is equal to 9 MHz, which is also in good agreement with the measured B=8 MHz.

The considered mechanism of phaser generation under conditions of spatial disequilibrium corresponds to a monotonic decrease of the spectrum intensity with increasing distance from the center of the resonance lines. This monotonic decrease, as noted above, is observed only at small excesses above the generation threshold. The fact that the decrease is no longer monotonic at high pump levels (Figs. 6 and 7) demonstrates the limited character of this analysis. The irregularities and the dips in the spectrum show that at high pump levels the resonance line acquires a fine structure which apparently leads to the aforementioned spectral disequilibrium, when individual regions of lines that are quite close in frequency make independent contributions to the generation.

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<sup>1)</sup>Brief reports of some results of this work were published earlier. <sup>19,10</sup>

- <sup>2)</sup>In the pulsed measurements, the resonant properties of the rod can be disregarded, since  $\tau v < l_0$  ( $\tau$  is the pulse duration and  $l_0$  is the length of the rod).
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# Dynamic damping of dislocations in ferromagnets

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Dynamic damping of dislocations by spin waves in ferromagnets is investigated. The dependence of the magnon damping force on temperature and velocity is calculated. A general picture is presented of the temperature dependence of the dynamic damping force on dislocations in ferromagnets.

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### INTRODUCTION

Motion of dislocations in a crystal, which causes the process of plastic deformation, is limited, as is well known (see, for example,<sup>[1]</sup>), by two qualitatively different phenomena: the surmounting of barriers through

fluctuations, and dynamic damping caused by scattering of the energy of a dislocation by elementary excitations in the crystals (phonons, electrons, spin waves, etc.). Because of the fact that the density of an elementary-excitation gas increases with rise of temperature (electrons in a normal metal are an exception), the contribu-

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tion of dynamic damping of a dislocation increases, and calculation of it is necessary for an understanding of the processes that control plastic deformation. [1,2]

In ferromagnetic materials, moving dislocations must be damped because of interactions of their elastic fields with the magnetization of the ferromagnet.

The present paper is devoted to a study and analysis of the damping force caused by dissipation of the energy of a dislocation by magnons in ferromagnets<sup>1)</sup>. It is shown that at T=0 there is a critical dislocation velocity

$$v_c = 2(\Delta \Theta_c a^2)^{\frac{1}{2}}, \quad \Delta = 2\mu M_0 (\beta + H_0/M_0),$$

where  $\Delta$  is the activation energy of a spin wave,  $\mu$  is the Bohr magneton,  $M_0$  is the magnetic-moment density at saturation,  $\beta$  is the anisotropy constant,  $H_0$  is the external magnetic field,  $\Theta_c$  is the Curie temperature, and a is the lattice constant<sup>2</sup>). At velocities  $v < v_c$ , the damping force F on a dislocation is zero; but for  $v > v_c$ , it begins to increase; and for  $v \gg v_c$ , it is quadratic in the dislocation velocity:  $F \sim v^2$ . This peculiarity of magnetic damping is a consequence of the existence of an activation energy in the spectrum of quasiparticles of a ferromagnet and is due to the process of generation of spin waves by a dislocation when  $v > v_c$ .

For  $T \neq 0$ , damping exists at arbitrary dislocation velocities; but for  $T \ll \Delta$ , it remains exponentially small in the range  $v < v_c$ , while for  $T \gg \Delta$ , it increases with increase of temperature  $\sim (T/\Theta_c)^{5/2}$ ; then at dislocation velocities  $v \ll v_c$ , the damping force increases linearly with increase of v. For velocities  $v > v_c$  and temperatures  $T \ll \Delta$ , the damping is caused principally by generation of spin waves by the moving dislocation, and the character of the velocity dependence changes from a linear to a quadratic. For  $T \gg \Delta$  in the range  $v \gg v_c$ , the contribution of the mechanism of generation of spin waves is negligibly small in comparison with the contribution of the mechanism of scattering of magnons by dislocations; over a broad velocity range  $v \ll (T\Theta_c a^2)^{1/2}$ , the damping has a viscous character:  $F \sim v$ .

In the paper, an analysis is made of the role of the various mechanisms of dissipation of the energy of a dislocation in a ferromagnet, and a description is given of the general character of the temperature dependence of the dynamic damping force on dislocations.

