1435 (1969) [Sov. Phys. JETP 30, 777 (1970)].
[7]L. E. Hartmann and J. M. Luttinger, Phys. Rev. 151, 430 (1966).
[8]I. S. Gradshteĭn and I. M. Ryzhik, Tablitsy integralov, summ, ryadov i proizvedeniĭ (Tables of Integrals, Sums, Series, and Products), Fizmatgiz, 1963, p. 942 (§8.23) and p. 944 (§8.25) (translation, Academic Press, 1965, pp. 928 and 930).

Translated by W. F. Brown, Jr.

Dynamics of fluctuations at multicritical points

V. V. Prudnikov and G. B. Teĭtel'baum

Kazan' Physicotechnical Institute, USSR Academy of Sciences
(Submitted June 28, 1976)
Zh. Eksp. Teor. Fiz. 72, 308–316 (January 1977)

The dynamics of phase transitions at multicritical points is investigated. Systems whose free energy can be represented in a Ginzburg-Landau form with one order parameter are considered. The dependence of the critical dynamics on conservations laws in the system is considered. Different types of universal dynamical behavior are distinguished. The effect of many-particle excitations on the critical-damping frequency is established. The logarithmic corrections to the theory of dynamic scale invariance that arise from the interaction of fluctuations at a tricritical point are found. The possibility that these effects are manifested in nondegenerate metamagnets is discussed.

PACS numbers: 64.60.Kw

One of the most attractive features of critical phenomena is their universality. This consists in the fact that the critical exponents and a number of other quantities depend only on such characteristics of physical systems as the dimensionality, the number of components of the order parameter, and the symmetry. [1] In the dynamics of critical fluctuations the concept of universality takes on a narrower meaning—the conservation laws for the order parameter and the energy in the system also become important. (Here it is understood that the system is on a critical line in the space of the thermodynamic variables and external fields.)

However, curves of continuous transitions can terminate at certain points, passing into phase-coexistence curves. Such behavior occurs at certain points on the phase diagrams of a ^{3}He–^{4}He mixture, metamagnets, and compressible magnets. [2–10] Such points are critical for several phases at once (ordered, disordered and mixed), and the number σ of phases determines the order of the critical points. It has been established that the static properties of phase transitions change substantially at these points. [11–13] The changes that the dynamics of fluctuations can undergo at multicritical points are unknown. The role of the conservation laws at such points has also not been elucidated. The present paper is devoted to examining this group of questions.

1. At an ordinary critical point ($\sigma=2$) two-particle excitations of the energy density can decay into one-particle excitations of the order parameter ψ, changing the damping of the latter. [14] But in the case of higher critical points the fluctuations of ψ may be coupled nonlinearly with excitations consisting of more than two fluctuations. We show below that if such excitations are not damped they determine the dynamics of the critical fluctuations in many respects.

The local field $\rho_k(\mathbf{x})$ of excitations consisting of k particles can be constructed in the form of a product of k fluctuating fields of the n-component order parameter $\psi(\mathbf{x})$. In the general case the quantity ρ_k is a tensor of rank k. Its components (or their linear combinations) should be such that the operations of the symmetry group of the free energy do not change them. This ensures the conservation of the total field ρ, equal to the integral of the local field $\rho_k(\mathbf{x})$ over the volume (henceforth we shall speak simply of the conservation of the field $\rho_k(\mathbf{x})$).

We shall consider a free energy that is isotropic in the space of the components of the order parameter, when the field ρ_k is characterized by a tensor contracted with respect to the maximum possible number of pairs of indices. As a result, for even k the field ρ_k is a scalar field and for odd k it is a vector field with n components. For many-particle excitations to be manifested in the critical dynamics it is important that, in the equations of motion, the nonlinearities due to the coupling of one-particle and many-particle excitations do not fall off as we go to large lengths (small frequencies); this imposes restrictions on the scaling dimensions of the nonlinearities. This requirement is inherently linked with the condition that the inclusion of many-particle excitations in the equations of motion should not change the equilibrium distribution function of the field $\psi(x)$.

