Metastability exchange in a mixture of He^3 and He^4 isotopes

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Analytic expressions are obtained for the magnetic resonance line widths of 2^3S_1 metastable helium atoms in a He^3-He^4 mixture by taking into account metastability exchange. The cross sections for metastability exchange, $\sigma_1 = \sigma(\text{He-He}^3)$ and $\sigma_2 = \sigma(\text{He}^3\text{-He}^4)$ are found in experiments on optical orientation and magnetic resonance of 2^3S_1 metastable helium atoms in a He^3-He^4 isotope mixture for various temperatures in the range 77–300°K. The temperature dependences of these cross sections and their relative differences $\Delta\sigma/\sigma_1$ are determined. An explanation is given for the behavior of the cross sections σ_1 and σ_2 and of their temperature dependence.

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Processes of exchange of metastability for the He^3 isotope have been considered in a number of works.[1–3] The cross section σ for exchange of metastability in the collisions of He^3 atoms in $1\,^1S_0$- and $2\,^3S_1$-states was determined from the experimental data. The determination of σ is of great interest, since it makes it possible to obtain information on the interaction potential $V(r)$ of two helium atoms, one of which is in the ground state $1\,^1S_0$ and the other in the metastable state $2\,^3S_1$.

Optical orientation and magnetic resonance of $2\,^3S_1$ metastable atoms of helium in mixtures of He^3 and He^4 isotopes were investigated in Ref. 4. The measurements were carried out at room temperature. It followed from the results that the cross sections were the same within the limits of 10% experimental error: $\sigma(\text{He}^3\text{–He}^3) = \sigma(\text{He}^3\text{–He}^4)$. The hypothesis was advanced that these cross sections should be equal because the nuclear spins do not change their orientation in collisions of two helium atoms and have no effect on the result of the collision, since $\tau\Delta W \ll h/2\pi$, where τ is the collision time, ΔW the splitting of the hyperfine structure and h is Planck's constant.

The statement that these cross sections must be equal was not self-evident,[5] because of the difference in energy of excitation of the metastable $2\,^3S_1$ state in the isotopes He^3 and He^4. This difference can lead to the result that, under certain experimental conditions, the cross sections for the exchange of metastability between atoms of one and the same type or between atoms of different helium isotopes turn out to be quite different. Earlier[5] the cross sections $\sigma_1 = \sigma(\text{He}^3\text{–He}^3)$ and $\sigma_2 = \sigma(\text{He}^3\text{–He}^4)$ were measured at the temperature of liquid nitrogen by us. These were determined from the magnetic resonance line widths for the $2\,^3S_1$ state with optical orientation of the atoms in mixtures of He^3–He^4 isotopes. It was established that these cross sections were essentially different at liquid-nitrogen temperature ($\sigma_2/\sigma_1 \approx 0.6$).

As a consequence of the presence of a potential barrier in the formation of the molecule He$(1\,^1S_0)\cdot$He$(2\,^3S_1)$, which is intermediate in the case of metastability exchange,[6] the cross section of this process, σ, measured for the He^3 isotope, turned out to depend significantly on the temperature of the gas.[1–2] Therefore, there is great interest in the determination of the values of the cross sections of metastability exchange between atoms of different helium isotopes for different temperatures.

The present research is devoted to the study of the process of metastability exchange between helium atoms in a mixture of He^3 and He^4 isotopes at different temperatures (77–300 °K), to a study of the dependence of the relation between the cross sections σ_1 and σ_2 and the temperature, and also to a clarification of the problem of the nature of this dependence.

1. THEORETICAL ANALYSIS OF THE MAGNETIC RESONANCE OF HELIUM ATOMS IN THE METASTABLE $2\,^3S_1$ STATE IN THE CASE OF THEIR OPTICAL ORIENTATION IN A MIXTURE OF He^3 AND He^4 ISOTOPES.

Figure 1 shows the diagram of the energy levels of He^3 and He^4. For He^3, the ground state $1\,^1S_0$ has only the nuclear angular momentum ($I = \frac{1}{2}$) while the metastable state $2\,^3S_1$ contains two sublevels with hyperfine structure $F = \frac{3}{2}$ and $F = \frac{1}{2}$, separated 6600 MHz (0.22 cm^{-1}) from each other. The excitation energies of the metastable $2\,^3S_1$ states of He^3 and He^4 differ by approximately $\Delta = \Delta_1 = \Delta_2 \approx 6$ cm^{-1} (the estimate is made from the data of Ref. 7). In the working cell, which contains the He^3 and He^4 atoms, a weak gas discharge is excited and a certain number of atoms transform from the ground

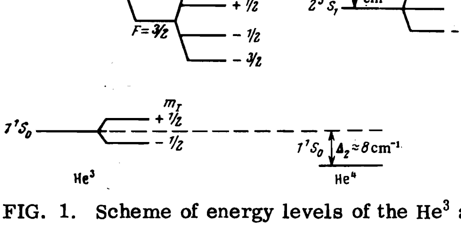

FIG. 1. Scheme of energy levels of the He^3 and He^4 atoms.

state to the metastable state (usually, the density of atoms in the ground state is $N \approx 10^{16}$ cm^{-3} and in the metastable state, $n \approx 10^{10}$ cm^{-3}). The helium atoms in the state $2\,^3S_1$ are optically oriented by the resonance line $\lambda = 10,830$ Å (the transformation $2\,^3S_1 \rightarrow 2\,^3P$). For this purpose, the cell is irradiated by circularly polarized light from a helium lamp. Exchange collisions between the atoms of metastable $2\,^3S_1$ and ground $1\,^1S_0$ states lead to considerable nuclear orientation of the ground level, too.

