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Translated by J. G. Adashko

Magnetostriction and thermal expansion of single crystals of the rare earth gadolinium-dysprosium alloys

S. A. Nikitin, D. Kim, and O. D. Chistyakov

Moscow State University; Metallurgy Institute, USSR Academy of Sciences
(Submitted May 10, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1610–1619 (October 1976)

The thermal expansion of single-crystal gadolinium-dysprosium alloys in a magnetic field is measured. The temperature dependence of the magnetostriction contributions to the thermal expansion is compared with that predicted by the theory in which isotropic two-ion exchange interactions and anisotropic one-ion magnetocrystal interactions are taken into account. It is found that the magnetostriction contribution to the thermal expansion along the c axis is mainly due to exchange interaction and, along the a axis, to both exchange and magnetocrystal interactions. It is shown that the derivatives of the exchange parameters with respect to interatomic distances are anisotropic and depend nonlinearly on the dysprosium concentration; an explanation of this may be the deformation of the Fermi surface in $Gd_{1-x}Dy_x$ alloys. The derivatives of the magnetocrystal energy with respect to interatomic distances depend linearly on the dysprosium concentration in accordance with the predictions of the one-ion theory.

PACS numbers: 65.70.+y, 75.80.+q

It has been established[1–4] that when heavy rare-earth metals (HREM) become magnetically ordered, giant magnetostriction deformations are produced in them. Nonetheless, the magnetostriction[5,6] and especially thermal expansion[7] of single crystals of HREM alloys with one another have not received much study. Investigations of the thermal expansion as well as of the temperature dependence of the spontaneous striction and its dependence on the magnetization, on the atomic number, and on the concentration of the fused components are needed in order to understand the nature of the giant magnetostriction deformations of REM and their alloys. Only a joint investigation of the magnetostriction induced by an external field and the spontaneous magnetostriction makes it possible in principle to separate the different contributions made to the magnetostriction deformation and to determine all the magnetostriction constants.

In this paper, principal attention is paid to an investigation of the dependence of the magnetostriction component to the thermal expansion on the temperature and on the concentration of the fused components, inasmuch as this contribution is more influenced by exchange interactions than the striction induced by an external field. We have investigated by a tensometric method the thermal expansion of polycrystalline samples of the alloys $Gd_{1-x}Dy_x$, prepared in accordance with a technology described earlier.[5] Measurement of the thermal expansion was carried out in a magnetic field, a procedure needed to separate the magnetostriction contribution to the thermal expansion and to determine the magnetostriction constants. The field was sufficient to destroy the helicoidal structure in the $Gd_{1-x}Dy_x$ alloys at $x > 0.5$ and to realize a single-domain state in the entire temperature region.

Figure 1 shows the thermal expansion of single-crystal $Gd_{1-x}Dy_x$ alloys along different crystallographic directions: in the basal plane (a axis) and for the hexagonal axis c. The a axis is the easy magnetization axis in $Gd_{1-x}Dy_x$ alloys. It is seen from the figure that in all the samples investigated by us magnetic ordering produces anomalies of the thermal-expansion curve with a reversal, in a number of cases, of the sign of the thermal expansion coefficient. These anomalies are attributed to the fact that below the magnetic-ordering temperature there arise giant magnetostriction deformations, which are superimpoed on the phonon part of the thermal expansion. If the thermal expansion is measured at $H = 0$, the value of the magnetostriction contribution depends both on the spontaneous striction and on the domain structure. On cooling below the magnetic-ordering tem-