We write the free energy of the system in the form of the functional

$$\frac{\mathscr{H}(\psi)}{T}=\int d^dx\left[\frac{1}{2}|\nabla\psi(\mathbf{x})|^2+\sum_{k=1}^{\sigma}u_{2k}(\psi\psi)^k\right], \qquad (1)$$

where d is the dimensionality of the system. We shall describe the dynamics of the model by the equation

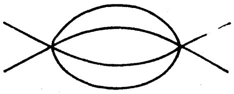

FIG. 1. Dynamic four-point function arising at a tricritical point.

$$\frac{\partial\psi_\alpha}{\partial t}=-\frac{\Gamma_0}{T}\frac{\delta\mathscr{H}}{\delta\psi_\alpha^*}+\Gamma_0 h_\alpha(\mathbf{x},t)+\eta^\alpha(\mathbf{x},t);\quad \alpha=1,2\dots n, \tag{2}$$

where $h(\mathbf{x}, t)$ is the magnetic field and the kinetic coefficient Γ_0 depends on the conservation laws for the order parameter. Thus, when the order parameter is conserved, $\Gamma_0 \to -\Gamma_0\nabla^2$; otherwise, $\Gamma_0 \sim$ const. The introduction of the random Gaussian force $\eta(\mathbf{x}, t)$ takes into account the influence of the heat reservoir. The average $\langle\eta^\alpha(\mathbf{x}, t)\rangle$ is equal to zero, and the correlation function

$$\langle\eta^\alpha(\mathbf{x},t)\eta^\beta(\mathbf{x}',t')\rangle=2\Gamma_0\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')\delta_{\alpha\beta}, \tag{3}$$

where the angular brackets denote averaging over the fluctuations of the random force.

For the subsequent analysis we shall use the renormalization-group method.[14,15] Here it must be remembered that the space dimensionality d_σ below which stable nontrivial solutions of the renormalization-group equations for the coupling constants of the fluctuating fields appear, changes at critical points of higher orders.[11] In the calculations we shall use an expansion in $\varepsilon_\sigma = d_\sigma - d$—the deviation of the dimensionality d of the system from $d_\sigma = 2\sigma/(\sigma-1)$. It is convenient to redefine the bare vertices u_{2k} by including in them the contributions, independent of the wave vector $\mathbf{q}$ and frequency ω, from all closed loops arising in the expansion in the coupling constant. In order to carry out the analysis at a critical point of order σ, at each stage of the calculations we can assume that the redefined vertices u_{2k} with $k<\sigma$ are equal to zero, since their specified values vanish in the critical region.

The interaction of the critical fluctuations leads to the appearance of purely dynamic vertices

$$u_{2k}\sim\omega^{[2k/(k-1)-d](k-1)/2}, \tag{4}$$

including some with less than 2σ legs (cf., e.g., Fig. 1). However, such vertices appear in at least second order in $u_{2\sigma}$. For small internal momenta the vertex $u_{2\sigma}\sim\varepsilon_\sigma$, and, confining ourselves to first order in ε_σ, we can neglect the contribution from vertices with $k<\sigma$. Passage to the region of internal momenta that are not small is characterized by the appearance of momentum dependence of $u_{2\sigma}$, so that, in place of the specified values $\sim\varepsilon_\sigma$, certain functions that are not proportional to ε_σ appear. Vertices u_{2k} with $k<\sigma$ constructed by means of such $u_{2\sigma}$ give nonsingular contributions to the self-energy parts of the dynamic correlators, and these contributions are cancelled in the subtraction accompanying the renormalization of the mass (critical temperature). Consequently the vertices u_{2k} with $k<\sigma$ do not lead to change of the critical dynamics.