## 1. THE HAMILTONIAN FOR INTERACTION OF SPIN WAVES WITH DISLOCATIONS IN FERROMAGNETS

We consider the scattering of spin waves by dislocations in ferromagnets. We write the Hamiltonian of this interaction in the form

$$\mathscr{H}_{int} = \int \lambda_{iklm} M_l M_m u_{ik}(\mathbf{r}, t) \, d\mathbf{r} + \int \gamma_{iklm} \frac{\partial \mathbf{M}}{\partial x_i} \frac{\partial \mathbf{M}}{\partial x_k} u_{lm}(\mathbf{r}, t) \, d\mathbf{r}, \qquad (1.1)$$

where  $u_{ik}(\mathbf{r}, t)$  is the dislocation deformation tensor;  $\lambda_{iklm}$  and  $\gamma_{iklm}$  are magnetostriction-constant tensors, of which the first describes magnetoelastic effects for uniform and the second for nonuniform magnetization; **M** is the magnetic-moment density vector; the integration extends over the volume V of the crystal. Hereafter we shall consider the isotropic case, for which

$$\begin{aligned} & \lambda_{iklm} = \lambda_0 \delta_{ik} \delta_{lm} + \lambda \left( \delta_{il} \delta_{km} + \delta_{im} \delta_{kl} \right), \\ & \gamma_{iklm} = \left( \Theta_c a^2 / \mu M_0 \right) \left[ \frac{1}{2} \left( \delta_{il} \delta_{km} + \delta_{im} \delta_{kl} \right) \beta_1 + \beta_2 \delta_{ik} \delta_{lm} \right], \end{aligned}$$

where  $\lambda_0$ ,  $\lambda$ ,  $\beta_1$ , and  $\beta_2$  are magnetoelastic constants.

We express the Hamiltonian (1.1) in terms of generation operators  $a_k^*$  and annihilation operators  $a_k$  of spin waves with wave vector k. It is well known (see, for example, <sup>[4]</sup>) that

$$M_{z} \approx (\mu M_{0}/2V)^{\frac{1}{2}} \sum_{\mathbf{k}} [a_{\mathbf{k}}e^{i\mathbf{k}\mathbf{r}} + a_{\mathbf{k}}^{+}e^{-i\mathbf{k}\mathbf{r}}],$$

$$M_{v} \approx i(\mu M_{0}/2V)^{\frac{1}{2}} \sum_{\mathbf{k}} [a_{\mathbf{k}}e^{i\mathbf{k}\mathbf{r}} - a_{\mathbf{k}}^{+}e^{-i\mathbf{k}\mathbf{r}}],$$

$$M_{z} = M_{0} - (\mu/V) \sum_{\mathbf{k},\mathbf{k}'} a_{\mathbf{k}}^{+}a_{\mathbf{k}'}e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}}.$$
(1.3)

If we then describe the elastic field of the dislocation in the form

$$u_{ik}(\mathbf{r},t) = u_{ik}(\mathbf{r}-\mathbf{v}t) = \sum_{a} u_{ik}(\mathbf{q}) e^{i(\mathbf{q}\mathbf{r}-\mathbf{s}t)}$$
(1.4)

(where  $u_{ik}(\mathbf{q})$  is the Fourier transform of the static deformation field of the dislocation, and where  $\omega = \mathbf{q} \cdot \mathbf{v}$ ), substitute (1.2)-(1.4) in (1.1), and retain terms quadratic in the spin-wave operators, we get<sup>3</sup>)

$$\mathscr{H}_{int} = \sum_{\mathbf{q}} \left\{ \Phi_{i}(\mathbf{k},\mathbf{q}) a_{\mathbf{k}+\mathbf{q}}^{\dagger} a_{\mathbf{k}} + \Phi_{2}(\mathbf{q}) a_{\mathbf{k}}^{\dagger} a_{-\mathbf{k}+\mathbf{q}}^{\dagger} \right\} e^{-i\omega t} + \text{h.c.}$$
(1.5)

where

$$\Phi_{1}(\mathbf{k},\mathbf{q}) = \frac{1}{2}\lambda\mu M_{0}[u_{xx}(\mathbf{q}) + u_{yy}(\mathbf{q}) - 2u_{zz}(\mathbf{q})]$$

$$+ \Theta_{c}a^{2}[\beta_{1}k_{l}(\mathbf{k}-\mathbf{q})_{m}u_{lm}(\mathbf{q}) + \beta_{2}(\mathbf{k}\cdot\mathbf{k}-\mathbf{q})u_{ll}(\mathbf{q})],$$

$$\Phi_{2}(\mathbf{q}) = \frac{1}{2}\lambda\mu M_{0}[u_{xx}(\mathbf{q}) - u_{yy}(\mathbf{q}) - 2iu_{xy}(\mathbf{q})].$$

