Behavior of a coulomb superconductor with inverse distribution in a magnetic field

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We consider the superconducting state that arises in a system with repulsion between the electrons under inverse distribution of the quasiparticles. The Ginzburg-Landau equations that describe the behavior of such a superconductor in a magnetic field are derived from the microscopic theory. This equation differs from the usual ones only in the sign in the expression for the current density. One-dimensional solutions of the obtained equations are investigated for the cases of a superconducting half-space and of thin films. The question of the vortical state of such a superconductor is considered.

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1. INTRODUCTION

The superconducting state in an electron system usually sets in when attraction exists between the electrons. This attraction can be due, for example, to electron–phonon interaction ("ordinary" superconductivity) or to interaction between electrons and excitons (excitonic superconductivity). Under substantial disequilibrium conditions, however, namely in case of inverse distribution of the quasiparticles, a superconducting state can arise also when there is repulsion between the electrons.[1–4] This interaction corresponds to the Coulomb interaction between the electrons (Coulomb superconductor).

Apart from the fact that such superconductors, like the excitonic ones, are of interest from the point of view of realizing high-temperature superconductivity,[5] they should have certain unusual properties. Thus, the sign of the superconducting current produced by turning on an external field is different from the sign of the ordinary current. There is therefore no Meissner effect in superconductors with inverse distribution, and the external field penetrates into the superconductor and experiences spatial oscillations. This raises the question of deriving the equations that describe a Coulomb superconductor in a magnetic field.

An ordinary superconductor is described by the Ginzburg–Landau phenomenological equations, which contain two constants: the effective charge and the mass. Gauge-invariance requires that the charge should be a multiple of the elementary charge, but the mass remains arbitrary and should be determined from experiment. It might appear therefore that since Coulomb superconductors produce a paramagnetic response to an external field, it is necessary in this case to use a negative effective mass in the Ginzburg–Landau equations (this would correspond to a transition from the quasiparticle representation to the quasihole representation). But then, in addition to the reversal of the sign in the expression for the current density, the gradient term in the first equation also becomes negative. This indicates that in such a system, the inhomogeneous state becomes more favored even in the absence of an external field. It has been shown, however,[7] that in the case of a superconductor with inverse distribution the homogeneous state is stable (in the absence of a field).

It is of interest in this connection to derive from the microscopic theory the Ginzburg–Landau equations for a Coulomb superconductor. This is the main purpose of the present paper.

To produce an inverse distribution of the quasiparticles by means of an external source, a priming energy gap is necessary. We therefore consider below a semiconductor with non-coinciding band extrema and with a forbidden band exceeding the energy of the Debye phonon. To make the superconducting state possible, the gap must be smaller than a certain critical value.[1] In addition, a superconducting gap is produced in this case not at the Fermi level but near the extrema of the bands, where the density of states is small in the three-dimensional case, so that the analysis is carried out for two-dimensional systems with a constant state density that does not depend on the energy.

In Sec. 2 we derive for this model the Ginzburg–Landau equations for an inverse distribution of the quasiparticles (at $T=0$). The equation for Δ coincides with the usual equation, where the role of the temperature is played by the pump intensity μ, and the expression for the current density has a sign opposite to the ordinary one.

This result is explained by the fact that the first equation describes the behavior of the "wave function" of superconducting pairs, whereas the second equation contains the total current, which consists of the diamagnetic current of the pairs and the paramagnetic current of the quasiparticles. The quasiparticle current then turns out to be superconducting,[3] and it is double the diamagnetic current of the pairs in the considered case of a Fermi distribution. The solutions of the obtained equations are considered in the case when all the quantities depend on only one coordinate.

Section 3 is devoted to a superconducting half-space. In the case of weak fields, the solutions are oscillating, the fields varying with a period $2\pi\lambda$ and the gap with a period λ, where λ is the analog of the depth of penetration of the field in ordinary superconductors. Inclusion of the terms of higher order in the external field H_0 shows that the superconducting phase is concentrated near the surface, and that at larger distances the system apparently goes into the normal or vortical state.