We shall now elucidate what complications of the equations of motion can arise as a result of the interaction of the order parameter with many-particle excitations. In order to select, from the nonlinearities which then arise, those which are able to change the critical dynamics as compared with mean-field theory, we must take into account that only nonlinearities with scaling dimension $\Delta<d-\Delta_A$, where Δ_A is the dimension of the relaxing quantity, are important. Wegner[11] has calculated the scaling dimensions of quantities composed of products of fields $\psi(\mathbf{x})$ and has found their dependence on m (the number of pairs of indices over which the contraction is performed) and l (the number of free indices). From all the quantities of this kind, satisfying the inequality given above, we must select the fields whose coupling with the order parameter $\psi(\mathbf{x})$ does not change the equilibrium distribution of $\psi(\mathbf{x})$. At a critical point of order σ the fields $\rho_\sigma(\mathbf{x})$ for which $l+2m=\sigma$ are such fields. According to Wegner,[11] for fields transforming like $\sim|x|^{-\Delta_{ml}}$ under a change of scale, we have

$$\Delta_{ml}=(d-2)(m+{}^1/_2l)+2(\sigma-1)\varepsilon_\sigma g_{ml}(n)g_{\sigma 0}^{-1}(n), \tag{5}$$

$$g_{ml}(n)=\sum_j\frac{\sigma!m!(n/2+l+m-1)!(l+2m-2j)!}{(\sigma-2j)!(j!)^2(m-j)!(n/2+l+m-j-1)!(l+2m-\sigma)!}. \tag{6}$$

The binary correlator of the field ρ_σ depends on the wave vector $\mathbf{q}$ in the following way:

$$\langle\rho_\sigma(\mathbf{q})\rho_\sigma(\mathbf{q}')\rangle\sim g^{-y}\delta(\mathbf{q}+\mathbf{q}'),\quad y=d-2\Delta_{ml}. \tag{7}$$

2. We shall study the dependence of the fields ψ and ρ_σ on time. To obtain the conjugate thermodynamic forces appearing in the right-hand sides of the equations of motion, it is convenient to regard $\rho_\sigma(\mathbf{x}, t)$ as an independent field interacting with $\psi(\mathbf{x}, t)$ and write the following expression for the part of the entropy that depends on these fields:

$$S(\psi,\rho_\sigma)=-\int d^dx\left\{\frac{1}{2}|\nabla\psi|^2+w_{2\sigma}\psi^{2\sigma}+\gamma_\sigma\rho_\sigma\psi^\sigma+\frac{v_\sigma}{2}\rho_\sigma^2\right\}. \tag{8}$$

Here $w_{2\sigma}$, γ_σ and v_σ are certain constants. In this case the quantity $f\sim\exp[S(\psi, \rho_\sigma)]$ determines the joint distribution function of the fields ψ and ρ_σ in the equilibrium state. The wave vectors of the harmonics of ψ and ρ_σ are bounded from above by the values Λ and Λ_ρ, respectively. We note that the quantity ρ_σ averaged for a given configuration $\psi(\mathbf{x})$ coincides with its definition in terms of the field $\psi(\mathbf{x})$. The order-parameter distribution function, which is not difficult to obtain by averaging f over all possible configurations $\rho_\sigma(\mathbf{x})$, coincides at a critical point of order σ with the order-parameter equilibrium distribution of the type $e^{-\mathscr{H}/T}$ at the same point, if the quantities $u_{2\sigma}$ and $w_{2\sigma}$ are connected by the relation

$$u_{2\sigma}=w_{2\sigma}-\gamma_\sigma^2/2v_\sigma. \tag{9}$$

Introducing the thermodynamic forces by means of the entropy (8), we write the equation for $\psi(\mathbf{x}, t)$ in the form

$$\frac{\partial\psi_\alpha}{\partial t}=\Gamma_0\left(\frac{\delta S}{\partial\psi_\alpha^*}+h_\alpha(\mathbf{x},t)\right)+\eta^\alpha(\mathbf{x},t),\quad \alpha=1,2\dots n, \tag{10}$$

and the equation for the conserved field $\rho_\sigma(\mathbf{x})$ as

$$\frac{\partial\rho_\sigma}{\partial t}=-\lambda_0^\sigma\nabla^2\left(\frac{\delta S}{\partial\rho_\sigma}-\tau_\sigma(\mathbf{x},t)\right)+\xi_\sigma(\mathbf{x},t). \tag{11}$$