The processes of exchange of metastability in optically oriented helium atoms, which are of interest to us, take place in collisions of the following three types:

$$\begin{aligned}&1)\ \mathrm{He}^{3*}+\mathrm{He}^{3}\rightarrow\mathrm{He}^{3}+\mathrm{He}^{3*},\\&2)\ \mathrm{He}^{3*}+\mathrm{He}^{4}\rightarrow\mathrm{He}^{3}+\mathrm{He}^{4*},\\&3)\ \mathrm{He}^{4*}+\mathrm{He}^{3}\rightarrow\mathrm{He}^{4}+\mathrm{He}^{3*}\end{aligned}\tag{1}$$

(the asterisks denote atoms in the metastable $2\,^3S_1$ state).

Let the density of He^3 and He^4 atoms in the cell be equal to N. Here the density of He^3 atoms is αN and the density of He^4 atoms is $(1-\alpha)N$. The fraction of metastable atoms out of their total number is β and this quantity is determined by the intensity of the discharge. For an atom in the $2\,^3S_1$ state, the probabilities of participation in one of the processes (1) given above, per unit time, are equal to the following:

$$1/\tau_1=\alpha N v_1\sigma_1,\quad 1/\tau_2=(1-\alpha)Nv_2\sigma_2,\quad 1/\tau_3=\alpha N v_2\sigma_2.\tag{2}$$

Here τ_1, τ_2, τ_3, are the lifetimes of the He^3 and He^4 atoms in the $2\,^3S_1$ state relative to processes of metastability exchange, v_1 and v_2 are the mean thermal relative velocities for the atoms He^3–He^3 and He^3–He^4, respectively, σ_1 and σ_2 are the cross sections of metastability exchange for atoms of the same and different helium isotopes.

For He^3 and He^4 atoms in the ground state $1\,^1S_0$, the probabilities of participation in processes of metastability exchange (1) are equal to

$$1/T_1=\alpha\beta N v_1\sigma_1,\quad 1/T_2=(1-\alpha)\beta N v_2\sigma_2,\quad 1/T_3=\alpha\beta N v_2\sigma_2,\tag{3}$$

where T_1 and T_3 are the lifetimes of the He^3 atoms for the first and third of the processes (1), T_2 is the lifetime of He^4 atoms.

We introduce the density matrix of the system, ρ, consisting of three parts: $\rho_{m1}(6\times 6)$—for the metastable $2\,^3S_1$ state of He^3; $\rho_{m2}(3\times 3)$—for the metastable $2\,^3S_1$ state of He^4, and $\rho_f(2\times 2)$—for the ground $1\,^1S_0$ state of He^3. We consider the evolution of these matrices in collisions of the three types of Eq. (1).

1) $\mathrm{He}^{3*}+\mathrm{He}^3\rightarrow\mathrm{He}^3+\mathrm{He}^{3*}$. In this case, the change in the density matrix can be represented as

$$\rho_{m1}+\rho_f\rightarrow\rho_f'+\rho_{m1}'.$$

From the definition of metastability exchange, ρ_f' and ρ_{m1}' are of the form[8]

$$\rho_f'=\mathrm{Tr}_e\rho_{m1},\qquad \rho_{m1}'=\rho_f\otimes\mathrm{Tr}_n\rho_{m1},\tag{4}$$

where Tr_e and Tr_n are the partial traces over the electron and nuclear variables, and $\otimes$ denotes the direct product of the matrices.

2) $\mathrm{He}^{3*}+\mathrm{He}^4\rightarrow\mathrm{He}^3+\mathrm{He}^{4*}$. In this process we have $\rho_{m1}\rightarrow\rho_f''+\rho_{m2}''$, where

$$\rho_f''=\mathrm{Tr}_e\rho_{m1},\qquad \rho_{m2}''=\mathrm{Tr}_n\rho_{m1}.\tag{5}$$

3) $\mathrm{He}^{4*}+\mathrm{He}^3\rightarrow\mathrm{He}^4+\mathrm{He}^{3*}$. Now $\rho_f+\rho_{m2}\rightarrow\rho_{m1}'''$, where

$$\rho_{m1}'''=\rho_f\otimes\rho_{m2}.\tag{6}$$

Then the evolution of the density matrix of the assembly of atoms will be determined by the following expressions:

$$\begin{gathered}d\rho_{m1}=-\frac{dt}{\tau_1}\rho_{m1}+\frac{dt}{\tau_1}\rho_f\otimes\mathrm{Tr}_n\rho_{m1}-\frac{dt}{\tau_2}\rho_{m1}+\frac{dt}{\tau_3}\rho_f\otimes\rho_{m2},\\ d\rho_{m2}=-\frac{dt}{\tau_3}\rho_{m2}+\frac{dt}{\tau_2}\mathrm{Tr}_n\rho_{m1},\\ d\rho_f=-\left(\frac{1}{T_1}+\frac{1}{T_3}\right)\rho_f+\left(\frac{1}{T_1}+\frac{1}{T_2}\right)\mathrm{Tr}_e\rho_{m1}.\end{gathered}\tag{7}$$

In what follows, we shall use the method proposed in Ref. 3 of transformation from the equations for the density matrix to the equations for the quantities observed experimentally. We shall assume that there is no coherence of the hyperfine structure in the assembly of atoms of He^3, inasmuch as the case of a weak magnetic field is considered.

