in the region $\bar{\nu} < |\Delta_1| < \bar{\nu}(qd)^{1/2}$. The formula for the resonant, nonoscillating term $\Phi_y^{(1)}$ coincides with the expression (5.4).

Under conditions when the coupling takes place in a region that is further from the ACR, $1 > |\Delta_1| > \bar{\nu}(qd)^{1/2}$, the curves $\Gamma_y(H)$ and $D_y(H)$ have the same character as in the case (3.28) of an arbitrary dispersion law for the electrons. The extremal values of D_y and Γ_y are equal: $\Gamma_{y\,\mathrm{ext}} \sim D_{y\,\mathrm{ext}} \sim \Phi_0 \Delta_1^2/\nu$.

1) In this case, as is well known,[9] the resonance should not be very "sharp," in order that the condition $qR(p_x) \approx qR_{\max}(\nu/\Omega)^{1/2} \gg 1$ be satisfied.

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Transition of second order in the field in a two-dimensional Heisenberg ferromagnet

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It is shown that a second-order phase transition occurs near the point $h_c = \lambda$ (λ is the anisotropy constant) in a two-dimensional Heisenberg ferromagnet with anisotropy of the easy plane type. The magnetic susceptibility is infinite below the transition point in a weak field parallel to the plane. The field dependences of the magnetic moment $n_z(h_z)$ are determined above and below the transition point.

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We consider a planar ferromagnetic Heisenberg magnet with easy-axis anisotropy. We shall show that in such a system, at sufficiently low temperatures, a second-order phase transition takes place at a definite value of the magnetic field perpendicular to the plane.

We consider first the case $T = 0$. The Hamiltonian is given by

$$H = \int \left(\frac{\lambda n_z^2}{2} - h_z n_z \right) \frac{d^2x}{a^2}. \tag{1}$$

Here $\lambda > 0$ is the anisotropy constant, h_z is the magnetic field perpendicular to the plane of the magnetic (the XY plane), and a is the lattice constant. It is easily seen that the minimum energy corresponds to the value (Fig. 1)

$$n_z = \begin{cases} h_z/\lambda, & h_z \leqslant \lambda, \\ 1, & h_z > \lambda. \end{cases}$$

The phase transition manifests itself in a change of the magnetic susceptibility χ_{XY} relative to an infinitesimally weak field h_X or h_Y parallel to the plane. The susceptibility is infinite below the phase-transition point ($h_z < \lambda$), since the spontaneous moment has a component parallel to the plane. The susceptibility $\chi_{XY} = 1/(h_z - \lambda)$ above the transition point is finite.

We show now that at sufficiently low $T \neq 0$ the transition takes place as before. The Hamiltonian now contains exchange terms

$$H = \int \left[\frac{1}{2} J(\partial_\mu \mathbf{n})^2 + \frac{\lambda}{2a^2} n_z^2 - \frac{h_z n_z}{a^2} \right] d^2x, \tag{2}$$

where $J > 0$ is the exchange constant. Consider the case $h_z \ll \lambda$. We shall show that $\chi_{XY} = \infty$. We change over to the variables φ and α; then

$$\mathbf{n} = ((1-g^2\varphi^2)^{1/2} \cos g\alpha,\ (1-g^2\varphi^2)^{1/2} \sin g\alpha,\ g\varphi).$$

Here $g^2 = T/J \ll 1$. The Hamiltonian takes the form

$$\frac{H}{T} = \int \left[\frac{1}{2} \frac{(\nabla\varphi)^2}{1-g^2\varphi^2} + \frac{1}{2}(1-g^2\varphi^2)(\nabla\alpha)^2 + \frac{1}{2} m^2\varphi^2 - \left(\frac{h\varphi}{g}\right) \right] d^2x,$$

where $m^2 = \lambda/J$, $h = h_z/J$, and the lattice constant is set equal to unity.

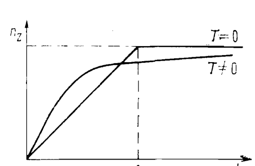

FIG. 1.