Here $\tau_\sigma(\mathbf{x}, t)$ is the field conjugate to $\rho_\sigma(\mathbf{x}, t)$, λ_0^σ is the transport coefficient of the σ-particle excitations, and $\xi_\sigma(\mathbf{x}, t)$ is a random force such that

$$\langle\xi_\sigma(\mathbf{x},t)\rangle=0,\quad \langle\xi_\sigma(\mathbf{x},t)\xi_\sigma(\mathbf{x}',t')\rangle =-2\lambda_0^\sigma\nabla^2\delta(\mathbf{x}-\mathbf{x}')\delta(t-t'). \tag{12}$$

In the case when the field ρ_σ is not conserved, it is sufficient to consider Eq. (2) only. As a result we can distinguish four types of dynamic behavior:

I. ψ is not conserved, ρ_σ is not conserved.

II. ψ is conserved, ρ_σ is not conserved.

III. ψ is not conserved, ρ_σ is conserved.

IV. ψ is conserved, ρ_σ is conserved.

This classification differs from that introduced in[14] in that, instead of the energy conservation laws, the conservation laws for the fields $\rho_\sigma(\mathbf{x})$ are important in it.

To analyze these cases we shall consider the dynamic correlation functions of the order parameter ($G(\mathbf{q}, \omega)$) and the field ρ_σ ($D(\mathbf{q}, \omega)$), which have the meaning of the responses to the corresponding conjugate fields $h(\mathbf{x}, t)$ and $\tau_\sigma(\mathbf{x}, t)$. Iterative solution of Eqs. (10) and (11) enables us to write them as

$$G^{-1}(\mathbf{q},\omega)=G_0^{-1}(\mathbf{q},\omega)+\Sigma(\mathbf{q},\omega), \tag{13}$$

$$D^{-1}(\mathbf{q},\omega)=D_0^{-1}(\mathbf{q},\omega)+\Pi(\mathbf{q},\omega); \tag{14}$$

$$G_0(\mathbf{q},\omega)=[-i\omega/\Gamma_0+q^2]^{-1},\ D_0(\mathbf{q},\omega)=[-i\omega/\lambda_0^\sigma q^2+v_\sigma]^{-1}$$

are the bare correlators at the critical point. The self-energy parts $\Sigma(\mathbf{q}, \omega)$ and $\Pi(\mathbf{q}, \omega)$ are expansions in the coupling constants $u_{2\sigma}$ and γ_σ. The graphs of first order in ε_σ are given in Fig. 2.

To determine the dynamic critical indices by means of formulas (13) and (14) it is necessary to know those fixed values of the constants $u_{2\sigma}$, γ_σ, and v_σ that ensure scale-invariant behavior in the critical region and are of a certain order in ε_σ. As follows from a dimensional analysis, the frequency dependence of the corresponding vertices is given by the factor $\omega^{\mathrm{const}\cdot\varepsilon_\sigma}$; therefore, to within terms $\sim\varepsilon_\sigma$, the fixed values of $u_{2\sigma}$, γ_σ, and v_σ calculated from the static renormalization-group relations suffice for us. These relations relate the force constants $u_{2\sigma}$, γ_σ, and v_σ to their values obtained after a decrease of the cutoffs Λ and Λ_ρ by a factor of b and a subsequent change of scale.[14] Having carried out these operations, in lowest order in ε_σ and for $b\gg 1$ we have (see the graphs in Fig. 3)

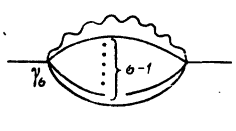

FIG. 2. Self-energy part $\Sigma(\mathbf{q}, \omega)$ in the approximation linear in ε_σ. The wavy lines correspond to $D_0(\mathbf{q}, \omega)$ and the smooth lines to $G_0(\mathbf{q}, \omega)$. The internal integration is performed over wave vectors with lengths from Λ/b to Λ and over frequencies from $-\infty$ to ∞.

