Polarization of exciton luminescence in an external magnetic field[1)]

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The circular polarization of the exciton luminescence from germanium with emission of LA-phonons in a magnetic field is measured. The degree of polarization and the nature of its variation are found to depend strongly on the direction of the magnetic field. The polarization of exciton luminescence with emission of LA and TO phonons and the selection rules for indirect transitions in germanium are calculated on the basis of general symmetry considerations. It is shown that the regularities observed in experiment can be explained only if the diamagnetic splitting of the exciton levels, which is quadratic in the magnetic field, is taken into account. Comparison of the theoretical and experimental dependences of the polarization on magnetic field strength yields the g-factor for holes in the exciton is determined, $g_1 = -1.6$, as well as the diamagnetic-shift constant and diamagnetic-splitting constant for exciton levels, $\lambda_2 = 2$ meV/kOe and $\lambda_3 = 1.5$–2.5 meV/kOe.

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1. INTRODUCTION

The orientation of the electron and hole spins by an external magnetic field leads to polarization of recombination radiation of a semiconductor.

Investigation of the polarization of the exciton luminescence in a magnetic field makes it possible to determine the fine structure of the exciton levels, as well as the sign and magnitude of the electron and hole g factors. The character of the splitting of the exciton levels in a magnetic field depends on the ratio of the exchange (Δ_{ex}), crystal (Δ_{cr}), and Zeeman (Δ_H) splittings. In germanium, where estimates indicate that the exchange splitting does not exceed 3×10^{-2} meV in magnetic fields $H>5$ kOe we have $\Delta_H \gg \Delta_{ex}$ and the magnetic moments of the electrons and holes are independently oriented. Owing to the crystal splitting, the g-factors of the holes in the two split states with $j_z=\pm\frac{3}{2}$ and $j_z=\pm\frac{1}{2}$ are strongly anisotropic, and their magnitude depends on the ratio of Δ_{cr} and Δ_H, and also on the exciton energy E(K); the g-factor of the germanium electrons in each of the extrema is also strongly anisotropic. Therefore the direction of the orientations of the spins of the electrons and holes in germanium does not coincide with the direction of the magnetic field, while the Zeeman splitting depends on the orientation of the field relative to the crystal axes. In strong magnetic fields $H>30$ kOe, when the magnetic radius $(\hbar c/eH)^{1/2}$ approaches the Bohr radius of the exciton, the magnetic field causes a strong diamagnetic shift. Owing to the large anisotropy of the effective masses of the electron this shift is anisotropic also in the case of non-equivalent positions of the extrema relative to the magnetic field the energy of excitons with electrons located in different valleys are different. The strong magnetic field gives rise also to an anisotropic diamagnetic splitting of the excitonic levels quadratic in the field, and this splitting competes with the crystal splitting. All this leads to a considerable anisotropy of the radiation polarization.

Luminescence in germanium is accompanied by emission of a phonon, which can carry away an angular momentum. The polarization of the radiation is therefore determined by the type of the phonon that takes part in the process, and also by the channel that predominates in the indirect transitions. The degree of thermal orientation of the excitons in a magnetic field depends on the ratio of the lifetime to the electron and hole spin relaxation time. The lifetime of the indirect excitons in germanium is $\tau\approx10^{-5}$–10^{-6} and is much longer than the hole spin relaxation time, which does not exceed 10^{-9} sec at helium temperatures. In weak magnetic fields the main spin-relaxation mechanism of the electrons in the excitons is their exchange interaction with the holes, and even at $\Delta_{ex}\approx10^{-6}$ eV we have $\tau_{se}=\hbar/\Delta_{ex}\lesssim10^{-8}$ sec (at $\Delta_{ex}\tau_{sh}/\hbar\gtrsim1$ the value is $\tau_{se}\approx\tau_{sh}$). In strong magnetic fields, however, at $\Delta_H>\Delta_{ex}$, the value of τ_{se} increases like $\tau_{se}=\tau_{sh}(\Delta_H/\Delta_{ex})^2$. In this case τ_{se} can be determined by the time τ_{um} of the intervalley transitions, inasmuch as the strong anisotropy of the g factors of the electrons and germanium causes τ_{se} and τ_{um} to be of the same order of magnitude. The value of τ_{um} in germanium depends on the type of the impurity centers and, according to[3–5], at a concentration $N_{As}\gtrsim5\cdot10^{12}$ cm^{-3} we have $\tau_{um}\lesssim10^{-6}$ sec $<\tau$. We shall therefore assume that the excitons in germanium have time to acquire a thermal-equilibrium distribution.

