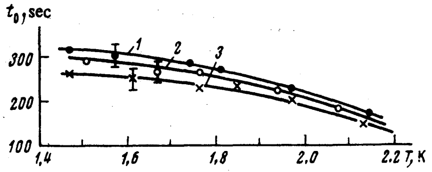

FIG. 5. Temperature dependences of the relaxation time of the vortices after the rotation is stopped. Curve 1—pure He^4, curve 2—solution with $C=3.03$ at.%, curve 3—solution with $C=5.7$ at.% He^3.

by Andronikashvili and Tsakadze[15] and was subsequently used many times by the Tbilisi group to investigate a great variety of relaxation phenomena in rotating helium II (due, e.g., to changes in the temperature[16] or in the rotary speed[17]).

Figure 5 shows the dependence of the relaxation time t_0 on the solution temperature. Curve 1 pertains to pure He^4, curve 2 to a solution with an He^3 concentration $C=3.03$, and curve 3 to a solution with $C=5.7$ at.%. In all cases, a transition was effected from a rotation at an angular velocity $\tilde{\omega}_0$ to an immobile stage.

Examination of Fig. 5 shows that as the He^3 concentration in the solution increases the relaxation times t_0 decrease in the entire temperature range from ~1.5 to 2.13 K. It must be assumed that, just as in the case of the critical velocity, this is caused by the fact that the He^3 particles dissolved in the He^4 assume the role of the normal component of the liquid.

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[1]R. P. Feynman, Prog. Low Temp. Phys. Amsterdam **1**, 2 (1955).
[2]É. L. Andronikashvili, Yu. G. Mamaladze, and Dzh. S. Tsakadze, Tr. Inst. Fiz. Akad. Nauk Gruz. SSR **7**, 59 (1960).
[3]E. L. Andronikashvili and Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. **37**, 562 (1959) [Sov. Phys. JETP **10**, 397 (1960)].
[4]E. L. Andronikashvili, K. B. Mesoed, and Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. **46**, 157 (1964) [Sov. Phys. JETP **19**, 113 (1964)].
[5]Yu. G. Mamaladze, Seventh All-Soviet Congress on Low Temp. Phys., Tez. Dokl., Char'kov, 1960.
[6]L. P. Pitaevskiĭ, Candidate's Dissertation, USSR Acad. Sci., 1958.
[7]G. E. Hall, Abstracts, Fourth All-Soviet Congress on Low Temp. Physics, Moscow, 1958.
[8]L. S. Reut and I. Z. Fisher, Zh. Eksp. Teor. Fiz. **55**, 722 (1968) [Sov. Phys. JETP **28**, 375 (1969)].
[9]T. Ohmi, T. Tsuneto, and T. Usui, Proc. Theor. Phys. **41**, 1395 (1969).
[10]Dzh. S. Tsakadze, Soobshch. Akad. Nauk Gruz. SSR **74**, 309 (1974).
[11]É. L. Andronikashvili, Yu. G. Mamaladze, S. G. Matinyan, and Dzh. S. Tsakadze, Usp. Fiz. Nauk **73**, 3 (1961) [Sov. Phys. Usp. **4**, 1 (1961)].
[12]Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. **65**, 617 (1973) [Sov. Phys. JETP **38**, 304 (1974)].
[13]G. A. Gamtsemlidze, Zh. Eksp. Teor. Fiz. **37**, 950 (1959) [Sov. Phys. JETP **10**, 678 (1960)].
[14]Z. Sh. Nadirashvili and Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. **64**, 1672 (1973) [Sov. Phys. JETP **37**, 844 (1973)].
[15]É. L. Andronikashvili and Dzh. S. Tsakadze, Fifth All-Soviet Congress on Low Temp. Physics, Tbilisi, 1958.
[16]G. V. Gudzhabidze and Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. **50**, 55 (1966) [Sov. Phys. JETP **23**, 37 (1966)].
[17]Z. Sh. Nadirashvili and Dzh. S. Tsakadze, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 77 (1973) [JETP Lett. **18**, 43 (1973)].

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Nonlinear cyclotron resonance in metals

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Nonlinear reflection at the second harmonic frequency in the case of the anomalous skin effect is considered for a metal located in a magnetic field parallel to its surface. It is shown that the nonlinearity is much greater in this case than in the absence of the magnetic field. The amplitude of the reflected second harmonic undergoes cyclotron resonance oscillations and increases additionally when $\omega = 1/2 l \Omega_m$, where ω is the electromagnetic field frequency, Ω_m is the extremal cyclotron frequency, and l is an integer.

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The generation of higher harmonics of an electromagnetic field in conductors has been studied experimentally and theoretically in a number of papers.[1-5] Harmonic generation in the presence of a magnetic field, however, has been previously studied only under conditions of the normal skin effect, at low frequencies $\omega\tau \ll 1$[4,5] (ω is the frequency of the electromagnetic wave, τ is the relaxation time). In the present paper we consider the nonlinear reflection, at the frequency of the second harmonic, from a metal situated in a magnetic field parallel to its surface, in the case of the anomalous skin effect, when the inequalities

$$\frac{\delta}{v_F\tau} \ll 1, \quad \frac{\delta\omega}{v_F} \ll 1, \quad \frac{\delta}{r_H} \ll 1, \tag{1}$$

are satisfied, where δ is the skin depth, v_F is the Fermi velocity, r_H is the Larmor radius. The magnetic field **H** is assumed to be nonquantizing but strong:

$$\Omega\tau \gg 1, \qquad (2)$$

where Ω is the cyclotron frequency. It is also assumed that there are no open trajectories. The dispersion law of the electrons is not specified. As shown below, the amplitude of the reflected second harmonic is much greater in this case than in the absence of the field, and undergoes cyclotron-resonance oscillations and increases additionally upon satisfaction of the condition $\omega = \frac{1}{2} l\Omega$, where l is an integer. The only case considered is that of practical importance, when the resonance is not too sharp and the anomaly parameter is a very small parameter of the problem.

THE NONLINEAR CONDUCTIVITY TENSOR IN A STRONG MAGNETIC FIELD

The kinetic equation for the electron distribution function in a strong magnetic field is written in the form

$$\frac{\partial f}{t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{\partial f}{dt_1}\frac{dt_1}{dt} + e\mathbf{E}(\mathbf{r},t)\mathbf{v}\frac{\partial f}{\partial \varepsilon} + e\Big(E_z(\mathbf{r},t) + \frac{1}{c}[\mathbf{v}\times\mathbf{H}(\mathbf{r},t)]_z\Big)\frac{\partial f}{\partial p_z} = -\frac{f-f_0}{\tau}, \qquad (3)$$

where t_1 is the time of motion along the trajectory, ε is the energy, p_z is the projection of the momentum on the direction of the constant magnetic field **H**, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$ are the alternating electric and magnetic fields, and f_0 is the equilibrium distribution function.