In Sec. 4 we investigate the properties of thin superconducting films. These films turn out to be paramagnetic with positive magnetic moments. The phase transition in the magnetic fields is of second order independently of the film thickness.

An expression is obtained for the parameter $\varkappa = \lambda/\xi$ in terms of the characteristics of the system. Estimates yield a value $\varkappa \gg 1$. One can therefore expect a vortical state to be produced in the case of a Coulomb superconductor even in weak fields. Section 5 is devoted to a discussion of such a state.

2. GINZBURG-LANDAU EQUATIONS FOR A COULOMB SUPERCONDUCTOR

We consider the model proposed by Kirzhnits and Kopaev[1] and constituting an indirect semiconductor with narrow forbidden band E_g, in which an external source has produced an inverse population characterized by a chemical potential μ, which is the same for both bands. The Hamiltonian of such a system is

$$\mathscr{H}=\sum_{\alpha=1}^{2}\left\{\sum_{\mathbf{p}}\varepsilon_\alpha(\mathbf{p})a_{\alpha\mathbf{p}}^{+}a_{\alpha\mathbf{p}}+\frac{g_0}{2}\sum_{\mathbf{p}}a_{\alpha\mathbf{p}}^{+}a_{\alpha-\mathbf{p}}^{+}a_{\alpha-\mathbf{p}}a_{\alpha\mathbf{p}}\right\}+\frac{g_1}{2}\sum_{\mathbf{p}}a_{1\mathbf{p}}^{+}a_{1-\mathbf{p}}^{+}a_{2-\mathbf{p}}a_{2\mathbf{p}}+\text{H. c.},\tag{1}$$

where $\varepsilon_1(\mathbf{p})=\mathbf{p}^2/2m+E_g$ and $\varepsilon_2(\mathbf{p})=-(\mathbf{p}-\mathbf{w})^2/2m-E_g$ are the dispersion laws for the conduction and valence bands, respectively, and g_0 and g_1 are the constants of the intraband and interband interactions ($g_0>0$).

In the absence of an external field, at $T=0$, the equations of motion for the Green's function take the form[3]:

$$\begin{aligned}&(\omega-\varepsilon_1(\mathbf{p}))G_1(\omega,\mathbf{p})-i\Delta F_1^{+}(\omega,\mathbf{p})=1,\\&(\omega+\varepsilon_2(\mathbf{p}))F_1^{+}(\omega,\mathbf{p})+i\Delta^{+}G_1(\omega,\mathbf{p})=0.\end{aligned}\tag{2}$$

Analogous equations are written also for the second band. In the presence of an external field, the system (2) is rewritten in the following manner:

$$\begin{aligned}&\left[\omega+\frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}}-ie\mathbf{A}(\mathbf{r})\right)^2-E_g\right]G_1(\omega,\mathbf{r},\mathbf{r}')-i\Delta(\mathbf{r})F_1^{+}(\omega,\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'),\\&\left[-\omega+\frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}}+ie\mathbf{A}(\mathbf{r})\right)^2-E_g\right]F_1^{+}(\omega,\mathbf{r},\mathbf{r}')-i\Delta^{+}(\mathbf{r})G_1(\omega,\mathbf{r},\mathbf{r}')=0.\end{aligned}\tag{3}$$

Here and below we use the system of units $\hbar=c=1$. The superconducting gap Δ is obtained from the equation

$$\Delta^{+}(\mathbf{r})=g_0\int\frac{d\omega}{2\pi}F_1^{+}(\omega,\mathbf{r},\mathbf{r})+g_1\int\frac{d\omega}{2\pi}F_2^{+}(\omega,\mathbf{r},\mathbf{r}).\tag{4}$$