2. SELECTION RULES

The symmetry group for the indirect exciton in germanium is the group L, which is equivalent to the point group $D_{3d}=C_{3v}\cdot i$. The selection rules for the indirect transitions can be obtained by starting from general symmetry considerations analogous to the method of invariants in the construction of the spectrum.[6] In germanium, the wave functions of the electrons S are transformed in accordance with the representation $L_1(A_1^+)$, while the wave functions of the hole X, Y, Z transform in accordance with the representation $\Gamma_{25}'(A_1^+ + E^+)$, and the polarization vectors e_x, e_y, e_z transform in accordance with the representation $D_1^- = A_1^- + B_1^-$. Therefore for

TABLE I.

Phonon	Electron, hole		
	S_{x+}	S_{x-}	S_{z_1}
LA	$-\frac{1}{\sqrt{2}}\eta e_+U_z$	$\frac{1}{\sqrt{2}}\eta e_-U_z$	λe_zU_z
TO	$\alpha e_zU_+ - \gamma^* U_-e_-$	$\alpha e_zU_- - \gamma U_+e_+$	$\frac{3}{\sqrt{2}}(-U_+e_- + U_-e_+)$

the longitudinal LA phonons U_z, which transform in accordance with the representation $L_2'(A_1^-)$, there are two independent constants that determine the selection rules, since the product $A_1^+(A_1^+ + E^+)(A_1^- + E^-)A_1^-$ contains the single representation A_1^+ twice. For the transverse TO phonons U_x and U_y, which transform in accordance with the representation $L_3'(E^-)$, the product

$$A_1^+(A_1^+ + E^+)(A_1^- + E^-)(A_1^+ + E^-)$$

contains A_1^+ three times, i.e., the selection rules are determined by three constants.[2)]

In the basis of the electron functions S and the hole functions Z, $X_\pm = \mp(X \pm iY)/\sqrt{2}$ the selection rules in the coordinate system (x, y, z) connected with the axes of a given extremum take the form represented in Table I, where $e_\pm = e_x \pm ie_y$, $U_\pm = \mp(U_x \pm iU_y)/\sqrt{2}$, the constants η, λ, α, and β are real, and the constant γ is real under the condition that the x axis is chosen in one of the planes σ_v of the given extremum. On the other hand, if the x axis makes an angle φ with this plane, then $\gamma = |\gamma| e^{-3i\varphi}$. For the ground state of the exciton, which is classified in accordance with the spin indices $j_z = \pm\frac{3}{2}$ of the functions ψ_j and $s_z = \pm\frac{1}{2}$ of the electronic functions φ_s, we obtain from Table I the selection rules listed in Table II and III (the functions ψ_j were chosen in a canonical basis).

If the transition with emission of an LA phonon proceeds only via the Γ_2' band, then $\eta = \lambda$. In this case Table II is valid in any coordinate frame, provided that the functions ψ_j and φ_s correspond to definite values of j_z and s_z in this coordinate frame. On going through the L_3' band, $\lambda = 0$, and on going through L_2' we have $\eta = 0$. We note that for the lower states $j_z = \pm\frac{3}{2}$ the selection rules, according to Table II, are determined by a single constant η. If the transition with emission of a TO phonon goes only through L_3', then $\alpha = 0$, while on going through L_3' we have $\beta = \gamma = 0$. On going through the band Γ_{15} we have $\alpha = \beta = \gamma$ (transition with emission of a TO phonon through the nearest band Γ_2' is forbidden).

For a substitution-type impurity, the local symmetry of which is the group T_d, the potential $V(\mathbf{r})$ transforms in accordance with the representation A_1 of the group C_{3v}, which is a meet of the groups T_d and D_{3d}. Therefore the selection rules for zero-phonon lines connected with scattering by this impurities take the same form as for the LA phonon, i.e., are determined by Table I.