The orientation of the sublevels of the hyperfine structure of He^3—$F=\frac{3}{2}$ and $F=\frac{1}{2}$—are determined in the following way:

$$\langle\mathbf{F}\rangle_{3/2}=\mathrm{Tr}(P_{3/2}\rho_{m1}P_{3/2}\mathbf{F}),\qquad \langle\mathbf{F}\rangle_{1/2}=\mathrm{Tr}(P_{1/2}\rho_{m1}P_{1/2}\mathbf{F}),\tag{8}$$

where $P_{3/2}$ and $P_{1/2}$ are the projection operators, which separate from the matrix ρ_{m1} those parts which are connected either with $F=\frac{3}{2}$ or with $F=\frac{1}{2}$.

The electron orientation of the $2\,^3S_1$ state of He^4 is

$$\langle\mathbf{S}\rangle_{m2}=\mathrm{Tr}(\mathbf{S}_{m2}\rho_{m2}).\tag{9}$$

The nuclear orientation of the $1\,^1S_0$ state of He^3 is

$$\langle\mathbf{I}\rangle_f=\mathrm{Tr}(\mathbf{I}\rho_f).\tag{10}$$

Starting out from Eqs. (7), we obtain the following equations of evolution of the mean values of the observed quantities (8)–(10) because of the processes of metastability exchange (1):

$$\begin{gathered}\frac{d}{dt}\langle\mathbf{F}\rangle_{3/2}=-\left(\frac{4}{9\tau_1}+\frac{1}{\tau_2}\right)\langle\mathbf{F}\rangle_{3/2}+\frac{10}{9\tau_1}\langle\mathbf{F}\rangle_{1/2}+\frac{5}{6\tau_3}\langle\mathbf{S}\rangle_{m2}+\frac{10}{9}\left(\frac{1}{\tau_1}+\frac{1}{\tau_3}\right)\langle\mathbf{I}\rangle_f,\\ \frac{d}{dt}\langle\mathbf{F}\rangle_{1/2}=\frac{1}{9\tau_1}\langle\mathbf{F}\rangle_{3/2}-\left(\frac{7}{9\tau_1}+\frac{1}{\tau_2}\right)\langle\mathbf{F}\rangle_{1/2}+\frac{1}{6\tau_3}\langle\mathbf{S}\rangle_{m2}-\frac{1}{9}\left(\frac{1}{\tau_1}+\frac{1}{\tau_3}\right)\langle\mathbf{I}\rangle_f,\\ \frac{d}{dt}\langle\mathbf{S}\rangle_{m2}=\frac{2}{3\tau_2}\langle\mathbf{F}\rangle_{3/2}+\frac{4}{3\tau_2}\langle\mathbf{F}\rangle_{1/2}-\frac{1}{\tau_3}\langle\mathbf{S}\rangle_{m2},\\ \frac{d}{dt}\langle\mathbf{I}\rangle_f=\frac{1}{3}\left(\frac{1}{T_1}+\frac{1}{T_2}\right)\langle\mathbf{F}\rangle_{3/2}-\frac{1}{3}\left(\frac{1}{T_1}+\frac{1}{T_2}\right)\langle\mathbf{F}\rangle_{1/2}-\left(\frac{1}{T_1}+\frac{1}{T_3}\right)\langle\mathbf{I}\rangle_f.\end{gathered}\tag{11}$$

The coefficients $\frac{4}{9}$ and $\frac{7}{9}$ in Eqs. (11) are connected with the partial conservation of the orientation in the states with $F=\frac{3}{2}$ and $F=\frac{1}{2}$, respectively, in collisions of He^3 atoms with one another.[3] The coefficient -1 in these same equations is due to the fact that, total loss of orientation in the metastable state of He^3 takes place in the collision of an He^3 atom with an He^4 atom.

We take into account the effect of the constant magnetic field H_0, the pump light, the alternating radiofrequency field $H_1 \cos\omega t$, and also the effect of the discharge and of the diffusion of atoms, similar to what was done in Ref. 3. Then the evolution of the longitudinal orientations is described by a set of linear differential equations of the form

$$\frac{d}{dt}\begin{pmatrix}\langle F_z\rangle_{3/2}\\ \langle F_z\rangle_{1/2}\\ \langle S_z\rangle_{m2}\\ \langle I_z\rangle_f\end{pmatrix}=[A]\begin{pmatrix}\langle F_z\rangle_{3/2}\\ \langle F_z\rangle_{1/2}\\ \langle S_z\rangle_{m2}\\ \langle I_z\rangle_f\end{pmatrix}+\begin{bmatrix}\Phi_1/\tau_{p1}\\ \Phi_2/\tau_{p2}\\ \Phi_3/\tau_{p3}\\ 0\end{bmatrix}, \tag{12}$$

with

$$= \overset{[A]}{\begin{bmatrix} -\left(\frac{4}{9\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_{p1}}+\frac{1}{\tau_{r1}}\right) & \frac{10}{9\tau_1} & \frac{5}{6\tau_3} & \frac{10}{9}\left(\frac{1}{\tau_1}+\frac{1}{\tau_3}\right) \\ \frac{1}{9\tau_1} & -\left(\frac{7}{9\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_{p2}}+\frac{1}{\tau_{r1}}\right) & \frac{1}{6\tau_3} & -\frac{1}{9}\left(\frac{1}{\tau_1}+\frac{1}{\tau_3}\right) \\ \frac{2}{3\tau_2} & \frac{4}{3\tau_2} & -\left(\frac{1}{\tau_3}+\frac{1}{\tau_{p3}}+\frac{1}{\tau_{r3}}\right) & 0 \\ \frac{1}{3}\left(\frac{1}{T_1}+\frac{1}{T_2}\right) & -\frac{1}{3}\left(\frac{1}{T_1}+\frac{1}{T_2}\right) & 0 & -\left(\frac{1}{T_1}+\frac{1}{T_3}+\frac{1}{T_r}\right) \end{bmatrix}},$$

where Φ_1, Φ_2, Φ_3 are constant quantities, which are determined by the pump conditions; τ_{p1}, τ_{p2}, τ_{p3} are the times of pumping by circularly polarized light of the resonant wavelength; τ_{r1} and τ_{r3} are the relaxation times determined mainly by the diffusion of metastable atoms of He^3 and He^4 to the walls of the absorption cell; T_r is the relaxation time of nuclear spins.