We now calculate the derivative

$$\frac{dt_1}{dt} = \frac{\partial t_1}{\partial \mathbf{p}}\frac{d\mathbf{p}}{dt} = 1 + \frac{\partial t_1}{\partial \mathbf{p}}\Big(e\mathbf{E}(\mathbf{r},t) + \frac{e}{c}[\mathbf{v}\times\mathbf{H}(\mathbf{r},t)]\Big). \qquad (4)$$

The vector $\partial t_1/\partial \mathbf{p}$ is determined from the equations of motion in a constant magnetic field:

$$\frac{d\mathbf{p}}{dt} = \frac{e}{c}[\mathbf{v}\times\mathbf{H}]. \qquad (5)$$

It is obvious that the vector $\partial t_1/\partial \mathbf{p}$ is not determined uniquely, since the choice of the initial time t_1 along each trajectory is arbitrary. We shall reckon the time t_1 from the plane $p_y = 0$. Then, as follows from (5),

$$t = \frac{c}{eH^2}\int_{\mathbf{p}'}^{\mathbf{p}} \frac{[\mathbf{v}\times\mathbf{H}]}{v_\perp^2} d\mathbf{p}, \qquad (6)$$

where $v_\perp^2 = v_x^2 + v_y^2$. The integral in (6) is taken along the trajectory from a point $\mathbf{p}'$ in the plane $p_y = 0$ to some other point $\mathbf{p}$.

We consider the expression

$$\frac{c}{eH^2}\oint_C \frac{[\mathbf{v}\times\mathbf{H}]}{v_\perp^2} d\mathbf{p}, \qquad (7)$$

where C is a close contour in the plane $p_z = \text{const}$ (see Fig. 1). Two sides of the contour C are segmants of the trajectories passing through the points **p** and **p** + Δ**p**. By calculating (7), it is not difficult to obtain

$$\frac{\partial t_1(\mathbf{p})}{\partial p_\alpha} = \frac{c}{eH^2}\left\{\frac{[\mathbf{v}(\mathbf{p})\times\mathbf{H}]_\alpha}{v_\perp^2(\mathbf{p})} v_\alpha(\mathbf{p}) \frac{[\mathbf{v}(\mathbf{p}')\times\mathbf{H}]_x}{v_\perp^2(\mathbf{p}')v_x(\mathbf{p}')} + v_\alpha(\mathbf{p})\int_{\mathbf{p}'}^{\mathbf{p}} \frac{dl}{v_\perp}\Big[\nabla\Big(\frac{[\mathbf{v}\times\mathbf{H}]}{v_\perp^2}\Big]_z\right\}, \qquad (8)$$

where $\alpha = x, y$; dl is the element of length of the electron trajectory in momentum space. For arbitrary $\partial t_1/\partial p_z$, we get directly from (6):

$$\frac{\partial t_1}{\partial p_z} = \frac{c}{eH}\int_{\mathbf{p}'}^{\mathbf{p}} \frac{dl}{v_\perp^2}\frac{\partial v_\perp}{\partial p_z}. \qquad (9)$$

In the case of an isotropic and quadratic spectrum, it follows from (8) and (9) that

$$\frac{\partial t_1}{\partial \mathbf{p}} = \frac{c}{eH^2}\frac{[\mathbf{v}\times\mathbf{H}]}{v_\perp^2} \qquad (10)$$

Let the fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$ have the time dependence

$$\mathbf{E}(\mathbf{r},t) = e^{-i\omega t}\mathbf{E}(\mathbf{r}) + \text{c.c.}, \quad \mathbf{H}(\mathbf{r},t) = e^{-i\omega t}\mathbf{H}(\mathbf{r}) + \text{c.c.}$$

We expand the functions $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ in Fourier integrals:

$$\mathbf{E}(\mathbf{r}) = \int d^3\mathbf{k}\,\mathbf{E}(\mathbf{k})e^{i\mathbf{k}\mathbf{r}}, \quad \mathbf{H}(\mathbf{r}) = \int d^3\mathbf{k}\,\mathbf{H}(\mathbf{k})e^{i\mathbf{k}\mathbf{r}}.$$

The nonlinear conductivity tensor $\sigma_{\alpha\beta\gamma}(\mathbf{k}_1, \mathbf{k}_2)$ is defined in the following manner:

$$j_\alpha^{(2)}(\mathbf{k}, 2\omega) = \int d^3\mathbf{k}_1\, d^3\mathbf{k}_2\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\sigma_{\alpha\beta\gamma}(\mathbf{k}_1,\mathbf{k}_2)E_\beta(\mathbf{k}_2)E_\gamma(\mathbf{k}_1), \qquad (11)$$

where $j_\alpha^{(2)}(\mathbf{k}, 2\omega)$ is the Fourier component of the nonlinear current at the second-harmonic frequency.

Solving the kinetic equation by the iteration method, we obtain an expression for the nonlinear conductivity tensor $\sigma_{\alpha\beta\gamma}(\mathbf{k}_1, \mathbf{k}_2)$ in the case when $\mathbf{k}_1 \parallel \mathbf{k}_2 \perp \mathbf{H}$. We shall assume that $k_1 \sim k_2 \sim \delta^{-1}$. We expand the distribution function f in a series in powers of the field:

$$f = e^{-i\omega t}\int d^3\mathbf{k}_1 f^{(1)}\exp(i\mathbf{k}_1\mathbf{r}) + e^{-2i\omega t}\int d^3\mathbf{k}_1\, d^3\mathbf{k}_2 f^{(2)}\exp\{i(\mathbf{k}_1+\mathbf{k}_2)\mathbf{r}\} + \ldots \qquad (12)$$

In the linear approximation, we obtain

$$i(\mathbf{k}_1\mathbf{v}-\omega-i\tau^{-1})f^{(1)} + \frac{\partial f^{(1)}}{\partial t_1} = -e\mathbf{E}(\mathbf{k}_1)\mathbf{v}\frac{\partial f_0}{\partial \varepsilon}. \qquad (13)$$

Solving Eq. (13), we obtain[6]

$$f^{(1)} = -e\frac{\partial f_0}{\partial \varepsilon}\int_{t_1}^{t_1+T} dt_2\mathbf{E}(\mathbf{k}_1)\mathbf{v}(t_2) \times \exp\Big\{i\int_{t_1}^{t_2}(\mathbf{k}_1\mathbf{v}-\omega-i\tau^{-1})dt'\Big\}[\exp\{-iT(\omega+i\tau^{-1})\}-1]^{-1}, \qquad (14)$$

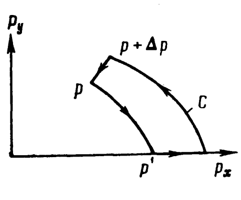

FIG. 1.

where $T=2\pi/\Omega$ is the period of rotation of the electron in the magnetic field.