3. EXCITON SPECTRUM

The Hamiltonian $\mathcal{H}(\mathbf{H}, \mathbf{K})$, describing the splitting of the terms of the ground state in a magnetic field, in a coordinate frame tied with the principal axes of the given extremum, is given by

$$\mathcal{H} = \mathcal{H}_{cr} + \mathcal{H}_H + \mathcal{H}_{dm}. \tag{1}$$

The first terms describe the crystal and Zeeman splittings:

$$\mathcal{H}_{cr} = \frac{\Delta_{cr}}{2}\left(J_z^2 - \frac{5}{4}\right), \tag{2}$$

$$\mathcal{H}_H = \mu_0\{g_1(\mathbf{JH}) + [g_\parallel s_z H_z + g_\perp(s_xH_x + s_yH_y)]\}. \tag{3}$$

We have left out here small terms containing j_i^3, and disregard also the anisotropy of the g factor of the holes, resulting from the spin-orbit interaction on account of the anisotropy of the exciton wave function. $\mathcal{H}_{dm}$ includes the isotropic and anisotropic shifts as well as the diamagnetic splitting

$$\mathcal{H}_{dm} = \lambda_1 H^2 + \lambda_2(3H_z^2 - H^2) + \lambda_3((JH)^2 - {}^5/_4 H^2). \tag{4}$$

The next term in (4), which describes the diamagnetic splitting, has been written out in a spherical approximation. In the general case it is determined by six constants:

$$\begin{aligned}\mathcal{H}_{dm,\ spl} = (J_z^2 - {}^5/_4)(\lambda_3 H_z^2 + \lambda_4 H_\perp^2) + \lambda_5(J_+^2 H_-^2 - J_-^2 H_+^2)\\ + \lambda_6([J_zJ_+]H_-H_z + [J_zJ_-]H_+H_z)\\ + \lambda_7(J_+^2 H_+ H_z + J_-^2 H_- H_z) + \lambda_8([J_zJ_-]H_+^2 + [J_zJ_-]H_-^2),\end{aligned} \tag{5}$$

where

$$[J_iJ_k] = {}^1/_2(J_iJ_k + J_kJ_i), \quad H_\pm = H_x \pm iH_y, \quad J_\pm = J_x \pm iJ_y.$$

We have calculated the constants λ_i by perturbation theory, using, just as in[7], the one-column functions $f(\mathbf{r})\psi_i^h\varphi_e^e$ as the zeroth approximation and taking into account the term

$$-\frac{\hbar^2}{m_0}\frac{e^2}{c^2}\left[\gamma_2\sum_i J_i^2A_i^2 + \gamma_3\sum_{i\neq j}[J_iJ_j]A_iA_j\right]$$

in first-order perturbation energy. If we choose for $f(\mathbf{r})$ the rotational function

$$f(\mathbf{r}) = C\exp\left\{-\left(\frac{z^2}{a_\parallel^2} + \frac{x^2+y^2}{a_\perp^2}\right)^{1/2}\right\} \tag{6}$$

then

$$\begin{aligned}&\lambda_1 = {}^2/_3\delta[(a_\perp^2 + a_\parallel^2)(\gamma_1 + \gamma_\perp) + a_\perp^2(\gamma_1 + \gamma_\parallel)],\\ &\lambda_2 = {}^1/_3\delta[a_\perp^2(\gamma_\perp - \gamma_\parallel) + (a_\perp^2 - a_\parallel^2)(\gamma_1 + \gamma_\perp)],\\ &\lambda_3 = 2\delta\gamma_3 a_\perp^2, \quad \lambda_4 = -\delta\gamma_3(2a_\perp^2 - a_\parallel^2), \quad \lambda_5 = {}^1/_6\delta(\gamma_2 + 2\gamma_3)a_\parallel^2,\\ &\lambda_6 = {}^2/_3\delta(2\gamma_2 + \gamma_3)a_\perp^2, \quad \lambda_7 = {}^1/_3\sqrt{2}\delta(\gamma_3 - \gamma_2)a_\perp^2, \quad \lambda_8 = {}^1/_3\sqrt{2}\delta(\gamma_3 - \gamma_2)a_\parallel^2.\end{aligned} \tag{7}$$

