In closing, the authors cordially thank M. I. Kaganov and V. D. Natsik for discussion of the results of the research, and V. L. Sobolev for help in carrying out some of the calculations.

[1]Energy losses by "Cerenkov" radiation of magnons during motion of a dislocation were considered by Babkin and Kravchenko.[3]

[2]We use a system of units in which $\hbar = 1$ and $k = 1$.

[3]It can be shown that the Hamiltonian (1.5) must contain terms linear in the operators a_k and a_k^*. In actuality, these terms drop out by virtue of the condition that the energy be a minimum (see[5]).

[1]V. I. Al'shitz and V. L. Indenbom, Usp. Fiz. Nauk 115, 3 (1975) [Sov. Phys. Usp. 18, 1 (1975)].

[2]M. I. Kaganov, V. Ya. Kravchenko, and V. D. Natsik, Usp. Fiz. Nauk 111, 655 (1973) [Sov. Phys. Usp. 16, 878 (1974)].

[3]G. I. Babkin and V. Ya. Kravchenko, Pis'ma Zh. Eksp. Teor. Fiz. 13, 30 (1971) [JETP Lett. 13, 19 (1971)].

[4]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, Spinovye volny (Spin Waves), Nauka, 1967 (translation, North-Holland, 1968).

[5]V. G. Bar'yakhtar, M. A. Savchenko, and V. V. Tarasenko, Zh. Eksp. Teor. Fiz. 51, 936 (1966) [Sov. Phys. JETP 24, 623 (1967)].

Translated by W. F. Brown, Jr.

The conductivity of a quasi-one-dimensional metal at $T = 0$

A. A. Abrikosov and I. A. Ryzhkin

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted June 14, 1976)
Zh. Eksp. Teor. Fiz. 72, 225–237 (January 1976)

The longitudinal and transverse conductivities of a quasi-one-dimensional metal containing impurity centers are calculated for $T = 0$. It is assumed that the energy spectrum is given by expression (2), where $|\alpha| \ll \epsilon_F$. It is found that for $\alpha l_2'/v \gg 1$ the longitudinal conductivity is described by the "three-dimensional" formula (28), where l_2 is the mean free path of $p_o \to -p_o$ transitions, and S is the area of the xy-section of the unit cell. In the case of where $\alpha l_2'/v \ll 1$, the order of magnitude of the conductivity is expressed for a short-range potential by formula (42), where l_1 is the mean free path for $p_o \to p_o$ transitions.

PACS numbers: 72.15.Nj

1. INTRODUCTION

In a preceding paper[1] (referred to below as I) we discussed a method of deriving the characteristics of a one-dimensional metal containing random impurity centers. As was pointed out, the true aim of this method was not the solution of the one-dimensional problems with a random potential, which may also be solved by another method (see[2]), but of quasi-one-dimensional ones in which the motion of the electrons is not purely one-dimensional and there is relatively slow motion in the transverse direction.

In the present paper we shall examine the conductivity of a metal with an energy spectrum

$$\varepsilon(\mathbf{p}) = \varepsilon(p_z) + \alpha(p_x, p_y) \tag{1}$$

$(\alpha \ll \varepsilon_F)$ and containing random impurity centers at $T = 0$. Referring the energy to the chemical potential, we have

$$\varepsilon - \mu \equiv \xi(\mathbf{p}) = v(|p_z| - p_0) + \alpha(p_x, p_y), \quad \int \alpha\, dp_x\, dp_y = 0 \tag{2}$$

(the integral is taken over the area of the xy-cross-section of the Brillouin zone). If a more concrete estimate is needed we shall use the formula for strong coupling for a rectangualr cell in the plane:

$$\alpha(p_x, p_y) = -\alpha_x \cos(p_x a_x) - \alpha_y \cos(p_y a_y). \tag{3}$$

It is obvious that for sufficiently large α the problem becomes truly three-dimensional and localization effects should not come into play. In this event the usual kinetic equation is applicable, which is equivalent to neglecting diagrams with intersection of the impurity lines (see[3]). The criterion for their neglect is $\tau\Delta\varepsilon \gg 1$, where $\Delta\varepsilon$ is the characteristic energy, and τ the time between collisions. In the present case the role of $\Delta\varepsilon$ is played by α, while $\tau = l_2/v$ (see I), i.e., the problem becomes three-dimensional when

$$\alpha l_2/v \gg 1. \tag{4}$$

It is evident from this that there is a region $\alpha l_2/v \ll 1$ in which, on the one hand, static conductivity must exist, and on the other hand, localization effects must be strongly in evidence. A rough estimate of the conductivity in this case can be obtained from a diffusion analysis. As a result of collisions, an electron diffuses first of all in the xy-plane. The relevant diffusion coefficient is of the order of

$$D_\perp \sim v_\perp^2 \tau. \tag{5}$$

In I it was shown that two times τ exist: the time τ_1 for the processes without appreciable change in the z-component of the momentum ("forward" scattering) and the time τ_2 for processes in which p_z undergoes a transition from the neighborhood of p_0 to the neighborhood of

$-p_0$ ("back" scattering). Localization is determined by τ_2 only. In formula (5) τ takes both processes into account, since a transverse component of momentum arises for both of them, i.e., $\tau = (\tau_1^{-1} + \tau_2^{-1})^{-1}$. As regards the transverse velocity, it is

$$v_\perp = \partial\alpha/\partial p \sim \alpha a, \tag{6}$$

where a is the transverse period.

The distance ρ in the transverse direction negotiated by a particle in the time t is $\rho \sim (D_\perp t)^{1/2}$. If the impurity potential is short-range, i.e., there is in reality no correlation of the potentials in neighboring threads, then it is sufficient for the particle to cross into the neighboring thread for it to get into new localization conditions. Since the localization radius is of the order of $l_2 = v\tau_2$, the particle is displaced in this event along z by a distance of the order of l_2. According to the foregoing, the average time necessary for this is $t \sim a^2/D_\perp$. Consequently,

$$D_\parallel \sim l_2^2/t \sim (l_2/a)^2 D_\perp. \tag{7}$$

The conductivity in a degenerate system is related to the diffusion coefficient by the well known relationship

$$\sigma = e^2 v(\mu) D. \tag{8}$$

Substituting (5), (6), and (7), and taking into account that in our case

$$v = (\pi v S)^{-1}, \tag{9}$$

where S is the area of the xy-section of the unit cell ($S = a_x a_y$), we obtain

$$\sigma_\perp \sim e^2\alpha^2 a^2 (l_1^{-1} + l_2^{-1})^{-1}/\pi v^2 S, \tag{10a}$$

$$\sigma_\parallel \sim e^2\alpha^2 l_2^2 (l_1^{-1} + l_2^{-1})^{-1}/\pi v^2 S. \tag{10b}$$

If the interaction is long-range, i.e., its radius of action $r_0 \gg a$, violation of the correlation will only occur for $\rho \sim r_0$. This leads to $t \sim r_0^2/D_\perp$, $D_\parallel \sim D_\perp (l_2/r_0)^2$, and

$$\sigma_\parallel \sim e^2\alpha^2 (a/r_0)^2 l_2^2 (l_2^{-1} + l_1^{-1})^{-1}/\pi v^2 S.$$

We shall verify below that expression (10a) is an accurate result, while (10b) is somewhat approximate, apparently because of some inaccuracy in the concept of a displacement by l_2 in a time t.