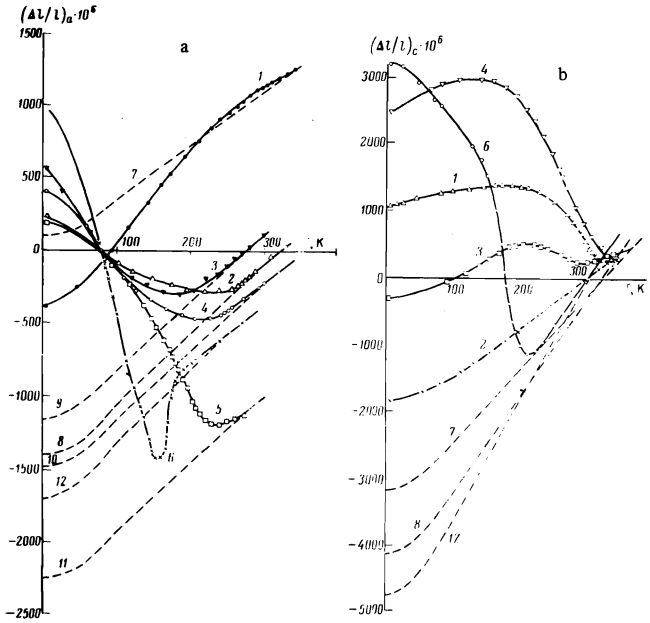

FIG. 1. Thermal expansion, measured in a magnetic field applied along the axes a and c for the alloys $Gd_{1-x}Dy_x$: 1—$x=0$; 2—$x=0.183$; 3—$x=0.296$; 4—$x=0.372$; 5—$x=0.696$; 6—$x=1.0$; 7–12—phonon contributions to the thermal expansion for $x=0$; 0.183; 0.296; 0.372; 0.696; 1.0, respectively. a) $(\Delta l/l)_a$—thermal expansion along the a axis at $H=15$ kOe; b) $(\Delta l/l)_c$—thermal expansion along the c axis at $H=50$ kOe.

perature, the spontaneous magnetization increases, and this is accompanied by spontaneous striction. The magnetostriction contribution to the thermal expansion is small near the temperature of the transition from the paramagnetic state into the magnetically-ordered state Θ, but increases strongly with decreasing temperature.

If a sufficiently strong magnetic field is applied along the easy magnetization axis, then the magnetization vector in all the regions of the crystal will be directed along the field. Thus, a one-domain state is realized in the crystal in this case. If the sample temperature is not very close to Θ, then the influence of the external field on the magnetization and the magnetostriction deformation of the single-domain state can be neglected. Thus, the magnetostriction contribution of the thermal expansion of the sample situated in a magnetic field coincides here practically with the spontaneous striction of the single-domain crystal.

To determine the phonon part of the thermal expansion below the magnetic-transition point, we use the values of the thermal-expansion coefficient in the paramagnetic region of the investigated crystals, where there is no magnetic contribution at $H=0$, and also the temperature dependence of the thermal expansion of single-crystal lutecium,[8] which can be chosen as $(\Delta l/l)_{ph}$ for HREM, with correction for the change of the Debye temperature. Lutecium has electronic and crystallographic structures analogous to those of HREM, but the lutecium atom has no magnetic moment, and consequently lutecium remains paramagnetic down to the lowest temperatures. When separating the phonon part of the thermal expansion $(\Delta l/l)_{ph}$, a correction was introduced for the distance between the Debye temperatures of lutecium and the $Gd_{1-x}Dy_x$ alloys, and it was assumed that the largest elongation $(\Delta l/l)_{ph}$ is a universal function of T/Θ_D, where Θ_D is the Debye temperature.

The phonon part of the thermal expansion $(\Delta l/l)_{ph}$ obtained in this manner is shown in Fig. 1 by thin dashed lines. The electronic contribution to the thermal expansion of HREM can be neglected[9,14] in comparison with $(\Delta l/l)_{ph}$ and the magnetostriction contribution $(\Delta l/l)_{mag}$ to the thermal expansion, which can be obtained from the experimental curve of the thermal expansion $\Delta l/l$ by means of the formula

$$(\Delta l/l)_{mag}=(\Delta l/l)-(\Delta l/l)_{ph}. \quad (1)$$

The magnetostriction contributions to the thermal expansion of the alloys $Gd_{1-x}Dy_x$ at 4.2 °K, determined in this manner as functions of the dysprosium content, are shown in Fig. 2, while Figs. 3–5 show their temperature dependences.

An analysis of the magnetostriction deformations will be carried out on the basis of the formulas obtained by Clark *et al.*,[10] who have shown that the relative magnetostriction change of the length for rare-earth hexagonal crystals can be written in the cylindrical-symmetry approximation in the form

$$\lambda=\lambda_1^{\alpha,0}(\beta_x^2+\beta_y^2)+\lambda_2^{\alpha,0}\beta_z^2+\lambda_1^{\alpha,2}(\beta_x^2+\beta_y^2)(\alpha_z^2-1/3)+\lambda_2^{\alpha,2}\beta_z^2(\alpha_z^2-1/3)$$
$$+\lambda^{\gamma,2}\{1/2(\beta_x^2-\beta_y^2)(\alpha_x^2-\alpha_y^2)+2\beta_x\beta_y\alpha_x\alpha_y\}+2\lambda^{\varepsilon,2}\{\beta_x\alpha_x+\alpha_y\beta_y\}\beta_z\alpha_z, \quad (2)$$

where $\lambda_i^{\eta,l}$ are the magnetostriction constants, while α_i and β_i are respectively the direction cosines of the magnetization vectors and of the deformation direction.