Khokhlachev[1] has shown that in the absence of a magnetic field, at distances larger than $1/m$, the fluctuations of the z-component of the spin are not correlated, $\langle\varphi\rangle = 0$, and the magnet behaves like a planar one with renormalized temperature

$$T_R = T\left[1 - \frac{T}{2\pi J}\ln(1/m)\right]^{-1}.$$

In the presence of a magnetic field, we shift the origin of φ, viz., $\varphi \to n_z/g + \varphi$. In the first-order approximation in T we have $n_z = h/m^2$. We expand the Hamiltonian in powers of g. We have

$$\begin{aligned} H/T = {}^1/_2\int [\,&(\nabla\varphi)^2/(1-n_z^2) + (1-n_z^2)(\nabla\alpha)^2 + m^2\varphi^2 \\ &+ g^2\varphi^2(\nabla\varphi)^2/(1-n_z^2)^2 - g^2\varphi^2(\nabla\alpha)^2 \\ &+ 2gn_z\varphi((\nabla\varphi)^2/(1-n_z^2)^2 - (\nabla\alpha)^2)\,]d^2x. \end{aligned} \tag{3}$$

We leave out for the time being the terms linear in g, allowance for which is equivalent to introducing corrections in n_z. The Hamiltonian then takes the same form as in Khokhlachev's paper, albeit with different coefficients. The appearance of the magnetic field has led, first, to the appearance of an average magnetic moment n_z parallel to the z axis, and second to a change of the correlation radius of the component φ:

$$r_c' = r_c(1-n_z^2)^{-1/2}.$$

The φ correlator now takes the form

$$G_{\varphi\varphi} = \frac{1-n_z^2}{q^2 + m^2(1-n_z^2)}.$$

In addition, the effective temperature for the XY components of the spin is increased on account of the factor $(1-n_z^2)$ in front of $(\nabla\alpha)^2$.

At scales larger than r_c', the fluctuations of the z component become immaterial. The behavior of the XY components is described by a planar model with a temperature

$$T_R(h) = T/(1-n_z^2)\left(1 - \frac{T}{2\pi J}\ln\frac{1}{m}\right).$$

The magnetic susceptibility is $\chi_{XY} = \infty$. The discarded terms that are linear in g

$$-2n_z g\varphi\left[(\nabla\alpha)^2 - \frac{(\nabla\varphi)^2}{(1-n_z^2)^2}\right]$$

introduce a temperature-dependent correction in the temperature. We obtain this correction with the aid of the Gell-Mann–Low equations. We carry out the averaging in a region of dimension e^ξ:

$$\left\langle(\nabla\alpha)^2 - \frac{(\nabla\varphi)^2}{(1-n_z^2)^2}\right\rangle_{e^{-\xi}<q<1} = \int_{e^{-\xi}<q<1} \frac{q^2 d^2q}{(2\pi)^2(1-n_z^2)q^2} - \int_{e^{-\xi}<q<1} \frac{q^2 d^2 q}{(2\pi)^2(1-n_z^2)^2[(q^2/(1-n_z^2))+m^2]} = \frac{m^2}{2\pi}\xi.$$

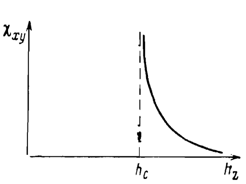

FIG. 2.

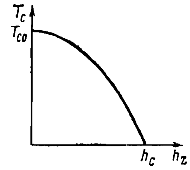

FIG. 3.

The letter q stands for the wave vector of the fluctuations. The increase of n_z due to this field yields

$$\Delta(n_z/g) = \frac{2n_z g(m^2/2\pi)\xi}{2m^2} = (gn_z/2\pi)\xi.$$

From this we get the Gell-Mann–Low equation for n_z:

$$d\ln n_z/d\xi = g^2/2\pi.$$

Integration of this equation over the interval $1 < \xi < \ln(1/m)$ yields

$$n_z = h/m^2\left(1 - \frac{T}{2\pi J}\ln\frac{1}{m}\right).$$

Discarding the terms linear in g did not alter the qualitative aspect of the phenomena.