In what follows we present a rigorous theory of conductivity based on the method developed in I.

2. THE GREEN FUNCTIONS AND THE GENERAL FORMULA FOR LONGITUDINAL STATIC CONDUCTIVITY

The Green function in the static potential $V(\mathbf{r})$ will depend on the two coordinates $\mathbf{r}$ and $\mathbf{r}'$ and on the time difference $t - t'$. Let us introduce a Fourier transformation with respect to $t - t'$ (i.e., ω) and to xy, $x'y'$. The Green function will then have the form $G(z, z', \mathbf{p}, \mathbf{p}', \omega)$, where $\mathbf{p} = (p_x, p_y)$. Such a Green function satisfies the

Dyson equation

$$\omega G + i v\sigma_3 \frac{\partial G}{\partial z} - \alpha(\mathbf{p})G$$
$$= (2\pi)^2 \delta(\mathbf{p} - \mathbf{p}')\delta(z - z') + \int \hat{\varphi}(\mathbf{k}, z)G(\mathbf{p} - \mathbf{k}, \mathbf{p}', z, z')\frac{d^2 k}{(2\pi)^2}. \tag{11}$$

Here, as in I, a 2×2-matrix Green function is introduced, the index 1 denoting the vicinity of p_0, and the index 2, that of $-p_0$. The field $\hat{\varphi}$ is the matrix

$$\hat{\varphi}(\mathbf{k}, z) = \eta(\mathbf{k}, z) + \tfrac{1}{2}\zeta(\mathbf{k}, z)\sigma^{(+)} + \tfrac{1}{2}\zeta^*(\mathbf{k}, z)\sigma^{(-)}.$$

Here $\sigma^{(\pm)} = \sigma_1 \pm i\sigma_2$, where σ_1, σ_2, and σ_3 in (11) are Pauli matrices. The fields η and ζ are random Gaussian fields which are averaged according to the rule

$$\langle \eta(\mathbf{k}, z)\eta(\mathbf{k}', z')\rangle = (2\pi)^2\delta(k_x + k_x')\delta(k_y + k_y')\delta(z - z')D_1(\mathbf{k}),$$

$$\langle \zeta(\mathbf{k}, z)\zeta^*(\mathbf{k}', z')\rangle = (2\pi)^2\delta(k_x + k_x')\delta(k_y + k_y')\delta(z - z')D_2(\mathbf{k}). \tag{12}$$

This description corresponds to the Born approximation (see I).

To eliminate $\delta(\mathbf{p} - \mathbf{p}')$ from the right-hand side of (11), we return to the coordinate by the Fourier procedure, but with respect to the variable $\mathbf{p}' - \mathbf{p}$. On doing this we obtain

$$\omega G + i v\sigma_3\frac{\partial G}{\partial z} - \alpha(\mathbf{p})G = \delta(z - z') + \int \hat{\varphi}(\mathbf{k}, z)G(\mathbf{p} - \mathbf{k}, \rho', z, z')e^{-i\mathbf{k}\rho'}d^2 k/(2\pi)^2. \tag{13}$$

This equation is of the same form as in I. The difference is that previously the potential was an operator only with respect to the "spin" indices, whereas it now also acts on the variables $\mathbf{p}$ and ρ', viz., it displaces $\mathbf{p}$ by $\mathbf{k}$ and multiplies G by $\exp(-i\mathbf{k}\cdot\rho')$. Nevertheless the equation may be solved symbolically. From (13) it follows that

$$G(z, z') = S(z, z_1)G(z_1, z') - iv^{-1}S(z, z')\sigma_3\theta(z - z')\theta(z' - z_1), \tag{14}$$

where the S-matrix has the form

$$S(z, z') = T_z \exp\left\{\frac{i}{v}\int_{z'}^{z} [\sigma_3(\omega - \alpha(\mathbf{p}) - [\eta(z_1)])\right.$$
$$\left. - [\zeta(z_1)]\sigma^{(+)}/2 + [\zeta^*(z_1)]\sigma^{(-)}/2]dz_1\right\}, \tag{15}$$
$$[\eta(z)] = \int \eta(\mathbf{k}, z)e^{-i\mathbf{k}\rho'}d^2 k/(2\pi)^2.$$

Equation (14) is solved taking account of the boundary conditions for $G_{\alpha\beta A}$ and $G_{\alpha\beta R}$ in the same way as in I, but with the difference that the operator character of S with respect to $\mathbf{p}$ and ρ' must always be borne in mind, i.e., the non-commutative nature of $S_{\alpha\beta}(z, z')$ and $S_{\gamma\delta}(z_1, z_1')$. However, the derivation in I is easily generalized to this case.

A more convenient representation of the Green functions is the following

$$G_{R\alpha\beta}(z > z') = -\frac{i}{v}S_{\alpha 1}^{-1}(\infty, z)[S_{11}^{-1}(\infty, -\infty)]^{-1}[S^{-1}(z', -\infty)\sigma_3]_{1\beta},$$

$$G_{R\alpha\beta}(z < z') = \frac{i}{v}S_{\alpha 2}(z, -\infty)[S_{22}(\infty, -\infty)]^{-1}[S(\infty, z')\sigma_3]_{2\beta}, \tag{16}$$

$$G_{A\alpha\beta}(z > z') = -\frac{i}{v}S_{\alpha 2}^{-1}(\infty, z)[S_{22}^{-1}(\infty, -\infty)]^{-1}[S^{-1}(z', -\infty)\sigma_3]_{2\beta},$$

A. A. Abrikosov and I. A. Ryzhkin   119

$$G_{A\alpha\beta}(z<z')=\frac{i}{v}S_{\alpha\cdot}(z,-\infty)[S_{11}(\infty,-\infty)]^{-1}[S(\infty,z')\sigma_3]_{1\beta}.$$

Here the notation is the following: $S_{\alpha 1}^{-1}$ must be understood in the sense that the component $\alpha 1$ is taken from the matrix inverse to (15). In relation to p and ρ' this is still an operator; $[S_{\alpha 1}^{-1}]^{-1}$ denotes the inverse of this operator.