From this we obtain the magnetostriction contribution along the axes a, b, and c when the magnetization vector is directed along the easy axis a:

$$\lambda_a=\lambda(a,a)=\lambda_1^{\alpha,0}-1/3\lambda_1^{\alpha,2}+1/2\lambda^{\gamma,2},$$
$$\lambda_b=\lambda(a,b)=\lambda_1^{\alpha,0}-1/3\lambda_1^{\alpha,2}-1/2\lambda^{\gamma,2}, \quad (3)$$
$$\lambda_c=\lambda(a,c)=\lambda_2^{\alpha,0}-1/3\lambda_2^{\alpha,2},$$

where the first index in the parentheses following λ shows the direction of the external magnetic field, and the second shows the direction in which the deformation was measured. The values of λ_a, λ_b, and λ_c at 4.2 °K are designated in Fig. 2 as λ_{0a}, λ_{0b}, and λ_{0c} respectively.

A theoretical analysis[10,11] shows that λ_a, λ_b, and λ_c depend on magnetostriction constants of two types, $\lambda_1^{\alpha,0}$ and $\lambda_2^{\alpha,0}$ are determined by two-ion terms of exchange origin. The corresponding contribution to the magnetostriction (see (2)) is isotropic and does not depend on the direction of the magnetization vector in space. The magnetostriction constants $\lambda_1^{\alpha,2}$, $\lambda_2^{\alpha,2}$ and $\lambda^{\gamma,2}$ are determined by one-ion terms that result from the dependence of the one-ion energy of the magnetic anisotropy on the deformation. In addition, these constants depend in principle on two-ion terms that can be appreciable in magnitude in the case when the exchange energy depends on the direction of the magnetization vector relative to the crystallographic axes (the anisotropy of the exchange interaction[12]). The contribution to the magnetostriction

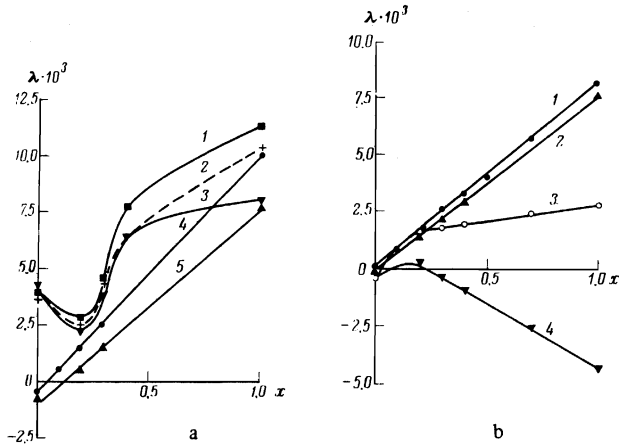

FIG. 2. Dependence of the magnetostriction constant on the dysprosium concentration for $Gd_{1-x}Dy_x$ alloys at 4.2 °K: a) 1—$\lambda_2^{\alpha,0}$ calculated from formula (3), 2—$\lambda_2^{\alpha,0}$ obtained by fitting the theoretical curves obtained from formulas (6) to the experimental data for $\lambda_c(T)$; 3—λ_{0c}—experimental values at 4.2 °K, 4—$\lambda_2^{\alpha,2}$—data of[6], 5—$\lambda_2^{\alpha,2}$ obtained by fitting the theoretical curves obtained from formulas (6) to the experimental data for $\lambda_c(T)$; b) 1—$\lambda^{\gamma,2}$ from the data of[5], 2—$(\frac{1}{2}\lambda^{\gamma,2} - \frac{1}{5}\lambda_1^{\alpha,2})$, 3—$\lambda_{0a}$, 4—$\lambda_1^{\alpha,0}$, calculated from formulas (3).

(see (2)) corresponding to the constants $\lambda_1^{\alpha,2}$, $\lambda_2^{\alpha,2}$ and $\lambda^{\gamma,2}$ is anisotropic.