Here $\delta = \mu_0^2(\gamma_1 + \gamma_\perp)/4E_i$; $a_\parallel$ and $a_\perp$ are the radii in units of a_h; E_i and a_h are the Bohr energy and the Bohr radius at $m^* = m_0/(\gamma_1 + \gamma_\perp)$, $\gamma_\parallel = m_0/m_\parallel^e$, $\gamma_\perp = m_0/m_\perp^e$; $m_\parallel^e$ and $m_\perp^e$ are the effective masses of the electrons; γ_1, γ_2, and γ_3 are parameters that determine the hole spectrum.[6] It is seen that the constant λ_3 in (4) is positive, since $\Delta_{cr} < 0$, therefore the diamagnetic splitting cancels the crystal splitting in part.[3)] We have left out of the Hamiltonian the term $\mathcal{H}(\mathbf{K})$, which determines the exciton

TABLE II.

LA phonon

s	j			
	$^3/_2$	$-^3/_2$	$^1/_2$	$-^1/_2$
$^1/_2$	0	$-\frac{1}{\sqrt{2}}\eta e_-U_z$	$\frac{1}{\sqrt{6}}\eta e_+U_z$	$-\sqrt{^2/_3}\lambda e_zU_z$
$-^1/_2$	$-\frac{1}{\sqrt{2}}\eta U_z e_+$	0	$\sqrt{^2/_3}\lambda e_zU_z$	$\frac{1}{\sqrt{6}}\eta e_-U_z$

spectrum $E(\mathbf{K})$ and includes a term that describes its splitting into two branches of light and heavy exciton.[7] Allowance for this term leads to an effective decrease of the crystal splitting by an amount $\approx\eta kT$ and yields corrections to the degree of circular polarization in a magnetic field, on the order of $\eta^2(kT/\Delta_{cr})^2$, where $\eta\approx 0.5-0.7$ according to[7]. We shall disregard these corrections in this paper.

4. POLARIZATION OF LUMINESCENCE WITH EMISSION OF LA PHONON

Using Table II, we can show that at $\lambda=\eta$ the degree of circular polarization of LA luminescence, for radiation propagating along the magnetic field, is given by

$$P_{\text{circ}}=\frac{2\langle J_\zeta\rangle+3\langle S_\zeta\rangle-4\langle\mathbf{S}\rangle\langle\mathbf{J}J_\zeta\rangle}{^3/_4+\langle J_\zeta^2\rangle+\langle\mathbf{S}\rangle\langle\mathbf{J}\rangle+4\langle J_\zeta\rangle\langle S_\zeta\rangle-4\langle\mathbf{S}\rangle\langle\mathbf{J}J_\zeta^2\rangle} \tag{8}$$

(the ζ axis is directed here along the magnetic field). In the derivation of (8) it was assumed that the exciton density matrix ρ^{ex} is the product of the electron and hole density matrices in each valley, an assumption valid of exchange interaction is neglected. The angle brackets denote averaging over each extremum l: $\langle\hat{A}\rangle_l=\mathrm{Tr}\langle\rho_l^{ex}A\rangle$, and the summation is over all extrema with allowance for their occupation.

A. Weak magnetic field, low temperatures

In a weak magnetic field, when the Zeeman and diamagnetic splittings are small in comparison with Δ_{cr}, at $T\ll\Delta_{cr}$, we can take into account only two lower states with $j_z=\pm\frac{3}{2}$.

To calculate the mean values of the components in (8), we first calculated the mean values of the components in the coordinate system (x, y, z) connected with the axes of the given extremum, after which we converted to the coordinate system (ξ, η, ζ), in which formula (8) is expressed. Account was taken here of the fact that when the coordinates are transformed the mean values of the products of the operators transform like the products themselves, and the components J_i and σ_i transform like components of a vector.