Since we are interested in the static conductivity, it is sufficient, as shown in I, for us to know G for $\omega=0$[1]:

$$\sigma_{zz}=\frac{e^2v^2}{2\pi}\int Sp_\sigma[G_R(z,z_1,p,\rho')\sigma_3 G_A(z_1,z,p',0)\sigma_3]$$
$$\times\exp[-i(p'-p)\rho']\,dp\,dp'\,d\rho'\,dz_1/(2\pi)^4. \tag{17}$$

The dependence on ρ' in G_R is due to the multipliers $\exp(-i\mathbf{k}\cdot\rho')$ in the potential components. The n-th order term for interaction with the potential is proportional to

$$\exp\left[-i\left(\sum_{i=1}^{n}\mathbf{k}_i\right)\rho'\right].$$

If this multiplier is substituted in (17) and integration is carried out with respect to ρ', we obtain

$$\mathbf{p}'=\mathbf{p}-\sum_{i=1}^{n}\mathbf{k}_i.$$

But according to (13) each component of potential does, after all, lead to the replacement of $\mathbf{p}$ by $\mathbf{p}-\mathbf{k}$. Consequently, we can assume that in this sense all the potential components that have already acted on G_R act on G_A. Taking into account that according to (12) after averaging to every $\mathbf{k}_i$ there corresponds a $-\mathbf{k}_i$, all the $\mathbf{k}$ components cancel out. This makes it possible to write the expression for conductivity in the form of a trace not only with respect to σ, but also with respect to $\mathbf{p}$, viz.,

$$\sigma_{zz}=\frac{e^2v^2}{2\pi}\int Sp_{\sigma p}[G_R(z,z_1)\sigma_3 G_A(z_1,z)\sigma_3]dz_1, \tag{18}$$

it being necessary to consider each impurity potential as a displacement operator with respect to $\mathbf{p}$: $\mathbf{p}\to\mathbf{p}-\mathbf{k}_i$.

Let us substitute in this the expressions (16) for G. After some easy transformations we obtain

$$\sigma_{zz}=\frac{e^2}{2\pi}L\,Sp_p\{[S_{11}^{-1}(L,0)]^{-1}[S_{11}(L,0)]^{-1}\}. \tag{19}$$

In this formula we have changed to the finite length L, i.e., $S_{11}(\infty,-\infty)\to S_{11}(L,0)$. The factor L appears as a result of the integration with respect to z_1.

Equation (19) can be presented in a different form due to the fact that the trace does not change on going over to a different complete system of functions. To be precise, let us carry out a Fourier transformation of $\alpha(p)$ with respect to $\mathbf{p}$:

$$\alpha(\mathbf{p})=N^{-1}\sum_i\alpha_i e^{-i\mathbf{p}\rho_i},\quad \alpha_i=S\int\alpha(\mathbf{p})e^{i\mathbf{p}\rho_i}d^2p/(2\pi)^2,$$

where the sum with respect to i is over lattice periods in the xy-plane, N is the number of cells in the xy-plane,

and S is the area of the xy-section of the unit cell. Let f be a certain function of $\alpha(\mathbf{p})$. Then on inserting the relation $\eta(\mathbf{k})f(\mathbf{p})=f(\mathbf{p}-\mathbf{k})\eta(\mathbf{k})$ in the Fourier components, we obtain

$$\eta(\mathbf{k})\sum_i f_i\exp\{-i\mathbf{p}\rho_i\}=\sum_i f_i\exp\{-i(\mathbf{p}-\mathbf{k})\rho_i\}\eta(\mathbf{k}),$$

or

$$f_i\eta(\mathbf{k})=\exp\{-i\mathbf{k}\rho_i\}\eta(\mathbf{k})f_i.$$

In other words, the function f is converted into an operator which, acting on the potential operators, multiplies them by $\exp(-i\mathbf{k}\cdot\rho_i)$. Bearing this in mind, we can write in place of (19)

$$\sigma_{zz}=\frac{e^2L}{2\pi SN}\sum_i\{[S_{11}^{-1}(L,0)]^{-1}[S_{11}(L,0)]^{-1}\}_i. \tag{20}$$

Let us note that

$$\alpha_{ax}=\alpha_{-ax}=\alpha_x/2,\quad \alpha_{ay}=\alpha_{-ay}=\alpha_y/2 \tag{21}$$

corresponds to the form (3). All the remaining α_i's are equal to zero.

In real calculations formula (19) is more convenient in the case of a short-range potential, since use can be made of the fact that $\bar\alpha=0$, the averaging being done over a range of momenta small in comparison to $1/r_0$, where r_0 is the radius of action of the potential. In the case of a long-range potential, on the other hand, formula (20) is more convenient. The point is that if α did not have an operator character, it would play the same role as ω, i.e., it would drop out of the result. Consequently, one can always substitute

$$\exp(-i\mathbf{k}\rho_i)\to\exp(-i\mathbf{k}\rho_i)-1\to-\mathbf{k}^2\rho_i^2,$$

if $r_0\gg\rho_i$, then $\mathbf{k}^2\cdot\rho_i^2\ll1$.

Thus, the problem consists in calculating expressions (19) and (20) for the different cases.

