5.3\cdot 10^{2}$	$6.9\cdot 10^{6}$	$4.8\cdot 10^{10}$	»	33.3
μC_6^{12}	−33	$1.66\cdot 10^{14}$	$3.1\cdot 10^{4}$	$7.9\cdot 10^{7}$	$5.35\cdot 10^{9}$	»	75.2
μO_8^{16}	−162	$5.3\cdot 10^{14}$	$5.5\cdot 10^{5}$	$4.5\cdot 10^{8}$	$0.97\cdot 10^{9}$	[5]	134
μNa_{11}^{23}	−727	$1.87\cdot 10^{15}$	$1.2\cdot 10^{7}$	$2.6\cdot 10^{9}$	$1.56\cdot 10^{7}$	[2]	

Note. We estimate the splitting $\hbar(2p\tfrac{3}{2})-\varepsilon(2p\tfrac{1}{2})$ for the field of a point nucleus: $\varepsilon(2p\tfrac{3}{2})-\varepsilon(2p\tfrac{1}{2})=m\mu c^2(Ze^2/\hbar c)^4/32$; $m\mu$ is the meson mass.

Below we give the results of estimates for a number of possible processes which appear to us to the most dangerous:

1) configuration mixing in the electron shell of a mesic atom ($Z \geq 2$);

2) configuration interaction of μLi with conduction electrons;

3) $E1$ conversion of the $2s\frac{1}{2} \to 2pj$ transition in μBe, μB, μC, ... on electrons of the conduction band;

4) the Stark effect of the field of the atom in a collision with a meson;

5) the Stark effect of the intra-atomic field in the collision of μH with a hydrogen atom.

Naturally the list given above does not exhaust all possible effects of the medium. Thus, in the capture of a mesic atom into a molecule a Stark effect is possible due to the intramolecular electric field arising in vibrational and rotational excitations of the molecule. For a mesic atom situated in an ionized gas one should make an estimate of the Stark effect of the fluctuation field. Specific effects also occur in the case of condensed media. However for $Z \geq 3$ the binding energy of the $1s$ electron is significantly greater than the ionization energy of the atoms of the medium. Consequently in condensed media the mesic ion ($Z \geq 3$) must be partially neutralized. It would appear to us that in such a situation the most violent effect will be configuration mixing in the electron shell of a partially neutralized mesic atom.

§2. EFFECT OF CONFIGURATION MIXING IN THE ELECTRON SHELL OF A MESIC ATOM ($Z \geq 2$)

1. Suppose that a mu-meson is captured into a $|2s\frac{1}{2}\rangle$ orbit in the field of a nucleus of $Z \geq 2$ and that partial or complete neutralization of the mesic ion has taken place, i.e., $N \neq 0$ $(0 < N \leq Z-1)$ electrons are situated in the electron shell of the mesic atom. Since the radii of the $|2pj\rangle$ and $|2s\frac{1}{2}\rangle$ orbits of a meson are significantly (by a factor $m/m_\mu \sim 1/207$) smaller than the radii of electron orbits, then in the first approximation we regard the electron shell of a mesic atom as the shell of an atom with nuclear charge $(Z-1)$, and we regard the interaction between a meson and the electrons as a perturbation. We expand this interaction in terms of multipoles and retain for future use only the dipole–dipole interaction:

$$H_{\text{int}} = \sum_{i=1}^{N} e^2 \left(\frac{1}{|\mathbf{r}_i - \mathbf{r}|} - \frac{1}{r_i} \right) \approx e^2 \frac{4\pi}{3} \sum_{i=1}^{N} \sum_{m} \frac{r}{r^2} Y_{1m}^*(\mathbf{r}) Y_{1m}(\mathbf{r}_i), \quad (3)$$

where $\mathbf{r}$ is the position vector of the meson; $\mathbf{r}_i$ is the position vector of the i-th electron.

The dipole–dipole interaction leads to mixing of the configurations of the system meson plus electron shell with the meson in the $|2p\frac{1}{2}\rangle$ and $|2p\frac{3}{2}\rangle$ states and thereby opens the channel for the radiative transition of the meson to the $|1s\frac{1}{2}\rangle$ orbit, and this distorts the observed effect of the neutral interaction.

2. We introduce a system of states of configurations of the electron shell of a mesic atoms: $|\{k\}J_k\nu_k\rangle$, where where $\{k\}$ is the index of the configuration (occupation numbers of the electron orbits); $J_k\nu_k$ are the angular momentum and the component of the angular momentum. We denote the energy of such a state by $E(\{k\}J_k)$. Further we consider a group of meson orbits which for the sake of brevity we denote below by:

$$|j_0\mu_0\rangle = |2s^1/_2\mu_0\rangle, \quad |j_1\mu_1\rangle = |2p^1/_2\mu_1\rangle,$$
$$|j_3\mu_3\rangle = |2p^3/_2\mu_3\rangle, \quad |j_2\mu_2\rangle = |1s^1/_2\mu_2\rangle,$$

where μ_i is the component of the angular momentum j_i of the meson; $\varepsilon(nlj) = \varepsilon(j)$ is the energy of the (nlj) level of the mesic atom.

The total system (meson plus shell electrons) is directed in terms of the set of basic states with a fixed total angular momentum F $(\mathbf{F} = \mathbf{j} + \mathbf{J})$ in the following manner:

$$|\{k\}J_k; j_k; Ff\rangle = \sum_{\mu_k\nu_k} (j_k J_k \mu_k \nu_k | Ff) \, |n_k l_k j_k \mu_k\rangle \, |\{k\} J_k \nu_k\rangle \quad (4)$$

with the unperturbed values of the energy

$$\varepsilon(n_k l_k j_k) + E(\{k\}J_k). \quad (5)$$

The "switching on" of the dipole–dipole interaction (3) leads to mixing of the basic states (4), and the mesic atom with a meson captured into the $|2s\frac{1}{2}\rangle$ orbit is described by the superposition

$$\Psi_{F_1f_1}(2s^1/_2) = a(\{0\}J_0; 2s^1/_2; F_1) \, |\{0\}J_0; 2s^1/_2; F_1f_1\rangle$$
$$+ \sum_{j_k} \delta_W(\{0\}J_0; 2pj_k; F_1) \, |\{0\}J_0; 2pj_k; F_1f_1\rangle$$
$$+ \sum_{\{k\}J_k} \sum_{j_k} a(\{k\}J_k; 2pj_k; F_1) \, |\{k\}J_k; 2pj_k; F_1f_1\rangle, \quad (6)$$

where $j_k = \frac{1}{2}$ and $\frac{3}{2}$; the amplitude of the principal configuration of the electron shell $a(\{0\}J_0; 2s\frac{1}{2}; F_1) \approx 1$; the sum contains configurations of the electron shell connected to the principal one by an electric dipole transition. We have denoted by δ_W the amplitude of the admixture of the $|2pj_k\rangle$ orbits of the meson due to the weak neutral interaction. The electron configuration is unchanged by this interaction. We obtain the values of $a(\{k\}J_k; 2pj_k; F_1)$ to the first order of perturbation theory in terms of H_{int}:

$$a(\{k\}J_k; 2pj_k; F_1) = \frac{\langle \{k\}J_k; 2pj_k; F_1 | H_{\text{int}} | \{0\}J_0; 2s^1/_2; F_1 \rangle}{\varepsilon(2s^1/_2) - \varepsilon(2pj_k) + E(\{0\}J_0) - E(\{k\}J_k)}. \quad (7)$$