The determination of the various magnetostriction contributions to the thermal expansion is a rather complicated task. These contributions can be estimated by investigating the magnetostriction and the thermal expansion as functions of the concentration of the fused components, the magnetization, and the temperature. It was established experimentally[5,6] that the magnetostriction constants $\lambda^{\gamma,2}$ and $\lambda_2^{\alpha,2}$, determined from measurements of the magnetostriction with rotation of the magnetization vector in the basal plane ($\lambda^{\gamma,2}$) and relative to the axis with ($\lambda_2^{\alpha,2}$), depends linearly on the dysprosium concentration in the case of the $Gd_{1-x}Dy_x$ alloys. A similar dependence on the concentration can be suggested for the constant $\lambda_1^{\alpha,2}$ which is of the same type as the constants $\lambda_2^{\alpha,2}$ and $\lambda^{\gamma,2}$.

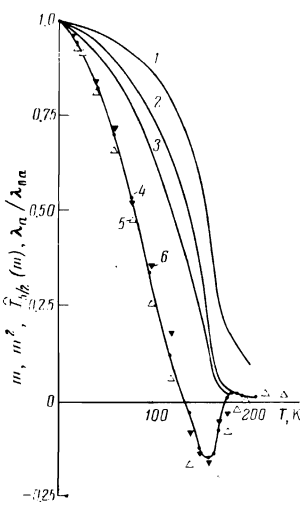

FIG. 3. Temperature dependences of magnetization and of the magnetostriction contribution to the thermal expansion along the *a* axis in a field $H=15$ kOe applied along the *a* axis for Dy: 1—m, relative magnetization 2—m^2, 3—$I_{5/2}(m)$, 4—λ_a/λ_{a0}—experimental data; 5—λ_a/λ_{0a}, data of[10], 6—λ_a/λ_{0a}, theoretical data calculated from formula (6).

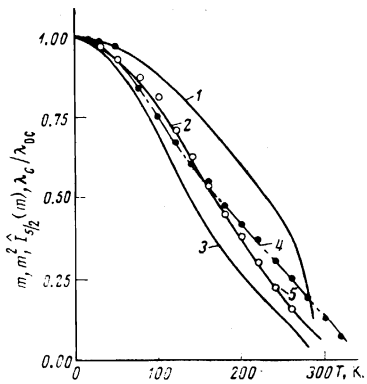

FIG. 4. Temperature dependence of the magnetization and of the magnetostriction contribution to the thermal expansion for the alloy $Gd_{0.183}Dy_{0.817}$ along the *c* axis in a field $H=50$ kOe applied along the *a* axis: 1—m, relative magnetization, 2—m^2, 3—$\hat{I}_{5/2}(m)$, 4—λ_c/λ_{0c}—experimental data, 5—λ_c/λ_{0c}—theoretical data calculated from formula (6).

Starting from the values obtained by us for the magnetostriction contributions to the thermal expansion λ_{0a}, λ_{0b}, λ_{0c} and the previously determined constants $\lambda^{\gamma,2}$, $\lambda_2^{\alpha,2}$ of the alloys $Gd_{1-x}Dy_x$,[5,6] and also from the values of the magnetostriction constant $\lambda_1^{\alpha,2}$ (see[13]), we have determined by means of formulas (3) the magnetostriction constants $\lambda_1^{\alpha,0}$ and $\lambda_2^{\alpha,0}$ (see Fig. 2). The constants $\lambda_1^{\alpha,0}$ and $\lambda_2^{\alpha,0}$ depend nonlinearly on the dysprosium concentration, whereas for the constants $\lambda_1^{\alpha,2}$, $\lambda_2^{\alpha,2}$, $\lambda^{\gamma,2}$ this dependence is linear. At $x \approx 0.2$, the concentration plot has a minimum for $\lambda_2^{\alpha,0}$ and a maximum for $\lambda_1^{\alpha,0}$ (see Fig. 2).

As shown by Tonegawa,[14] the exchange magnetostriction in REM depends on the singularities of the Fermi surface.

According to the theoretical data,[15] the Fermi surface of gadolinium differs noticeably from the Fermi surface of dysprosium and other HREM. Therefore one can expect in $Gd_{1-x}Dy_x$ deformations of the Fermi surface, and consequently a nonlinear change in the magnetostriction constants of exchange origin $\lambda_1^{\alpha,0}$ and $\lambda_2^{\alpha,2}$.

