Velocity of domain boundary motion

V. M. Eleonskiĭ, N. N. Kirova, and N. E. Kulagin

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Several types of simple waves of the magnetic-moment vector, connected with the motion of domain boundaries, are investigated within the framework of the Landau-Lifshitz equations. It is shown that the existence of a limiting velocity of propagation of a simple wave, found by Akhiezer and Borovik [Sov. Phys. JETP 25, 885 (1967)], is not due to the explicit form of the particular analytic solution that they obtained, but is a consequence of asymptotic boundary conditions of a definite type. "Fast" simple waves are investigated in the limit of a nondissipative medium. It is shown that propagation of a simple wave with a velocity above the limiting value is related to precession of the magnetic moment about the anisotropy axis on approach to the regions of uniform magnetization. An analytical solution corresponding to a "fast" simple wave of the magnetic moment is found for a medium with large uniaxial anisotropy energy and small saturation magnetization.

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1. One of the first papers on the theory of simple waves connected with the motion of domain boundaries is that of I. A. Akhiezer and A. E. Borovik.[1] They showed, in particular, that in a nondissipative magnetic medium the spectrum of velocities of a moving domain boundary is continuous and bounded from above, if the rotating magnetic-moment vector in the domain boundary remains in a constant plane in a coordinate system that moves with the boundary. The possible velocity values below the limiting value are determined by the orientation of the plane of rotation of the magnetic moment. In a dissipative medium, as was shown by Walker,[2] the orientation of the plane of rotation of the magnetic-moment vector is determined by the value of the external magnetic field.

It will be shown below, on the basis of the Landau-Lifshitz[3] equations, that the existence of an upper limit to the velocity of propagation of a simple wave of the magnetic-moment vector (for example, a moving domain boundary) is not due to the explicit form of the particular analytic solution obtained in[1,2], but is an exact consequence of asymptotic boundary conditions of a definite type. Specifically: if the magnetic-moment vector, on approach to a region of uniform magnetization, asymptotically approaches a plane of rotation with a constant orientation in space (in a reference system attached to the simple wave), then for magnetic media with anisotropy of the "axis of easy magnetization" type, the velocity of propagation of a simple wave, U, is bounded from above by the quantity

$$U_m=u_0(\sqrt{1+\varepsilon}-1), \tag{1.1}$$

which is determined by the parameter $\varepsilon=2\pi M_s^2/K_1$ of the medium and the characteristic velocity $u_0=2|\gamma| \times (AK_1)^{1/2}M_s$. Here M_s is the saturation magnetization, K_1 is the uniaxial-anisotropy constant, A is the exchange energy, and γ is the gyromagnetic ratio.

The asymptotic boundary conditions mentioned above essentially exclude precession of the magnetic moment about the anisotropy axis in the regions of uniform magnetization. In a number of papers[4,5] containing indications of the possible realization of domain-boundary motions with speeds exceeding the limiting speed (1.1), consideration was given to the question of the existence of solutions of the Landau–Lifshitz equations that would correspond to "fast" isolated simple waves of the magnetic moment.

In the present paper, on the basis of a qualitative analysis of the Landau-Lifshitz equations and of numerical calculations, it is shown that in the nondissipative limit, there are solutions of the type of a "fast" isolated magnetic-moment wave, whose velocity of propagation may be larger than the limiting velocity found earlier. A peculiarity of such waves is a broadening of domain boundaries (more accurately, an increase of a characteristic dimension of the forward and rear wave front) with increase of velocity, in contrast to the "dynamic compression" of a plane Bloch domain boundary in motion at velocities less than (1.1). The asymptotic boundary conditions for these fast waves correspond to precession of the magnetic moment about the anisotropy axis in the regions of uniform magnetization. For a problem degenerate with respect to the parameter ε, an explicit form of the solution is found. For $\varepsilon=0$, the spectrum of velocities of simple waves that correspond to a moving structure of the isolated-domain type is bounded from above by the value $U_m=2u_0$. A characteristic feature of the degenerate solution is that the angular velocity of precession in the region of uniform magnetization asymptotically approaches a constant value, determined by the velocity of the "fast" wave. For $U\to\tilde{U}_m$, the simple isolated wave degenerates to a small-amplitude wave with almost uniform precession of the magnetic moment about the anisotropy axis. For nonvanishing but small ε, the solution found, as is shown by qualitative and numerical analyses, retains its characteristic features.