The calculation yielded

$$P_{\text{circ}}=\sum_l A_l\exp(-\delta_l/T)\Big/\sum_l B_l\exp(-\delta_l/T), \tag{9}$$

where

$$A_l=\cos\theta_l\left[\frac{g_\parallel}{g_l}\cos\theta_l\,\mathrm{th}\left(\frac{1}{2}g_lH^*\right)-\mathrm{th}\left(\frac{3}{2}g_l\cos\theta_lH^*\right)\right], \tag{10}$$

$$B_l=\frac{1}{2}(1+\cos^2\theta_l)\left[1-\frac{g_\parallel}{g_l}\cos\theta_l\,\mathrm{th}\left(\frac{1}{2}g_lH^*\right)\mathrm{th}\left(\frac{3}{2}g_l\cos\theta_lH^*\right)\right].$$

Here θ_l is the angle between the magnetic field and the principal axis z of the extremum l; $H^*=\mu_0H/T$; g_l is the average g-factor of the electron in the l valley:

$$g_l=(g_\parallel^2\cos^2\theta_l+g_\perp^2\sin^2\theta_l)^{1/2}; \tag{11}$$

is the anisotropic diamgnetic shift of the level $j_z=\pm\frac{3}{2}$ of the given valley, and equals, according ot (4),

$$\delta_l=(\lambda_2+^3/_4\lambda_3)(3\cos^2\theta_l-1)H^2. \tag{12}$$

It is seen from (10) that in the linear region, where $\frac{1}{2}g_lH^*\ll 1$ and $\frac{3}{2}g_lH^*\ll 1$ and $\delta_l\ll T$ the value of P_{circ} does not depend on the magnetic field direction. This isotropy is preserved also when account is taken of the mixing of the states with $j_z=\pm\frac{1}{2}$ on account of the terms linear in H in first-order perturbation theory. Taking these corrections into account we have in the linear region

$$P_{\text{circ}}(\mathbf{H})=\frac{1}{4}\left[g_\parallel-3g_\perp\left(1-\frac{4}{3}\frac{T}{\Delta_{cr}}\right)\right]\frac{\mu_0H}{T}. \tag{13}$$

It is seen that these corrections lead to the appearance of a temperature-independent contribution to P_{circ}.

If the temperature T is low enough and the magnetic field is strong, so that $\frac{1}{2}g_lH^*\gg 1$ and $\frac{3}{2}g_lH^*\gg 1$, and also $\delta_l\gg T$, but $T\ll\Delta_{cr}$ and $\lambda_3H^2\ll|\Delta_{cr}|$, then only the lowest extrema with the minimal value $\cos^2\theta_l=\cos^2\theta_m$ make a contribution to the radiation. In this case P_{circ} depends essentially on the value of θ_m:

$$P_{\text{circ}}^{\text{sat}}=-2\frac{g_\perp}{|g_\parallel|}\frac{\cos\theta_m}{1+\cos^2\theta_m}. \tag{14}$$

As seen from (10) A_l and B_l do not reverse sign when θ_l is replaced by $\pi-\theta_l$, so that we always assume in (14) and henceforth that $0\leqslant\theta_l\leqslant\pi/2$ and $\cos\theta_l\geqslant 0$.

B. Arbitrary magnetic fields

In the general case of arbitrary fields and temperatures, the polarization was calculated by formula (8). The eigenvalues and eigenfunctions of the Hamiltonian (1)–(4) were obtained by solving the corresponding system of equations with a computer. The results of these calculations, and also of calculations by means of the approximate formulas (9) and (10), are shown in Figs. 1a–1c and are discussed in Sec. 6.

5. POLARIZATION OF LUMINESCENCE WITH EMISSION OF TO PHONON

Since the selection rules for the TO luminescence are essentially anisotropic, it is necessary in this case, using Table III, to calculate first the total radiation intensity

$$I=\sum d_{\alpha\beta}'e_\alpha e_\beta^*$$

(e is the polarization vector) for each of the extrema in the coordinate system (x, y, z) connected with the axes of the extremum, and then find the components $w_\mp=d_{++}$