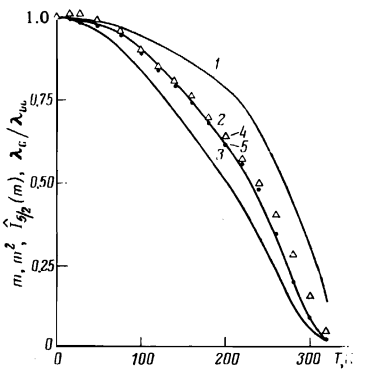

FIG. 5. Temperature dependence of the magnetization and of the magnetostriction contribution to the thermal expansion along the *c* axis for gadolinium in a field $H=15$ kOe applied along the *a* axis: 1—m, relative magnetization, 2—m^2, 3—$I_{5/2}(m)$, 4—λ_c/λ_{0c}—experimental data, 5—λ_c/λ_{0c}—theoretical data calculated from formula (6).

The linear dependence of the constants $\lambda^{\gamma,2}$ and $\lambda_2^{\alpha,2}$ on the dysprosium concentration indicate that the one-ion terms that result from the dependence of the one-ion energy of the magnetic anisotropy on the deformation make the largest contributions to the constants $\lambda^{\gamma,2}$ and $\lambda_2^{\alpha,2}$. The two-ion terms of exchange origin should vary like the square of the concentration of the fused components, just as the energy of the exchange interaction itself. Thus, the contribution of the two-ion terms to the constants $\lambda^{\gamma,2}$ and $\lambda_2^{\alpha,2}$ is small. The anisotropic increment to the exchange energy, which should exist, in accordance with the theory,[12] seems to have little influence on the magnetostriction constants $\lambda_2^{\alpha,2}$ and $\lambda^{\gamma,2}$.

It was shown in the molecular-field-theory approximation[13,16] that the magnetostriction constants can be expressed in terms of the derivatives of the exchange integral and of the constants of the magnetocrystalline energy $V_{2\varepsilon}^0$ and $V_{2\varepsilon}^{2c}$ with respect to the interatomic distances

$$\lambda_1^{\alpha,0}=m^2s^2k\left[(S_{11}+S_{12})\frac{\partial I/k}{\partial \log a}+S_{13}\frac{\partial I/k}{\partial \log c}\right]$$

$$\lambda_2^{\alpha,0}=m^2s^2k\left[2S_{13}\frac{\partial I/k}{\partial \log a}+S_{33}\frac{\partial I/k}{\partial \log c}\right],$$

$$\lambda_1^{\alpha,2}=-\frac{3}{2}\left[(S_{11}+S_{12})\frac{\partial V_{2\varepsilon}^0}{\partial \log a}+S_{13}\frac{\partial V_{2\varepsilon}^0}{\partial \log c}\right] \qquad (4)$$

$$\lambda_2^{\alpha,2}=-\frac{3}{2}\left[2S_{13}\frac{\partial V_{2\varepsilon}^0}{\partial \log a}+S_{33}\frac{\partial V_{2\varepsilon}^0}{\partial \log c}\right],$$

$$\lambda^{\gamma,2}=-\frac{2}{C_{11}-C_{12}}\frac{\partial V_{2\varepsilon}^{2c}}{\partial \log a}. \qquad (5)$$

Here m is the relative magnetization, s is the spin, S_{ij} are the elastic pliability constants, C_{ij} are the elastic constants, k is Boltzmann's constant, and a, b, and c are the crystal-lattice constants. The values of S_{ij} and C_{ij} are tabulated in a paper by Bartholin.[13]

Using formulas (4) and the obtained experimental data, we have calculated the change of the exchange integral and of the magnetocrystal energy constants with changing interatomic distance for the alloys $Gd_{1-x}Dy_x$. It is seen from Fig. 6 that the derivatives of the magnetocrystalline energy constants with respect to the interatomic distances (curves 3, 4, 5) depend linearly on the dysprosium concentration, in agreement with the one-ion theory. The quantities $\partial(I/k)/\partial \log a$ and $\partial(I/k)/\partial \log c$ depend linearly on the dysprosium concentration, with $\partial(I/k)/\partial \log a$ reversing sign near the equiatomic composition. For pure metals, the values of $\partial(I/k)/\partial \log a$ and $\partial(I/k)/\partial \log c$ agree with the values obtained from measurements of the shifts of the Curie and Neel points under the influence of a unilateral stress.[16] The exchange integral changes more strongly when the interatomic distances change along the hexagonal c axis than in the basal plane. To the contrary, the magnetocrystal energy is more sensitive to deformations in the basal plane ($\partial V_{2\varepsilon}^0/\partial \log a > \partial V_{2\varepsilon}^0/\partial \log c$).