3. THE LIMIT OF APPLICABILITY OF THE KINETIC EQUATION

Let us examine the S-matrix (15) for $\omega=0$. On changing to the "interaction representation," we have

$$S(z,z')=\exp\left\{-\frac{i}{v}\alpha(\mathbf{p})\sigma_3 z\right\}\tilde S(z,z')\exp\left\{\frac{i}{v}\alpha(\mathbf{p})\sigma_3 z'\right\},$$

$$\tilde S(z,z')=T_z\exp\left\{\frac{i}{v}\int\left(-[\bar\eta(z_1)]\sigma_3-[\bar\xi(z_1)]\frac{\sigma^{(+)}}{2}+[\bar\xi^*(z_1)]\frac{\sigma^{(-)}}{2}\right)dz_1\right\},$$

$$[\bar\eta(z_1)]=\int\eta(\mathbf{k},z_1)\exp\{i[\alpha(\mathbf{p})-\alpha(\mathbf{p}-\mathbf{k})]\sigma_3 z_1/v\}P_\mathbf{k}\,d^2k/(2\pi)^2, \tag{22}$$

$$[\bar\xi(z_1)]=\int\zeta(\mathbf{k},z_1)\exp\{i[\alpha(\mathbf{p})+\alpha(\mathbf{p}-\mathbf{k})]\sigma_3 z_1/v\}P_\mathbf{k}\,d^2k/(2\pi)^2,$$

$$[\bar\xi^*(z_1)]=\int\zeta^*(\mathbf{k},z_1)\exp\{-i[\alpha(\mathbf{p})+\alpha(\mathbf{p}-\mathbf{k})]\sigma_3 z_1/v\}P_\mathbf{k}\,d^2k/(2\pi)^2,$$

where $P_\mathbf{k}$ is an operator displacing $\mathbf{p}$ by $\mathbf{k}$.

Let us, as in I, change from integrals with respect to z in the exponents to sums and use the notation

A. A. Abrikosov and I. A. Ryzhkin   120

$$-\frac{i}{v}[\tilde\zeta(z_i)]\Delta=\tilde\zeta_i, \quad \frac{i}{v}[\tilde\zeta^{\cdot}(z_i)]\Delta=\tilde\zeta_i^{\cdot}, \quad -\frac{1}{v}[\tilde\eta(z_i)]\Delta=\tilde\eta_i,$$

where Δ is a normalizing segment which will later tend to zero. Then (see I)

$$S(z,z')=T_z\prod_i\exp[i\tilde\eta_i\sigma_s+\tilde\zeta_i\sigma^{(+)}/2+\tilde\zeta_i^{\cdot}\sigma^{(-)}/2]$$

$$=\prod_i(1-\eta_i^2/2+|\zeta_i|^2/2)T_z\prod_i(1+i\tilde\eta_i\sigma_3+\tilde\zeta_i\sigma^{(+)}/2+\tilde\zeta_i^{\cdot}\sigma^{(-)}/2).$$

We have expanded the exponents and kept only terms not higher than η_i^2, ζ_i, ζ_i^*, $|\zeta_i|^2$ in order. It must be taken into account that in the $|\zeta_i|^2$'s which enter into the products in i it is sufficient to keep only those products which "link together," i.e., $\zeta(k)\zeta^*(-k)$ with their appropriate multipliers. It is not difficult to see that these multipliers cancel and as a result

$$\tilde\eta_i^2\to\eta_i^2\to\frac{\Delta}{v^2}\int D_1(\mathbf k)\,d^2k/(2\pi)^2=\frac{\Delta N_i}{v^2}\int|U(k_z=0,\mathbf k)|^2\,d^2k/(2\pi)^2=\Delta/l_1,$$

$$|\tilde\zeta_i|^2\to|\zeta_i|^2\to\frac{\Delta}{v^2}\int D_2(\mathbf k)\,d^2k/(2\pi)^2=\frac{\Delta N_i}{v^2}\int|U(k_z=2p_0,\mathbf k)|^2\,d^2k/(2\pi)^2=\Delta/l_2$$

(23)

is obtained. On substituting the $S(z,z')$ expression in (19) the external multipliers $\exp[i\alpha(\mathbf p)z/v]$ cancel and an expression is left which, in place of the full S-matrices, contains reduced ones $\bar S$.

Let us examine the expression

$$Q(z)=S\cdot\mathrm{Sp}_\mathbf p\{[\bar S_{11}^{-1}(z,0)]^{-1}[\bar S_{11}(z,0)]^{-1}\}.$$ (24)

which enters in σ_{zz} (19) as a function of z. Changing from integrals with respect to z_1 to sums, we separate out the field operators with the largest z. Expanding as in I with respect to these operators and keeping terms not higher than the second order in η and the first order in $|\zeta|^2$, we obtain

$$Q(z)=Q(z-\Delta)(1+\eta_i^2-|\zeta_i|^2)-S\cdot\mathrm{Sp}_\mathbf p\{[\bar S_{11}^{-1}]^{-1}$$
$$\times\bar S_{12}-\tilde\zeta_i^{\cdot}[\bar S_{11}^{-1}]^{-1}[\bar S_{11}]^{-1}\tilde\zeta_i\bar S_{21}[\bar S_{11}]^{-1}\}-\eta_i^2 Q(z-\Delta).$$ (25)

Here all the $\bar S$'s in the curly brackets are $\bar S(z-\Delta,0)$. Notice that η_z^2 drops out of this relationship.

Before averaging with respect to $\tilde\zeta$, let us note that according to formula (22) $\tilde\zeta$ has phase multipliers which depend on α. In the general case such a phase multiplier is equal to

$$\exp\{i[\alpha(\mathbf p)+\alpha(\mathbf p-\mathbf k)-\alpha(\mathbf p_1)-\alpha(\mathbf p_1-\mathbf k)]z/v\}.$$

If the "intersections" of the impurity lines are ignored, account must be taken of the fact that the term in the brace brackets in (25) breaks up into two independent averages; in this event $\mathbf p=\mathbf p_1$. On averaging with respect to ζ and introducing the notation $t=z/l_2$, we obtain

$$\partial Q/\partial t=-Q(P+1), \quad P=S\cdot\mathrm S_\mathbf p\{\bar S_{21}[\bar S_{11}]^{-1}[\bar S_{11}^{-1}]^{-1}\bar S_{12}^{-1}\}.$$ (26)

Transposing the matrices in P under the trace sign and carrying out some elementary transformations, we obtain $P=Q-1$. Substituting this in (26) and solving the equation, we obtain

$$Q=(t+t_0)^{-1}.$$ (27)

Here t_0 is found, in principle, from the boundary conditions for $t=0$. However, whatever t_0 may be, $Q\approx t^{-1}$ for sufficiently large values of t. On substituting this in formula (19), we find

$$\sigma_{zz}=e^2l_2/2\pi S.$$ (28)

This answer could also be derived from the kinetic equation.