In the homogeneous case, Δ is described by the expression

$$\Delta=[\Delta_0(\Delta_0-2E_g)]^{1/2},\quad \Delta_0=\frac{2\mu^2}{\omega_p}\exp\left(-\frac{2}{N_0(g_0+g_1)}\right),$$

where ω_p is the electron plasma density, and N_0 is the density of states, which is assumed to be independent of the energy, i.e., we consider quasi-two-dimensional systems. It is seen therefore that there exists a critical value of the pump intensity μ_c equal to

$$\mu_c=(\omega_pE_g)^{1/2}\exp(1/N_0(g_0+g_1)),\tag{5}$$

below which Δ vanishes. At pump intensities close to critical, Δ is small:

$$\Delta=2^{1/2}E_g\left(\frac{\mu-\mu_c}{\mu_c}\right)^{1/2}.$$

In this case, at $T=0$ and $(\mu-\mu_c)/\mu_c\ll 1$, we can use Gor'kov's derivation of the Ginzburg–Landau equations for temperatures close to critical.[8]

Expanding the system (3) up to third order in Δ, we obtain from (4) the first of the sought equations in the form

$$\begin{aligned}&\left\{\frac{1}{6}\left(\frac{\partial}{\partial\mathbf{r}}+2ie\mathbf{A}(\mathbf{r})\right)^2P+Q-\frac{1}{g_0+g_1}-|\Delta(\mathbf{r})|^2R\right\}\Delta^{+}(\mathbf{r})=0,\\&P=i\int\frac{d\omega}{2\pi}\int G^0(-\omega,\mathbf{s}-\mathbf{r})G^0(\omega,\mathbf{s}-\mathbf{r})(\mathbf{s}-\mathbf{r})^2ds,\\&Q=i\int\frac{d\omega}{2\pi}\int G^0(-\omega,\mathbf{s}-\mathbf{r})G^0(\omega,\mathbf{s}-\mathbf{r})ds,\\&R=i\int\frac{d\omega}{2\pi}\int G^0(-\omega,\mathbf{s}-\mathbf{r})G^0(\omega,\mathbf{s}-\mathbf{l})G^0(-\omega,\mathbf{m}-\mathbf{l})G^0(\omega\ \mathbf{m}-\mathbf{r})\,dl\,dm\,ds.\end{aligned}\tag{6}$$

Here G^0 is the Green's function for a semiconductor in the absence of an external field and with allowance for the inverse distribution. Its Fourier component is of the form

$$G^0(\omega,\mathbf{p})=\{\omega-\varepsilon_1(\mathbf{p})+i\delta\,\mathrm{sign}[\varepsilon_1(\mathbf{p})-\mu]\}^{-1},\tag{7}$$

where the circuit around the poles takes into account the inverse distribution of the quasiparticles. Calculating the integrals P, Q, and R with the Green's function (7), we obtain an equation for Δ:

$$\left\{\frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}}-2ie\mathbf{A}(\mathbf{r})\right)^2+\frac{1}{\eta}\left(\ln\frac{\mu}{\mu_c}-\frac{\mu_c^2-2E_g^2}{8\mu_c^2E_g^2}|\Delta(\mathbf{r})|^2\right\}\Delta(\mathbf{r})=0,\tag{8}$$

where the coefficient η is equal to

$$\eta=(\mu_c^2+2\mu_cE_g-4E_g^2)/6\mu_c^2E_g.\tag{9}$$

The current density is determined by the expression[9]

$$\mathbf{j}(\mathbf{r})=\left\{\frac{ie}{m}(\nabla_{\mathbf{r}'}-\nabla_{\mathbf{r}})\int\frac{d\omega}{2\pi i}G(\omega,\mathbf{r},\mathbf{r}')-\frac{2e^2}{m}\mathbf{A}(\mathbf{r})\int\frac{d\omega}{2\pi i}G(\omega,\mathbf{r},\mathbf{r}')\right\}\tag{10}$$