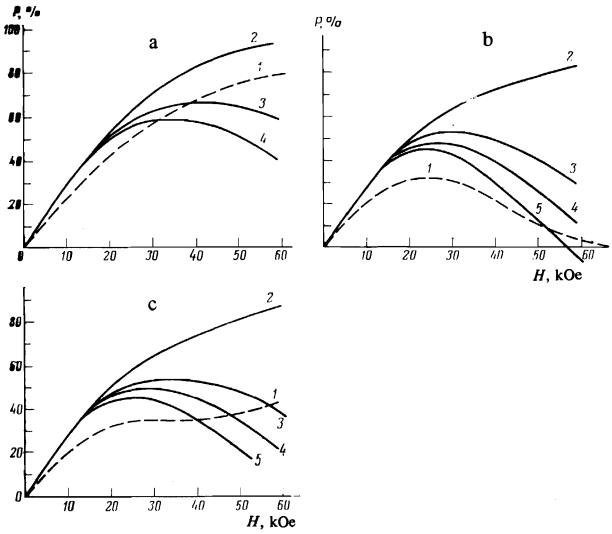

FIG. 1. Theoretical plots of the degree of circular polarization of exciton radiation vs the magnetic field at $T = 4.2$ °K; $\Delta_{cr} = 0.8$ meV; $\lambda_2 = 2$ meV/kOe^2; $g_1 = -1.6$; $g_\parallel = 0.90$; $g_\parallel = 1.92$: 1—calculation by formulas (9)–(12); 2—with account of the mixing of the $\pm\frac{3}{2}$ and $\pm\frac{1}{2}$ terms by the Zeeman splitting, $\lambda_3 = 0$; 3–5—with allowance for the Zeeman and diamagnetic splitting: 3—$\lambda_3 = 1.5$ meV/kOe^2, 4—$\lambda_3 = 2$ meV/cOe^2, 5—$\lambda_4 = 2.54$ meV/kOe^2. a) H ∥ ⟨100⟩, b) H ∥ ⟨110⟩, c) H ∥ ⟨111⟩.

$\mp d_{--}$ in the coordinate system (ξ, η, ζ) connected with the magnetic field. This calculation is facilitated by the fact that these components do not depend on the choice of the axes ξ and η perpendicular to H, so that when calculating the contribution made to $w_\mp$ by each extremum it is convenient to choose these axes in different manners, such as to make the ξ axis lie in the $(z\zeta)$ plane.

As seen from Table III, transitions from the lower state $j_z = \pm\frac{3}{2}$ are determined only by the constants α and γ. A comparison of the experimental and theoretical dependences of the degree of linear polarization on the deformation[13] shows that the constant $|\alpha|$, which is connected with the transition through the farther zone, is small in comparison with $|\beta| + |\gamma|$. On the other hand, one can expect the constant $|\gamma|$, which vanishes in the isotropic approximation, to be smaller than $|\beta|$. We therefore retain in the forthcoming formulas the terms of order $(\mu_0 H/\Delta_{cr})\beta\gamma$ and $(\mu_0 H/\Delta_{cr})^2\beta^2$, which arise when account is taken of the mixing of the states $\pm\frac{3}{2}$ and $\pm\frac{1}{2}$ in first and second orders of perturbation theory as a result of the Zeeman splitting.

When these terms are taken into account, P_{circ} is determined by formula (9) with

$$A_l = 2|\gamma|^2 \cos\theta_l \left[\operatorname{th}\left(\frac{3}{2} g_1 \cos\theta_l H^*\right) - \frac{g_\parallel}{g_l} \cos\theta_l \operatorname{th}\left(\frac{1}{2} g_l H^*\right)\right], \quad (15)$$

$$B_l = \left[|\gamma|^2 (1+\cos^2\theta_l) + \frac{1}{4}\alpha^2 \sin^2\theta_l \right]\left[1 - \frac{g_\parallel}{g_l}\cos\theta_l \right.$$

$$\times \left. \operatorname{th}\left(\frac{3}{2} g_1 H^* \cos\theta_l\right) \operatorname{th}\left(\frac{1}{2} g_l H^*\right)\right] - \beta|\gamma| \sin^3\theta_l \cos 3\psi$$

$$\times (g_1 \mu_0 H/\Delta_{cr}) \left[\operatorname{th}\left(\frac{3}{2} g_1 H^* \cos\theta_l\right) - \frac{g_\parallel}{g_l}\cos\theta_l \operatorname{th}\left(\frac{1}{2} g_l H^*\right)\right]$$

$$+ \frac{1}{2}\beta^2 (1+\cos^2\theta_l) \sin^2\theta_l (g_1 \mu_0 H/\Delta_{cr})^2$$

$$\times \left[1 - \frac{g_\parallel}{g_l}\cos\theta_l \operatorname{th}\left(\frac{3}{2} g_1 H^* \cos\theta_l\right) \operatorname{th}\left(\frac{1}{2} g_l H^*\right)\right].$$