2. For solutions of the simple stationary-wave type

$$\theta(\xi)=\theta(x-ut), \quad \varphi(\xi)=\varphi(x-ut), \quad d\varphi/d\xi\equiv\omega(\xi)$$

(where θ and φ are the polar and azimuthal angles of the magnetic-moment vector in a spherical coordinate system whose axis coincides with the anisotropy axis), the

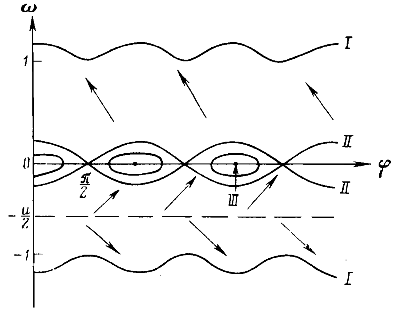

FIG. 1. Contour curves $h = \text{const}$ in the plane $\theta = 0$; I, contour curves $h = 0$; II, contour curves $h = 1$; III, points corresponding to $h = 1 + \epsilon$.

Landau-Lifshitz system of equations has the following form:

$$\frac{d^2\theta}{d\xi^2} - (1+\omega^2+\varepsilon\cos^2\varphi)\sin\theta\cos\theta - h_z\sin\theta = u\omega\sin\theta - \alpha u\frac{d\theta}{d\xi},$$
$$\frac{d\omega}{d\xi}\sin^2\theta + 2\omega\frac{d\theta}{d\xi}\sin\theta\cos\theta + \varepsilon\sin\varphi\cos\varphi\sin^2\theta$$
$$= -u\sin\theta\frac{d\theta}{d\xi} - \alpha u\omega\sin^2\theta. \qquad (2.1)$$

Here $U = u_0 u$ is the velocity of the simple wave; the variable $\xi = x - ut$ is referred to the characteristic dimension of a Bloch wall, $(A/K_1)^{1/2}$; the external magnetic field h_z, parallel to the anisotropy axis, is referred to the anisotropy field $2K_1/M_s$; α is Gilbert's dissipation parameter.[6]

An important property of the system (2.1) for $\alpha = 0$ is the existence of a first integral

$$\mathscr{H} = \left(\frac{d\theta}{d\xi}\right)^2 + \omega^2\sin^2\theta - \sin^2\theta(1+\varepsilon\cos^2\varphi) + 2h_z\cos\theta = \text{const}. \qquad (2.2)$$

We note that (2.2) is identical with the corresponding expression for the first integral for walls at rest[7]; this is a consequence of the gyroscopic properties of the system (2.1) when $\alpha = 0$.

The equilibrium positions of the system (2.1), corresponding to uniform magnetization parallel and antiparallel to the external field (or to the anisotropy axis), correspond to the values

$$\theta \to 0, \quad \mathscr{H} \to +2h_z \quad \text{as} \quad \xi \to -\infty,$$
$$\theta \to \pi, \quad \mathscr{H} \to -2h_z \quad \text{as} \quad \xi \to +\infty.$$

Determination of the asymptotic behavior of the values of φ and ω requires, just as in the static case,[7] an additional investigation.

3. For the nondissipative case, we shall investigate the asymptotic behavior of the integral curves for $\theta \to 0$ and for $\theta \to \pi$, not assuming in general that the plane of rotation of the magnetic-moment vector, which is determined by the angle φ, is constant. We note that when $\alpha \to 0$ in (2.2), it is necessary that $h_z \to 0$, since otherwise there is, for $\alpha = 0$, no simple wave corresponding to an asymptotic transition from $\theta = 0$ to $\theta = \pi$. In fact, for $h_z \neq 0$ we have

$$\mathscr{H}(0) = 2h_z \neq \mathscr{H}(\pi) = -2h_z.$$

Thus on setting $h_z = 0$ in (2.2), we get

$$(d\theta/d\xi)^2 = \sin^2\theta \times (1+\varepsilon\cos^2\varphi - \omega^2). \qquad (3.1)$$

On substituting (3.1) in the second equation of the system (2.1), we get the following equation:

$$\frac{d\omega}{d\xi} + 2\left(\operatorname{sign}\frac{d\theta}{d\xi}\right)\left(\omega\cos\theta + \frac{u}{2}\right)(1+\varepsilon\cos^2\varphi - \omega^2)^{1/2} + \varepsilon\cos\varphi\sin\varphi = 0. \qquad (3.2)$$