3. For a $|1s\frac{1}{2}\rangle$ meson orbit the admixture of such a configuration is approximately by three orders of magnitude smaller than for a $|2s\frac{1}{2}\rangle$ orbit since the difference in the energies is

$$|\varepsilon(2pj_k) - \varepsilon(1s^1/_2)| \approx Z^2 (m_\mu/m)^1/_2 (me^4/\hbar^2) \approx 2{,}1 Z^2 \text{ [keV]} \quad (8)$$

(m_μ is the meson mass, m is the electron mass), while for the $|2s\frac{1}{2}\rangle$ and the $|2pj_k\rangle$ meson orbits in the region of $Z \leq 11$ the difference in energies is comparable with or less than the binding energy of the $1s$ electron:

$$|\varepsilon(2s^1/_2)-\varepsilon(2pj_k)|\leqslant Z^2\,{}^1/_2(me^4/\hbar^2)=13.6Z^2\ [\text{eV}]. \tag{9}$$

Consequently, if the meson is in a $|1s\tfrac{1}{2}\rangle$ orbit, then one can neglect the configuration mixing in the system and use for the final states the pure states

$$|\{0\}J_0;\,1s^1/_2;\,F_2f_2\rangle=\sum_{\mu_2\nu_0}(j_2J_0\mu_2\nu_0|F_2f_2)\,|j_2\mu_2\rangle\,|\{0\}J_0\nu_0\rangle \tag{10}$$

of energy $\varepsilon(1s\tfrac{1}{2})+E(\{0\}J_0)$ and the states

$$|\{k\}J_k;\,1s^1/_2;\,F_2f_2\rangle=\sum_{\mu_2\nu_k}(j_2J_k\mu_2\nu_k|F_2f_2)\,|j_2\mu_2\rangle\,|\{k\}J_k\nu_k\rangle \tag{11}$$

of energy $\varepsilon(1s\tfrac{1}{2})+E(\{k\}J_k);\ \{k\}\neq 0,\ |j_2\mu_2\rangle\equiv|1s\tfrac{1}{2}\mu_2\rangle$.

In the course of a radiative transition of a meson from $\Psi_{F_1f_1}(2s\tfrac{1}{2})$ into these final states the mesic atom emits a spectrum of quanta whose energies $\hbar\omega_0$ and $\hbar\omega_k$ correspond to the final states of the electron shell of the mesic atom,

$$\begin{aligned}\hbar\omega_0&\approx[\varepsilon(2s^1/_2)-\varepsilon(1s^1/_2)],\\ \hbar\omega_k&\approx[\varepsilon(2s^1/_2)-\varepsilon(1s^1/_2)]+[E(\{0\}J_0)-E(\{k\}J_k)].\end{aligned} \tag{12}$$

4. It is important to emphasize that the satellite lines $\hbar\omega_k$ practically do not contain any information concerning the weak interaction between the meson and the nucleus, since the state $|\{k\}J_k;\,2pj_k;F_1f_1\rangle$ is mixed by the weak interaction with the state $|\{k\}J_k;2s\tfrac{1}{2};F_1f_1\rangle$, and, therefore, when a quantum $\hbar\omega_k$ is emitted in the transition of the meson to the $|1s\tfrac{1}{2}\rangle$ orbit the effect of the weak interaction enters with a factor

$$[W(M1;\ 2s^1/_2\to 1s^1/_2)/W(E1;\ 2pj_k\to 1s^1/_2)]^{1/2}\ll 1. \tag{13}$$

The position of the lines $\hbar\omega_k$ can be obtained for each mesic atom with N electrons in the shell from the experimental data on the spectra of the ions of the corresponding elements $(Z-1)$. In the case of only a single electron in the shell of the mesic atom the principal configuration is $\{0\}=\{1s\tfrac{1}{2}\}^1$ while the excited configurations are $\{k\}=\{n_kp_k\}^1$, and from this we obtain

$$\hbar\omega_k=\hbar\omega_0-\frac{1}{2}\frac{me^4}{\hbar^2}(Z-1)^2\left(1-\frac{1}{n_k^2}\right), \tag{14}$$

so that for the neutral μHe the closest satellite line is shifted from $\hbar\omega_0=8.2$ keV by an amount ≈ 10.2 eV, while for $(\mu\text{Li})^+$ it is shifted by an amount ~ 40 eV with $\hbar\omega_0=18.6$ keV. An increase in the number of electrons in the shell of the mesic atom leads to an increase in the number of satellite lines and to a decrease in the magnitude of the shift of the nearest $\hbar\omega_k$; thus, for two-electron mu-atoms we have the following shifts of the closest lines and the energy $\hbar\omega_0$:

μ-atom	μLi	$(\mu\text{Be})^+$	$(\mu\text{B})^{2+}$	$(\mu\text{C})^{3+}$	$(\mu\text{N})^{4+}$	$(\mu\text{O})^{5+}$
$\hbar\omega_0-\hbar\omega_k$, eV:	21.2	62.5	124	206	308	430
$\hbar\omega_0$, keV	18.6	33.3	52.4	75.2	103	134

5. The configurations $\{k\}$ differ from $\{0\}$ by the fact that one of the electrons from the state with the principal quantum number N_0, orbital quantum number L_0 and total angular momentum I_0 has been transferred to the state $|N_kL_kI_k\rangle$, where $L_k=L_0\pm 1$. Utilizing nonrelativistic functions for the meson states we obtain the configuration element of the dipole–dipole interaction

$$\begin{aligned}&\langle\{k\}J_k;\,2pj_k;\,F_1|H_{\text{int}}|\{0\}J_0;\,2s^1/_2;\,F_1\rangle\\ &=-\frac{e^2}{a_0}3\sqrt{3}\frac{m}{m_\mu}\frac{1}{Z}\left\langle N_kL_kI_k\left|\frac{1}{x^2}\right|N_0L_0I_0\right\rangle\xi_{F_1}(\{k\}J_kL_kI_kj_k;\ \{0\}J_0L_0I_0j_0),\end{aligned} \tag{15}$$

where ξ_{F_1} is the configuration factor of the order of magnitude of unity for states connected by allowed $E1$-transitions; for example, for a one-electron configuration we have

$$\xi_F=\frac{1}{\sqrt{3}}(L_0100|L_k0)\,(-)^{j_k-1/2}u\left(I_0\frac{1}{2}1L_k;\,L_0I_k\right)u\left(\frac{1}{2}F1I_k;\,I_0j_k\right), \tag{16}$$