The equation for $f^{(2)}$ is of the form

$$i(\mathbf{kv}-2\omega-i\tau^{-1})f^{(2)}+\partial f^{(2)}/\partial t_1 =-\frac{e}{c}[\mathbf{v}\times\mathbf{H}(k_2)]\frac{\partial t_1}{\partial \mathbf{p}}\frac{\partial f^{(1)}}{\partial t_1}-\frac{e}{c}[\mathbf{v}\times\mathbf{H}(k_2)]_z\frac{\partial f^{(1)}}{\partial p_z}, \qquad (15)$$

where $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$. In (15), we have taken into account only the nonlinearity due to the Lorentz force, since it predominates in the situation considered. The role of a nonlinearity that is quadratic in the electric field will be discussed later.

We get from Eq. (15)

$$f^{(2)}=-\frac{e}{c}\int_{t_1}^{t_1+T}dt_2\left\{[\mathbf{v}(t_2)\times\mathbf{H}(\mathbf{k}_2)]\frac{\partial t_2}{\partial\mathbf{p}}\frac{\partial f^{(1)}(t_2)}{\partial t_2}+[\mathbf{v}(t_2)\times\mathbf{H}(\mathbf{k}_2)]_z\frac{\partial f^{(1)}(t_2)}{\partial p_z}\right\}$$
$$\times\exp\left\{i\int_{t_1}^{t_2}(\mathbf{kv}-2\omega-i\tau^{-1})dt'\right\}(\exp\{-iT(2\omega+i\tau^{-1})\}-1)^{-1}. \qquad (16)$$

We consider first the contribution made to the nonlinearity by the term proportional to $\partial f^{(1)}/\partial t_2$. Calculating the nonlinear current $j_\alpha^{(2)}(\mathbf{k}, 2\omega)$, we obtain

$$j_\alpha^{(2)}(\mathbf{k},2\omega)=\frac{e^3}{c}\frac{2}{(2\pi)^3}\left|\frac{eH}{c}\right|\int dp_z\, dt_1 v^\alpha(t_1)$$
$$\times\int_{t_1}^{t_1+T}dt_2\exp\left\{i\int_{t_1}^{t_2}(\mathbf{kv}-2\omega-i\tau^{-1})dt'\right\}[\mathbf{v}(t_2)\times\mathbf{H}(\mathbf{k}_2)]\frac{\partial t_2}{\partial\mathbf{p}}\left\{i(\mathbf{k}_1\mathbf{v}(t_2)-\omega-i\tau^{-1})\right.$$
$$\times\int_{t_2}^{t_2+T}dt_3\mathbf{E}(\mathbf{k}_1)\mathbf{v}(t_3)\exp\left\{i\int_{t_2}^{t_3}(\mathbf{k}_1\mathbf{v}-\omega-i\tau^{-1})dt'\right\}(\exp\{-iT(\omega+i\tau^{-1})\}-1)^{-1}$$
$$\left.-\mathbf{E}(\mathbf{k}_1)\mathbf{v}(t_2)\right\}(\exp\{-iT(2\omega+i\tau^{-1})\}-1)^{-1}_{\varepsilon=\varepsilon_F}, \qquad (17)$$

where ε_F is the Fermi energy.

As is seen from (17), the nonlinear response at the frequency of the second harmonic increases in resonance fashion upon satisfaction of the condition $\omega=\frac{1}{2}l\Omega_m$, where Ω_m is the extremal cyclotron frequency and l is an integer. We shall call the resonance even if l is even and odd if l is odd.

We calculate the integrals with respect to t_1, t_2, and t_3 in (17). For definiteness, we assume that the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are parallel to the y axis. We first consider the following integral

$$\int_0^T dt_1 v^\alpha(t_1)\exp\left\{-i\int_0^{t_1}(\mathbf{kv}-2\omega-i\tau^{-1})dt'\right\}\int_{t_1}^{t_1+T}dt_2 ik_1v^y(t_2)[\mathbf{v}(t_2)\times\mathbf{H}(\mathbf{k}_2)]\frac{\partial t_2}{\partial\mathbf{p}}$$
$$\times\exp\left\{i\int_0^{t_2}(\mathbf{k}_2\mathbf{v}-\omega)dt'\right\}\int_{t_2}^{t_2+T}dt_3\mathbf{E}(\mathbf{k}_1)\mathbf{v}(t_3)\exp\left\{i\int_0^{t_3}(\mathbf{k}_1\mathbf{v}-\omega-i\tau^{-1})dt'\right\}. \qquad (18)$$

The inner integral over t_2 can be written in the following fashion:

$$\frac{k_1}{k_2}\int_{t_1}^{t_1+T}dt_2F(t_2)\exp\{-i\omega t_2\}\frac{d}{dt_2}\left(\exp\left\{i\int_0^{t_2}\mathbf{k}_2\mathbf{v}dt'\right\}\right)\int_{t_2}^{t_2+T}dt_3\Phi(t_3). \qquad (19)$$

Integrating by parts, we obtain

$$\frac{k_1}{k_2}\left(F(t_2)\exp\left\{-i\omega t_2+i\int_0^{t_2}\mathbf{k}_2\mathbf{v}dt'\right\}\int_{t_2}^{t_2+T}dt_3\Phi(t_3)\right)\Big|_{t_1}^{t_1+T}-\frac{k_1}{k_2}\int_{t_1}^{t_1+T}dt_2F(t_2)$$
$$\times\exp\left\{-i\omega t_2+i\int_0^{t_2}\mathbf{k}_2\mathbf{v}dt'\right\}(\Phi(t_2+T)-\Phi(t_2))$$
$$-\frac{k_1}{k_2}\int_{t_1}^{t_1+T}dt_2\frac{d}{dt_2}(F(t_2)\exp\{-i\omega t_2\})\exp\left\{i\int_0^{t_2}\mathbf{k}_2\mathbf{v}dt'\right\}\int_{t_2}^{t_2+T}dt_3\Phi(t_3). \qquad (20)$$

Substituting (20) in (17), we obtain double and triple integrals with respect to time, which can be calculated or estimated at $r_H/\delta\gg1$ by the stationary-phase method. At sufficiently large values of the parameter r_H/δ (the exact criteria will be given below), the principal contribution is made by those terms which contain only double integration with respect to time. The calculation of these double integrals is performed in the same way as in the linear theory (see, for example, Refs. 6, 7).