There are no terms in Eqs. (11), (12) which are connected with alignment in the $2\,^3S_1$ state, since they are small in our case.

The effect of the radiofrequency magnetic field can be taken into account in a manner similar to what was done in Ref. 9, namely, by the introduction into the matrix $[A]$ of diagonal terms of the form

$$\begin{aligned} F_1(\omega_1)&=-\left(\frac{2\omega_1}{3}\right)^2\frac{4/9\tau_1+1/\tau_2+1/\tau_{p1}+1/\tau_{r1}}{(4/9\tau_1+1/\tau_2+1/\tau_{p1}+1/\tau_{r1})^2+(\omega-2\omega_m/3)^2},\\ F_2(\omega_1)&=-\left(\frac{4\omega_1}{3}\right)^2\frac{7/9\tau_1+1/\tau_2+1/\tau_{p2}+1/\tau_{r1}}{(7/9\tau_1+1/\tau_2+1/\tau_{p2}+1/\tau_{r1})^2+(\omega-4\omega_m/3)^2},\\ F_3(\omega_1)&=-\omega_1^2\frac{1/\tau_3+1/\tau_{p3}+1/\tau_{r3}}{(1/\tau_3+1/\tau_{p3}+1/\tau_{r3})^2+(\omega-\omega_m)^2} \end{aligned} \tag{13}$$

where $\omega_1=\gamma H_1$, $\omega_m=\gamma H_0$; γ is the gyromagnetic ratio of He^4 in the $2\,^3S_1$ state. It has been assumed here that at resonance with one of the metastable sublevels the effect of the radiofrequency field on the helium atom in the ground state is negligibly small.

Magnetic resonance at the sublevel $F=\frac{3}{2}$ of He^3

In this case, $\omega\approx\frac{2}{3}\omega_m$. Then $F_3(\omega_1)\approx F_2(\omega_1)\approx 0$. Since the conditions τ/τ_r and $\tau/\tau_p \ll 1$ are satisfied in the experiment, only terms of the form Φ/τ_p are left. From the solution of the set of equations (12) for the stationary case, we can determine the halfwidth of the resonance line:

$$\Delta\omega_{3/2}=\left[\frac{(4/9+a)^2}{\tau_1^2}+\frac{10}{3}\left(\frac{4}{9}+a\right)\frac{1+b}{1+a}\frac{T_r}{T_1}\left(\frac{2}{3}\omega_1\right)^2\right]^{1/2}, \tag{14}$$

where

$$a=\tau_1/\tau_2,\qquad b=\tau_1/\tau_3.$$

The first two terms determine the degree of destruction of orientation of the sublevel with $F=\frac{3}{2}$ in exchange collisions of the first and second types of (1). The second term characterizes the line broadening as a result of the effect of the transverse radiofrequency field. We can now determine the longitudinal and transverse relaxation times:

$$T_1^*=\frac{10}{3}\frac{1+b}{1+a}\tau_1\frac{T_r}{T_1},\quad T_2^*=\frac{\tau_1}{4/9+a}. \tag{15}$$

Magnetic resonance at the sublevel $F=\frac{1}{2}$ of He^3

Here $\omega\approx\frac{4}{3}\omega_m$ and the halfwidth of the magnetic resonance line is determined by the expression

$$\Delta\omega_{1/2}=\left[\frac{(7/9+a)^2}{\tau_1^2}+\frac{1}{3}\left(\frac{7}{9}+a\right)\frac{1+b}{1+a}\frac{T_r}{T_1}\left(\frac{4}{3}\omega_1\right)^2\right]^{1/2}. \tag{16}$$

The longitudinal and transverse relaxation times here are

$$T_1^*=\frac{1}{3}\frac{1+b}{1+a}\frac{T_r}{T_1}\tau_1,\qquad T_2^*=\frac{\tau_1}{7/9+a}. \tag{17}$$

Magnetic resonance of He^4 in the $2\,^3S_1$ state

The resonance conditions are satisfied at $\omega\approx\omega_m$. The halfwidth of the resonance line is equal to

$$\Delta\omega_{He^4}=\left[\frac{1}{\tau_3^2}+\frac{8}{3}a\frac{1+b}{1+a}\frac{T_r}{T_1}\omega_1^2\right]^{1/2}. \tag{18}$$

The first term determines the destruction of orientation of the metastable state as a result of metastability exchange with atoms of He^3 in the ground $1\,^1S_0$ state. The relaxation times are

$$T_1^*=\frac{8}{3}a\frac{1+b}{1+a}\frac{T_r}{T_1}\tau_3,\qquad T_2^*=\tau_3. \tag{19}$$

Equations (14), (16), and (18) were used by us for the determination of the cross sections of metastability exchange from experimental data on the magnetic resonance in the $2\,^3S_1$ states of He^3 and He^4. These equations were applied jointly with the expressions (2), (3), which, upon extrapolation to zero amplitude of the radiofrequency field, leads to the result:

$$\begin{aligned} \Delta\omega'_{3/2}&=4/9\,\alpha N v_1\sigma_1+(1-\alpha)Nv_2\sigma_2,\\ \Delta\omega'_{1/2}&=7/9\,\alpha N v_1\sigma_1+(1-\alpha)Nv_2\sigma_2,\\ \Delta\omega'_{He^4}&=\alpha N v_2\sigma_2. \end{aligned} \tag{20}$$

The cross sections σ_1 and σ_2 were also found from the experimental line widths with the help of these expressions.