$u(abcd;ef)$ is the normalized Racah function tabulated in[8]; $a_0=\hbar^2/me^2=0.529\times 10^{-9}$ cm; $x=r/a_0$. The electron radial elements can be calculated using hydrogen functions for the case of one electron in the shell:

$$\begin{aligned}\left\langle 2p\left|\left(\frac{1}{x}\right)^2\right|1s\right\rangle&=\frac{4}{9\sqrt{6}}(Z-1)^2\approx 0.182(Z-1)^2,\\ \left\langle 3p\left|\left(\frac{1}{x}\right)^2\right|1s\right\rangle&=\frac{1}{4\sqrt{6}}(Z-1)^2\approx 0.102(Z-1)^2,\\ \left\langle 4p\left|\left(\frac{1}{x}\right)^2\right|1s\right\rangle&=\frac{2}{5}\sqrt{\frac{5}{3}}\left(\frac{3}{5}\right)^4(Z-1)^2\approx 0.067(Z-1)^2.\end{aligned} \tag{17}$$

Another estimate can be obtained by taking the value of the electron radial elements $\langle N_kL_kI_k|(1x^2)|N_0L_0I_0\rangle$, obtained for a neutral mesic atom with the functions for the electron shell obtained numerically within the framework of the Hartree-Fock-Slater method. The results are reproduced below in Table II for the region $1\leqslant Z\leqslant 5$, where relativistic effects can be neglected.

TABLE II.

Electron radial elements	μ-atom*			
	μLi (He)	μBe (Li)	μB (Be)	μC (B)
$\langle 2p\|(\frac{1}{x})^2\|1s\rangle$	0.208	0.314	0.804	1.67
$\langle 3p\|(\frac{1}{x})^2\|1s\rangle$	0.120	0.184	0.390	0.582
$\langle 4p\|(\frac{1}{x})^2\|1s\rangle$	0.0795	0.122	0.242	0.343
$\langle 2p\|(\frac{1}{x})^2\|2s\rangle$	−0.0297	−0.0682	−0.124	−0.206
$\langle 3p\|(\frac{1}{x})^2\|2s\rangle$	$-2.83\cdot 10^{-3}$	$-2.49\cdot 10^{-2}$	$-3.93\cdot 10^{-2}$	$-4.45\cdot 10^{-2}$
$\langle 4p\|(\frac{1}{x})^2\|2s\rangle$	$1.34\cdot 10^{-4}$	$-1.40\cdot 10^{-2}$	$-2.13\cdot 10^{-2}$	$-2.32\cdot 10^{-2}$
$\langle 3s\|(\frac{1}{x})^2\|2p\rangle$	$-4.57\cdot 10^{-3}$	$-1.28\cdot 10^{-2}$	$-1.82\cdot 10^{-2}$	$-2.32\cdot 10^{-2}$
$\langle 3d\|(\frac{1}{x})^2\|2p\rangle$	$2.29\cdot 10^{-2}$	$2.33\cdot 10^{-2}$	$2.43\cdot 10^{-2}$	$2.32\cdot 10^{-2}$

*The analogous atom is indicated in brackets.

Utilizing these data and the values of the differences in the energies of the $|2s\tfrac{1}{2}\rangle$ and $|2pj_k\rangle$ meson orbits and noting that

$$\varepsilon(2p^3/_2)-\varepsilon(2p^1/_2)\approx{}^1/_{32}m_\mu c^2(Ze^2/\hbar c)^4\approx 0.94Z^4\ [\text{eV}], \tag{18}$$

we obtain for the amplitudes of the $\{k\}$-configurations quantities of the order of magnitude $a(\{k\})\approx 10^{-4}-10^{-3}$,

TABLE III.

F_1	Configuration $\{[2pJ_k]^1\} \mid 2pj_k\rangle$	ξ_{F_1}	Amplitude of the admixture $a(\{k\}J_k;\ 2pj_k;\ F_1)$
0	$\{[2p^1/_2]^1\} \mid 2p^1/_2\rangle$	$-1/3$	$-1.31\cdot10^{-3}$
0	$\{[2p^3/_2]^1\} \mid 2p^3/_2\rangle$	$-\sqrt{2/9}$	$-1.86\cdot10^{-3}$
1	$\{[2p^1/_2]^1\} \mid 2p^1/_2\rangle$	$-1/9$	$-4.37\cdot10^{-4}$
1	$\{[2p^1/_2]^1\} \mid 2p^3/_2\rangle$	$+\sqrt{8/9}$	$1.24\cdot10^{-3}$
1	$\{[2p^3/_2]^1\} \mid 2p^1/_2\rangle$	$-\sqrt{8/9}$	$-1.24\cdot10^{-3}$
1	$\{[2p^3/_2]^1\} \mid 2p^3/_2\rangle$	$-\sqrt{10/9}$	$-1.38\cdot10^{-3}$

which is considerably greater than the value of the amplitude $\delta_W \approx (10^{-7} - 5\times10^{-7})$.

As an example we consider the ion $(\mu Li_3^6)^+$ with its electron in the initial configuration $\{0\} = \{[1s\tfrac{1}{2}]^1\}$. Here the differences in the energies of the meson orbits are equal to

$$\varepsilon(2p^1/_2) - \varepsilon(2s^1/_2) = 1.1\ \text{eV}\ ,\ \varepsilon(2p^3/_2) - \varepsilon(2p^1/_2) \approx 0.76\ \text{eV}.$$

The configuration—the electron in the $1s\tfrac{1}{2}$ orbit and the meson in the $|2s\tfrac{1}{2}\rangle$ orbit—gives rise to a group of states closely spaced in energy of the system with total angular momentum $F_1 = 0$ and $F_1 = 1$. The nearest satellite line of energy $\hbar\omega_1 \approx \hbar\omega_0 - 41$ [eV] is produced by states of the electron configuration $\{[2p\tfrac{1}{2}]^1\}$ and $\{[2p\tfrac{3}{2}]^1\}$ with the meson occupying the $|2p\tfrac{1}{2}\rangle$ or the $|2p\tfrac{3}{2}\rangle$ orbit. For this simple case the results of calculations of the configuration factor and of the mixing of the states are summarized in Table III.