The contribution to the nonlinearity from the term proportional to $\partial f^{(1)}/\partial p_z$ in Eq. (15) is calculated analogously. As a result, we obtain that the nonlinear conductivity $\sigma_{\alpha\beta}$ has a part that depends smoothly on $\mathbf{k}_1$ and $\mathbf{k}_2$, as well as a part that undergoes geometric resonance oscillations, and is much less smooth. The principal contribution to the smooth part of $\sigma_{\alpha\beta\gamma}$, the only part of interest to us, is obtained from Eq. (17). Omitting all the cumbersome calculations, we put down the final result:

$$\sigma_{\alpha\beta\gamma}(\mathbf{k}_1,\mathbf{k}_2)=\frac{c}{\omega H}k\left(\tilde\sigma_{\alpha\beta\gamma}(\mathbf{k},2\omega)-\frac{k_1}{k}\tilde\sigma_{\alpha\beta\gamma}(\mathbf{k}_1,\omega)\right). \qquad (21)$$

The tensor $\tilde\sigma_{\alpha\beta\gamma}(\mathbf{k},\omega)$ is of the form

$$\tilde\sigma_{\alpha\beta\gamma}(\mathbf{k},\omega)=\frac{e^2}{4\pi^2|\mathbf{k}|}\int_0^{2\pi}d\varphi\frac{n_\alpha n_\beta n_\gamma}{K(\varphi)n_x}\operatorname{cth}\{-iT(\omega+i\tau^{-1})\}, \qquad (22)$$
$$(Q=\pi/2)$$

where n_α is a vector normal to the Fermi surface, $K(\varphi)=K(\pi/2,\varphi)$ is the Gaussian curvature of the Fermi surface, Q and φ are the polar and azimuthal angles of the normal vector n_α (the polar axis coincides with the y axis).

In obtaining (21) and (22), we have also taken the following equality into account:

$$\left([\mathbf{v}\times\mathbf{H}(\mathbf{k}_2)]\frac{\partial t}{\partial\mathbf{p}}\right)_{v_y=0}=\frac{c}{\omega}\left([\mathbf{v}\times[\mathbf{k}_2\times\mathbf{E}(\mathbf{k}_2)]]\frac{\partial t}{\partial\mathbf{p}}\right)$$
$$=-\left(\frac{ck_2}{\omega}\right)\frac{c}{eH}\left(\frac{\mathbf{vE}(\mathbf{k}_2)}{v_x}\right)_{v_y=0}, \qquad (23)$$

which follows from Eq. (8). It follows from (22) that $\sigma_{\alpha\beta x}(\mathbf{k},\omega)=\tilde\sigma_{\alpha x\beta}(\mathbf{k},\omega)=\tilde\sigma_{x\alpha\beta}(\mathbf{k},\omega)=\sigma_{\alpha\beta}(\mathbf{k},\omega)$, where $\sigma_{\alpha\beta}(\mathbf{k},\omega)$ is the linear conductivity tensor. If $\alpha=\beta=\gamma=z$, then the integrand in (22) has at $n_x=0$ a singularity that must be integrated in the sense of its principal value. It is also seen from Eq. (22) that the contribution of longitudinal fields to the transverse current is insignificant, as in the linear theory.

Estimating the terms discarded in the derivation of (21), it is not difficult to establish that their contribution is small if the inequalities

$$(r_H/\delta)^{1/2}\gg1,\quad (r_H/\delta)^{1/2}\gg\omega/\Omega \qquad (24)$$

are satisfied.

Furthermore, some limitations exist on the range of applicability of Eq. (21). Actually, as follows from (21), the nonlinear response at resonance, as also in the linear case, increases by a factor of $\Omega\tau$ in the case of a square-law dispersion and by a factor of $(\Omega\tau)^{1/2}$ in the case of a nonquadratic dispersion law. It follows from the inital equation (17) that the nonlinear current increases unboundedly as $\tau\to\infty$ in even resonance, and is proportional to $(\Omega\tau)^2$ in the quadratic case and proportional to $(\Omega\tau)^{3/2}$ in the nonquadratic case. Consequently, the terms that were discarded in obtaining Eq. (21) have a stronger singularity in even resonance than those taken into account. Therefore, in order that the expression (21) remain valid even at $\omega=l\Omega_m$, the parameter $\omega\tau$ should not be too large. Estimates show that the inequality

$$\omega\tau<(r_H/\delta)^{1/2}. \tag{25}$$

must be satisfied. A similar restriction appears also in the range of frequencies $\omega\ll\Omega$, since the specified situation is also resonant. It can be established that even in this case, upon satisfaction of (24) and the following condition:

$$\left|\frac{\Omega}{\omega+i\tau^{-1}}\right|\left(\frac{\delta}{r_H}\right)^{1/2}\ll 1, \tag{26}$$

formula (21) remains valid. We note that the satisfaction of the inequality (26) is generally not essential. In some special cases, for example in the case of an isotropic and quadratic dispersion law, the validity of the expression (21) in the range of frequencies $\omega\ll\Omega$ is assured by satisfaction of the conditions (24) and satisfaction of the inequality (26) is not required. It can be shown that the validity of the formula (21) for the terms $\sigma_{\alpha xy}$, in the case of an arbitrary dispersion law, is also not connected with satisfaction of condition (26).

We now estimate the magnitude of the nonlinearity. Let $\omega\sim\Omega$ but assume no cyclotron resonance. Then we get from (21) and (22)

$$\sigma_{\alpha\beta\gamma}\sim\frac{e^3}{\omega}\frac{p_F}{4\pi^2}\delta\left(\frac{r_H}{\delta}\right), \tag{27}$$

where p_F is the Fermi momentum.

This estimate is valid both for quadratic and nonquadratic dispersion laws. In the absence of a magnetic field, the nonlinearity depends significantly on the anisotropy of the dispersion law (we have in mind a transverse current). If the dispersion law is of the form $\varepsilon=\frac{1}{2}\alpha_{ij}p_ip_j$ with an anisotropy of the order of unity, then, according to Ref. 3,

$$\sigma_{\alpha\beta\gamma}\sim\frac{e^3}{\omega}\frac{p_F}{4\pi^2}\delta\left(\frac{\omega\delta}{v_F}\right). \tag{28}$$

In the case of an isotropic dispersion law, the transverse current generally vanishes. Comparing (27) with (28), we see that in the given situation, the nonlinearity in a magnetic field is $(r_H/\delta)^2\sim(v_F/\omega\delta)^2$ times greater than the nonlinearity without a magnetic field. Besides, the nonlinearity in the magnetic field increases additionally upon satisfaction of the condition of cyclotron resonance $\omega=\frac{1}{2}l\Omega_m$.