Similarly, expanding the system (3) up to second order in and substituting the functions G_1 and G_2 in (10), we obtain

$$\mathbf{j}(\mathbf{r})=\left\{\frac{ie}{m}\left(\Delta\frac{\partial\Delta^{+}}{\partial\mathbf{r}}-\Delta^{+}\frac{\partial\Delta}{\partial\mathbf{r}}\right)-\frac{4e^2|\Delta|^2}{m}\mathbf{A}(\mathbf{r})\right\}C,\quad C=-n(\mu_c^2-2E_g^2)/8\mu_c^2E_g^2.\tag{11}$$

Here $n=(2m\mu_c)^{3/2}/3\pi^2$ is the concentration of the nonequilibrium electrons.

We introduce the "wave function" $\psi(r)$ defined by the formula

$$\psi(\mathbf{r})=\left(\frac{\mu_c^2-2E_g^2}{8\mu_c^2}n\right)^{1/2}\frac{\Delta(\mathbf{r})}{E_g}. \tag{12}$$

Then Eqs. (8) and (11) take the form

$$\left\{\frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}}-2ie\mathbf{A}(\mathbf{r})\right)^2+\frac{1}{\eta}\left[\frac{\mu-\mu_c}{\mu_c}-\frac{1}{n}|\psi|^2\right]\right\}\psi(\mathbf{r})=0,$$
$$\mathbf{j}(\mathbf{r})=\frac{ie}{m}\left(\psi^*\frac{\partial\psi}{\partial\mathbf{r}}-\psi\frac{\partial\psi^*}{\partial\mathbf{r}}\right)+\frac{4e^2}{m}|\psi|^2\mathbf{A}(\mathbf{r}). \tag{13}$$

It is seen from these equations that the first is similar to the Ginzburg–Landau equation for an equilibrium superconductor, where the role of the temperature is played by the pump intensity μ. The sign of the current density is reversed from the usual one, for the reasons indicated above.

3. SUPERCONDUCTING HALF-SPACE

We consider a superconducting space $z>0$ bordering on a vacuum. The external field is directed parallel to the boundary. In this case all quantities depend only on a single coordinate z.

We introduce in the usual manner the new dimensionless variables by means of the formulas

$$\mathbf{r}'=\mathbf{r}/\lambda,\quad \psi'=\psi/\tilde{\psi},\quad \mathbf{A}'=\mathbf{A}/2^{1/2}\tilde{H}\lambda,\quad \mathbf{H}'=\mathbf{H}/2^{1/2}\tilde{H},$$
$$\tilde{\psi}=\left(\frac{\mu-\mu_c}{\mu_c}n\right)^{1/2},\quad \lambda=\left(\frac{m}{16\pi e^2\tilde{\psi}^2}\right)^{1/2},\quad \xi=\left(\frac{\eta n}{2m\tilde{\psi}^2}\right)^{1/2}, \tag{14}$$
$$\tilde{H}=(2^{1/2}e\xi\lambda)^{-1}=2(\pi/\eta n)^{1/2}\tilde{\psi}^2.$$

The field $\mathbf{H}$ is directed along the y axis, the vector potential $\mathbf{A}$ and the current $\mathbf{J}$ are directed along the x axis, while the function ψ can be regarded as real. Then Eqs. (13) with allowance for Maxwell's equations assume in terms of the new variables the form:

$$\frac{1}{\varkappa^2}\frac{d^2\psi}{dz^2}+(1-A^2)\psi-\psi^3=0,$$
$$d^2A/dz^2+\psi^2A=0, \tag{15}$$

where $\varkappa=\lambda/\xi$. We have left out all the primes here, since we shall be using henceforth, unless specially stipulated, the dimensionless variables.

Equations (15) must be supplemented with the boundary conditions

$$z=0:\ H=dA/dz=H_0,\quad d\psi/dz=0, \tag{16}$$

where H_0 is an external field parallel to the surface and perpendicular to the conducting layers.[1)] The last condition means that we neglect the fact that the recombination rate on the semiconductor surface is larger than in the volume, and the condition $\mu>\mu_c$ may not be satisfied. This can be done if the diffusion length over which the Fermi quasilevel at the surface varies is less than the characteristic lengths ξ and λ of Eqs. (15).