6. We examine the angular distribution of quanta emitted by a mesic atom polarized beforehand as the meson makes the $2s\tfrac{1}{2} \to 1s\tfrac{1}{2}$ transition. Suppose that the polarized meson is captured into a $|2s\tfrac{1}{2}\rangle$ orbit and that an electron configuration with $N \neq 0$ is formed in the shell of the mesic atom. Such a system is described by the superposition $\Psi_{F_1 f_1}$ of states (6):

$$\Psi_{F_1}(2s^1/_2) = \sum_{f_1} b_{f_1}(F_1)\,\Psi_{F_1 f_1}(2s^1/_2), \tag{19}$$

with the amplitudes $b_{f_1}(F_1)$ fixing the spin-tensor describing the occupancy of the level $|F_1\rangle$

$$\rho_{00}(F_1) = \sum_{f_1} |b_{f_1}(F_1)|^2 \tag{20}$$

and the spin-tensor describing the polarization of the mesic atom

$$\rho_{10}(F_1) = \sum_{f_1} (F_1 1 f_1 0 | F_1 f_1)\,|b_{f_1}(F_1)|^2, \tag{21}$$

the existence of which is due to the initial polarization of the captured meson. The spin-tensors of higher ranks are in this case equal to zero. As the result of the radiative transition of the meson into a $|1s\tfrac{1}{2}\rangle$-orbit the mesic atom goes over into one of the final states of total angular momentum F_2 which belongs either to the group of states with initial electron configuration $|\{0\}J_0\rangle$, or to the group with the altered electron configuration $|\{k\}J_k\rangle$; in the former case the quantum $\hbar\omega_0$ is emitted, in the latter case the quantum $\hbar\omega_k$ shifted in energy is emitted. If the angular momenta of the electron shell J_0, J_k are not equal to zero then the weak neutral interaction of the meson with the nucleus mixes the meson state $|2s\tfrac{1}{2}\rangle$ with the states $|2p\tfrac{1}{2}\rangle$ and $|2p\tfrac{3}{2}\rangle$, and this manifests itself in the interference term of the angular distribution of the unshifted quantum $\hbar\omega_0$. The probability of the transition $2s\tfrac{1}{2} \to 1s\tfrac{1}{2}$ per unit time with the emission of the quantum $\hbar\omega_0$ at an angle θ (solid angle element $d\Omega$) to the direction of polarization of the mesic atom contains terms proportional to $\rho_{00}(F_1)$ and $\rho_{10}(F_1)$:

$$\begin{aligned} &W(\hbar\omega_0;\ \theta;\ F_1(2s^1/_2) \to F_2\{0\}J_0;\ (1s^1/_2))\,d\Omega/4\pi \\ &= [W_0(\hbar\omega_0;\ F_1 \to F_2)\,\rho_{00}(F_1) + W_1(\hbar\omega_0;\ F_1 \to F_2)\,\rho_{10}(F_1)\cos\theta]\,d\Omega/4\pi. \end{aligned} \tag{22}$$

Here the following quantities have been introduced

$$\begin{aligned} W_0(\hbar\omega_0; F_1 \to F_2) &= |a_0(\{0\}J_0; 2s^1/_2; F_1)|^2 [u(F_1 J_0 1^1/_2; {}^1/_2 F_2)]^2 W(M1; 2s \to 1s) \\ &+ \Big| \sum_{(2pj_k)} \delta_W(\{0\}J_0; 2pj_k; F_1)\, u(F_1 J_0 1^1/_2; j_k F_2) \Big|^2 W(E1; 2p \to 1s), \end{aligned} \tag{23}$$

$$\begin{aligned} W_1(\hbar\omega_0; F_1 \to F_2) &= 3\sqrt{2}\,[W(M1; 2s \to 1s)\, W(E1; 2p \to 1s)]^{1/2} \\ &\times \operatorname{Im}\Big\{ \sum_{(2pj_k)} \delta_W(\{0\}J_0; 2pj_k; F_1)\, a^*(\{0\}J_0; 2s^1/_2; F_1) \\ &\times u(F_1 J_0 1^1/_2; {}^1/_2 F_2)\, u(F_1 J_0 1^1/_2; j_k F_2)\, u(1 F_2 1 F_1; F_1 1) \Big\}, \end{aligned} \tag{24}$$

where the rates of the meson $E1$ and $M1$ transitions have been separated (cf., formulas (2)). In contrast to the $\hbar\omega_0$ quantum the angular distribution of the $\hbar\omega_k$ quantum shifted in energy is isotropic and proportional only to the tensor $\rho_{00}(F_1)$:

$$\begin{aligned} &W(\hbar\omega_k;\ \theta;\ F_1(2s^1/_2) \to F_2\{k\}J_k(1s^1/_2))\,d\Omega/4\pi \\ &= W(\hbar\omega_k;\ F_1 \to F_2\{k\}J_k)\,\rho_{00}(F_1)\,d\Omega/4\pi. \end{aligned} \tag{25}$$

Here we have introduced the abbreviated notation:

$$\begin{aligned} &W(\hbar\omega_k; F_1 \to F_2\{k\}J_k) \\ &= \Big| \sum_{(2pj_k)} a(\{k\}J_k; 2pj_k; F_1)\, u(F_1 J_k 1^1/_2; j_k F_2) \Big|^2 W(E1; 2p \to 1s). \end{aligned} \tag{26}$$

The quantum detector has a finite energy resolving power $\Delta\hbar\omega$, and if together with the $\hbar\omega_0$ quanta of the satellite lines $\hbar\omega_k$ fall into the acceptance band of the detector $\hbar\omega_0 - \Delta\hbar\omega \leq \hbar\omega \leq \hbar\omega_0 + \Delta\hbar\omega$, then α—the anisotropy coefficient of the observed angular distribution of the quanta $w(\theta) = 1 + \alpha\cos\theta$—is determined also by the contribution of these lines

$$\begin{aligned} \alpha = \sum_{F_1 F_2} \rho_{10}(F_1)\, W_1(\hbar\omega_0; F_1 \to F_2) \Big[&\sum_{F_1 F_2} \rho_{00}(F_1)\, W_0(\hbar\omega_0; F_1 \to F_2) \\ &+ \sum_{F_1 F_2} \sum_{\{k\}J_k} W(\hbar\omega_k; F_1 \to F_2\{k\}J_k)\,\rho_{00}(F_1) \Big]^{-1}. \end{aligned} \tag{27}$$

A similar formula holds also for circular polarization of the quanta.

For example, in the case of $(\mu Li_3^6)^+$ the nearest satellite line has the intensity $|a|^2 W(E1; 2p \to 1s) = 1.04\times10^7$ sec^{-1}, while $W(M1) = 30$ sec^{-1}. Therefore, if the detector does not separate out the satellite line, i.e., $\Delta\hbar\omega \gtrsim 40$ eV, then the observed effect of neutral currents is weakened by approximately five orders of magnitude (10^5). From this follows the limitation on the energy resolving power of the quantum detector $\Delta\hbar\omega/\hbar\omega < 10^{-3}-10^{-4}$, and it is not possible in the experiment to let the mesic ion be completely neutralized, since as the number of electrons N in the shell of the mesic atom increases the magnitude of the shift of the satellite line $\hbar\omega_1$ closest to $\hbar\omega_0$ is significantly diminished.