As follows from formula (1.5), interaction of spin waves with dislocations is caused both by processes of emission and absorption of a phonon by a magnon, and by processes of fusion of two magnons into a phonon and splitting of a phonon into two magnons. Each of these processes is characterized by the appropriate amplitude  $\Phi_1(\mathbf{k}, \mathbf{q})$  or  $\Phi_2(\mathbf{q})$ . The simple time dependence of the Hamiltonian of interaction of a moving dislocation with spin waves,

$$\mathcal{H}_{int} = \sum_{\mathbf{q}} A(\mathbf{q}) e^{-i\omega t}$$

allows us to treat the energy acquired by spin waves from moving dislocations as energy of absorption of quanta during various transition processes in the magnon system.

## 2. RATE OF DISSIPATION OF ENERGY OF A MOVING DISLOCATION IN A FERROMAGNET

The rate of dissipation of energy per unit length of a moving dislocation is determined by the expression

$$W = -\frac{1}{L} \sum_{\mathbf{k},\mathbf{q}} \omega(\mathbf{q}) v_{\mathbf{k},\mathbf{k}+\mathbf{q}}, \qquad (2.1)$$

where L is the length of the dislocation line, and where

 $\nu_{\mathbf{k},\mathbf{k}+\mathbf{q}}$  is the probability of a transition in the magnon system from a state with wave vector  $\mathbf{k}$  to a state with wave vector  $\mathbf{k}+\mathbf{q}$ , during which a quantum  $\omega(\mathbf{q})$  is absorbed (radiated).

The dissipation of the energy of a dislocation is determined, as was mentioned above, by two types of scattering processes, each of which is characterized by its own transition probability. By calculating the respective probabilities and substituting the expressions for them in formula (2.1), we find

$$W = 2\pi \sum_{\mathbf{k},\mathbf{q}} \omega(\mathbf{q}) \{ |\Psi_{1}(\mathbf{k},\mathbf{q})|^{2} (n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{q}}) \delta(\varepsilon_{\mathbf{k}} + \omega - \varepsilon_{\mathbf{k}+\mathbf{q}}) + |\Phi_{2}(\mathbf{q})|^{2} (1 + n_{\mathbf{k}} + n_{-\mathbf{k}+\mathbf{q}}) \delta(\varepsilon_{\mathbf{k}} + \varepsilon_{-\mathbf{k}+\mathbf{q}} - \omega) \}, \qquad (2.2)$$

where  $n_{\mathbf{k}} = [\exp(\varepsilon_{\mathbf{k}}/T) - 1]^{-1}$  is the number of spin waves in the state with energy  $\varepsilon_{\mathbf{k}}$ .

We shall suppose that the external magnetic field  $H_0$ is large enough so that the energy of a magnon may be considered to be  $\varepsilon_{\mathbf{k}} = \Delta + \Theta_c(ak)^2$ .<sup>[4]</sup> Then on going over from summation to integration in formula (2. 2) and integrating, we obtain after simplification

$$W = W_{1} + W_{2},$$

$$W_{1} = \frac{V}{8\pi^{2}\Theta_{c}a^{2}} \sum_{\mathbf{q}} \frac{\omega(\mathbf{q})}{q} \int_{0}^{\infty} k \, dk \, (n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{q}}) \int_{0}^{2\pi} |\Phi_{1}(\mathbf{k},\mathbf{q})|^{2} \, d\Psi,$$

$$\Omega = |(\omega - \Theta_{c}(aq)^{2})/2\Theta_{c}a^{2}q|,$$

$$W_{2} = \frac{VT}{4\pi(\Theta_{c}a^{2})^{2}} \sum_{\mathbf{q}} \frac{\omega(\mathbf{q})}{q} |\Phi_{2}(\mathbf{q})|^{2} \ln \frac{\mathrm{sh}\left(\varepsilon_{+}/2T\right)}{\mathrm{sh}\left(\varepsilon_{-}/2T\right)},$$
(2.3)

where

$$\varepsilon_{\pm}(\mathbf{q},\boldsymbol{\omega}) = -\frac{\omega}{2} \pm \frac{q}{2} \{ [2\omega - 4\Delta - \Theta_c(aq)^2] \Theta_c a^2 \}^{\frac{1}{2}},$$

and where  $\Psi$  is the azimuthal angle of the vector **k**.