Let us now determine what error we made in neglecting the intersections. For this purpose let us put $Q(z)$ with $\eta=0$ in the following form (see I):

$$Q(z)=S\cdot\prod_i(1+|\zeta_i|^2)\mathrm{Sp}_\mathbf p\Big\{\Big[1+\sum\tilde\zeta_i\tilde\zeta_m^{\cdot}+\sum\tilde\zeta_i\tilde\zeta_m^{\cdot}\tilde\zeta_n\tilde\zeta_k^{\cdot}+\dots\Big]^{-1}$$
$$\times\Big[1+\sum\tilde\zeta_{l_1}\tilde\zeta_{m_1}^{\cdot}+\sum\tilde\zeta_{l_1}\tilde\zeta_{m_1}^{\cdot}\tilde\zeta_{n_1}\tilde\zeta_{k_1}^{\cdot}+\dots\Big]^{-1}\Big\},$$

where in the first square bracket $l<m<n<k\dots$, and in the second $l_1>m_1>n_1>k_1\dots$.

Let us expand the square brackets with respect to ζ and ζ^*. The general term of such an expansion will have the form

$$\mathrm{Sp}_\mathbf p\{\tilde\zeta_l\tilde\zeta_m^{\cdot}\tilde\zeta_n\tilde\zeta_k^{\cdot}\tilde\zeta_p\tilde\zeta_q^{\cdot}\},$$

where $\tilde\zeta$ and $\tilde\zeta^*$ must alternate. If we substitute formula (22) for ζ and $\tilde\zeta^*$, we shall obtain a phase multiplier of the type

$$\exp\{i[\alpha(\mathbf p)z_l+\alpha(\mathbf p-\mathbf k_1)(z_l-z_m)-\alpha(\mathbf p-\mathbf k_1-\mathbf k_2)(z_m-z_n)$$
$$+\alpha(\mathbf p-\mathbf k_1-\mathbf k_2-\mathbf k_3)(z_n-z_k)-\dots-\alpha(\mathbf p-\dots)z_q]/v\}.$$

Taking into account that on averaging with respect to ζ the coordinates z_l must coincide in pairs, then we will of necessity get phase multipliers of the type

$$\exp\Big\{\frac{i}{v}\Big[\sum_{slm}\alpha(\mathbf p_i)(z_l-z_m)\Big]\Big\}.$$ (29)

On averaging the expressions depending on several z_l's with respect to ζ, it always turns out that these expressions decrease exponentially for $z_i-z_k\sim l_2$. An example of this is the calculation carried out in I, and the computations of $Q(z)$ for small values of α in the next section of this paper.

Consequently, a typical phase multiplier is $\exp[i\alpha(\mathbf p)l_2/v]$. Since integration with respect to $\mathbf p$ is implied in the full expression, we can carry it out in two stages. We shall first average over the interval $|\Delta p|\ll\pi/a$, and then carry out a full integration with respect to $\mathbf p$. If $\alpha l_2/v\gg1$, then $|\Delta p|\ll\pi/a$ can always be chosen so that $|\Delta p\cdot\partial\alpha/\partial p|l_2/v$ is large since $\partial\alpha/\partial p\sim\alpha a$. But the phase multiplier will then give zero on averaging with respect to Δp. If, on the other hand, $\alpha l_2/v\ll1$, then the phase multiplier is as a first approximation equal to unity.

Consequently, in the first limiting case the intersections can be neglected and result (28) is obtained, while in the second limiting case there is a purely one-dimensional situation.

4. LONGITUDINAL CONDUCTIVITY IN THE CASE $\alpha l_2/v \ll 1$

The case $\alpha l_2/v \ll 1$ is the most interesting since, in the first place, it corresponds to a real quasi-one-dimensional substance with internal disorder, and in the second, it makes it possible to see how the finiteness of α leads to the occurrence of finite conductivity. It is not possible to compute expression (19) for non-commuting ζ operators, and therefore the first thing that enters one's head is to expand with respect to α. Care must, however, be exercised here as σ may prove to be non-analytical in the neighborhood of $\alpha = 0$. We shall see below that this is in fact so, but in principle this could be guessed just from the fact that the conductivity of an infinite specimen is equal to zero for $\alpha = 0$, i.e., for a value of α as small as one likes, the first-order term will be infinite with respect to the null-order term. This difficulty can be avoided if a sample of finite length L is considered. We shall therefore consider L finite and find the first terms in the expansion of σ with respect to α.

The null order was determined in I, and it turned out that

$$\sigma_{zz}^{(0)} = {}^1\!/_4 \pi^{1/2} e^2 l_2^{1/2} L^{-1/2} \exp(-L/4l_2). \tag{30}$$

The next term will obviously be proportional to

$$\overline{\alpha^2} = S \cdot \int \alpha^2(\mathbf{p}) d^2p/(2\pi)^2,$$

since $\overline{\alpha} = 0$.

On expanding $S(z, z')$ with respect to α in the form (15) and substituting in expression (24), we obtain

$$Q = \frac{S}{v^2} \mathrm{Sp}_\mathbf{p} \int_0^z dz_1 \int_0^z dz_2 \{ [S_{11}^{-1}]^{-1}[S^{-1}(z_1, 0)\hat{\alpha}S^{-1}(z, z_1)]_{11}[S_{11}^{-1}]^{-1}[S_{11}]^{-1}$$

$$\times [S(z, z_2)\hat{\alpha}S(z_2, 0)]_{11}[S_{11}]^{-1} - [S_{11}^{-1}]^{-1}[S^{-1}(z_1, 0)\hat{\alpha}S^{-1}(z, z_1)]_{11}[S_{11}^{-1}]^{-1}$$

$$\times [S^{-1}(z_2, 0)\hat{\alpha}S^{-1}(z, z_2)]_{11}[S_{11}^{-1}][S_{11}]^{-1} + \theta(z_1 - z_2)[S_{11}^{-1}]^{-1}$$

$$\times [S^{-1}(z_2, 0)\hat{\alpha}S^{-1}(z_1, z_2)\hat{\alpha}S^{-1}(z, z_1)]_{11}[S_{11}^{-1}]^{-1}[S_{11}]^{-1}$$

$$- [S_{11}^{-1}]^{-1}[S_{11}]^{-1}[S(z_1, z_1)\hat{\alpha}S(z_1, 0)]_{11}[S_{11}]^{-1}[S(z, z_2)\hat{\alpha}S(z_2, 0)]_{11}[S_{11}]^{-1}$$

$$+ \theta(z_1 - z_2)[S_{11}^{-1}]^{-1}[S_{11}]^{-1}[S(z, z_1)\hat{\alpha}S(z_1, z_2)\hat{\alpha}S(z_2, 0)]_{11}[S_{11}]^{-1}\}, \tag{31}$$

where $\hat{\alpha} = \alpha\sigma_3$, while $S_{11} = S_{11}(z, 0)$. Note should be taken of the following. Since each ζ (or η) operator exchanges $\mathbf{p}$ for $\mathbf{p} - \mathbf{k}$, then having taken, on averaging with respect to ζ, the sequence order, let us say, of $\zeta\hat{\alpha}\zeta^*\hat{\alpha}$, we obtain

$$\int |\zeta(\mathbf{k})|^2 \alpha(\mathbf{p})\alpha(\mathbf{p}-\mathbf{k}) d^2k/(2\pi)^2.$$