We now obtain the expression for the nonlinear conductivity tensor $\sigma_{\alpha\beta\gamma}$ near the even resonance in the case in which

$$\omega\tau>(r_H/\delta)^{1/2}. \tag{29}$$

As earlier, the chief contribution to the monotonic portion of the nonlinear conductivity $\sigma_{\alpha\beta\gamma}$ is obtained from Eq. (17), wherein upon satisfaction of the inequality (29), only those terms of (17) which contain two resonance factors are significant. Since we assume the frequency ω to be close to the resonant frequency, we can then replace ω in (17) by $l\Omega$ everywhere except in factors that describe cyclotron resonance. This allows integrals of the form $\int_0^{+T}$ to be replaced by integrals of the form $\int_0^T$. After some transformations, we get from Eq. (17),

$$\sigma_{\alpha\beta\gamma}(\mathbf{k}_1,\mathbf{k}_2)=\frac{e^3k_2}{\omega}\frac{2}{(2\pi)^3}\left|\frac{eH}{c}\right|\int dp_zv_{2l}{}^{\alpha}(-\mathbf{k})I_{-l}{}^{\beta}(\mathbf{k}_1,\mathbf{k}_2)v_{-l}{}^{\gamma}(\mathbf{k}_1)$$
$$\times(\exp\{-iT(\omega+i\tau^{-1})\}-1)^{-1}(\exp\{-iT(2\omega+i\tau^{-1})\}-1)^{-1}, \tag{30}$$

where

$$v_{-l}{}^{\gamma}(\mathbf{k}_1)=\int_0^T dt\,v^{\gamma}(t)\exp\{i\mathbf{k}_1\mathbf{r}(t)-il\Omega t\},$$

$$I_{-l}{}^{\beta}(\mathbf{k}_1,\mathbf{k}_2)=\int_0^T dt\,i(k_1v^{\nu}-\Omega)\left(\frac{\partial t}{\partial p_\nu}v^{\beta}-\frac{\partial t}{\partial p_\beta}v^{\nu}\right)\exp\{i\mathbf{k}_2\mathbf{r}(t)-il\Omega t\},$$

$$\mathbf{r}(t)=\int_0^t\mathbf{v}(t')\,dt'.$$

After integration by parts, $I_{-l}^{\beta}(\mathbf{k}_1,\mathbf{k}_2)$ can be represented in the form

$$I_{-l}{}^{\beta}(\mathbf{k}_1,\mathbf{k}_2)=\frac{1}{m^*}\left(\bar{w}_{-l}{}^{\beta}(\mathbf{k}_2)+\frac{k_1}{k_2}\tilde{w}_{-l}{}^{\beta}(\mathbf{k}_2)\right), \tag{31}$$

where

$$\bar{w}_{-l}{}^{\beta}(\mathbf{k}_2)=\int_0^T dt\,\bar{w}^{\beta}(t)\exp\{i\mathbf{k}_2\mathbf{r}(t)-il\Omega t\},$$

$$\tilde{w}_{-l}{}^{\beta}(\mathbf{k}_2)=\int_0^T dt\,\tilde{w}^{\beta}(t)\exp\{i\mathbf{k}_2\mathbf{r}(t)-il\Omega t\},$$

m^* is the cyclotron mass. We get the following expressions for the functions $\bar{w}^{\beta}(t)$ and $\tilde{w}^{\beta}(t)$:

$$\bar{w}^{\beta}(t)=-\frac{eH}{c}\left(\frac{\partial t}{\partial p_\nu}v^{\beta}-\frac{\partial t}{\partial p_\beta}v^{\nu}\right),$$
$$\tilde{w}^{\beta}(t)=\frac{eH}{c}\left\{il\left(\frac{\partial t}{\partial p_\nu}v^{\beta}-\frac{\partial t}{\partial p_\beta}v^{\nu}\right)-\frac{1}{\Omega}\frac{\partial}{\partial t}\left(\frac{\partial t}{\partial p_\nu}v^{\beta}-\frac{\partial t}{\partial p_\beta}v^{\nu}\right)\right\}. \tag{32}$$

The matrix elements $v_l^{\alpha}(\mathbf{k})$, $\bar{w}_{-l}^{\beta}(\mathbf{k}_2)$, and $\tilde{w}_{-l}^{\beta}(\mathbf{k}_2)$ can be calculated by the method of stationary phase. For simplicity, we shall assume that there are only two points on the electron trajectory where $\mathbf{k}\cdot\mathbf{v}=0$, the vicinity of which also makes the principal contribution to the integral. We denote them by $t_{(1)}$ and $t_{(2)}$. For $\bar{w}_{-l}^{\beta}(\mathbf{k}_2)$ we obtain

$$\bar{w}_{-l}{}^{\beta}(\mathbf{k}_2)=\sum_{i=1,2}\left|\frac{2\pi}{k_2v'_{(i)}}\right|^{1/2}\bar{w}_{(i)}\exp\left\{i\frac{\pi}{4}\operatorname{sign}(\mathbf{k}_2\mathbf{v}_{(i)})\right\}\exp\{i\mathbf{k}_2\mathbf{r}(t_{(i)})-il\Omega t_{(i)}\},$$
$$\mathbf{v}'=d\mathbf{v}/dt.$$

The remaining matrix elements have the same form. Substituting the expressions for the matrix elements in formula (30), we get the following for the parts of the nonlinear conductivity that depend monotonically on $\mathbf{k}_1$ and $\mathbf{k}_2$:

$$\sigma_{\alpha\beta\gamma}(\mathbf{k}_1,\mathbf{k}_2)=\frac{k_2}{|k_1k_2k|^{1/2}}K_{\alpha\beta\gamma}+\frac{k_1}{|k_1k_2k|^{1/2}}L_{\alpha\beta\gamma}, \tag{33}$$

where

$$K_{\alpha\beta\gamma}=\frac{e^3}{\omega}\frac{2^{1/2}}{(2\pi)^{1/2}}\sum_{i=1,2}\int dp_z\frac{v^{\alpha}_{(i)}\bar{w}^{\beta}_{(i)}v^{\gamma}_{(i)}}{|v'_{y(i)}|^{1/2}}\Omega(\exp\{-iT(\omega+i\tau^{-1})\}-1)^{-1}$$
$$\times(\exp\{-iT(2\omega+i\tau^{-1})\}-1)^{-1},$$
$$\Omega=|eH/m^*c|. \tag{34}$$

The expression for $L_{\alpha\beta\gamma}$ is obtained from the formula (34) by the replacement of $\bar{w}^{\beta}$ by $\tilde{w}^{\beta}$.