The appearance of two terms in (2.3) is caused by the two types of scattering processes: the first of these describes the part of the damping that is due to radiation and absorption of phonons by magnons and exists at arbitrary dislocation velocities. The second term is due to fusion of two magnons into a phonon and splitting of a phonon into two magnons. As follows from the formula, these processes make a contribution only beginning with velocity  $v \ge v_c = 2(\Delta \Theta_c a^2)^{1/2}$ . The expression (2.3) describes the dependence of the damping force of the magnons on temperature and velocity.

## The case $v \ll (T\Theta_c a^2)^{1/2}$

The damping of dislocations in this case is determined by the first term of formula (2.3), in which the expression for the amplitude of scattering of spin waves by straight-line dislocations is conveniently written in the form<sup>[5]</sup>

$$\Phi_{i}(\mathbf{k},\mathbf{q}) = -\frac{\pi bi}{qV} \lambda \mu M_{0} \varkappa (\mathbf{q}_{0},\mathbf{b},\tau) \delta(\mathbf{q}\tau) + \frac{2\pi bi}{qV} \Theta_{c} \beta_{0}(ak)^{2} \varphi(\mathbf{q}_{0},\mathbf{b},\tau,\mathbf{k}_{0}) \delta(\mathbf{q}\tau),$$
(2.4)

where  $\beta_0 \sim \beta_1 \sim \beta_2 \approx 1$ ;  $\varkappa$  and  $\varphi$  are certain functions of order of magnitude unity, dependent on the directions of the wave vectors **q** and **k**, of the Burgers vector **b**, and

of the unit vector  $\tau$  tangent to the axis of the dislocation. Noting that at low temperatures,  $T \ll \Delta$ , the chief contribution to the damping of dislocations comes from the first term of the expression (2.4), we express formula (2.3) in the form

where  $C_1$  is a constant of order of magnitude unity,  $\mathbf{q}_{\perp}$  is the projection of the wave vector on the plane perpendicular to  $\boldsymbol{\tau}$ , and  $\chi$  is the azimuthal angle of the vector  $\mathbf{q}_{\perp}$  in the plane plane perpendicular to  $\boldsymbol{\tau}$ .

After integrating in formula (2.5), we find for the damping force on dislocations,  $F_s = \dot{W}/v$ , the following expression:

$$F_{s} = B_{s}^{(1)} (T/\Theta_{c})^{\frac{1}{2}} e^{-\Delta/T} v, \quad B_{s}^{(1)} = C_{1} (4\pi)^{-2} \lambda^{2} (\mu M_{0}/\Theta_{c})^{2} b^{2} a^{-5}.$$
 (2.6)

The coefficient of damping of a dislocation by spin waves because of relativistic magnetoelastic interaction is  $B_s^{(1)} \approx \lambda^2 \cdot 10^{-14} \text{ g/cm}^{-1} \text{ sec}^{-1}$ . For ferromagnets with  $\lambda \approx 5.10^2$ we have  $B_s^{(1)} \approx 10^{-9} \text{ g/cm}^{-1} \text{ sec}^{-1}$ .

At temperatures  $T \gg \Delta$ , the damping force is determined chiefly by the second term of the amplitude  $\Phi_1(\mathbf{k}, \mathbf{q})$  of (2.4), which is due to magnetoelastic interaction of exchange type. In this case the rate of energy loss  $\hat{W}$  is again described by formula (2.5), in which

$$\Lambda = \beta^2 b^2 C_2 / 4\pi (2\pi)^{-\alpha} \Theta_{\epsilon} a^2, C_2 \approx 1,$$
  
$$\mathcal{F}(\epsilon_{u}, \omega) \approx \int_{-\infty}^{\infty} d\epsilon \left(\epsilon - \epsilon_0\right)^2 [n(\epsilon) - n(\epsilon + \omega)],$$

hence we have for the damping force on a dislocation

$$F_{s}=B_{s}^{(1)}(T/\Theta_{c})^{s/2}v,$$

$$B_{s}^{(2)}=(2\pi)^{-3}\pi^{t/2}C_{2}\beta_{0}^{-2}b^{2}a^{-5}.$$
(2.7)

The coefficient of magnon damping because of magnetoelastic interaction of exchange type is  $B_s^{(2)} \approx \beta^2 \cdot 10^{-4}$  $g/cm^{-1} sec^{-1}$ . For ferromagnets with  $\beta \sim 10$ , we have  $B_s^{(2)} \approx 10^{-2} g/cm^{-1} sec^{-1}$ .