Let us consider the potential as short-range, i.e., let us assume that its radius of action is $r_0 \ll a$. In this case integration with respect to $\mathbf{k}$ within the limits of the cell of the reciprocal lattice is in fact done for constant $|\zeta(\mathbf{k})|^2$, and since $\overline{\alpha}(\mathbf{p}) = 0$, this integral vanishes. This means that non-null terms must have a sequence order of the averaging operators of $\zeta\hat{\alpha}\hat{\alpha}\zeta^*$ or $\hat{\alpha}\zeta\zeta^*\hat{\alpha}$. In other words, the ζ and η operators situated between two $\hat{\alpha}$'s and those which stand outside $\hat{\alpha}\ldots\hat{\alpha}$, are averaged in-

dependently. After this one can forget the operator character of ζ in the sense of translations of $\mathbf{p}$, and the trace in $\mathbf{p}$ will simply denote

$$\int \alpha^2(\mathbf{p}) d^2p/(2\pi)^2 = \overline{\alpha^2}/S$$

On removing the brackets in (31) and taking what has been said above into account, we get

$$Q = \overline{(\alpha/v)^2} \int_0^z dz_1 \int_0^z dz_2 \{ 2\theta(z_1-z_2) [S_{11}(z, z_1)S_{11}(z_2, 0) + S_{12}(z, z_1)S_{21}(z_2, 0)]$$

$$\times \exp\{(z_1-z_2)(l_2^{-1}-l_1^{-1})/2\}/S_{11}^2 S_{22} -$$

$$- 2[S_{11}(z, z_2)S_{11}(z_1, 0)/S_{11}][S_{11}(z, z_1)S_{11}(z_2, 0)/S_{11}^2 S_{22}]$$

$$- 2[S_{12}(z, z_2)S_{21}(z_1, 0)/S_{11}][S_{12}(z, z_1)S_{21}(z_2, 0)/S_{11}^2 S_{22}]$$

$$+ 2[S_{11}(z, z_2)S_{21}(z_1, 0)/S_{11}][S_{12}(z, z_1)S_{11}(z_2, 0)/S_{11}^2 S_{22}]$$

$$+ 2[S_{12}(z, z_2)S_{11}(z_1, 0)/S_{11}][S_{11}(z, z_1)S_{21}(z_2, 0)/S_{11}^2 S_{22}]$$

$$+ [S_{22}(z, z_1)S_{11}(z, z_2)/S_{11}S_{22}][S_{22}(z_1, 0)S_{11}(z_2, 0)/S_{11}S_{22}]$$

$$+ [S_{12}(z, z_2)S_{21}(z, z_1)/S_{11}S_{22}][S_{12}(z_1, 0)S_{21}(z_2, 0)/S_{11}S_{22}]$$

$$- [S_{22}(z, z_1)S_{12}(z, z_2)/S_{11}S_{22}][S_{22}(z_1, 0)S_{21}(z_2, 0)/S_{11}S_{22}]$$

$$- [S_{21}(z, z_1)S_{11}(z, z_2)/S_{11}S_{22}][S_{11}(z_2, 0)S_{12}(z_1, 0)/S_{11}S_{22}]\}. \tag{32}$$

(Here and below the square brackets denote averaging over the fields η and ζ in accordance with formula (12).) In deriving this formula use was made of the properties of $S_{\alpha\beta}$ found in I as these quantities are now purely one-dimensional, i.e., they do not have an operator character. In (32) each expression contained in the square brackets is averaged independently. Use was also made of the invariant nature of each bracket with respect to the substitution $\zeta^* \rightleftarrows \zeta$ and of the property

$$[S_{\alpha\beta}(z_1, z_2)] = \delta_{\alpha\beta}\exp\{(z_1-z_2)(l_2^{-1}-l_1^{-1})/2\}.$$

Of the terms derived, all those containing in any bracket a different index figure 1 and 2 in the numerator vanish. In the terms left it is easy to find averages of the type $[S_{\alpha 1}(z_1, 0)S_{1\beta}(z, z_2)/S_{11}(z, 0)]$, since these are precisely those combinations which enter into the components $G_{\alpha\beta A}$ (see I). After this

$$\left\{ 2\exp\{-(z_1-z_2)(l_2^{-1}+l_1^{-1})/2\}\theta(z_1-z_2) \left[\frac{2S_{12}(z, z_1)S_{21}(z_2, 0)}{S_{11}^2 S_{22}} - \frac{S_{11}(z_1, z_2)}{S_{11}S_{22}} \right] \right.$$

$$+ \left[\frac{S_{22}(z, z_1)S_{11}(z, z_2)}{S_{22}S_{11}} \right]\left[\frac{S_{22}(z_1, 0)S_{11}(z_2, 0)}{S_{11}S_{22}} \right]$$

$$\left. + \left[\frac{S_{12}(z, z_2)S_{21}(z, z_1)}{S_{11}S_{22}} \right]\left[\frac{S_{12}(z_1, 0)S_{21}(z_2, 0)}{S_{11}S_{22}} \right] \right\} \tag{33}$$

is left in the brace brackets of expression (32).

First of all, let us analyze the role of the field η. If one changes to the interaction representation with respect to η (see I), then since on averaging with respect to ζ compensation of the ζ and ζ^* multipliers $\exp(\pm 2i\eta z/v)$ occurs, it is sufficient to take only the external multipliers $\exp(i\eta z/v)$ into account. It is not difficult to see that after averaging with respect to η the same dependence on l_1 appears in all the terms of expression (33), viz., the multiplier $\exp\{-(z_1-z_2)/l_1\}$. Bearing this in mind, we can carry out all the future computations without taking account of the field η.