Here ψ is the azimuthal angle between one of the planes of the given extremum and the plane passing through the axes z and ζ. If the components of H along the principal axes of the crystal are H_1, H_2, and H_3, then we have for the ⟨111⟩ extremum

$$\cos\psi = \frac{H_1 + H_2 - 2H_3}{2(H^2 - H_1H_2 - H_1H_3 - H_2H_3)^{1/2}}.$$

(H_1 is replaced by $-H_1$ for the ⟨$\bar{1}$11⟩ extremum, etc.) It must be assumed here that $\sin\theta > 0$. It is seen that from the initial slope of $P(H)$

$$P_{circ} = \frac{1}{4}(3g_1 - g_\parallel)\frac{\mu_0 H}{T} \Big/ \left(1 + \frac{1}{8}\frac{\alpha^2}{|\gamma|^2}\right) \quad (16)$$

we can determine $\alpha/|\gamma|$. If $|\alpha| \ll |\gamma|$, the initial slope for the TO luminescence is of the same magnitude as for the LA luminescence (see (13), but is of opposite sign.

The function $P_{circ}(H)$ in strong fields, as seen from (15), depends on the ratio of the constants $|\gamma|$ and $|\beta|$. If $|\gamma| \ll |\beta|$, as H increases, the mixing of the states $j_z = \pm\frac{3}{2}$ and $j_z = \pm\frac{1}{2}$ by the magnetic field should cause the radiation intensity to increase and the degree of polarization to decrease, inasmuch as at $\alpha = \gamma = 0$, as seen from Table III, the radiation is always unpolarized, regardless of the term splitting by the magnetic field.

6. RESULTS OF EXPERIMENT AND THEIR DISCUSSION

The measurements of the degree of circular polarization of the excition radiation in germanium were carried out in a Faraday geometry on samples with shallow impurity center concentrations $5 \cdot 10^{12}$–$5 \cdot 10^{13}$ cm^{-3}. The experimental procedure was the same as in[2,14].

Figures 2a–2c show the experimental dependences of the degree of circular polarization of the LA line of the radiation of a free exciton on the value of the magnetic field for three orientations, H ∥ ⟨100⟩, H ∥ ⟨110⟩, H ∥ ⟨111⟩.

In the case H ∥ ⟨100⟩ all the extrema of the conduction bands are equivalent, there is no anisotropic diamagnetic shift of the exciton levels, and $P(H)$ increases monotonically, almost reaching saturation when the Zeeman splitting exceeds kT.

In the orientation H ∥ ⟨110⟩ there are two extrema, ⟨111⟩ and ⟨11$\bar{1}$⟩ with light transverse electron mass $m_{c1} = 0.099\, m_0$ and two extrema, ⟨$\bar{1}$11⟩ and ⟨1$\bar{1}$1⟩ with

TABLE III.

	TO phonon			
s	j			
	$3/2$	$-3/2$	$1/2$	$-1/2$
$1/2$	0	$\gamma U_+ e_+ - \frac{1}{2}\alpha U_- e_z$	$\frac{1}{\sqrt{3}}\left(\gamma^* U_- e_- - \frac{\alpha}{2} U_+ e_z\right)$	$\frac{\beta}{\sqrt{3}}(U_+ e_- - U_- e_+)$
$-1/2$	$-\gamma^* U_- e_- + \frac{\alpha}{2} U_+ e_z$	0	$\frac{\beta}{\sqrt{3}}(-U_+ e_- + U_- e_+)$	$\frac{1}{\sqrt{3}} \times \left(-\gamma U_+ e_+ + \frac{\alpha}{2} U_- e_z\right)$