Recognizing that $v'_{y(i)}=eHc^{-1}v^2_{x(i)}/\rho_{(i)}$, where $\rho_{(i)}$ is the radius of curvature of the orbit in $\mathbf{p}$ space at the point of stationary phase $t_{(i)}$, and using the equality $dp_z\rho_{(i)}/v^2_{x(i)}$, $\equiv d\varphi/K(\varphi)v^2$, the formula (34) can be transformed to

$$K_{\alpha\beta\gamma}=\frac{e^3}{\omega}\frac{2^{1/2}}{(2\pi)^{1/2}}\left|\frac{c}{eH}\right|^{1/2}\int_0^{2\pi}\frac{d\varphi}{vK(\varphi)}(\rho(\varphi))^{1/2}$$
$$\times\frac{n^{\alpha}\bar{w}^{\beta}n^{\gamma}}{m^*|n_z|}(\exp\{-iT(\omega+i\tau^{-1})\}-1)^{-1}(\exp\{-iT(2\omega+i\tau^{-1})\}-1)^{-1}, \tag{35}$$
$$\rho(\varphi)=\rho(\pi/2,\varphi).$$

In the case of quadratic and isotropic dispersion law, we obtain from Eqs. (10) and (32):

$$\bar{w}^{\beta}_{(i)}=\frac{v^{\beta}_{(i)}}{v^{x}_{(i)}},\quad \tilde{w}^{\beta}_{(i)}=-il\frac{v^{\beta}_{(i)}}{v^{x}_{(i)}}. \tag{36}$$

We have considered here only the nonlinearity due to the Lorentz force, and have not taken into account the nonlinearity that is quadratic in the field $\mathbf{E}$. However, it can be shown that in our case the nonlinearity due to the Lorentz force actually predominates at all times. One of the reasons for this is that the force $e\mathbf{E}$ is smaller than the Lorentz force by a factor $(\delta\omega/v_F)^{-1}$. The nonlinearity that is quadratic in the field $\mathbf{E}$ becomes significant only near the cyclotron resonance $\omega=\frac{1}{2}l\Omega_m$ if the cyclotron frequency Ω depends on the energy, and the parameter $\Omega\tau$ is sufficiently large. Estimates show that the nonlinearity due to the Lorentz force always predominates if the inequality $\Omega\tau<(r_H/\delta)$ is satisfied.

NONLINEAR REFLECTION FROM A METAL SURFACE

Let an electromagnetic wave of frequency ω impinge on a metal that fills the halfspace $y>0$. The magnetic field is parallel to the surface of the metal and is directed along the z axis. We calculate the amplitude of the reflected wave at the frequency of the second harmonic. From Maxwell's equations, we obtain

$$\mathbf{E}^{(2)}(0)=[\mathbf{H}^{(2)}(0)\times\mathbf{n}]=-\frac{4\pi}{c}(\mathbf{I}^{(2)}(2\omega)+\mathbf{I}^{(1)}(2\omega)), \tag{37}$$

where $\mathbf{E}^{(2)}(0)$ and $\mathbf{H}^{(2)}(0)$ are the amplitudes of the electric and magnetic fields of the second harmonic at the surface of the metal, $\mathbf{n}$ is the vector normal to the surface, $\mathbf{I}^{(2)}(2\omega)$ is the total nonlinear current at the frequency of the second harmonic, $\mathbf{I}^{(1)}(2\omega)$ is the linear current at the second-harmonic frequency.

Taking into account the relation

$$\mathbf{I}^{(1)}(2\omega)=\left(\frac{4\pi}{c}\hat{\xi}(2\omega)\right)^{-1}\mathbf{E}^{(2)}(0), \tag{38}$$

where $\hat{\xi}(2\omega)$ is the surface impedance at the second harmonic frequency, we get from (37)

$$\mathbf{E}^{(2)}(0)=-\frac{4\pi}{c}\frac{\hat{\xi}(2\omega)}{1+\hat{\xi}(2\omega)}\mathbf{I}^{(2)}(2\omega). \tag{39}$$

Since $\xi\ll1$ in the case of the anomalous skin effect, we have

$$\mathbf{E}^{(2)}(0)=-\frac{4\pi}{c}\hat{\xi}(2\omega)\mathbf{I}^{(2)}(2\omega),$$
$$I^{(2)}_{\alpha}(2\omega)=\int_0^{\infty}dy\,j^{(2)}_{\alpha}(y,2\omega), \tag{40}$$

where $j^{(2)}_{\alpha}(y,2\omega)$ is the nonlinear current density.

The exact calculation of the nonlinear current $\mathbf{I}^{(2)}(2\omega)$ is extremely complicated, both for specular and diffuse reflection. It is known from linear theory that, in the case of diffuse reflection, if we neglect numerical factors of the order of unity, we can in practice consider the problem in an unbounded space. However, it is necessary to replace the conductivity tensor of the unbounded medium $\sigma_{\alpha\beta}$ by the modified tensor $\sigma'_{\alpha\beta}$ (see, for example, Ref. 7). The difference between $\sigma'_{\alpha\beta}$ and $\sigma_{\alpha\beta}$ is that the factor in $\sigma'_{\alpha\beta}$ which describes the cyclotron resonance is taken into account only at those stationary-phase points $\mathbf{k}\cdot\mathbf{v}=0$ for which the orbit can be a resonant one in real space. In the case when there are only two stationary-phase points on the electron trajectory $\sigma'_{\alpha\beta}$ is obtained from $\sigma_{\alpha\beta}$ by replacement of the resonance factor $\coth\{T(-i\omega+\tau^{-1})\}$ by $(1-\exp\{T(+i\omega-\tau^{-1})\})^{-1}$; consequently,

$$\sigma_{\alpha\beta}'=\frac{e^2}{4\pi^2|k|}\int_0^{2\pi}\frac{d\varphi}{K(\varphi)}n_{\alpha}n_{\beta}\frac{1}{1-\exp\{T(i\omega-\tau^{-1})\}}, \tag{41}$$
$$Q=\pi/2.$$

We consider here also an unbounded space, modifying $\sigma_{\alpha\beta}$ and the nonlinear conductivity $\sigma_{\alpha\beta\gamma}$ in the above fashion. The result thus obtained corresponds to the rigorous solution of the problem in the case of diffuse reflection to within a numerical factor of the order of unity.