As shown by Callen and Callen,[11] the exchange two-ion constants $\lambda_1^{\alpha,0}$ and $\lambda_2^{\alpha,0}$ are proportional to the square of the magnetization in a wide temperature interval. The temperature dependence of the one-ion magnetostriction constant $\lambda_1^{\alpha,2}$, $\lambda_2^{\alpha,2}$, $\lambda^{\gamma,2}$ is determined by the reduced hyperbolic Bessel function $I_{5/2}(m)$. Thus, the

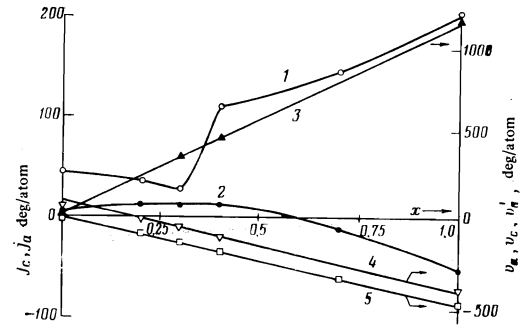

FIG. 6. Dependence of the change of the exchange integral and of the parameters of the magnetocrystal energy (deg/atom) with interatomic distances on the concentration of the dysprosium in the alloys $Gd_{1-x}Dy_x$: 1—$j_c=\partial(I/k)/\partial \log c$, 2—$j_a=\partial(I/k)/\partial \log a$, 3—$v_a=\partial V_{2\varepsilon}^0/\partial \log a$, 4—$v_c=\partial V_{2\varepsilon}^0/\partial \log c$, 5—$v_a'=\partial V_{2\varepsilon}^{2c}/\partial \log a$.

temperature dependences of the magnetostriction contribution to the thermal expansion can be described by the following formulas

$$\lambda_a(T)=\lambda_1^{\alpha,0}m^2-(^1/_3\lambda_1^{\alpha,2}-^1/_2\lambda^{\gamma,2})I_{5/2}(m),$$

$$\lambda_b(T)=\lambda_1^{\alpha,0}m^2-(^1/_3\lambda_1^{\alpha,2}+^1/_2\lambda^{\gamma,2})I_{5/2}(m), \qquad (6)$$

$$\lambda_c(T)=\lambda_2^{\alpha,0}m^2-^1/_3\lambda_2^{\alpha,2}I_{5/2}(m).$$

We have used these formulas to calculate the temperature dependence of the magnetostriction contributions to the thermal expansion for the $Gd_{1-x}Dy_x$ alloys (see Figs. 3–5). It should be noted that a comparison of the theory[11] with experiment in a wide temperature interval from 4.2 °K to the temperature of the magnetic ordering has not yet been carried out even for gadolinium and dysprosium. We have established that the theory agrees well with the experimental data for the magnetostriction contribution to the thermal expansion along the c axis in the entire range of compositions and along the a axis for pure dysprosium (see Figs. 3, 4, 5). By fitting the theoretical curves corresponding to formulas (6) to the experimental data on $\lambda_c(T)$ we calculated the magnetostriction constants $\lambda_2^{\alpha,0}$ and $\lambda_2^{\alpha,2}$. The values of the constants $\lambda_2^{\alpha,0}$ and $\lambda_2^{\alpha,2}$ obtained in this manner for the $Gd_{1-x}Dy_x$ alloys agree with the values determined by the methods described above (see Fig. 2).

In the magnetostriction contribution to the thermal expansion along the c axis, the principal role is played by the exchange two-ion term $\lambda_2^{\alpha,0}m^2$, which in accordance with the Callen's theory is proportional to the square of the magnetization: the value of the magnetostriction constant $\lambda_2^{\alpha,2}$ of one-ion origin is small in comparison with $\lambda_2^{\alpha,0}$. This conclusion can be drawn for dysprosium and for all $Gd_{1-x}Dy_x$ alloys, including gadolinium (see Figs. 2 and 5).