7. The capture of electrons into the shell of the mesic atom, together with the effect discussed above of the shifted lines $\hbar\omega_k$, leads to a decrease in the total yield of quanta since a new competing channel is opened up of $E0$ conversion in the $2s\to 1s$ meson transition with the ejection of an electron into the continuous spectrum of states. In the case of a single $1s$ electron in the shell of a light mesic atom $(1\leqslant Z\leqslant 10)$ we obtain for $E0$ conversion in the Born approximation

$$W(E0;2s\to 1s)=\frac{e^2}{\hbar a_0}\frac{2\sqrt{3}}{9}\left(\frac{Z-1}{Z}\right)^3\left(\frac{m_\mu}{m}\right)^{1/2}\left(\frac{m}{m_\mu}\right)^4\langle 1s|x^2|2s\rangle^2, \qquad (28)$$

where

$$\langle 1s|x^2|2s\rangle=-2^9\cdot 3^{-5}\sqrt{2}\approx -3,$$
$$e^2/\hbar a_0=4.134\cdot 10^{16}\ \mathrm{sec}^{-1},\ a_0=\hbar^2/me^2, \qquad (29)$$

or

$$W(E0;2s_{1/2}\to 1s_{1/2})\approx\left(\frac{Z-1}{Z}\right)^3\cdot 1.1\cdot 10^9\ [\mathrm{sec}^{-1}]. \qquad (30)$$

The probability of $E0$ conversion depends very little on the nuclear charge, the role of this process falls off significantly with increasing Z since the probabilities of the radiative $M1$ and $E1$ transitions increase.

§3. CONFIGURATION MIXING OF THE STATES OF μLi IN A METAL

Suppose that $(\mu\mathrm{Li})^{2+}$ is formed in a metallic target by means of the capture of a μ meson into a $|2s\frac{1}{2}\rangle$ orbit. We estimate the effect of configuration mixing for conduction band electrons utilizing the free electron model which appears to be sufficient to estimate the order of magnitude. For the principal configuration $\{0\}$ of the conduction band electrons we take the momentum sphere filled up to the Fermi momentum

$$p_0=\pi[3N/\pi]^{1/3}. \qquad (31)$$

In accordance with[9] we have $N=4.6\times 10^{22}\ \mathrm{cm}^{-3}$ for metallic Li in the free electron model, while the energy of the Fermi-level is $E_0=4.72$ eV. For the mesic ions $(\mu\mathrm{Li}_3^6)^{2+}$ and $(\mu\mathrm{Li}_3^7)^{2+}$ in the group of meson orbits considered below

$$|2s^1/_2\mu_0\rangle\equiv|j_0\mu_0\rangle,\quad |2p^1/_2\mu_1\rangle\equiv|j_1\mu_1\rangle,\quad |2p^3/_2\mu_3\rangle\equiv|j_3\mu_3\rangle$$

the $|2s\frac{1}{2}\rangle$ level lies below the $|2pj_k\rangle$ levels and therefore for the mesic atom in the $|2s\frac{1}{2}\rangle$ level the only open channel is $E0$ conversion with the ejection of a conduction band electron. In the free electron model we have for the probability of this process

$$W(E0;2s^1/_2\to 1s^1/_2)\approx\frac{e^2}{\hbar a_0}\cdot 2\cdot 3^{1/3}\pi\left(\frac{1}{Z}\right)^3\left(\frac{m}{m_\mu}\right)^4\left(\frac{m_\mu}{m}\right)^{1/2}(Na_0^3). \qquad (32)$$

From this in the case of metallic lithium with $Na_0^3 = 6.8\times 10^{-3}$ we obtain $W(E0)=0.88\times 10^6\ \mathrm{sec}^{-1}$.

On the other hand the presence of free electrons leads to mixing of the configurations in the system: $(\mu\mathrm{Li})^{2+}$ plus conduction band electrons; to the initial configuration $\{0\}|2s\frac{1}{2}\mu_0\rangle$ there are admixed configurations in which the meson exists in $|2p\frac{1}{2}\mu_1\rangle$ or $|2p\frac{3}{2}\mu_3\rangle$ states, while the electron makes the transition from the states $|\mathbf{p}_1\sigma_1\rangle$ of the filled momentum sphere (σ is the spin component) into the state $|\mathbf{p}_2\sigma_2\rangle$ outside the Fermi sphere.

For the amplitude of the admixed state

$$|[\mathbf{p}_2\sigma_2]^1[\mathbf{p}_1\sigma_1]^{-1}\{0\}\rangle|2pj_k\mu_k\rangle, \qquad (33)$$

utilizing first order perturbation theory in terms of H_{int} (3) we obtain

$$a([\mathbf{p}_2\sigma_2]^1[\mathbf{p}_1\sigma_1]^{-1}\{0\}|2pj_k\mu_k\rangle)=i\frac{1}{V}\delta_{\sigma_1\sigma_2}\left[\sqrt{3}\frac{m}{m_\mu}\frac{1}{Z}\frac{4\pi^2}{\sqrt{\pi}}\right]$$
$$\times(1j_0m'\mu_0|j_k\mu_k)Y^*_{1m'}(\mathbf{q})\frac{1}{q[p_2^2-p_1^2+Q_k^2]}, \qquad (34)$$

where V is the normalization volume, $\mathbf{q}=\mathbf{p}_1-\mathbf{p}_2$ and for μLi

$$Q_k^2=(2m/\hbar^2)[\varepsilon(2pj_k)-\varepsilon(2s^1/_2)]>0. \qquad (35)$$

Further we neglect the weakly manifested angular correlation between the transferred momentum $\mathbf{q}$ and the propagation vector of the $\hbar\omega$ quantum emitted in the transition of a meson to the $|1s\frac{1}{2}\rangle$ orbit, and make an overall estimate of the magnitude of the admixture of states in which the meson is situated in the $|2pj_k\rangle$ orbit.

With this in mind we introduce the quantity $S(2pj_k;y)$. According to the definition

$$S(2pj_k;y)=\sum_{\sigma_1\sigma_2}\sum_{\mu_0\mu_km'}V^2(2\pi)^{-6}$$
$$\times\oint d\Omega_{p_1}\oint d\Omega_{p_2}\int_0^{p_0}dp_1p_1^2\int_{p_0}^{\mathscr{P}}dp_2p_2^2|a([\mathbf{p}_2\sigma_2]^1[\mathbf{p}_1\sigma_1]^{-1}\{0\}|2pj_k\mu_k\rangle)|^2, \qquad (36)$$

where the maximum value of the momentum of the electron outside the Fermi sphere $\mathscr{P}=p_0y$ is regarded as a variable quantity. The existence of the admixtures (33) is manifested in the radiative transition of a meson to the $|1s\frac{1}{2}\rangle$ orbit in the fact that along with the $\hbar\omega_0$ quanta shifted quanta $\hbar\omega$ are emitted whose energy is distributed continuously below the limit $\hbar\omega_0$. We estimate the total intensity of the shifted quanta of energy $\hbar\omega_0-E_0y^2\leqslant\hbar\omega\leqslant\hbar\omega_0$ using the total magnitude of the admixture of the states (33):