Let the external field H_0 be weak. Then the approximate solutions can be obtained by putting

$$\psi=1+\varphi,\quad |\varphi|\ll 1. \tag{17}$$

In this case, accurate to terms φA and φ^2, Eqs. (15) take the form

$$\frac{1}{\varkappa^2}\frac{d^2\varphi}{dz^2}=2\varphi+A^2,\quad \frac{d^2A}{dz^2}+A=0. \tag{18}$$

The vector potential can be chosen such that at $z=0$ we have $A=0$. In this case the system (18) is immediately integrated and the solution, with (17) taken into account, takes the form

$$A=H_0\sin z,\quad H=dA/dz=H_0\cos z,$$
$$\psi=1-\frac{H_0^2}{4}\left(1-\frac{\varkappa^2}{\varkappa^2+2}\cos 2z\right). \tag{19}$$

It is seen from these expressions that the external field penetrates in the interior of the superconductor, undergoing oscillations with a period $2\pi\lambda$, and the quantity ψ also oscillates with a period $\pi\lambda$. The maximum values of ψ then correspond to the maximum values of H. The reason is that the superconductivity is destroyed not by the magnetic field but by the currents produced in the superconductor under the influence of this field. Indeed, the first equation of the system (15) contains the quantity A^2 (or j^2 if account is taken of the second equation), and ψ decreases when these quantities increase. At large values of A^2, the first equation of (15) has only a trivial solution. The average value of ψ is therefore maximal at points corresponding to the current $j=0$, the mean value being

$$\psi=1-H_0^2/4. \tag{20}$$

To find the solution in the next approximation, it is necessary to substitute the obtained solutions (19) again in (15) and take into account the terms of the next orders in H_0. Thus, accurate to terms $\sim H_0^3$ we obtain at $\varkappa\ll 1$

$$H=H_0\cos z+{}^1/_4H_0^3 z\sin z. \tag{21}$$

It is seen from this expression that at large distances from the boundary the field H_0, together with the vector potential A, becomes large, and the procedure of finding the solutions in the form (17) is no longer suitable. This indicates that the superconducting phase with the solution (19) will be concentrated near the surface, but at large distances the system will apparently be in the normal state, or possibly go over into the vortical state discussed in Sec. 5.

4. SUPERCONDUCTING FILMS

The one-dimensional equations (15) remain valid also in the case of superconducting films. To find the solutions we use the boundary conditions

$$z=\pm d:\ H=\frac{dA}{dz}=H_0,\quad \frac{d\psi}{dz}=0, \tag{22}$$

where d is half the film thickness. For sufficiently thin films and small $\varkappa$ we can assume that ψ varies little with changing z. We can therefore put

$$\psi=\psi_0+\varphi,\quad |\varphi|\ll\psi_0. \tag{23}$$

Equations (15) then assume in first-order approximation the form

$$\frac{1}{\varkappa^2}\frac{d^2\varphi}{dz^2}-(3\psi_0^2-1)\varphi=\psi_0(\psi_0^2-1)+\psi_0A^2,$$
$$d^2A/dz^2+\psi_0^2A=0. \tag{24}$$

From the second equation, taking (22) into account, we obtain

$$A=\frac{H_0\sin\psi_0z}{\psi_0\cos\psi_0d},\quad H=\frac{H_0\cos\psi_0z}{\cos\psi_0d}. \tag{25}$$

This solution is valid for films of thickness $d<\lambda$. In the opposite case, the denominator in (25) can vanish. This is apparently connected with the fact that for thicker plates the method of finding solutions in the form (23) with boundary condition (22) is not suitable. In this case terms of next order must be taken into account in the expansion of (3).