2. EXPERIMENTAL METHOD

The experimental investigations were carried out on apparatus whose block diagram is shown in Fig. 2. The

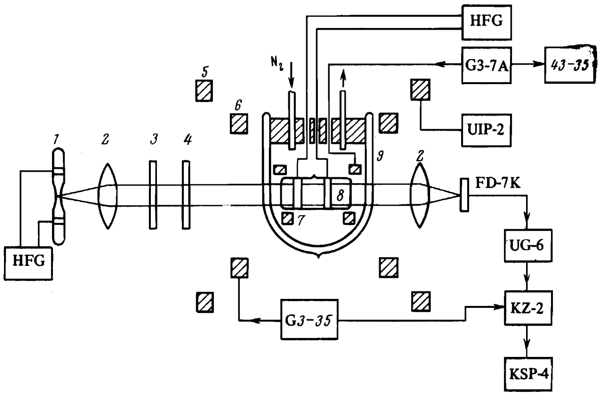

FIG. 2. Block diagram of the experimental arrangement for the study of optical orientation of atoms of helium in the temperature range 77–300 °K. 1—capillary helium tube, 2—lenses, 3—PPI-1 polaroid, 4—quarterwave mica plate, 5—set of Helmholtz coils, 6—modulation field coils, 7—radiofrequency field coils, 8—absorption cell, 9—Dewar vessel.

radiation of the electrodeless helium tube 1 was gathered by condenser 2 and then was transformed into a circularly polarized beam by means of the polaroid 3 and quarter wave plate 4. Weak radiofrequency discharge was excited by the generator HFG in the absorption chamber 8. This discharge transformed some of the helium atoms from the ground $1\,^1S_0$ state to the metastable $2\,^3S_1$ state. The circularly polarized light of the resonance wavelength ($\lambda = 10,830$ Å) brought about orientation of the helium atoms in the metastable state. Destruction of the optical orientation of the He^3 and He^4 in this case was accomplished by the resonant radiofrequency magnetic field, created by the generators G3-7A at a frequency ~ 3 MHz, which was measured by the frequency meter Ch3-35. The change in the pump light intensity at the instant of resonance was recorded by the silicon photodetector FS-7K. The signal from the output of the photodetector was fed to selective amplifier U2-6, operating in a band of 80 Hz, and a synchronous detector K3-2. The reference signal at the K3-2 was provided from the field-modulation generator G3-35. The derivative of the absorption signal was recorded on the tape of an automatic recorder KSP-4. The bandwidth of the entire recording system amounted to 1 Hz. Modulation of the constant magnetic field was accomplished at a frequency of 400 Hz.

We used absorption cells filled with a mixture of the isotopes He^3 and He^4 of various concentrations, but with a constant total pressure 0.4 Torr (at 300 °K). The partial pressures of He^3 were 0.05, 0.1, 0.2, 0.3 and 0.35 Torr. The cells were filled with the helium isotopes by a quartz leak valve, and were monitored by a thermocouple manometric transducer PMT-2 calibrated against helium. The error in the determination of the final pressure when the absorption cells were sealed off from the vacuum system amounted to ~ 8% of the total pressure.

The cell was placed in a weak constant magnetic field H_0, generated by three mutually perpendicular pairs of Helmholtz coils, connected to the rectifier UIP-2. The calculated relative inhomogeneity of the field of such a system amounted to 6×10^{-4}. The frequency of the excited gas discharge in the absorption cell was 30 MHz. The capillary tube of the pump 1 was filled with He^4 at a pressure of 6 Torr and excited by the separate HFG generator. The use of two HFG generators made it possible to vary the intensity of the discharge in the cell over wide limits, without varying the intensity of the light of the tube. A polaroid PPI-1 and quarterwave plate $\lambda/4$ made of mica were used to obtain circular polarization of the pump light.

The derivatives of the absorption signals were recorded at different temperatures in the range from 77 to 300 °K. For this purpose, the absorption cell was placed in a Dewar vessel through which nitrogen gas was passed (Fig. 2). The temperature was regulated by changing the flow of the nitrogen passing through the vessel, and was measured by a copper-constantan thermocouple. The stability of the temperature point during the measurement time was not worse than 3 °K.

The widths of the magnetic resonance lines were determined from the distance between the maximum and minimum of the derivative of the resonance curve of the absorption, with account of the Lorentzian line shape. The recording of the signals was carried out at various values of the amplitude of the radiofrequency field and the pump light intensity, which made it possible to determine the values of the magnetic-resonance line widths by extrapolating of these parameters to zero. To estimate the effect exerted on the magnetic resonance line width by such factors as the nonuniformity of the constant magnetic field, the gas discharge, and the diffusion of the metastable atoms to the walls of the absorption cell, we recorded the signals of the magnetic resonance of He^4 atoms in the $2\,^3S_1$ state for an absorption cell containing only the He^4 isotope. The error in the determination of the line width with account of all possible sources of error amounted in the mean to ± 5%.