An entirely different picture is observed for the magnetostriction contribution to the thermal expansion along the a axis. It is seen from Fig. 4 that for dysprosium the curve corresponding to this contribution along the a axis is not described by only the one-ion or two-ion constants, since it lies lower than either the curve described by the function $I_{5/2}(m)$ or the curve described by

the function m^2. We note that our measurements agree with the known experimental results[10] for the magnetostriction contribution made to the thermal expansion of dysprosium (Fig. 5, points 5). Starting from (6), this phenomenon can be attributed to the fact that the sum of the magnetostriction one-ion constants $(-\frac{1}{3}\lambda_1^{\alpha,2}+\frac{1}{2}\lambda^{\gamma,2})>0$ in the magnetostriction contribution to the thermal expansion along the axes a and b is comparable in absolute magnitude with the exchange two-ion constant $\lambda_1^{\alpha,0}(\lambda_1^{\alpha,0}<0)$. The quantity $\lambda_1^{\alpha,0}m^2$ depends on the temperature like m^2, and therefore the curve decreases more slowly than the curve described by the quantity $(-\frac{1}{3}\lambda_1^{\alpha,2}+\frac{1}{2}\lambda^{\gamma,2})I_{5/2}(m)$, which varies like m^3 in a wide range of temperatures (with the exception of temperatures close to the transition point Θ). This gives rise to the reversal of the sign of the striction when the transition point Θ is approached, as indeed observed in the dysprosium crystal along the a axis (see Fig. 3).

Thus, the exchange interaction in dysprosium, just as in gadolinium, causes a contraction of the a axis. In dysprosium, however, a large magnetostriction contribution to the thermal expansion is made also by the magnetocrystal interaction (orbit–crystal field), which causes expansion along the a axis. This latter contribution to the thermal expansion is larger than the former at low temperatures, resulting in $\lambda_{0a}>0$.

With increasing gadolinium content in the alloys, λ_{0a} decreases as a result of the decrease of the magnetocrystal energy (see Figs. 1a and 2b). In pure gadolinium $\lambda_{0a}<0$, since the entire contribution due to the magnetocrystal interaction is negligible (the orbital angular momentum of gadolinium is $L=0$) and the negative contribution to the thermal expansion due to exchange interactions predominates.

In gadolinium and gadolinium–dysprosium alloys with large gadolinium contents, the temperature dependence of the magnetostriction contribution to the thermal expansion along the a axis deviates noticeably from the theoretical curves that can be plotted on the basis of formulas (6). A particularly strong discrepancy between theory and experiment is observed for gadolinium. It can be assumed that the complicated temperature dependence of the uniaxial magnetic anisotropic constant of gadolinium,[17] which is likewise not described by the theory of one-ion anisotropy, comes into play here.

The results of the present investigation show that thermal expansion and magnetostriction can be used to obtain information on the character of the dependence of the exchange and magnetocrystalline interaction on the interatomic distances in REM and their alloys. The linear dependence of the derivatives of the magnetocrystalline energy with respect to the interatomic distances on the dysprosium concentration in the $Gd_{1-x}Dy_x$ alloys is a fact that indicates that the contribution of the anisotropic one-ion magnetocrystal energy to the spontaneous magnetostriction and to the thermal expansion predominate over the contribution that can come in principle from the anisotropic two-ion terms of exchange origin. For this latter contribution one should expect a quadratic dependence on the dysprosium concentration. In addition, one can also conclude that the one-ion magnetoelastic terms have a weak dependence of the deformation of the Fermi surface in $Gd_{1-x}Dy_x$ alloys.

The exchange integral and its derivatives with respect to the interatomic distances depend in a complicated manner on the singularities of the energy spectrum of the conduction electrons, on the density of states at the Fermi level, and on the concentration of the conduction electrons.[18] One can therefore assume that the observed nonlinear dependence of the derivatives of the exchange integrals with respect to the interatomic distances on the dysprosium concentration is due to deformation of the Fermi surface in the $Gd_{1-x}Dy_x$ alloys. The transition from the ferromagnetism in gadolinium and in $Gd_{1-x}Dy_x$ alloys at $x<0.5$ to helicoidal antiferromagnetism at $x>0.5$[5,20] can also be attributed, in accordance with the Dzyaloshinskiĭ theory,[19] to deformation of the Fermi surface. The derivatives of the exchange integrals with respect to the interatomic distances, according to our data, reveal a strong anisotropy along the crystallographic directions. It follows therefore that the theory of indirect exchange in the isotropic approximation cannot describe the magnetoelastic phenomena in REM and their alloys.