We introduce the function

$$h = \left(\frac{d\theta}{d\xi}\Big/\sin\theta\right)^2 = 1 + \varepsilon\cos^2\varphi - \omega^2, \qquad (3.3)$$

by means of it, Eq. (3.2) can be rewritten in the form

$$\frac{dh}{d\xi} = 4\left(\operatorname{sign}\frac{d\theta}{d\xi}\right)\left(\omega\cos\theta + \frac{u}{2}\right)\omega\sqrt{h}. \qquad (3.4)$$

We note that Eq. (3.4), or equivalently (3.2), is an exact consequence of the system (2.1) under consideration. Therefore, although the phase space of the system (2.1) is four-dimensional, it is convenient to investigate the behavior of the integral curves in the three-dimensional space $(\theta, \varphi, \omega)$. For $\theta \to 0$, Eq. (3.4) takes the form

$$\frac{dh}{d\xi} = 4\left(\operatorname{sign}\frac{d\theta}{d\xi}\right)\left(\omega + \frac{u}{2}\right)\omega\sqrt{h}. \qquad (3.5)$$

Figure 1 shows the contour curves $h = \text{const}$ in the (ω, φ) plane. Since by definition $h \geq 0$, the region in which the integral curves are located is bounded by the curves

$$\omega = \pm(1+\varepsilon\cos^2\varphi)^{1/2}. \qquad (3.6)$$

The derivative $dh/d\xi$ in (3.5) changes sign on crossing of the lines $\omega = 0$ and $\omega = -u/2$. On reflection of an integral curve from a limiting curve (3.6), there is a change of sign of the derivative $d\theta/d\xi$, and this entails a change of sign of $dh/d\xi$.

We shall consider a simple wave, supposing that for $\theta \to 0$, $d\theta/d\xi$ keeps its sign. In this case, sign $(d\theta/d\xi) = +1$ for $\xi \to -\infty$. For a certain ξ, let the integral curve in question be in the region

$$-(1+\varepsilon\cos^2\varphi)^{1/2} < \omega < -u/2.$$

In accordance with (3.5), we conclude that $dh/d\xi > 0$ and that for $\xi \to -\infty$ the integral curve crosses the contour curves $h = \text{const}$, shown in Fig. 1, in the direction of the downward-pointing arrows. When $-u/2 < \omega < 0$, we conclude that $dh/d\xi < 0$ and that for $\xi \to -\infty$ the integral curve crosses the contour curves $h = \text{const}$ in the direction of the upward-pointing arrows. Consequently, in accordance with the general properties of differential equations, in the neighborhood of $\omega = -u/2$ there must exist a separatrix curve, and

$$\omega = -u/2 + O(\varepsilon, \varphi)$$

for $\theta \to 0$.

When the integral curve reaches the line $\omega=0$, two cases are possible: 1) the integral curve falls into an equilibrium position $\omega=0$, $\varphi(-\infty)=\varphi_-$; 2) the integral curve crosses the line $\omega=0$. Since $dh/d\xi>0$ in the upper ω, φ half-plane, for $\xi \to -\infty$ the integral curve crosses the contour curves in the direction of the upward-pointing arrows and goes out to the boundary $h=0$ of the region.

Thus for $\theta \to 0$ the following asymptotic behavior is possible for the integral curve that describes the magnetic-moment distribution in a simple wave: 1) there is an equilibrium position $\omega=0$, $\varphi(-\infty)=\varphi_-$, corresponding to establishment of a constant orientation of the plane of rotation of the magnetic moment for $\theta \to 0$; 2) there is a separatrix "entrance" to the state $\theta=0$, corresponding to a precession of the magnetic moment, for which $\omega \to -u/2 + O(\varepsilon, \varphi)$.