$$W(y)\approx W(E1;2p\to 1s)[S(2p^1/_2;y)+S(2p^3/_2;y)]. \qquad (37)$$

After relatively simple calculations we represent $S(2pj_k;y)$ in the form

$$S(2pj_k;y)=\rho_{00}(j_0)(2j_k+1)\frac{12}{\pi^2}\left(\frac{m}{m_\mu}\frac{1}{Z}\right)^2\mathscr{F}(\beta_k^2;y), \qquad (38)$$

$$\mathscr{F}(\beta^2;y)=\frac{1}{\beta^2}\int_1^y d\xi\,\xi\left\{\frac{\xi}{(\xi^2+\beta^2)^{1/2}}\ln\frac{(\xi^2+\beta^2)^{1/2}+1}{(\xi^2+\beta^2)^{1/2}-1}-\frac{\xi^2-1}{\xi^2-1+\beta^2}\ln\frac{\xi+1}{\xi-1}\right\}, \qquad (39)$$

$$\beta_k^2=[\varepsilon(2pj_k)-\varepsilon(2s^1/_2)]/E_0. \qquad (40)$$

In the range of interest to us $1 \geq \beta^2 \geq 0.1$ and $1.2 \leq y \leq 4$ the function $\mathcal{F}(\beta^2; y)$ varies between the limits from 0.2 to 2.3 and practically reaches saturation for $\mathcal{P} = 2p_0$ ($y = 2$).

Utilizing the data of Table I for $(\mu Li_3^6)^{2+}$ and $(\mu Li_3^7)^{2+}$ and the value of the Fermi energy $E_0 = 4.72$ eV, we obtain the probability of emission per unit time for the shifted quanta of energy $\hbar\omega_0 - 20$ eV $\leq \hbar\omega \leq \hbar\omega_0$:

$$W(y=2) = 1.94 \cdot 10^8 \text{ sec}^{-1} \text{ for } (\mu Li_3^6)^{2+},$$
$$W(y=2) = 1.50 \cdot 10^8 \text{ sec}^{-1} \text{ for } (\mu Li_3^7)^{2+}. \quad (41)$$

In accordance with formula (27) we have for the coefficient α taking into account the effect of the shifted quanta

$$\alpha = \frac{\rho_{10}(2s^1/_2) W_1(\hbar\omega_0)}{\rho_{00}(2s^1/_2)[W_0(\hbar\omega_0) + W(y)]}, \quad (42)$$

where W_1 and W_0 are obtained from (23) and (24) by setting $J_0 = J_k = 0$, $F_1 = F_2 = \frac{1}{2}$. The value of the upper limit $\mathcal{P} = p_0 y$ is determined by the energy resolving power of the quantum detector. If $\Delta\hbar\omega \approx 20$ eV, then for $(\mu Li)^{2+}$ ions in a metal the value of α is approximately by seven orders of magnitude smaller than for an isolated mesic ion $(\mu Li)^{2+}$, since $W_0(\hbar\omega_0) \approx 30$ sec^{-1}, while $W(y=2) \approx 10^8$ sec^{-1}.

Consequently observation of the effect of the neutral interaction between the meson and the Li nucleus utilizing metallic targets is practically impossible. Improvement of the resolution $\Delta\hbar\omega$ to a value of ~ 2 eV does not lead to any significant change in the coefficient α.

§4. CONVERSION OF THE E1 TRANSITION $2s\frac{1}{2} \to 2pj_k$ IN A MESIC ATOM ON CONDUCTION BAND ELECTRONS IN A METAL

In the case of μBe the $|2s\frac{1}{2}\rangle$ level lies above the $|2p\frac{1}{2}\rangle$ level, while for $Z \geq 5$ the $|2s\frac{1}{2}\rangle$ level lies higher also than the $|2p\frac{3}{2}\rangle$ level, and therefore in the formation of a mesic atom in metals with $Z \geq 4$ a conversion transition $|2s\frac{1}{2}\rangle \to |2pj_k\rangle$ ($j_k = \frac{1}{2}; \frac{3}{2}$) is possible with the ejection of an electron from the filled Fermi sphere. In order to estimate the probability of $E1$ conversion we utilize the free electron model and the Born approximation. Neglecting the contribution of the vector potential and taking into account only the dipole–dipole interaction (3), we obtain

$$W(E1; 2s^1/_2 \to 2pj_k) = \frac{e^2}{\hbar a_0}(2j_k+1)\, 12\pi \left(\frac{3}{\pi}\right)^{1/3} (Na_0^3)^{1/3} \left(\frac{1}{Z}\frac{m}{m_\mu}\right)^2 \mathcal{F}(\beta_k^2), \quad (43)$$

where

$$\mathcal{F}(\beta^2) = \frac{1}{2}\left[(1+\beta^2)\ln\frac{(1+\beta^2)^{1/2}+1}{(1+\beta^2)^{1/2}-1} - (1+\beta^2)^{1/2}\right], \quad (44)$$
$$\beta_k^2 = [\varepsilon(2s^1/_2) - \varepsilon(2pj_k)]/E_0 > 0, \quad (45)$$

where E_0 is the energy of the Fermi-level; N is the number of free electrons per cm^3; $a_0 = 5.29 \times 10^{-9}$ cm.

The function $\mathcal{F}(\beta^2)$ is close to unity over a wide range of values of β^2. For a density $N \approx 10^{22}$ cm^{-3} and $Z = 4-5$ we obtain in accordance with (43) the estimate $W_{conv}(E1) \approx 10^{11}$ sec^{-1}, and this practically prohibits conducting an experiment utilizing metallic targets.

§5. STARK EFFECT OF ATOMIC FIELDS OF THE MEDIUM

1. Assume that when a meson is captured into a $|2s\frac{1}{2}\rangle$ orbit a mesic ion is formed devoid of electrons ($N = 0$), and that no capture of electrons into the shell occurs during its lifetime. Under these conditions the only background mechanism can be the interaction of the mesic atom with the electric field of the medium $\mathbf{E}(t)$:

$$H_{\text{int}} = -(er\mathbf{E}(t)), \quad (46)$$

which mixes the $|2s\frac{1}{2}\rangle$ state with the closely lying $|2p\frac{1}{2}\rangle$ and $|2p\frac{3}{2}\rangle$ states, and this leads to a radiative transition of the meson into the $|1s\frac{1}{2}\rangle$ orbit with the emission of a quantum $\hbar\omega_0$ with practically no shift in energy. We further denote by $\langle W_{st}(\mathbf{E})\rangle$ the rate of this Stark radiation process in the field of the medium $\mathbf{E}(t)$. Taking the Stark effect into consideration we have for the coefficient of angular distribution of the quanta $\hbar\omega_0$

$$\alpha = \frac{\rho_{10}(2s^1/_2) W_1(\hbar\omega_0)}{\rho_{00}(2s^1/_2)[W_0(\hbar\omega_0) + \langle W_{st}(\mathbf{E})\rangle]}, \quad (47)$$

where $W_1(\hbar\omega_0)$ and $W_0(\hbar\omega_0) \approx W(M1; 2s \to 1s)$ are defined by formulas (23) and (24) in which $J_0 = J_k = 0$, $F_1 = F_2 = \frac{1}{2}$.