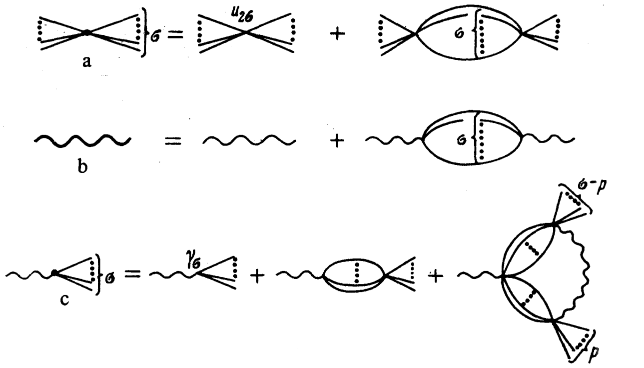

FIG. 3. Static recursion relations (without the combinatoric factors) for the quantities a) $u_{2\sigma}$, b) v_σ, c) γ_σ. Graphs with $p=1, 2, \ldots, \sigma-1$ must be taken into account.

$$u_{2\sigma}'=b^{\varepsilon_\sigma(\sigma-1)}\{u_{2\sigma}-u_{2\sigma}^2 2^{\sigma-1}g_{\sigma0}(n)B_\sigma\ln b\}, \tag{15}$$

$$\gamma_\sigma'=b^{[\varepsilon_\sigma(\sigma-1)+y]/2}\{\gamma_\sigma-u_{2\sigma}\gamma_\sigma 2^\sigma B_\sigma g_{ml}(n)\ln b-\gamma_\sigma^3 v_\sigma^{-1}A_\sigma(l)B_\sigma\ln b\}, \tag{16}$$

$$v_\sigma'=b^y\{v_\sigma-\gamma_\sigma^2A_\sigma(l)B_\sigma\ln b\}, \tag{17}$$

where

$$B_\sigma=\pi^{-\sigma}2^{1-2\sigma}\nu^{-1}\Gamma^{\sigma-1}(\nu),\quad \nu=(\sigma-1)^{-1},$$

$$A_\sigma(a)=\sigma!\prod_{j=a}^{(\sigma-2+l)/2}\frac{n+2j}{2j+1}.$$

From these equations we obtain the following fixed values of the quantities $u_{2\sigma}$, γ_σ and v_σ (we mark them by an asterisk):

$$u_{2\sigma}^*=\varepsilon_\sigma\nu^{-1}2^{1-\sigma}B_\sigma^{-1}g_{\sigma0}^{-1}(n), \tag{18}$$

$$\gamma^{*2}/v_\sigma^*=yA_\sigma^{-1}(l)B_\sigma^{-1}. \tag{19}$$

Moreover, by means of (16) we can confirm the relation (7). The solution with $\gamma^*\neq 0$ is stable only when $y>0$. In the opposite case, when $y\leqslant 0$, i.e., the static correlator of the field ρ_σ does not diverge, $\gamma^*=0$ and the order parameter is not coupled to the field ρ_σ. All the fixed values are independent of the bare coupling constants.