The investigation for $\theta \to \pi$ also indicates two possible types of asymptotic behavior of the integral curves:

$$\omega \to 0, \quad \varphi \to \varphi_+, \text{ or } \quad \omega \to -u/2 + O(\varepsilon, \varphi).$$

We shall investigate in more detail the asymptotic behavior of the integral curves when

$$\varphi \to \varphi_\pm \equiv \varphi(\pm\infty), \quad \omega \to 0$$

for $\xi \to \pm\infty$ and $\theta \to 0, \pi$. From Eq. (3.2) it follows that

$$\left(\operatorname{sign}\frac{d\theta}{d\xi}\right) u (1+\varepsilon\cos^2\varphi_\pm)^{1/2} + \varepsilon\cos\varphi_\pm \sin\varphi_\pm = 0. \tag{3.7}$$

Thus the velocity of a simple wave is connected with $\varphi_\pm$ by the relations

$$u^2 = \varepsilon^2 \cos^2\varphi_\pm \sin^2\varphi_\pm / (1+\varepsilon\cos^2\varphi_\pm). \tag{3.8}$$

Consequently $\varphi_+ = \varphi_- + n\pi$. We note that the value of u in (3.8) is bounded from above, just as for a simple plane wave (whose magnetic-moment vector is everywhere alined in a certain constant plane), by the value

$$u_m = (1+\varepsilon)^{1/2} - 1.$$

Thus we have obtained the important result: if the asymptotic behavior of the magnetic-moment distribution in a simple wave is such that, in a system of coordinates attached to it, there is a plane of "entrance" to and "exit" from an equilibrium position corresponding to uniform magnetization, then the velocity of this simple wave is bounded from above by the value (1.1). The analytic solution found by Akhiezer and Borovik is the special case $\varphi(\xi) \equiv \varphi_\pm = \text{const}$. Therefore motions with velocities greater than the limiting velocity (1.1) may be possible if, for $\theta \to 0$ or π, there is a precession of the magnetic moment and $\omega \neq 0$.

4. For the problem degenerate with respect to the parameter ε ($\varepsilon \to 0$), the system (2.1) permits the existence of a first integral, because of the fact that in this case φ is a cyclic variable:

$$\omega \sin^2\theta = \mu_0 + u\cos\theta. \tag{4.1}$$

Here $\mu_0 = \text{const}$ is the constant of the first integral. If the constants in the first integrals (4.1) and (2.2) are so chosen that $\theta \to 0$ and $\omega \to \text{const}$ for $\xi \to \pm\infty$, the system to be integrated has the form

$$\left(\frac{d\theta}{d\xi}\right)^2 = (1-\omega^2)\sin^2\theta, \quad \omega\sin^2\theta = -u(1-\cos\theta). \tag{4.2}$$

The solution of this system (4.2) can be written in the form

$$\cos\theta = u\frac{1+p\,\mathrm{th}^2\zeta}{1-p\,\mathrm{th}^2\zeta} - 1, \quad \omega = -\frac{u}{1+\cos\theta}. \tag{4.3}$$

Here the following notation has been introduced:

$$p = (2-u)/(2+u); \quad \zeta = \xi(1-u^2/4)^{1/2}.$$

The solution (4.3) describes a moving, isolated domain, at both ends of which the magnetic-moment vector, on reaching an equilibrium position, precesses about the anisotropy axis. In other words, at the tails of the isolated simple magnetic-moment wave, associated spin waves are excited. The width of the transitional layer of the wave front is larger than the Bloch wall width in the ratio $(1-u^2/4)^{-1/2}$ and increases with increase of the velocity. We note that this result may explain qualitatively the experimental data on broadening of the wall of a magnetic domain during motion with a velocity larger than the limiting velocity (1.1).[4]

According to (4.3), for $\xi \to \pm\infty$ we have $\theta \to 0$, $\omega \to -u/2$, which agrees with the conclusions reached in the investigation of the asymptotic behavior of the integral curves of the exact problem ($\varepsilon \neq 0$). For $u \to 2$, we have $\cos\theta \to 1$, and there is a wave with almost uniform magnetization and with precession of the magnetic moment about the anisotropy axis. We note that for $\varepsilon=0$ there is also an isolated wave connected with the separatrix branch for $\theta=\pi$.