Formulas (2.6) and (2.7) describe the temperature dependence of the damping force on dislocations in the limit of low velocities,  $v \ll (T\Theta_c a^2)^{1/2}$ . In these formulas there are still undetermined constants  $C_1$ ,  $C_2 \sim 1$ . These constants can be calculated for specific dislocations.

If a straight-line screw dislocation, oriented along the axis of anisotropy, moves in such a way that the vector **v** makes an angle  $\alpha$  with the dislocation axis  $\tau$ , then  $C_2 = \pi/4$ , and for the low-temperature case  $T \ll \Delta$  we find the following expression for the magnon damping force on the dislocation:

$$F_{s} = \begin{cases} B_{s}^{(2)} \sin^{2} \alpha (T/\Theta_{c})^{\frac{1}{2}} e^{-\Delta/T} v, & v \sin \alpha \ll (T\Theta_{c}a^{2})^{\frac{1}{2}}, \\ 2B_{s}^{(2)} (\Theta_{c}a)^{\frac{2}{2}} v^{-1} (T/\Theta_{c})^{\frac{1}{2}} e^{-\Delta/T}, & v \sin \alpha \gg (T\Theta_{c}a^{2})^{\frac{1}{2}}. \end{cases}$$
(2.8)

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FIG. 1. Schematic representation of the velocity dependence of the magnon damping force on dislocations (in the limiting case  $T \ll \Delta$ ) for two orientations of the dislocations.

In the case  $T \gg \Delta$ , the expression for the damping force on a dislocation is described, over a wide velocity range  $v \ll (T \Theta_{\mu} a^2)^{1/2}$ , by the function

$$F_{s} = \pi^{-\frac{1}{2}} \zeta^{(3/2)} B_{s}^{(2)} \sin^{2} \alpha (T/\Theta_{c})^{3/2} v, \qquad (2.9)$$

where  $\zeta(x)$  is Riemann's zeta function. A schematic representation of the velocity dependence of the magnon damping force on dislocations for the low-temperature case  $T \ll \Delta$  is shown in Fig. 1.

If a straight screw dislocation, located in the basal plane, moves in such a way that v makes an angle  $\alpha$ with the axis  $\tau$  and an angle  $\beta$  with the anisotropy axis, then  $C_1 = 8\pi^{-1/2}$ , and the damping force is determined by the formula  $(T \ll \Delta)$ 

$$F_{\star} = \begin{cases} B_{\star}^{(1)} (T/\Theta_{c})^{u_{t}} e^{-\Delta/T} v (\sin^{2} \alpha + 2 \cos^{2} \beta), & v \sin \alpha \ll v_{\perp}, \\ 4B_{\star}^{(1)} (T/\Theta_{c})^{u_{t}} e^{-\Delta/T} (\Theta_{c} \alpha)^{2} v^{-1}, & v_{\perp} \ll v \sin \alpha \ll v_{c}, \end{cases} (2.10)$$

where  $v_{\perp}$  is the projection of the vector **v** on the plane perpendicular to  $\tau$ .

At temperatures  $T \gg \Delta$ , the magnon damping force is described, over a wide velocity range, by the formula

$$F_{s} = \zeta \left( \frac{3}{2} \right) B_{s}^{(2)} \left( T/\Theta_{c} \right)^{\frac{3}{2}} \left( 3\sin^{2}\alpha + 2\cos^{2}\beta \right) v.$$
 (2.11)

## 3. TEMPERATURE DEPENDENCE OF THE DYNAMIC DAMPING FORCE ON DISLOCATIONS IN FERROMAGNETS

Dynamic damping of dislocations in ferromagnetic metals is caused by the presence of three channels for scattering of the energy of moving dislocations: electron, phonon, and magnon. Each of these damping mechanisms is characterized by its own damping coefficient and temperature variation of the frictional force during motion of a dislocation in the ferromagnet. Therefore a comparison of the contributions of the various components of the damping of a dislocation will enable us to establish bounds to the temperature intervals within which each of the mechanisms considered is dominant. The electronic damping force on dislocations is a temperature-independent quantity and is described by the expression

$$F_{v}=B_{N}v, \quad B_{N}=m^{2}b^{2}\lambda_{1}^{2}q_{m}/(2\pi)^{n}$$
 (3.1)