First of all, let us note that in the second and third terms the case $z_1 > z_2$ is equivalent to the case $z_2 > z_1$. Indeed, each of the averages which occur here can be subjected to a formal transformation $\zeta \rightleftarrows \zeta^*$, after which

122   Sov. Phys. JETP, Vol. 45, No. 1, January 1977

A. A. Abrikosov and I. A. Ryzhkin   122

$z_1 > z_2$ goes over into $z_2 > z_1$. Consequently, it is sufficient to consider, for example, the case $z_1 > z_2$. Let us calculate

$$A_1(z, z_1, z_2) = [S_{11}(z_1, 0)S_{22}(z_2, 0)/S_{11}(z, 0)S_{22}(z, 0)]. \quad (34)$$

In analogy with the procedure in I, one can write equations connecting the various A_{1n}'s:

$$A_{1n} = \left[\frac{S_{12}{}^n(z, 0)S_{21}{}^n(z, 0)}{S_{11}^{n+1}(z, 0)S_{22}^{n+1}(z, 0)} S_{11}(z_1, 0)S_{22}(z_2, 0) \right]. \quad (35)$$

This system tallies with the one which was obtained for C_n in I:

$$\partial A_{1n}/\partial t = n^2 A_{1(n-1)} + (n+1)^2 A_{1(n+1)} - \{2n(n+1)+1\}A_{1n}, \quad (36)$$

where $t = z/l_2$. For $t = t_1 = z_1/l_2$

$$A_{1n} \to P_n(z_1, z_2)\exp\{-(t_1-t_2)/2\},$$

$$P_n(z_1, z_2) = \left[\frac{S_{12}{}^n(z_1, 0)S_{21}{}^n(z_1, 0)}{S_{11}{}^n(z_1, 0)S_{22}^{n+1}(z_1, 0)} S_{22}(z_2, 0) \right]\exp\{(t_1-t_2)/2\}. \quad (37)$$

These quantities are defined by the system of equations

$$\partial P_n/\partial t_1 = n^2 P_{n-1} + n(n+1)P_{n+1} - \{n^2+n(n+1)\}P_n. \quad (38)$$

Finally, for $z_1 \to z_2$, the quantities P_n change into the B_n's found in I:

$$B_n(z_2) = [S_{12}{}^n(z_2, 0)S_{21}{}^n(z_2, 0)/S_{11}{}^n(z_2, 0)S_{22}(z_2, 0)].$$

In I it was shown that $B_n(z_2)$ contains a part which is independent of z_2, viz., $B_n(z_2) = 1$, and a part which decreases exponentially at distance l_2. We only need the first part. From equation (38) it is evident that $P_n = 1$ satisfies both the equation and the boundary condition. On changing to the equation for A_{1n}, we see that the solution will be

$$A_{1n} = \exp[-(t_1-t_2)/2].$$

In particular, $A_{10} = A_1$ which we have been working out will be of this type.

Substituting this result in the second term of formula (33), taking account of the multiplier $\exp[-(z_1 - z_2)/l_1]$ from the field η, and integrating with respect to z, we get

$$2\int_0^z dz_1 \int_0^{z_1} dz_2 \exp[-(z_1-z_2)(l_1{}^{-1}+l_2{}^{-1})] \approx 2z(l_1{}^{-1}+l_2{}^{-1})^{-1}$$

(the factor 2 is due to the two regions $z_1 > z_2$ and $z_1 < z_2$).

To work out the third term of formula (33) it is necessary to find the averages

$$A_{2n} = \left[\frac{(S_{12}(z, 0)S_{21}(z, 0))^n}{(S_{11}(z, 0)S_{22}(z, 0))^{n+1}} S_{12}(z_1, 0)S_{21}(z_2, 0) \right].$$

For $z \to z_1$ the quantity A_{2n} becomes $K_n(z_1, z_2)$:

$$K_n(z_1, z_2) = \left[\frac{S_{12}^{n+1}(z_1, 0)S_{21}{}^n(z_1, 0)}{S_{11}^{n+1}(z_1, 0)S_{22}^{n+1}(z_1, 0)} S_{21}(z_2, 0) \right].$$

Finally, for $z_1 \to z_2$, $K_n \to B_{n+1}(z_2)$. Analogously to what was done for A_{1n}, we get

$$A_{2n} = A_{1n} = \exp[-(t_1-t_2)/2]. \quad (39)$$

Consequently the contribution of the third term coincides with that of the second.

It is considerably more difficult to find the contribution of the first term in (33). The corresponding computation gives $2l_1 l_2{}^2/(l_1+l_2)$. But if one considers that $z \gg l_2$, the contribution of this term is small in comparison to those found earlier. It is for this reason that we shall not set out here the long calculation leading to this result (unfortunately, we have not succeeded in demonstrating the smallness of the contribution of the first term compared with the others without a full calculation). Substituting the results for the second and third terms in (33) in the formula for the conductivity, we get

$$\sigma_{zz}^{(1)} = 2\pi^{-1}e^2\overline{\alpha^2}v^{-2}(l_1{}^{-1}+l_2{}^{-1})^{-1}L^2/S,$$
$$\overline{\alpha^2} = S\int\alpha^2(\mathbf{p})\,d^2p/(2\pi)^2 = (\alpha_x{}^2+\alpha_y{}^2)/2. \quad (40)$$

Consequently, formulae (30) and (40) describe the null and first orders in $\overline{\alpha^2}$ for conductivity with finite L. The calculation of the subsequent orders in α presents considerable difficulties, not to mention the finding of a general expression, and without this it is not possible to find the limit $L \to \infty$. In order to find an answer we shall use the hypothesis relating to the existence of a correlation length, i.e., of a value $L_c(\alpha)$ above which the sample can be considered infinite. In the spirit of similarity theory one can assume that for $L \approx L_c(\alpha)$ all the terms of the series for σ_{zz} become of one order. Comparing expressions (30) and (40), we get from this

$$L_c \approx 4l_2 \ln(v^2/l_2{}^2\overline{\alpha^2}). \quad (41)$$

Let us note that for $\alpha l_2/v \ll 1$ we have $L_c \gg l_2$, which justifies neglecting the first term in (33) in comparison with the others, even for $L \approx L_c$. From formula (40) it is evident that in this case σ_{zz} reaches a value of the order of

$$\sigma_{zz} \sim \frac{32e^2}{\pi S}\overline{\alpha^2}\left(\frac{l_2}{v}\right)^2 \frac{l_1 l_2}{l_1+l_2}\ln^2\frac{v^2}{\overline{\alpha^2}l_2{}^2}, \quad (42)$$

which can also be considered the conductivity of an infinite sample.