Taking the above into consideration, we obtain for the nonlinear current $I^{(2)}(2\omega)$

$$I^{(2)}_{\alpha}(2\omega)=\int_0^{\infty}dy\,j^{(2)}_{\alpha}(y,2\omega)=\int_{-\infty}^{\infty}dk\frac{j^{(2)}_{\alpha}(k,2\omega)}{i(k+i0)}$$
$$=\int_{-\infty}^{\infty}dk\,dk_1\,dk_2\frac{\delta(k-k_1-k_2)}{i(k+i0)}\sigma'_{\alpha\beta\gamma}(k_1,k_2)E_{\beta}(k_2)E_{\gamma}(k_1), \tag{42}$$

where $\sigma'_{\alpha\beta\gamma}$ is the modified nonlinear conductivity tensor.

We shall only consider the case in which the tensor

$\sigma_{\alpha\beta\gamma}$ is described by (21) and (22). If there are only two stationary-phase points on the electron trajectory, then, as in the linear case, the tensor $\sigma'_{\alpha\beta\gamma}$ is obtained from $\sigma_{\alpha\beta\gamma}$ by replacement of factors of the type $\coth\{T(-i\omega+\tau^{-1})\}$ by $(1-\exp\{T(i\omega-\tau^{-1})\})^{-1}$: We note, however, that such a modification of the nonlinear tensor $\sigma_{\alpha\beta\gamma}$ is in fact insignificant. For simplicity, we shall assume that the tensor $\xi_{\alpha\beta}(\omega)$ can be reduced to diagonal form, and that the vector $\mathbf{E}^{(1)}(0)$ coincides with one of the eigenvectors of the surface impedance with eigenvalue $\xi_1(\omega)$. Consequently,

$$E_\alpha^{(1)}(k)=i\frac{\omega}{c}\frac{1}{\xi_1(\omega)}\frac{E_\alpha^{(1)}(0)}{k^2-ib_1/|k|}, \tag{43}$$

where $b_1/|k|$ is one of the eigenvalues of the tensor $4\pi c^{-2}\omega\sigma'_{\alpha\beta}$. We write the tensor $\sigma'_{\alpha\beta\gamma}$ in the form

$$\sigma'_{\alpha\beta\gamma}(k_1,k_2)=A_{\alpha\beta\gamma}\frac{k}{|k|}+B_{\alpha\beta\gamma}\frac{k_1}{|k_1|}. \tag{44}$$

The explicit expressions for the quantities $A_{\alpha\beta\gamma}$ and $B_{\alpha\beta\gamma}$ can easily be obtained from (21) and (22). After several transformations, we get from formulas (40) and (42)–(44)

$$E_\alpha^{(2)}(0)=\frac{4\pi}{c}\left(\frac{\omega}{c}\right)^2\frac{\xi_{\alpha s}(2\omega)}{\xi_1^2(\omega)b_1(\omega)}[A_{s\beta\gamma}c_1+B_{s\beta\gamma}c_2]E_\beta^{(1)}(0)E_\gamma^{(1)}(0), \tag{45}$$

where c_1 and c_2 are constants, and $|c_1|\sim|c_2|\sim 1$. We shall not calculate c_1 and c_2, since we are not concerned with numerical factors of the order of unity.

Let the frequency $\omega\sim\Omega$, but let there be no cyclotron resonance. In this case,

$$\frac{A_{s\beta\gamma}}{b_1}\sim\frac{B_{s\beta\gamma}}{b_1}\sim\frac{c}{4\pi}\left(\frac{c}{\omega}\right)^2\frac{1}{H},\quad \xi\sim\frac{\delta\omega}{c}.$$

We then obtain from (45)

$$E^{(2)}(0)\sim\frac{c}{\delta\omega}\frac{1}{H}(E^{(1)}(0))^2. \tag{46}$$

We now consider cyclotron resonance. We shall assume that all the non-vanishing components of the tensors $\xi_{\alpha s}$, $A_{s\beta\gamma}$, $B_{s\beta\gamma}$ and the quantity $b_1(\omega)$ are resonant. Such a situation exists in the case of an isotropic and quadratic dispersion law, and also in the case of a nonquadratic dispersion law with resonance at a noncentral cross section. Near the even resonance $\omega=l\Omega_m$, the quantities $b_1(\omega)$, $A_{s\beta\gamma}$ and $B_{s\beta\gamma}$ increase while $\xi_1(\omega)$ and $\xi_{\alpha s}(2\omega)$ decay. It is not difficult to see that the ratios $A_{s\beta\gamma}/b_1$ and $B_{s\beta\gamma}/b_1$ remain finite at resonance, equal in order of magnitude to their values far from resonance. Thus, we see that at even resonance, in the case of a fixed value of the field of the fundamental frequency on the surface of the sample, the amplitude of the second harmonic increases as ξ_1^{-1}. Consequently, the amplitude of the second harmonic increases in the given case by a factor of $(\Omega\tau)^{1/6}$ in the case of a nonquadratic dispersion law and by $(\Omega\tau)^{1/3}$ in the quadratic case. Near the odd resonance $\omega=(l+\frac{1}{2})\Omega_m$, the quantities $b_1(\omega)$, $\xi_1(\omega)$ and $B_{s\beta\gamma}$ remain finite, and $\xi_{\alpha s}(2\omega)$ and $A_{s\beta\gamma}$ have singularities. The amplitude of the second harmonic increases in this case by a factor $(\Omega\tau)^{1/3}$ in the nonquadratic case and by $(\Omega\tau)^{2/3}$ in the quadratic case.

We have thus shown that the presence of a magnetic field parallel to the metal surface increases strongly the coefficient of nonlinear reflection of the electromagnetic wave in comparison to its value in the absence of a field. The amplitude of the reflected second harmonic at a fixed value of the field of fundamental frequency on the metal surface greatly increases upon satisfaction of the condition of nonlinear cyclotron resonance $\omega=\frac{1}{2}l\Omega_m$. The relative value of the peaks can be significantly different, depending on whether the resonance is even, $\omega=l\Omega_m$ or odd, $\omega=(l+\frac{1}{2})\Omega_m$. The period of oscillations of the second harmonic amplitude $\Delta(1/H)$ is equal to

$$\Delta(1/H)=e/2\omega m^*c.$$

It is seen that it is smaller by half than the period of oscillations in the surface impedance at the frequency ω. Consequently, the determination of the extremal orbit diameter by the method of cutoff of the nonlinear cyclotron resonance at the second harmonic yields half the error of the usual method of cutoff of cyclotron resonance in the surface impedance at frequency ω. The method of cutoff of the nonlinear cyclotron resonance at higher harmonics would allow a significant increase in the accuracy of the measurements.