In conclusion, we thank K. P. Belov for a discussion of the results and E. M. Savitskii and V. F. Terekhova for supplying the alloy single crystals.

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Translated by J. G. Adashko

Helicons in indium single crystals in the intermediate state

I. P. Krylov

Institute of Physics Problems, USSR Academy of Sciences
(Submitted May 11, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1620–1633 (October 1976)

Free and forced helicon oscillations in high-purity indium single crystals are investigated experimentally. The dependence of the helicon dispersion law and damping on the magnitude and direction of the magnetic field is studied. In agreement with the basic law of macroscopic electrodynamics, the dispersion law and damping of helicons are found to be independent of the structure of the intermediate-state domains. The resonance frequency of the oscillations in the plate do not depend on the concentration of the normal phase and decrease with inclination of the magnetic field relative to the normal to the plate surface. The damping of helicons in the intermediate state is independent of the inclination of the magnetic field. These facts are in agreement with Andreev's theory. (Zh. Eksp. Teor. Fiz. **51**, 1510, 1966; Sov. Phys. JETP. **24** 1019, 1967).

PACS numbers: 75.60.Fk, 76.90.+d

In 1966, Andreev derived the equations of macroscopic electrodynamics of superconductors in the intermediate state.[1] In Andreev's theory, the electromagnetic fields were averaged over distances greatly exceeding the dimensions of the normal and superconducting domains that are produced in the sample volume. In this macroscopic approach, to describe the field distributions there is no need to know the structure of the intermediate state, the shape of the domains, the rate of motion of the boundaries, etc. For Andreev's equations[1] to be valid it is important only that free motion of the domain walls between the phases be possible, i.e., that there be no pinning by the sample defects.

It was shown in[1] that in a pure uncompensated metal in the intermediate state, just as in the normal state, circularly polarized low-frequency electromagnetic waves—helicons—can propagate in a constant magnetic field. The condition that ensures weak damping of the helicons is a large electron mean free path l in comparison with the characteristic dimensions R of the electron orbits in the magnetic field. Helicons in the intermediate state were first observed experimentally by Maxfield and Johnson in polycrystalline indium samples.[2] A more thorough investigation was then carried out with lead samples.[3] In both cases, however, they used pure-quality samples of low-purity metals at $l < R$. This caused a strong damping of the helicons in the intermediate state, which led to difficulties in the interpretation of the experimental results and to apparent discrepancies with the theory.

We report here a detailed experimental investigation of helicons in high-purity indium single crystals. We investigated the dispersion and the damping of the helicons as functions of the magnitude and direction of the magnetic field. All the experimental results obtained in the intermediate state are in full agreement with Andreev's theory.[1]

HELICON OSCILLATIONS IN A PLATE (THEORY)

1. *Dispersion law.* We consider the distribution of alternating low frequency fields in an unbounded plate of thickness d in the intermediate state. We introduce a system of Cartesian coordinates $(x\eta\zeta)$ such that the surfaces of the plate are defined by the equation $\zeta = \pm d/2$. We introduce also a coordinate system (xyz), which is rotated through an angle θ about the x axis relative to the other system, so that the z axis is directed along the constant magnetic field $\mathbf{H}_0$ inside the plate. In the intermediate state $|\mathbf{H}_0| = H_c$ (H_c is the critical field) and the connection between the angle θ and the angle φ between the external constant field $\mathscr{H}$ and the ζ axis depends on the value of the field. From the condition that the tangential components of the field intensity be continuous, we get

$$\sin\theta = (\mathscr{H}/H_c)\sin\varphi. \tag{1}$$

The alternating increment to the constant field will be designated by the vector $\mathbf{H}_1$. We assume that $|\mathbf{H}_1| \ll H_c$, and that the dependence of the alternating field on the coordinates and on the time is given by the factor $e^{i\mathbf{kr}-i\omega t}$. Then as shown in[1], the following relation should hold between the projection k_z of the wave vector $\mathbf{k}$ on the z axis and the frequency ω:

$$\omega^2(\sigma_{xy}\sigma_{yx}-\sigma_{xx}\sigma_{yy}) - i\omega\frac{c^2k_z^2}{4\pi}(\sigma_{xx}+\sigma_{yy}) + \left(\frac{c^2k_z^2}{4\pi}\right)^2 = 0. \tag{2}$$