3. EXPERIMENTAL RESULTS

To find the cross sections $\sigma_1 = \sigma(He^3\text{–}He^3)$ and $\sigma_2 = \sigma(He^3\text{–}He^4)$, we determined the dependence of the magnetic resonance line width of optically oriented atoms of He^3 and He^4 in the metastable $2\,^3S_1$ state as a function of the percentage of He^3 in the mixture of the two helium isotopes at constant total gas pressure. Equations (20) were used for the calculation of σ_1 and σ_2. They were derived under the assumption that the alignment in the $2\,^3S_1$ states of He^3 and He^4 is small. The use of these formulas is perfectly valid under the conditions of our experiment, since the magnetic resonance signals in the case of optical orientation by unpolarized light were at least an order of magnitude weaker than the signals obtained for orientation by circularly polarized light.

Figure 3 shows the dependences of the magnetic resonance line widths on the concentration of He^3 for $T = 125$ °K (Fig. 3a) and for $T = 300$ °K (Fig. 3b). Corrections have already been introduced in the data of Fig. 3 for radiofrequency broadening, modulation effects, nonuniformity of the constant magnetic field H_0, the effect of discharge and the diffusion of the metastable atoms to the walls of the absorption cell.

TABLE I.

Temperature T, °K	Cross sections		σ_2/σ_1	Temperature T, °K	Cross sections		σ_2/σ_1
	σ_1, 10^{-16} cm²	σ_2, 10^{-16} cm²			σ_1, 10^{-16} cm²	σ_2, 10^{-16} cm²	
77	1.41±0.11	0.78±0,06	0.55±0.09	210	4.70±0.39	3.48±0.27	0.74±0.11
125	3.89±0.33	2,29±0.21	0.59±0.10	245	6.07±0,48	4,60±0.35	0,76±0.11
168	4.40±0.35	3.04±0.23	0.69±0.10	300	7.95±0.50	7.62±0,42	0,96±0.11

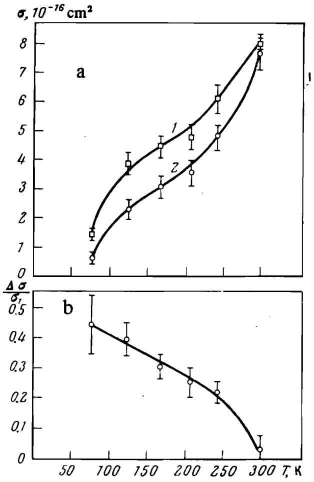

FIG. 4. Temperature dependence of the cross sections of metastability exchange: a) cross sections: 1) $\sigma_1 = \sigma(\mathrm{He}^3 - \mathrm{He}^3)$, 2) $\sigma_2 = \sigma(\mathrm{He}^3 - \mathrm{He}^3)$; b) relative difference of the cross sections $\Delta\sigma/\sigma_1 = (\sigma_1 - \sigma_2)/\sigma_1$.

As is seen in Fig. 3, the magnetic-resonance line widths in the two states of the hyperfine structure $F = \frac{3}{2}$ and $F = \frac{1}{2}$ and in the $^3S_1$ state of He^4 depend essentially on the He^3 concentration in the mixture of helium isotopes. The magnetic resonance line width of He^4 in the $2\,^3S_1$ state increases sharply with increase in the He^3 concentration in the mixture, both for $T = 125$ °K and for $T = 300$ °K. The magnetic resonance line width of He^3 in the $2\,^3S_1$ state for $F = \frac{1}{2}$ increases with increase in He^3 concentration at $T = 125$ °K and decreases at $T = 300$ °K. The line width of the He^3 for $F = \frac{3}{2}$ changes little at $T = 125$ °K and decreases materially with increase in the He^3 concentration at $T = 300$ °K. Such a change in the slope of the straight lines for He^3 in Fig. 3 with increase in temperature indicates a change in the relative values of the cross sections σ_1 and σ_2 in this temperature range. In the case of a constant ratio σ_2/σ_1, the transition from Fig. 3a to Fig. 3b could be brought about only by a change in the scale along the Δf axis.

The values of σ_1 and σ_2 were determined for six values of the temperature in the range 77–300 °K and are given in Table I. The error in the determination of σ_1 and σ_2 amounted to 8% in the mean. This error represents the maximum scatter of values of the calculated quantity with account of the sources of error pointed out above.

Figure 4a shows the dependences of σ_1 and σ_2 on the temperature. As is seen from Fig. 4a, in the case of an increase in the temperature from $T = 77$ °K to $T = 300$ °K, the absolute values of σ_1 and σ_2 increase monotonically. Two regions of more rapid growth of these quantities can be distinguished: 77–120 and 220–300 °K. At 300 °K, the cross sections σ_1 and σ_2 can be regarded as equal within the limits of error of the experiment.

Figure 4a shows the dependence of the relative difference $\Delta\sigma/\sigma_1 = (\sigma_1 - \sigma_2)/\sigma_1$ on the temperature. As follows from this figure, the quantity $\Delta\sigma/\sigma_1$ changes much more slowly in the temperature range from 77 to 300 °K than in the range from 200 to 300 °K.

4. DISCUSSION OF RESULTS

As is shown in the previous section, the cross sections of metastability exchange σ_1 and σ_2 differ significantly throughout the entire range of temperatures studied, with the exception of a small region near $T = 300$ °K. This experimental fact can be explained by considering the interaction of two helium atoms, one of which is in the ground $1\,^1S_0$ and the other in the metastable $2\,^3S_1$ state.