Different sources exist for the field in the medium $\mathbf{E}(t)$, but for a neutral gaseous medium the principal one is the dipole electric moment induced in the shell of the atom by the Coulomb field of the incident mesic ion ($Z \geq 2$), while for the μH-system the intra-atomic field is manifested directly in the collision process.

2. The estimates given below for the lower limit of $\langle W_{st}\rangle$ are obtained using a simplified model of the collision process (passage along straight line trajectories). They should be regarded as preliminary ones demonstrating the importance of this mechanism which limits significantly the conditions under which the experiment can be performed.

For the sake of brevity we introduce the notation for the group of states of the mesic atom selected below:

$$|2s^1/_2\mu_0\rangle \equiv |j_0\mu_0\rangle, \quad |2p^1/_2\mu_1\rangle \equiv |j_1\mu_1\rangle, \quad |2p^3/_2\mu_3\rangle \equiv |j_3\mu_3\rangle.$$

The operator $H_{\text{int}} = -(e\mathbf{r}\mathbf{E}(t))$ connects the $|j_0\mu_0\rangle$ and $|j_k\mu_k\rangle$ states ($k = 1, 3$):

$$\langle j_0\mu_0|H_{\text{int}}|j_k\mu_k\rangle = -\hbar b \sum_\nu (1j_0\nu\mu_0|j_k\mu_k) Q_\nu(t), \quad (48)$$

here we have introduced: the Stark constant

$$b = 3\frac{e^2}{\hbar a_0}\frac{m}{m_\mu}\frac{1}{Z} \approx 6 \cdot 10^{14} Z^{-1} [\text{sec}^{-1}], \quad a_0 = \frac{\hbar^2}{me^2}$$

and $Q_\nu(t)$ is the spherical component ($\nu = 0; \pm 1$) of the field expressed in atomic units:

$$\mathbf{E}(t) = |e/a_0^2|\mathbf{Q}(t). \quad (49)$$

For amplitudes of the occupancies of the states under consideration $C(j_0\mu_0 t)$ and $C(j_k\mu_k t)$ we obtain in the usual manner the system of equations in which radiation damping of the amplitudes of the $|2pj_k\rangle$ states $2\lambda = 1.29 \times 10^{11} Z^4$ sec^{-1} has been taken into account:

$$\frac{d}{dt}C(j_0\mu_0 t) = ib\sum_{j_k\mu_k}\sum_{\nu}(1j_0\nu\mu_0|j_k\mu_k)e^{-i\omega_k t-\lambda t}Q_\nu C(j_k\mu_k t), \tag{50}$$

$$\frac{d}{dt}C(j_k\mu_k t) = ib\sum_{\mu_0}\sum_{\nu}(1j_0\nu\mu_0|j_k\mu_k)e^{i\omega_k t+\lambda t}Q_\nu^* C(j_0\mu_0 t), \tag{51}$$

$$\omega_k = \omega_1, \omega_3; \quad \omega_k = [\varepsilon(2pj_k) - \varepsilon(2s^1/_2)]/\hbar. \tag{52}$$

For $Z \geqslant 2$ the frequencies ω_k are of the order of magnitude of $\gtrsim 10^{15}$ sec^{-1}, while the field $\mathbf{Q}(t)$ changes over a characteristic transit time $\tau = 2a_0/v$ which is equal to $\sim 10^{-13}$ sec for $v = 10^5$ cm/sec. Consequently $\omega_k\tau \gg 1$, and an expansion in terms of the parameters $1/\omega_k\tau \ll 1$ is possible. Integrating equation (51) by parts and substituting into (50) we obtain in first order with respect to $1/\omega_k\tau$

$$|C(j_0\mu_0 t)|^2 = |C(j_0\mu_0 0)|^2 \exp\left[-\eta\int_0^t |\mathbf{Q}|^2 dt'\right], \tag{53}$$

where

$$\eta = \frac{2}{3}\lambda\left(\frac{b^2}{\omega_1^2+\lambda^2} + \frac{2b^2}{\omega_3^2+\lambda^2}\right). \tag{54}$$

We define the average field $\langle|\mathbf{Q}|^2\rangle$ by means of the equation

$$\int_0^t |\mathbf{Q}(t')|^2 dt' = t\langle|\mathbf{Q}|^2\rangle$$

and from this determine the Stark rate of the radiative transition:

$$\langle W_{st}\rangle = \frac{2}{3}\lambda\left(\frac{b^2}{\omega_1^2+\lambda^2} + \frac{2b^2}{\omega_3^2+\lambda^2}\right)\langle|\mathbf{Q}|^2\rangle. \tag{55}$$

However, this estimate is valid for relatively weak fields which satisfy the condition

$$\frac{\lambda}{\omega_k}\frac{b^2}{\omega_k^2}|\mathbf{Q}^2| \ll 1, \tag{56}$$

while in the case of strong fields it is necessary to seek the solution of the system (50) and (51) more accurately (for example by the method of a canonical transformation).

3. In a single collision of a mesic atom with an atom of the medium the field $\mathbf{Q}(t)$ differs from zero over an interval of the order of $\tau \approx 2a_0/v$. Assume that the mesic atom passes with velocity v along a straight line trajectory with an impact parameter x with respect to the atom of the medium.

We introduce the quantity $S(x)$ which characterizes the change in population of the $|2s\frac{1}{2}\rangle$ level in a single collision:

$$|C(j_0\mu_0; T)|^2 = |C(j_0\mu_0, -T)|^2 \exp[-\eta S(x)], \tag{57}$$

where $(-T)$ is the instant "before collision," T is the instant "after collision" with $\mathbf{Q}(+T) = 0$. When condition (56) is satisfied along the trajectory we have for $S(x)$

$$S(x) = \lim_{T\to\infty}\int_{-T}^{+T}|\mathbf{Q}(xt')|^2 dt'. \tag{58}$$

We shall obtain the change in population $|C(j_0\mu_0 t)|^2$ along the path $L = vt$ when the mesic atom moves with velocity v in a medium of density ρ [at/cm^3] by averaging over all possible configurations of the atoms of the medium along the path of the mesic atom,

$$\langle|C(j_0\mu_0 t)|^2\rangle = |C(j_0\mu_0 0)|^2 \exp\left[-tv\rho\int_0^\infty (1-e^{-\eta S(x)})2\pi x\,dx\right] \tag{59}$$

and, consequently,

$$\langle W_{st}(\mathbf{E})\rangle = v\rho\int_0^\infty (1-e^{-\eta S(x)})2\pi x\,dx. \tag{60}$$