(*m* is the mass of the electron, and  $\lambda_1 \approx \varepsilon_F$ , where  $\varepsilon_F$  is the energy of electrons at the Fermi surface). For typical metals the coefficient of electronic damping of dislocations is  $B_N \approx 10^{-5} \,\mathrm{g/cm^{-1} \, sec^{-1}}$ . Comparison of

(3.1) with the expression for the magnon damping force on dislocations, (2.6) or (2.7) (we consider the respective expressions for slow dislocations), shows that in the very-low-temperature range the dynamic damping is caused chiefly by the conduction electrons. An estimate of the temperature at which the contributions of the two damping mechanisms become comparable gives

$$T_{c} = \Theta_{c} (B_{N}/B_{s}^{(2)})^{\frac{1}{2}}.$$
 (3.2)

For typical values  $B_N \approx 0.7 \cdot 10^{-5} \text{ g/cm}^{-1} \sec^{-1}$ ,  $B_s^{(2)} \approx 5 \cdot 10^{-2} \text{ g/cm}^{-1} \sec^{-1}$ ,  $\Theta_c \approx 10^3 \text{ K}$ , and the estimate (3.2) gives  $T_s \approx 25 \text{ K}$ .

Thus at temperatures  $T < T_e$  the chief contribution to the damping of dislocations comes from the conduction electrons. The estimate (3.2) gives a lower bound for the region of magnon damping. To determine an upper bound for this region, we compare the contributions due to the magnon and to the phonon mechanisms of damping. In accordance with the results of Al'shitz and Indenbom, <sup>[1]</sup> the phonon damping force is described by the expression

$$F_{ph} = \begin{cases} B_{ph}^{(h)} (T/\Theta_D)^2 \nu, & T \ll \Theta_D \\ B_{ph}^{(2)} (T/\Theta_D) \nu, & T \gg \Theta_D \end{cases},$$
(3.3)

where  $\Theta_D$  is the Debye temperature,  $B_{ph}^{(1)} \sim k_D^3 \approx 10^{-3}$  to  $10^{-4}$  g/cm<sup>-1</sup> sec<sup>-1</sup> is the coefficient of phonon damping of dislocations at low temperatures (the flutter-effect mechanism), and  $B_{ph}^{(2)} \approx \gamma b^{-3} (k_D b/2\pi)^5 \approx 10^{-2}$  to  $10^{-3}$  ( $\gamma \approx 10^2$  to  $10^3$ ,  $k_D \approx \pi/a$ ) is the coefficient of phonon damping at high temperatures (the phonon-wind mechanism).

Comparison of formulas (3.3) and (2.6), (2.8) shows that the region of magnon damping extends to the temperature

$$T_{s} = (B_{c}^{(2)}/B_{ph}^{(1)})^{2} (\Theta_{D}/\Theta_{c})^{3} \Theta_{c}.$$
(3.4)

From the estimate given, it follows that  $T_s \ll \Theta_c$ . Thus on setting  $\Theta_D \approx 3 \cdot 10^2$  K,  $\Theta_c \approx 1.6 \cdot 10^3$  K,  $B_s^{(2)} \approx 10^{-2}$  g/ cm<sup>-1</sup> sec<sup>-1</sup>, and  $B_{ph}^{(1)} \approx 10^{-3}$  g/cm<sup>-1</sup> sec<sup>-1</sup>, we find  $T_s \approx 10^2$  K. Thus the region of magnon damping is  $T_e \ll T \ll T_s$ .

The estimates made enable us to determine the range of the phonon contribution to the temperature dependence of the dynamic damping force on dislocations in ferromagnets. This range is  $T_s \ll T \ll \Theta_c$ . It is divided into two intervals, according to the character of the temperature dependence of the dynamic damping force, in accordance with (3.3).

Thus a general picture of the dynamic damping of dislocations in ferromagnets can be presented as follows. At very low temperatures,  $T \ll T_e$ , the conduction electrons are the principal channel for scattering of the energy of moving dislocations. Then, with increase of the temperature, the contribution of the magnon mechanism of damping of dislocations increases, and it becomes dominant in the temperature interval  $T_e \ll T$  $\ll T_e$ . On further increase of the temperature (up to  $\Theta_c$ ), the dynamic damping of dislocations is determined chiefly by the phonon mechanism. In closing, the authors cordially thank M. I. Kaganov and V. D. Natsik for discussion of the results of the research, and V. L. Sobolev for help in carrying out some of the calculations.