Substituting (25) into the first equation of (24), we can obtain the value of φ. Then, putting $\varphi=0$ at $z=0$, we can relate ψ_0 with the value of the external field. After performing all the calculations, we obtain

$$\varphi=-\frac{\psi_0(\psi_0^2-1)}{3\psi_0^2-1}[1-\operatorname{ch}\varkappa(3\psi_0^2-1)^{1/2}z]$$
$$-\frac{\varkappa H_0^2}{2\psi_0(3\psi_0^2-1)^{1/2}\cos^2\psi_0d}\left\{\frac{1-\operatorname{ch}\varkappa(3\psi_0^2-1)^{1/2}z}{\varkappa(3\psi_0^2-1)^{1/2}}\right.$$
$$\left.+\frac{\varkappa(3\psi_0^2-1)^{1/2}(\operatorname{ch}\varkappa(3\psi_0^2-1)^{1/2}z-\cos 2\psi_0z)}{4\psi_0^2+\varkappa^2(3\psi_0^2-1)}\right\}, \tag{26}$$

$$1-\psi_0^2=\frac{2H_0^2}{\{4\psi_0^2+\varkappa^2(3\psi_0^2-1)\}\cos^2\psi_0d}$$
$$\times\left\{1-\frac{\sin 2\psi_0d}{2\psi_0d}\frac{\varkappa(3\psi_0^2-1)^{1/2}d}{\operatorname{sh}\varkappa(3\psi_0^2-1)^{1/2}d}\right\}. \tag{27}$$

In the limiting case $\varkappa=0$, of course, φ is also equal to zero and

$$\psi_0^2(1-\psi_0^2)=\frac{H_0^2}{2\cos^2\psi_0d}\left(1-\frac{\sin 2\psi_0d}{2\psi_0d}\right). \tag{28}$$

Relations (25)–(28) coincide with the corresponding expressions for ordinary superconductors, [6] except that the hyperbolic functions are replaced by trigonometric functions.

It can be verified with the aid of (26) that $\varphi<0$. Therefore at $z=0$ the field H and the quantity ψ reach their maximum values. As already indicated, the reason is that the superconducting current differs from zero near the film surface and vanishes at its center (at $z=0$).

It is seen from (28) that in weak fields ψ_0 decreases with increasing H_0. In strong fields, when ψ_0 is small, (28) takes the form

$$H_0^2=\frac{3}{d^2}\left\{1-\left(1+\frac{4}{5}d^2\right)\psi_0^2\right\}. \tag{29}$$

We see therefore that with increasing field ψ_0 decreases monotonically and vanishes at a field value H_c equal to (in the standard variables)

$$H_c=\tilde{H}6^{1/2}\lambda/d. \tag{30}$$

Consequently, the phase transition in the magnetic field from the normal phase to the superconducting phase will always be of second order. On the other hand, in the case of the ordinary superconductors the round parentheses of (29) will contain a minus sign, as a result of which the transition predicted for films with thickness $d<5^{1/2}\lambda/2$ is of second order and that for $d>5^{1/2}\lambda/2$ is of first order. [6] The expression for the critical field in the case of a second-order transition coincides with (30), where $\tilde{H}$ is taken to be the thermodynamic critical field.

It is also possible to determine the magnetic moment of the film per unit surface, using the formula

$$M=\int_{-d}^{d}\frac{H(z)-H_0}{4\pi}dz. \tag{31}$$

Substituting here $H(z)$ from (25), we find that the magnetic moment turns out to be positive and equal to

$$M=\frac{H_0d}{2\pi}\left(\frac{\operatorname{tg}\psi_0d}{\psi_0d}-1\right). \tag{32}$$

Consequently, such films are paramagnetic.