This interaction is described by the potentials V_g and V_u, which correspond to the symmetric and antisymmetric states of the helium molecule He_2^* $[\mathrm{He}(1\,^1S_0) \cdot \mathrm{He}(2\,^3S_1)]$.[6,10] Figure 5 shows the potentials V_g and V_u obtained in Ref. 10. It is seen from this figure that the potentials V_g and V_u begin to diverge significantly at $r < 10a_0$, where a_0 is the radius of the first Bohr orbit. The cross section for metastability exchange is determined by the difference of these potentials, averaged over the time of effective interaction of the helium atoms, and multiplied by the time of interaction, i.e.,

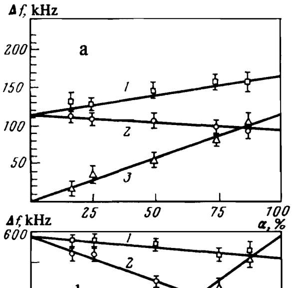

FIG. 3. Dependence of the magnetic resonance line widths of the optically oriented helium atoms in mixtures of He^3 – He^4 isotopes on the percentage content α of He^3 at a total pressure of 0.4 Torr; 1—resonance lines widths for $F = \frac{1}{2}$, $2\,^3S_1$ state of He^3; 2—resonance line widths of the $2\,^3S_1$ state of He^3, $F = \frac{3}{2}$: 3—resonance line widths of the $2\,^3S_1$ state of He^4: a) $T = 125$ °K, b) $T = 300$ °K.

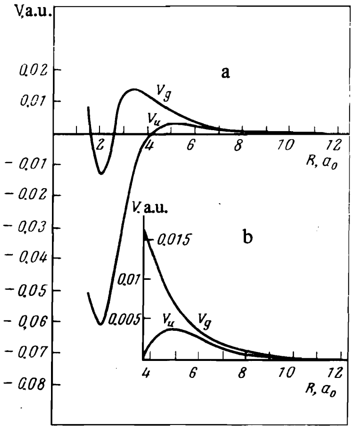

FIG. 5. Potential energies V_g and V_u of the symmetric and antisymmetric states of He $(1\,^1S_0) \cdot \mathrm{He}(2\,^3S_1)$ (from the data of Ref. 10); a) with the use of 12 basis functions; b) with the use of 20 basis functions.

by the time of stay of the two atoms in the region where the potentials V_g and V_u differ significantly. In the transition from low to higher temperatures the region of effective interaction and, consequently, the value of the mean value of the difference in the potentials $\Delta V = V_g - V_u$, increase appreciably (Fig. 5b). This leads to a sharp increase in the interaction cross sections σ_1 and σ_2 in the temperature range 77–120 °K. However, when two colliding atoms penetrate into the region of ever-increasing difference in the potentials (at $r < 8a_0$), the time of effective interaction begins to decrease considerably, which leads to a decrease in the rate of growth of the cross section of metastability exchange in the temperature range from 120 to 220 °K. Upon further increase in the temperature ($T > 220$ °K), the difference in the potentials ΔV increases so significantly, because of the slowing of the growth in the potential V_u, that a more rapid increase is observed in the cross sections σ_1 and σ_2. Since the potential barrier V_u amounts to ~ 0.1 eV,[10] it follows that at temperatures $T \leqslant 300$ °K only a small fraction of the atoms have sufficient kinetic energy to penetrate the barrier and form stable He_2^* molecules. Therefore, in this range of temperatures, we are still far from the temperature saturation, the possibility of which was pointed out in Ref. 1.

From among the researches on the determination of the transverse cross section σ_1 known at present, we can mention the reliable measurements at the temperatures 15–115 °K[2] and 300 °K.[4] In the range 77–115 °K, the values of σ_1 from Ref. 2 are somewhat smaller than those which are reported in the present work, but, because of the large errors in the measurements in this temperature range, we can assume that the agreement of the results is entirely satisfactory. Thus, at 113.8 °K, the value $\sigma_1 = (2.37 \pm 0.61) \cdot 10^{-16}$ cm^2 from Ref. 2 falls practically on the experimental curve 1 of Fig. 4a of the present paper. At 300 °K, the value $\sigma_1 = (7.6 \pm 0.4) \cdot 10^{-16}$ cm^2 given in Ref. 4 is very close to the value of σ_1 given above in the table.

In the case of resonance transfer of the energy of the excitation in the metastability exchange between helium atoms, the value of the cross section $\sigma_2 = \sigma(He^3 - He^4)$ should exceed somewhat the cross section $\sigma_1 = \sigma(He^3 - He^3)$, since the system with greater reduced mass ($He^3 - He^4$) stays a longer time in the region of effective interaction. However, the difference in the excitation energies of the metastable $2\,^3S_1$ states of the He^3 and He^4 isotopes, which is connected mainly with the difference in the masses of the He^3 and He^4 atoms (see Fig. 1), leads to a nonresonant character of the metastability exchange between atoms of different isotopes of helium. As a consequence, the cross section σ_2 becomes smaller than the cross section σ_1.

The relative difference of the cross sections σ_1 and σ_2 is determined to a significant extent by the time of stay of the system of two atoms He^3 and He^4 in the region of effective interaction. Since this time decreases by a factor of about two with increase in the temperature from 77 to 300 °K (the estimate is made on the basis of the results of Ref. 10), then, at temperatures close to room temperature, the values of σ_1 and σ_2 approach each other. This agrees with the behavior of the experimental curves shown in Fig. 4, and also with the result of Ref. 4, in which agreement of the values of σ_1 and σ_2 within the limits of 10% error in the experiment was established at 300 °K.