It is known that the impedance of a thin plate in a magnetic field parallel to its surface undergoes singularities of the cyclotron resonance at frequencies that are multiples of the frequency of rotation of the electrons, whose trajectory diameter $2r(p_z)$ is identical with the plate thickness d, i.e., upon satisfaction of the conditions

$$2r(p_z)=cD(p_z)/eH=d,\quad \omega=l\Omega(p_z), \tag{47}$$

where $D(p_z)$ is the diameter of the electron orbit in $\mathbf{p}$ space.

Cyclotron resonance at nonextremal orbits, defined by the conditions (47), allow us to find the function $m^*(D)$ or $m^*(p_z)$ if the shape of the Fermi surface is known. This effect was observed experimentally in Refs. 8 and 9. The theory of the size-effect cyclotron resonance was constructed in Refs. 10–12.

For an experimental study of the properties of electrons of nonextreme cross sections, nonlinear cyclotron resonance in the plate can also be used. This effect allows us to obtain more detail information on the function $m^*(D)$ or $m^*(p_z)$. Actually, from the expressions that we have obtained for the nonlinear conductivity, it follows that the nonlinear cyclotron resonance in a plate can be observed upon fulfilment of the conditions

$$2r(p_z)=d,\quad 2\omega=l\Omega(p_z). \tag{48}$$

For odd l, the frequencies defined by Eqs. (48) are not identical with those found from (47). Consequently, the number of resonances is doubled, and additional points appear on the $m^*(D)$ or $m^*(p_z)$ curve.

We note that although the linear impedance of the plate in an oblique field also has a singularity upon satisfaction of the conditions (48) with odd l[11]; however, the singularity here is very weak. The resonance contribution to the impedance is proportional to the square of the small parameter $\delta/r\varphi$, where φ is the angle of inclination of the magnetic field to the surface of the plate.[11] In thin plates, this contribution also contains an additional small parameter. Therefore, the singularities of the surface impedance at frequencies determined from (48) at odd l have not been observed experimentally to date. Of course, the dimensional nonlinear cyclotron resonance in an oblique field gives more detailed information on the electron spectrum than does the linear case.

Consideration of the problem of the nonlinear cyclotron resonance in a plate in the case in which reflection of the electrons from the boundary is not close to specular can be carried out if we use the results obtained above. This is to be the object of a separate investigation. Here we only note that the odd nonlinear resonance has a logarithmic character and at even l the singularity can become stronger if the parameter $\omega\tau$ is sufficently large.

[1]N. Bloembergen, R. K. Chang, S. S. Iha, and C. H. Lee, Phys. Rev. 174, 813 (1968).
[2]G. I. Leviev and E. G. Yashin, Pis'ma Zh. Eksp. Teor. Fiz. 10, 257 (1969) [JETP Lett. 10, 163 (1969)].
[3]G. I. Leviev, Fiz. Tverd. Tela (Leningrad) 12, 2131 (1970) [Sov. Phys. Solid State 12, 1691 (1971)].
[4]R. T. Bate and W. R. Wisseman, Phys. Rev. 181, 763 (1969).
[5]M. Ya. Azbel' and L. B. Dubovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. 5, 414 (1967) [JETP Lett. 5, 338 (1967)].
[6]M. Ya. Azbel' and E. A. Kaner, Zh. Eksp. Teor. Fiz. 32, 896 (1957) [Sov. Phys. JETP 730 (1957)]; Phys. Chem. Sol. 6, 113 (1958).
[7]R. Chambers, in the collection Fizika metallov (Physics of Metals), vol. 1: Electrons. (J. Ziman, ed.) Mir, 1972.
[8]A. P. Volodin, M. S. Khaĭkin, and V. S. Edel'man, Pis'ma Zh. Eksp. Teor. Fiz. 17, 491 (1973) [JETP Lett. 17, 353 (1973)]; Zh. Eksp. Teor. Fiz. 65, 2105 (1973) [Sov. Phys. JETP 38, 1052 (1074)].
[9]A. P. Volodin and M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. 70, 2006 (1976) [Sov. Phys. JETP 43, 1046 (1976)].
[10]M. A. Lur'e and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 66, 240 (1974) [Sov. Phys. JETP 39, 114 (1974)].
[11]M. A. Lur'e and V. G. Peschanskiĭ, Fiz. Nizk. Temp. 1, 1044 (1975) [Sov. J. Low Temp. Phys. 1, 502 (1975)].
[12]O. V. Kirichenko, M. A. Lur'e, and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 70, 337 (1975) [Sov. Phys. JETP 43, 175 (1975)].

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Amplification and generation of coherent phonons in ruby under conditions of spin-level population inversion

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The processes of amplification and generation of coherent phonons in ruby at a frequency 9.12 GHz following inversion of the spin-level populations by electromagnetic pumping at 23 GHz were investigated at temperatures 1.7–4.2°K. The resonant-longitudinal-phonon lifetime estimated from the threshold the gain of the hypersound excited in a crystal is $\tau_{ph} \gtrsim 2\times10^{-5}$ sec. A nonstationary effect, wherein the hypersound gain increases appreciably under conditions when the pump line is saturated on the wing is observed. This effect is interpreted on the basis of the thermodynamics of an electron-nuclear system made up of the Zeeman and dipole pools of the Cr^{3+} ions and the Zeeman pool of the Al^{27} nuclei. Stationary incoherent emission of phonons is realized. It is shown that the multimode character of this emission at the natural frequencies of the acoustic resonance and the narrow spectral radiation interval are due to the fact that phonon generation takes place under conditions of spatial disequilibrium in the case of a small excess above the pump threshold.

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1. INTRODUCTION

In connection with the successful use of hypersound waves in solid-state investigations, interest in the amplification and generation of these waves has greatly increased recently. Particular attention, however, has been paid to amplification under conditions of carrier drift in semiconductors, while amplification and generation of hypersonic waves based on stimulated emission by impurity paramagnetic centers in crystals have been practically neglected. The gist of these effects, predicted back in the early sixties independently by Townes,[1] Kopvillem and Korepanov,[2] and Kittel,[3] consists in the following:

A hypersonic wave in a crystal with nonmagnetic centers is resonantly absorbed, owing to the electron–phonon coupling, when the quantum of the elastic oscillations of the waves is equal to the spacing between the energy levels of the center in the magnetic field. If the