- <sup>1)</sup>Energy losses by "Cerenkov" radiation of magnons during motion of a dislocation were considered by Babkin and Kravchenko. <sup>[3]</sup>
- <sup>2</sup>)We use a system of units in which  $\hbar = 1$  and k = 1.
- <sup>3</sup>)It can be shown that the Hamiltonian (1.5) must contain terms linear in the operators  $a_k$  and  $a_k^*$ . In actuality, these terms drop out by virtue of the condition that the energy be a minimum (see<sup>[5]</sup>).
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# The conductivity of a quasi-one-dimensional metal at T=0

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The longitudinal and transverse conductivities of a quasi-one-dimensional metal containing impurity centers are calculated for T = 0. It is assumed that the energy spectrum is given by expression (2), where  $|\alpha| < \epsilon_F$ . It is found that for  $\alpha_2^l / \nu > 1$  the longitudinal conductivity is described by the "threedimensional" formula (28), where  $l_2$  is the mean free path of  $p_o \rightarrow -p_o$  transitions, and S is the area of the xy-section of the unit cell. In the case of where  $\alpha_2^l / \nu < 1$ , the order of magnitude of the conductivity is expressed for a short-range potential by formula (42), where  $l_1$  is the mean free path for  $p_o \rightarrow p_o$ transitions.

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### 1. INTRODUCTION

In a preceding paper<sup>[1]</sup> (referred to below as I) we discussed a method of deriving the characteristics of a one-dimensional metal containing random impurity centers. As was pointed out, the true aim of this method was not the solution of the one-dimensional problems with a random potential, which may also be solved by another method (see<sup>[2]</sup>), but of quasi-one-dimensional ones in which the motion of the electrons is not purely one-dimensional and there is relatively slow motion in the transverse direction.

In the present paper we shall examine the conductivity of a metal with an energy spectrum

$$\varepsilon(\mathbf{p}) = \varepsilon(p_z) + \alpha(p_x, \mathbf{p}_y) \tag{1}$$

 $(\alpha \ll \varepsilon_F)$  and containing random impurity centers at T = 0. Referring the energy to the chemical potential, we have

$$r - \mu = \xi(\mathbf{p}) = v(|p_z| - p_0) + \alpha(p_x, p_y), \quad \int \alpha \, dp_x \, dp_y = 0$$
(2)

(the integral is taken over the area of the xy-cross-section of the Brillouin zone). If a more concrete estimate is needed we shall use the formula for strong coupling for a rectangualr cell in the plane:

$$\alpha(p_x, p_y) = \alpha_x \cos(p_x a_x) \pm \alpha_y \cos(p_y a_y).$$
(3)

It is obvious that for sufficiently large  $\alpha$  the problem becomes truly three-dimensional and localization effects should not come into play. In this event the usual kinetic equation is applicable, which is equivalent to neglecting diagrams with intersection of the impurity lines (see<sup>[3]</sup>). The criterion for their neglect is  $\tau\Delta\varepsilon \gg 1$ , where  $\Delta\varepsilon$  is the characteristic energy, and  $\tau$  the time between collisions. In the present case the role of  $\Delta\varepsilon$  is played by  $\alpha$ , while  $\tau = l_2/v$  (see I), i.e., the problem becomes three-dimensional when

$$l_2/v \gg 1. \tag{4}$$

It is evident from this that there is a region  $\alpha l_2/v \ll 1$ in which, on the one hand, static conductivity must exist, and on the other hand, localization effects must be strongly in evidence. A rough estimate of the conductivity in this case can be obtained from a diffusion analysis. As a result of collisions, an electron diffuses first of all in the *xy*-plane. The relevant diffusion coefficient is of the order of

 $D_{\perp} \sim v_{\perp}^{2} \tau.$  (5)

In I it was shown that two times  $\tau$  exist: the time  $\tau_1$ for the processes without appreciable change in the zcomponent of the momentum ("forward" scattering) and the time  $\tau_2$  for processes in which  $p_z$  undergoes a transition from the neighborhood of  $p_0$  to the neighborhood of

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