For $v \leqslant 10^5$ cm/sec for mesic atoms ($Z \geqslant 2$) the usual order of magnitude is $2a_0\eta/v \leqslant 10^{-3}Z^{-2}$, and therefore only for small impact parameters $x \leqslant x_0 < a_0$ are the values of $\eta S(x) \sim 1$, while in the remaining interval $x > x_0$ we have $\eta S(x) < 1$. We separate out the region of strong fields $x \leqslant x_0$ and in order to obtain an estimate assume $S(x) = S(x_0)$ for $x \leqslant x_0$, while in the remaining interval $x > x_0$ we restrict ourselves to the linear term of the expansion in terms of $\eta S(x)$; in this manner we obtain the estimate

$$\langle W_{st}(\mathbf{E})\rangle \geqslant v\rho\left[\pi x_0^2(1-e^{-\eta S(x_0)}) + 2\pi\eta\int_{x_0}^\infty S(x)x\,dx\right]. \tag{61}$$

4. For the effect of polarization of a neutral atom of the medium by the Coulomb field of the mesic ion of charge Z_i ($Z_i = Z - 1$) in the case of straight line trajectories the magnitude of $S(x)$ is equal to

$$S(x) = \frac{35}{32}\pi\beta^2 Z_i^2 \frac{a_0}{v}\left(\frac{a_0}{x}\right)^8, \tag{62}$$

where β is the polarizability of the atom of the medium in units of a_0^3; the values of β are given, for example, in Smirnov's book.[10]

We take into account only the contribution of distant trajectories $x \geqslant x_0 \approx a_0$:

$$\langle W_{st}\rangle > \rho a_0^3 \frac{5\pi^2}{24}\beta^2 Z_i^2\left(\frac{a_0}{x}\right)^7 \lambda\left(\frac{b^2}{\omega_1^2+\lambda^2} + \frac{2b^2}{\omega_3^2+\lambda^2}\right). \tag{63}$$

From this for $(\mu\mathrm{He})^+$ ($\lambda = 1.03 \times 10^{12}$ sec^{-1}; $\omega_1 = 2.14 \times 10^{15}$ sec^{-1}; $\omega_3 = 2.36 \times 10^{15}$ sec^{-1}; $b = 3 \times 10^{14}$ sec^{-1}) in a helium medium ($\beta = 1.39$ (cf., [10]) with a density $\rho = 10^{19}$ at/cm^3 we obtain $\langle W_{st}\rangle > 3 \times 10^5$ sec^{-1}, which is significantly greater than $W_0(\hbar\omega_0) \approx 0.51$ sec^{-1}. Consequently, under these conditions the observed effect of a weak neutral interaction will be weakened by approximately six orders of magnitude. Comparing $\langle W_{st}\rangle$ and $W(M1; 2s \to 1s)$ we estimate the maximum allowable density of the medium $\rho \leqslant 10^{14}$ cm^{-3} for $(\mu\mathrm{He})^+$. For heavier atoms ($Z > 4$) the value of $\langle W_{st}\rangle$ falls off as Z^{-4}, since $b^2 \sim Z^{-2}$; $\lambda \sim Z^4$;

$\omega_k^2 \sim Z^8$, while $W(M1; 2s \to 1s)$ grows rapidly ($\sim Z^{10}$). Thus, the effect of $\langle W_{st} \rangle$ falls off; but also for an isolated mesic ion the effect of neutral interaction diminishes with increasing Z.

5. In contrast to other mesic neutral μH can penetrate deeply into the shell of the atom of the medium, and in this case the process of $E0$ conversion of the $2s\frac{1}{2} \to 1s\frac{1}{2}$ transition of μH in the shell of an atom of the medium is possible. In the Born approximation we obtain for the damping of the population of the $|2s\frac{1}{2}\rangle$ level of μH moving along a straight line trajectory in a hydrogen medium

$$\langle W(E0)\rangle \approx \pi\rho a_0^3 \frac{e^2}{\hbar a_0} 2\cdot 3^{1/2} \left(\frac{m}{m_\mu}\right)^4 \left(\frac{m_\mu}{m}\right)^{1/2}. \qquad (64)$$

Of greater significance turns out to be the Stark effect of the intra-atomic field. In the model utilizing straight line trajectories in the case of a collision of μH and an atom of hydrogen of the medium we obtain as a result of a numerical calculation assuming $x_0 = a_0/4$ where the condition of the weak field (56) is still satisfied

$$\langle W_{st}(E)\rangle > \rho a_0^3 14\pi\lambda \left(\frac{b}{\omega}\right)^2 \approx 1.15\cdot 10^{13} \rho a_0^3 \ [\text{sec}^{-1}].$$

For a density of $\rho = 10^{19}$ at/cm^3 $\langle W_{st}\rangle \geq 1.7\times 10^7$ sec^{-1}, while $W(M1; 2s \to 1s) = 4.6\times 10^{-4}$ sec^{-1}. Consequently, the Stark effect of the intra-atomic field of the atoms of the medium practically does not allow us to observe the effect of the neutral weak interaction in a μH system for medium densities of $\rho \geq 10^{11}$ at/cm^3. The estimates given above provide evidence of very rigorous limitations on the conditions of carrying out an experiment which are imposed by the Stark effect in the act of collision, and therefore the values of $\langle W_{st}(\mathbf{E})\rangle$ have to be calculated more accurately.

When a μH atom passes through a H_2 molecule the intra-atomic field acts on the average over a greater part of the trajectory than in the case of passage through an atom of hydrogen. Correspondingly the allowable density of H_2 gas must be lower than the one obtained in the estimate for atomic hydrogen.

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Translated by G. Volkoff

Measurement of the polarization correlation coefficient C_{nn}^{pp} in elastic *pp* scattering at 610 MeV

N. S. Borisov, L. N. Glonti, M. Yu. Kazarinov, Yu. M. Kazarinov, Yu. F. Kiselev, V. S. Kiselev, V. N. Matafonov, G. G. Macharashvili, B. S. Neganov, I. Strakhota, V. N. Trofimov, Yu. A. Usov, and B. A. Khachaturov

Joint Institute for Nuclear Research

(Submitted July 13, 1976)
Zh. Eksp. Teor. Fiz. 72, 405–410 (February 1977)

We measured the polarization correlation coefficient C_{nn} in elastic *pp* scattering at an energy 610 ± 10 MeV at four c.m.s. scattering angles (40, 67, 78, and 90°). We used in the experiments a polarized proton beam with maximum polarization 0.39 ± 0.02 and a polarized proton target of the frozen type. The maximum polarization of the target was 0.97 ± 0.04.

PACS numbers: 13.75.Cs

The main reasons that induced us to measure the correlations of the polarization in elastic *pp* scattering at 610 MeV were the following:

1. The ambiguity in the determination of the amplitude of the elastic *NN* scattering at 630 MeV. If it is assumed that the one-pion approximation is valid in elastic *NN* scattering starting with orbital angular momenta $l \geq 7$, then a phase-shift analysis at this energy yields at least two equally probable (in the sense of the χ^2 criterion) solutions.[1] The simplest way of discriminating between these solutions is to measure C_{nn}^{pp} with a relative error ~10%.

2. The previously observed maximum, at 600–700 MeV, in the energy dependence of C_{nn}^{pp} in scattering through an angle of 90° (c.m.s.), which may point to