We must emphasize once more that expression (42) is the consequence of the hypothesis adopted. Unfortunately, we have not succeeded in deriving σ_{zz} more strictly. The similarity of the formula obtained to expression I(93) for the purely one-dimensional case with $\omega_0 \neq 0$ is noteworthy (for further details see Conclusion).

5. THE TRANSVERSE CONDUCTIVITY

The transverse conductivity is found considerably more simply. In the general expression (17) the velocity operators $v\sigma_3$ are replaced by $\partial\alpha/\partial p_i$. On doing this we get

$$\sigma_{ik}^{\perp} = \frac{e^2}{2\pi}\text{Sp}_{p\sigma}\int_{-\infty}^{\infty}dz_1\left[G_R(z, z_1)\frac{\partial\alpha}{\partial p_i}G_A(z_1, z)\frac{\partial\alpha}{\partial p_k}\right]. \quad (43)$$

As previously, we shall analyze the simpler case of a short-range potential: $r_0 \ll a$. In this case the sequence order $\zeta_i \alpha \zeta_i^* \alpha$ is not permissible on averaging, and consequently

$$\sigma_{ik}^{\perp} = \frac{e^2}{2\pi} \mathrm{Sp}_p \left(\frac{\partial \alpha}{\partial p_i} \frac{\partial \alpha}{\partial p_k} \right) \int_{-\infty}^{\infty} dz_1 [G_{R\alpha\beta}(z,z_1)][G_{A\beta\alpha}(z_1,z)]. \quad (44)$$

is left.

It is not difficult to show that the averages of the G-functions depend on α only through the external multipliers $\exp(i\alpha\sigma_3 z/v)$ which cancel in (44). Indeed, if use is made of formula (16), then only $\Pi(1 + |\zeta_i|^2/2)$ need be taken into account in each S as there is nothing to average the other ζ's with. In this sense the situation does not differ from the purely one-dimensional case. If this is taken into account, we get, for example,

$$[G_{R11}(z > z_1)] = \exp(-i\alpha z/v)\left[\prod_i (1 - |\zeta_i|^2/2)\exp(i\eta_i) \right] \exp(i\alpha z'/v)$$
$$= \exp\{-i\alpha(z-z_1)/v\}\exp\{-(z-z_1)(l_1^{-1}+l_2^{-1})/2\}.$$

Consequently, the purely one-dimensional expressions

$$[G_{R11}(z,z_1)] = -(i/v)\exp\{-(z-z_1)(l_1^{-1}+l_2^{-1})/2\}\theta(z-z_1) = -[G_{A22}],$$
$$[G_{R22}(z,z_1)] = -(i/v)\exp\{-(z_1-z)(l_1^{-1}+l_2^{-1})/2\}\theta(z_1-z) = -[G_{A11}].$$

are left. Substituting this in (44), we get

$$\sigma_{ik}^{\perp} = \frac{e^2}{2\pi} \frac{1}{Sv^2} \overline{\frac{\partial \alpha}{\partial p_i} \frac{\partial \alpha}{\partial p_k}} \frac{l_1 l_2}{l_1+l_2}. \quad (45)$$

If use is made of formula (3) for α, we get

$$\sigma_{xx} = \frac{e^2}{2\pi} \frac{\alpha_x^2 a_x^2}{v^2 S} \frac{l_1 l_2}{l_1+l_2}, \quad \sigma_{yy} = \frac{e^2}{2\pi} \frac{\alpha_y^2 a_y^2}{v^2 S} \frac{l_1 l_2}{l_1+l_2}, \quad \sigma_{xy} = 0. \quad (46)$$

6. CONCLUSION

The formula (46) obtained for the transverse conductivity is identical with expression (10a) which results from a diffusion evaluation. The evaluating formula (10b) differs from (42) in the absence of the multiplier $\ln^2(v^2/l_2^2\alpha^2)$, which, as already noted, is a consequence of a somewhat inaccurate assumption made in the evaluation. Let us note, however, that the two mean free paths l_1 and l_2 enter into both formulae in the same way, confirming the correct physical interpretation of longitudinal conductivity.

Formula (42) is very reminiscent of the formula for the longitudinal conductivity of a purely one-dimensional metal in an a.c. field (see I, formula (93)), which is

quite natural. Both these formulae correspond to the well known qualitative evaluation due to Mott,[4] the role of the energy required to overcome the difference in levels in the random potential is played in one case by ω, and in the other by α. A linear term is naturally obtained only for $\omega \neq 0$ as $\bar{\alpha}(p) = 0$.

Consequently, making use of the expansion with respect to α, it is possible to find the longitudinal conductivity using the lacing hypothesis only with an accuracy to an unknown numerical coefficient. To check this hypothesis and to determine the coefficient it is necessary to manage without expanding with respect to α, which is not realistic in the framework of the method used. Another method is to examine a long-range potential. In doing this formula (20) has to be used. As has already been said in the appropriate place, one can in this case use the expansion with respect to $|\exp(i k \cdot \rho_i) - 1| \ll 1$ (since $k \sim r_0^{-1}$, $\rho_i \sim a$ and $r_0/a \gg 1$). As a first approximation it is sufficient to take account of the correction to one impurity line. If this line is separated out, then on the one side of it is the ρ_i-component from a certain expression, and on the other the ρ_i-component from another expression. But since

$$F_i = S \int F[\alpha(p)]\exp(i p \rho_i) d^2p/(2\pi)^2.$$

one can consider with logarithmic accuracy that on the left-hand side of the line separated out $\alpha_1 = \mathrm{constant}$, and on the right-hand side $\alpha_2 = \mathrm{constant}$, which play the role of different frequencies. Such a computational program proved in practice very complicated, but this route is apparently not without hope.

---

[1]Here and in what follows we shall assume the electrons to have zero spin. To take account of spin the whole expression for σ_{ik} must be multiplied by 2.

[1]A. A. Abrikosov and I. A. Ryzhkin, Zh. Eksp. Teor. Fiz. 71, 1916 (1976) [Sov. Phys. JETP 44, 1005 (1976)].

[2]V. L. Berezinskii, Zh. Eksp. Teor. Fiz. 65, 1251 (1973) [Sov. Phys. JETP 38, 620 (1974)].

[3]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, Metody kvantovoy teorii polya v statisticheskoy fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962, §32, Part 2 [Pergamon, 1965].

[4]N. F. Mott and E. A. Davis, Electronic Processes in Noncrystalline Materials, Oxford U.P., 1971 (Russ. Transl., Mir, 1974, p. 29).

Translated by N. G. Anderson