To elucidate the behavior of the separatrix integral curves for $\varepsilon \neq 0$, we shall investigate by means of Eq. (3.4) the behavior of the solutions in θ, φ, ω space. The surfaces $h=\text{const}$ are cylindrical surfaces with generators parallel to the θ axis, and the equation of the directing curves is

$$\omega = \pm(1+\varepsilon\cos^2\varphi - h)^{1/2}. \tag{4.4}$$

The intersection of these surfaces with the plane $\theta=0$ is shown in Fig. 1. For $\theta>0$, a change of sign of $dh/d\xi$ occurs on the plane $\omega=0$ and on the surface

$$\omega = -u/2\cos\theta, \tag{4.5}$$

which is a doubly connected cylindrical surface with generators parallel to the φ axis. Let an integral curve go out from the singular point $\theta=0$. The asymptotic be-

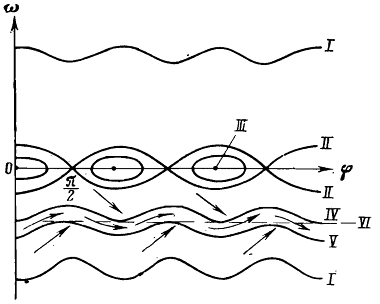

FIG. 2. Contour curves $h=\text{const}$ in the plane $\theta=\theta_0$; I, II, and III, same as in Fig. 1; IV, $h=h_+$; V, $h=h_-$; the straight line VI corresponds to the value $\omega=-(u/2)\cos\theta_0$.

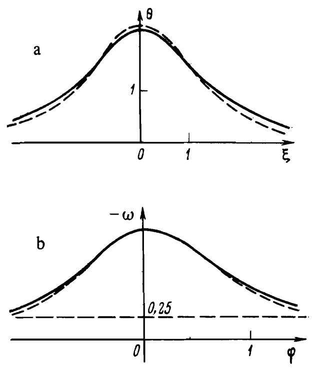

FIG. 3.

havior of $\omega(\xi)$ for $\varepsilon\neq 0$, but small, is easily found by perturbation theory. To terms linear in ε, we have

$$\omega(\xi)=-\frac{u}{2}-\frac{\varepsilon u}{8}\cos u\xi+\frac{\varepsilon}{4}\left(1-\frac{u^2}{4}\right)^{1/2}\sin u\xi.$$

If for some θ, $0<\theta<\pi/2$, we have $-u/2\cos\theta<\omega<0$, then $dh/d\xi<0$, and in the θ, φ, ω plane the integral curve is directed in the direction of decreasing h (that is, it recedes from the plane $\omega=0$). After an intersection with the surface (4.5), the sign of $dh/d\xi$ reverses, and the integral curve is directed in the direction of increasing $h=\text{const}$ (that is, it moves nearer to the plane $\omega=0$).

By considering the behavior of the solutions in a cross-sectional plane $\theta=\text{const}$ (Fig. 2), it is easily seen that after intersection of the surface (4.5) the integral curve cannot escape outside the limits $h_-\leqslant h\leqslant h_+$, where

$$h_-=1-u^2/4\cos^2\theta,$$
$$h_+=1+\varepsilon-u^2/4\cos^2\theta,$$

Consequently, in θ, φ, ω space an integral curve that goes out from the singular point $\theta=0$ is localized within the region

$$-(1+\varepsilon\cos^2\varphi)^{1/2}<\omega<-(1+\varepsilon\cos^2\varphi-h_0)^{1/2},$$

where $h_0=1+\varepsilon-u^2/4$. After intersection with the plane $\theta=\pi/2$, the derivative $dh/d\xi<0$, and the integral curve is directed in the direction of decreasing $h=\text{const}$, toward the limiting surface

$$\omega=-(1+\varepsilon\cos^2\varphi)^{1/2}.$$

By considering the behavior of an integral curve after reflection from this limiting surface, it is easily seen that in the absence of damping, there is no fast 180° wave with a $0\to\theta\to\pi$ transition.

We note that in the system (2.1) the effect of the gyroscopic term $u\omega\sin\theta$ is similar to the effect of an external magnetic field directed along the anisotropy axis. It is known that in the presence of an external magnetic field there is no separatrix solution, describing an isolated domain boundary, whereas there is a solution describing an isolated domain.

To verify the results of the qualitative analysis, a number of calculations were made with a computer. The method of finding the separatrix solutions is the same as that developed earlier for the static case.[7] Figure 3a shows the function $\theta(\xi)$ obtained for $\varepsilon=0.1$ and $u=0.5$. Figure 3b shows the function $\omega(\varphi)$ for the same values of the parameters. The dotted curve shows the solution of the degenerate problem, calculated in accordance with (4.3).

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