3. We consider now the behavior of the kinetic coefficients, which are determined by the relations

$$\Gamma^{-1}=\left.\frac{\partial G^{-1}}{\partial(-i\omega/q^2)}\right|_{\substack{\omega\to0\\ q\to0}}\quad\text{for a conserved order parameter }\psi.$$

$$\Gamma^{-1}=\left.\frac{\partial G^{-1}}{\partial(-i\omega)}\right|_{\substack{\omega\to0\\ q\to0}}\quad\text{for a nonconserved }\psi.$$

$$\lambda_\sigma^{-1}=\left.\frac{\partial D^{-1}}{\partial(-i\omega/q^2)}\right|_{\substack{\omega\to0\\ q\to0}}\quad\text{for a conserved field }\rho_\sigma.$$

In writing the recursion relations for these, we must take into account that under scale transformations the frequency changes as follows: $\omega\to\omega'=b^z\omega$.[1,14] To first order in ε_σ, using the diagrams of Fig. 2 we have, for $b\gg1$,

$$\Gamma'^{-1}=b^{2-z}\Gamma^{-1}\{1+\gamma_\sigma^2v_\sigma^{-1}\sigma A_\sigma(1)I(\mu_\sigma)\}, \tag{20}$$

$$\lambda'^{-1}=b^{2-z+y}\lambda^{-1}, \tag{21}$$

where

$$I(\mu_\sigma)=\frac{1}{(2\pi)^{d(\sigma-1)}}\int\limits_{\Lambda/b}^{\Lambda}\frac{d^dq_1\ldots d^dq_{\sigma-1}}{q_1^2q_2^2\ldots q^2_{\sigma-1}[q_1^2+\ldots+q^2_{\sigma-1}+\mu_\sigma(\mathbf{q}_1+\ldots+\mathbf{q}_{\sigma-1})^2]}$$

$$=\mu_\sigma^{-(1+\nu)}\ln bB_\sigma\Gamma^{-\sigma}(\nu)\int\limits_0^\infty dx\, x^{\nu-1}\exp\left(-\frac{x}{\mu_\sigma}\right)\gamma^{\sigma-1}(\nu,x),$$

$\mu_\sigma=\lambda_\sigma v_\sigma \Gamma^{-1}$ and $\gamma(\nu, x)$ is the incomplete gamma-function.

For case I, $\gamma^*=0$ (or $\lambda_0^\sigma\to\infty$), and therefore the fixed values of Γ are attained with $z=2+O(\varepsilon_\sigma^2)$. It has been shown by the authors[16] that the corrections to the value of the dynamical index determining the critical-damping frequency $\omega_q\sim q^z$ are numerically small. In case II, however, fluctuation corrections do not arise at all, and the exponent $z=4+O(\varepsilon_\sigma^2)$.

We turn to case III. Using the expressions (17), (20), and (21), we can write the following recursion relations for μ_σ:

$$\mu_\sigma'=\mu_\sigma[1-y\ln b+v_\sigma^{-1}\gamma_\sigma^2\sigma A_\sigma(1)I(\mu_\sigma)]. \tag{22}$$

Substitution of (19) into this leads to the following fixed values:

$$\mu^*=\infty,\quad \mu^*=0,\quad \mu^*=c,$$

where c is the solution of the equation

$$\ln b=\sigma I(\mu_\sigma)A_\sigma(1)A_\sigma^{-1}(l)B_\sigma^{-1}.$$

The point $\mu^*=\infty$ should be excluded from consideration, since it leads to the case I.

After investigating the remaining points for stability and substituting them into (20), we establish from the condition for the existence of nontrivial solutions for Γ that the exponent of the frequency of the critical damping of the order parameter for even σ is equal to

$$z=\begin{cases}2+y\sigma\nu^{2-\sigma}\Gamma^{1-\sigma}(\nu)n^{-1}, & n\geqslant 2,\\ 2+y, & n<2,\end{cases} \tag{23}$$

and for odd σ is equal to

$$z=2+y. \tag{24}$$

The frequency of the critical damping of the field ρ_σ is characterized by the index $z_\rho=2+y$. From this it can be seen that the coupling of ψ with ρ_σ has the result that the damping frequency of the order parameter decreases and is pulled toward the damping frequency of the field ρ_σ. This is manifested in a substantial increase in the value of z as compared with that in case I. To demonstrate the size of these deviations to first order in ε_σ we cite

$$\begin{aligned}y(\sigma=3)&=d-2\Delta_{11}=\varepsilon_3\cdot 4(n+9)/(3n+22),\\ y(\sigma=4)&=d-2\Delta_{20}=\varepsilon_4\frac{3(n^2+130n+1024)}{(3n^2+150n+1072)}.\end{aligned} \tag{25}$$

It can be seen that the index z depends essentially on the order of the critical point. For even σ we have put $l=1$ in formulas (23) and (25).