The foregoing analysis is valid at $T_R(h) \ll T_{c0} = 2\pi J/\ln(1/m)$—the critical temperature obtained by Khokhlachev.[1]

Let now $h_z > \lambda$. We use the variables φ_1 and φ_2

$$\mathbf{n} = (\varphi_1, \varphi_2, (1-\varphi_1^2-\varphi_2^2)^{1/2}).$$

In terms of these variables, that part of the Hamiltonian which does not depend on the gradients (1) takes the form

$$\frac{\lambda}{2} - h_z + \frac{h_z-\lambda}{2}(\varphi_1^2+\varphi_2^2) + o(\varphi_1^2, \varphi_2^2). \tag{4}$$

The combination $h_z - \lambda$ plays the same role as the magnetic field for a magnetic without anisotropy; this follows from a comparison of (4) with formula (5) of[2].

Using Khokhlachev's results[2] we obtain for the magnetization

$$n_z = 1 - \frac{T}{2\pi J}\ln\frac{J}{h_z-\lambda}$$

and for the susceptibility

$$\chi_{XY} = \frac{1}{h_z-\lambda}\left(1 - \frac{T}{2\pi J}\ln\frac{J}{h_z-\lambda}\right) < \infty.$$

Our method permits no analysis in the immediate vicinity of the point $h_z = \lambda$, but a comparison of the values of χ_{XY} on both sides of this point (Fig. 2) indicates that the transition is preserved also in a certain region $T \neq 0$. The bound for this region is given by the temperature obtained by Khokhlachev

$$T_{c0} = 2\pi J/\ln(1/m).$$

The variation of the phase-transition temperature can be qualitatively imagined to follow the results of Berezinskiĭ[3] for the XY components of the spin as functions of the magnetic field h_z (Fig. 3).

We turn on the dipole–dipole interaction in lieu of the anisotropy. At short distances this leads to the appearance of a mass $m^2 = 27\mu^2/a^3J$ (μ is the dipole moment) for the z component, and all the foregoing arguments remain valid. At large distances it is necessary to take into account the specific form of the interaction.

In fields $h < m^2$ it is necessary to introduce into the correlators for the XY components of the spin terms that correspond to the dipole–dipole interaction.[4] In fields $h > m^2$, a difference arises between the longitudinal and the transverse correlators of φ_1 and φ_2, i.e., the correlators of the fluctuations parallel and perpendicular to the vector $\mathbf{q}$

$$G_\parallel(\mathbf{q}) = [q^2 + (h - m^2)]^{-1},$$

$$G_\perp(\mathbf{q}) = [q^2 + 2R^{-1}q + (h - m^2)]^{-1},$$

where $R^{-1} = \pi\mu^2/a^2J$.[4]

In conclusion, I wish to thank V. L. Pokrovskiĭ for directing the work, as well as S. B. Khokhlachev and M. V. Feĭgel'man for a discussion of the results.

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ERRATA

Erratum: On the theory of collision-induced lines forbidden in Raman scattering [Sov. Phys. JETP 42, 982–985 (1975)]

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PACS numbers: 01.85.+s, 32.20.Dr

A number of subscripts have been left out of Eq. (8). The denominator of the first term of (8a) should be $\omega_{nj} - \omega_2$, the denominator of the second term should be $\omega_{nj} + \omega_1$, and in (8b) the denominators of the first and second terms should be $\omega_{nj} - \omega_1$ and $\omega_{nj} + \omega_2$, respectively.

Erratum: Collisionless emission of radiation by an inhomogeneous plasma [Sov. Phys. JETP 44, 546–553 (1976)]

B. E. Meĭerovich

PACS numbers: 52.25.Ps, 99.10.+y

On p. 547, left-hand column, line 23 from top, read "collective interaction" in lieu of "collision interaction."