5. VORTICAL STATE IN A COULOMB SUPERCONDUCTOR

We have considered above cases when one-dimensional equations were applicable. This is valid for relatively small values of the parameter $\varkappa$. Let us estimate the value of $\varkappa$ from the microscopic parameters of the system. Using relations (14) and (9), we obtain for $E_g\ll\mu_c$

$$\varkappa=(mE_g^{1/4}/e^4\omega_p^{3/4})^{1/2}\exp(-3/8g), \tag{33}$$

where $g=N_0(g_0+g_1)/2$ is the dimensionless coupling constant. From this expression it is seen that the parameter $\varkappa$ depends significantly on the coupling constant g. At the usual values $\omega_p=10$ eV, $E_g=0.01-0.1$ eV, and $m=(0.1-1)m_0$, where m_0 is the mass of the free electron, we have $\varkappa\sim1$ already at $g\sim0.2$. In the most interesting cases, however, when $g\sim0.5$ and more, we have $\varkappa>30$. It is then possible that the vortical state, which is not described by one-dimensional equations (15), will be produced already in sufficiently weak fields.

In the general case, Eq. (13) in dimensionless variables (14) take the form

$$\left(i\frac{\nabla}{\varkappa}+\mathbf{A}\right)^2\psi=\psi-|\psi|^2\psi,$$
$$\operatorname{rot}\operatorname{rot}\mathbf{A}=|\psi|^2\mathbf{A}+\frac{i}{2\varkappa}(\psi^*\nabla\psi-\psi\nabla\psi^*). \tag{34}$$

These equations differ from the ordinary Ginzburg–Landau equations only in the sign of the left-hand side of the second equation. It is easy to verify that Eqs. (34) go over into the ordinary equations if the depth of penetration λ is replaced by the imaginary quantity $i\lambda$. The same replacement must then be made in all the variables expressed in terms of λ (see (14)), i.e., $\varkappa$ must be replaced by $i\varkappa$, r by $-ir$, and H by iH. Bearing this in mind, we can analyze the vortical states in a

Coulomb superconductor by using the results obtained by Abrikosov for ordinary type-II superconductors.[10]

Let the external field H_0 be directed along the z axis, and let the superconductor fill all of the space. Assuming that the microscopic field H in the superconductor is parallel to the external field in fields close to H_{c2}, where H_{c2} is the upper critical field, equal to (in standard variables)

$$H_{c2}=\varkappa 2^{1/2}\tilde{H}, \tag{35}$$

we obtain

$$H=H_0+|\psi|^2/2\varkappa, \tag{36}$$

where the function ψ depends only on the coordinates x and y. The equation for ψ remains the same as in the ordinary superconductors. The solution for ψ must be chosen with allowance for the normalization[10]:

$$H_0/\varkappa=1-(1/2\varkappa^2+1)\overline{|\psi|^2}/\beta_A, \tag{37}$$

where $\beta_A=\overline{|\psi|^4}/\overline{|\psi|^2}$ is the geometric factor of the lattice and does not depend on H_0.

It is seen from (36) that the microscopic field H changes (in the xy plane) from a minimum value equal to H_0 in the core of the vortex filament to its maximum value reached in the intervals between the filaments, and equal to

$$H_{max}=H_0+(H_{c2}-H_0)2^{1/2}/(2\varkappa^2+1). \tag{38}$$

The magnetic moment is then positive and equal to

$$M=\frac{1}{4\pi}\frac{H_{c2}-H_0}{(2\varkappa^2+1)\beta_A}. \tag{39}$$

In (38) and (39) we used the ordinary variables. The value of the geometric factor β_A is determined by the form of the lattice made up of the system of vortex filaments. This problem must be solved under the condition that the corresponding thermodynamic potential be a minimum, a task beyond the scope of the present article.

We note in conclusion that, as can be easily verified from (34), there is no field component normal to the surface in a Coulomb superconductor. Therefore when an external field perpendicular to the surface (and perpendicular to the conducting layers) is applied to such a superconductor with a small value of the parameter $\varkappa$, an intermediate state should be observed, similar to the intermediate state in the case of London superconductors.

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