In case IV corrections to the value $z=4+O(\varepsilon_\sigma^2)$ do not arise.

For $\sigma>2$ our results characterize the fluctuation corrections that appear in planar models. For an ordinary critical point ($\sigma=2$) they coincide with the results obtained in[14].

The case when $d=d_\sigma$ merits special attention, since at a tricritical point it corresponds to a three-dimensional system. For $d=d_\sigma$ all the dynamical indices coincide with their mean-field values, but in case III the interaction of the fluctuations leads to the appearance of logarithmic corrections to the power laws. These are most important for the kinetic coefficients $\Gamma(\sim q^{z-2})$, which are constants in the mean-field theory. It is not difficult to find the powers of the logarithmic factors from formulas (23) and (24) for the fluctuation corrections to the dynamical exponents for $d<d_\sigma$. By using the fact that when the dimensionality d deviates from d_σ the long-wavelength singularities of the type $\ln q$ in the divergent diagrams are replaced by power singularities $\sim q^{\varepsilon_\sigma(1-\sigma)}$, we obtain for even σ

$$\Gamma\sim|\ln q|^{s(n)},\qquad s(n)=\begin{cases}s\sigma\nu^{2-\sigma}n^{-1}\Gamma^{1-\sigma}(\nu), & n\geqslant 2\\ s, & n<2\end{cases}, \tag{26}$$

and for odd σ

$$\Gamma\sim|\ln q|^s,\quad s=1-4g_{ml}(n)/g_{\sigma0}(n)>0. \tag{27}$$

For $s<0$, which, for $n>4$, occurs only for $\sigma=2$, $\Gamma=\text{const}$.

At a tricritical point we have

$$\Gamma\sim|\ln q|^{2(n+9)/(3n+22)}. \tag{28}$$

4. We shall discuss the connection between the models considered and specific physical substances, giving our attention principally to nondegenerate systems from the class of metamagnets. These are anisotropic antiferromagnets of the type $FeCl_2$,[4] $FeBr_2$,[5,6] $Ni(NO_3)_2\cdot 2H_2O$,[7] and others[8]; they are a set of planes weakly coupled by antiferromagnetic interaction, with a stronger ferromagnetic interaction within a single layer. Complicated phase transitions can occur in such systems as the temperature is lowered: a continuous transition from the paramagnetic to the antiferromagnetic state occurs first, and, in the presence of an external magnetic field, separation into a paramagnetic and an antiferromagnetic phase can begin, indicating the presence of a complex critical point. The same behavior in a magnetic field can be displayed by planar Ising systems in which, together with antiferromagnetic nearest-neighbor exchange, there is a sufficiently strong ferromagnetic interaction between next-nearest neighbors. Both types of metamagnets correspond to Ising models with two exchange integrals J_1 and J_2.[3] In studying the dynamics of such systems we must also take into account that each Ising spin is coupled with the lattice, and, therefore, irreversible equations of motion for the spins can be obtained only after averaging over the lattice variables. In this case, depending on the properties of the spin-lattice interaction, these equations will indicate whether the total spin is conserved or not. Moreover, in an isolated spin system there can be conservation of

both the energy and other quantities constructed from spin variables (these quantities should commute with the Hamiltonian). Averaging over distances larger than the lattice constant[13,14] transforms these quantities into the fields of the σ-particle excitations that we have considered, and the equations of motion take the form of (10) and (11), the conservation laws remaining as before. In this way one of the four cases of dynamical behavior that we have analyzed can be realized. After the smoothing the free energy takes the form (1), in which the coefficients u_{2k} will be functions of the temperature, the external magnetic field B, and the ratio of the exchange integrals. The values of T and B for which $u_2 = u_4 = 0$ correspond to a tricritical point. For the particular value $J_1/J_2 = 0.6$ there are certain T and B that ensure $u_2 = u_4 = u_6 = 0$ and correspond to a tetracritical point.[3] Since the magnitude of the exchange interaction is a function of pressure, metamagnets can exist in which it is possible to observe a tetracritical point by varying the pressure applied to the system. The experimental observation of multicritical points is discussed in[9,10].