There are a considerable number of researches devoted to the calculation or determination from the results of various experiments (elastic scattering, diffusion, optical orientation of the atoms) of the interaction potentials V_g and V_u, for example, Refs. 6, 10–14. In the region of long-range interaction (at $r > 7a_0$), the potentials obtained in the different experiments vary greatly. Starting out from the temperature dependences for σ_1 and σ_2, we can select the most satisfactory of these potentials. The important question here is how single-valued such a choice can be made. We can say the following regarding this question.

As was already pointed out above, the quantity σ_1 is determined by the integral $\int \Delta V dt$, taken over the region of effective interaction of the atoms. In first approximation, $\Delta V \sim \exp(-\alpha r)$, and the region of interaction is determined by the character of the behavior of the potential curves V_g and V_u at $r > r_{\min}$, where $r_{\min}$ is the distance of closest approach of the two helium atoms; this distance is determined by the kinetic energy and the impact parameter. We can therefore assume that the interaction potentials V_g and V_u, having sharp[12,13] and a smooth[10,14] decay, give close values of σ_1 if the potentials with a sharp falloff have a greater difference between V_g and V_u than the smooth potentials. It was proposed in Ref. 13 to use $\alpha = 2.23$, and the data of Ref. 10 can be approximated by the value $\alpha = 1.1$. Our preliminary estimates have shown that potentials V_g and V_u that differ so much in the long-range interaction region agree well with the temperature dependence of σ_1 given in Fig. 4a (curve 1). Thus, the use of the cross section σ_1 alone does not allow us to select the potentials V_g and V_u with the correct variation in the region $r > 7a_0$.

It should be noted that the analysis of the temperature dependence of σ_2 allows us to choose between the interaction potentials, since the value of σ_2 is very sensitive to the time the two helium atoms of the system spend in the region of effective interaction, i.e., to the steepness of the potential curves V_g and V_u in the region of their falloff. Of course, the choice of the potential that agrees best with the temperature dependence of σ_2 can be made only after solving the problem of the nonresonant transfer of the excitation in the $He^3 - He^4$ system, which should serve as the object of a special theoretical investigation. Preliminarily, it can be shown only that the smoother potentials evidently agree better with the experimental curve for σ_2 represented in Fig. 4.

Whereas there are at present theoretical calculations that connect the cross section σ_1 and its temperature dependence with the potentials V_g and V_u,[13,14] there are no such calculations for the interaction of He^3 and He^4 and the cross section σ_2. The results of these calculations allow us to determine more accurate potentials V_g and V_u of interaction of the atoms $He(1\,^1S_0)$ and $He(2\,^3S_1)$ from the values of σ_1 and σ_2 and their temperature dependences obtained in the present work.

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Induced Compton scattering by relativistic electrons

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The behavior of an electron moving with arbitrary velocity in a given random field of intense, low-frequency radiation is considered under conditions when the dominant electron-radiation interaction mechanism is induced Compton scattering. The evolution of the electron energy spectrum is investigated in the diffusion approximation, and the equilibrium spectrum of relativistic electrons in the field of radiation with a high brightness temperature is found. The induced light-pressure force acting on a moving electron and the rate of induced heating of an electron gas in an isotropic radiation field are calculated. It is shown that, in contrast to the well-known spontaneous retarding force, the direction of the induced force depends on the radiation spectrum. Radiation spectra for which the induced force accelerates an electron in a given direction right up to ultrarelativistic energies are found.

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INTRODUCTION

The process of induced Compton scattering of electromagnetic radiation by free electrons (see the review article[1]) can play an important role in astrophysics in the interaction of high-power radio emissions of pulsars, quasars, and other objects with the surrounding tenuous plasma,[2–4] as well as under laboratory conditions in the investigation of plasma heating by radiation from lasers, masers, and superhigh-frequency devices.[5–7] It is well-known that this process[8] leads to electron heating,[5,9] the appearance of an induced light-pressure force,[10,11] a change in the radiation spectrum[12] and, in particular, to the appearance in the continuous radiation spectrum of spectrally narrow components and solitons,[13] to the divergence or convergence of the radiation beam,[14] etc.

In the present paper we consider the question of the behavior of electrons in a given radiation field, i.e., we shall be interested in: 1) the distribution of the electrons over energy in a situation in which the plasma is sufficiently rarefied and the major role in the formation of the electron distribution over energy is played by scattering processes leading to the diffusion of the electrons in momentum space; 2) the induced light pressure acting on a free electron; 3) the heating of relativistic electrons during induced scattering of light.

For all the quantities characterizing the behavior of electrons in a statistical isotropic radiation field, we have obtained exact relativistic formulas that are valid for arbitrary electron energies and arbitrary radiation spectra. The obtained formulas are fairly simple and convenient for the computation of induced effects, and allow the investigation of both nonrelativistic and ultrarelativistic asymptotic behaviors.

To the question of the induced interaction of radiation with a relativistic plasma have been devoted a large number of papers.[15–17] The dependence, obtained in these papers, of the rate of heating of monoenergetic ultrarelativistic electrons on their energy is valid only for radiation spectra of a definite form. However, it is precisely for these spectra that the pattern of plasma heating is qualitatively different from the heating of monoenergetic electrons. This is connected with the presence in momentum space, as a result of induced scattering, of electron diffusion,[10] which, in these spectra, washes out the monoenergetic distribution much more rapidly than heats it.[18] At the same time, the pattern of electron heating in a wide class of spectra differs from the results obtained by Ochelkov and Charugin,[16] Dedkov,[17] and Blandford and Scharlemann.[18] In our paper we analyze the induced effects for arbitrary radiation spectra.