Summarizing, we can say that, besides the conservation laws for the order parameter, the conservation laws for coupled σ-particle excitations play a fundamental role in determining the dynamical properties of a system at a critical point of order σ. The presence of these excitations has a substantial effect on the transport processes. This leads us to distinguish four types of dynamical behavior in systems with the same static critical properties. In the framework of each of these four classes the dynamical behavior is universal, although it is true that in case III certain subtleties can arise in the presence of more than one conserved field ρ_σ.

We note also that the strong dependence of the dynamical indices on σ that appears in the most interesting case III may turn out to be useful for the experimental identification of unknown multicritical points.

In the present paper we have investigated only the stochastic equations of motion, which do not take into account the order-parameter precession that is characteristic for degenerate systems. When this precession is taken into account in the case of ^{3}He–^{4}He mixtures and degenerate magnets, corrections to mean-field theory arise on account of nonlinearities that do not depend on the order of the critical point. Consequently, the dynamical indices z are expressed in the same way for all σ, in terms of the static critical exponents at the corresponding critical point.

One of the authors (G. T.) takes the opportunity to thank V. L. Pokrovskiĭ and D. E. Khmel'nitskiĭ for a useful discussion.

[1] A. Z. Patashinskiĭ and V. L. Pokrovskiĭ, Fluktuatsionnaya teoriya fazovykh perekhodov (Fluctuation Theory of Phase Transitions), Nauka, M., 1975.
[2] R. B. Griffiths, Phys. Rev. Lett. **24**, 715 (1970).
[3] J. M. Kincaid and E. G. D. Cohen, Phys. Rep. **22C**, 59 (1975).
[4] I. S. Jacobs and P. E. Lawrence, Phys. Rev. **164**, 866 (1967).
[5] R. J. Birgeneau, W. B. Yelon, E. Cohen, and J. Makovsky, Phys. Rev. **B5**, 2607 (1972); W. B. Yelon and R. J. Birgeneau, Phys. Rev. **B5**, 2615 (1972).
[6] R. J. Birgeneau, G. Shirane, M. Blume, and W. C. Koehler, Phys. Rev. Lett. **33**, 1098 (1974).
[7] V. A. Schmidt and S. A. Friedberg, Phys. Rev. **B1**, 2250 (1970).
[8] D. P. Landau, B. E. Keen, B. Schneider, and W. P. Wolf, Phys. Rev. **B3**, 2310 (1971).
[9] W. B. Yelon, D. E. Cox, P. J. Kortman, and W. B. Daniels, Phys. Rev. **B9**, 4843 (1974).
[10] C. W. Garland and B. B. Weiner, Phys. Rev. **B3**, 1634 (1971).
[11] F. J. Wegner, Phys. Rev. **B6**, 1891 (1972); Phys. Lett. **54A**, 1 (1975).
[12] M. J. Stephen and J. L. McCauley, Jr., Phys. Lett. **44A**, 89 (1973).
[13] G. F. Tuthill, J. F. Nicoll, and H. E. Stanley, Phys. Rev. **B11**, 4579 (1975).
[14] B. I. Halperin, P. C. Hohenberg, and S. Ma, Phys. Rev. **B10**, 139 (1974).
[15] K. G. Wilson and J. Kogut, Phys. Rep. **12C**, 75 (1974).
[16] V. V. Prudinkov and G. B. Teitel'baum, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 330 (1976) [JETP Lett. **23**, 296 (1976)].

Translated by P. J. Shepherd