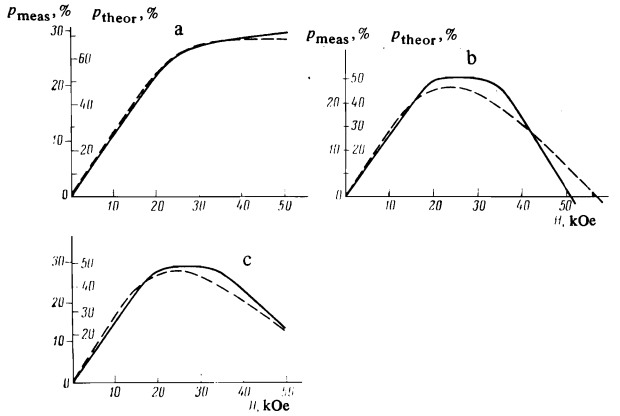

FIG. 2. Experimental plots of the degree of circular polarization of the exciton emission on the magnetic field (solid curve) and theoretical plots (dashed). $T = 4.2$ K, a) $\mathbf{H} \| \langle 100 \rangle$, $\lambda_3 = 1.5$, meV/kOe^2, b) $\mathbf{H} \| \langle 110 \rangle$, $\lambda_3 = 2.54$ meV/kOe^2, c) $\mathbf{H} \| \langle 111 \rangle$, $\lambda_3 = 2.54$ meV/kOe^2.

heavy mass $m_{c2} = 0.36\, m_0$. With increasing magnetic field the excitons go over to the extrema $\langle \bar{1}11 \rangle$ and $\langle 1\bar{1}1 \rangle$, in which the radiation for the lower of the split levels with $j_z = \pm \frac{3}{2}$, in the direction $\langle 110 \rangle$ perpendicular to the principal axis of these extrema, is fully linearly polarized.[13] Therefore as they become filled, as seen from (14), the circular polarization should drop to zero.

At $\mathbf{H} \| \langle 111 \rangle$, with increasing magnetic field, the extremum $\langle 111 \rangle$ with the light mass $m_{c1} = 0.082\, m_0$ becomes depleted, and all the excitons go over to the remaining three extrema with heavy mass $m_{c2} = 0.207\, m_0$, for which the value of P_{circ}^{sat}, according to (14), is less than for the extremum $\langle 111 \rangle$. Therefore in this case, too, one should expect a nonmontonic behavior of $P(\mathbf{H})$.

The experimental curves agree with the arguments advanced above and are qualitatively described by formulas (9)–(12) obtained on their basis (curves 1 on Figs. 1a–1c. Actually, however, expressions (9)–(12) no longer hold in fields exceeding 20–30 kOe, since they were obtained without allowance for the mixing of the exciton states $j_z = \pm \frac{3}{2}$ and $j_z = \pm \frac{1}{2}$, which become quite significant when the Zeeman splitting approaches Δ_{cr} in magnitude. Such a mixing greatly increases the degree of polarization in strong fields, as seen from curves 2 of Figs. 1a–1c, and leads to $P(\mathbf{H})$ dependences that differ greatly from the experimental ones.

The decrease of the polarization in strong fields at $\mathbf{H} \| \langle 111 \rangle$ and $\mathbf{H} \| \langle 100 \rangle$ and particularly the reversal of the sign of the polarization in the second case can be explained only by taking into account the diamagnetic splitting of the excitonic levels, which is connected with the splitting of the hole states. The corresponding theoretical curves, plotted for different values of the parameter of the diamagnetic splitting λ_3, are shown in Figs. 1a–1c. In the calculations,[4)] in accord with[15,16], it was assumed[5)] that $g_\parallel = 0.90$, $g_\perp = 1.92$, $g_1 = -1.6$, $\Delta_{cr} = 0.8$ meV and $\lambda_2 = 2.0 \cdot 10^{-4}$ meV/kOe^2 A comparison oi the theoretical and experimental plots of $P(\mathbf{H})$ has shown that the best agreement can be obtained at $\lambda_3 = 1.5$ meV/kOe^2 at the orientation $\mathbf{H} \| \langle 100 \rangle$ and at $\lambda_3 = 2.54$ meV/kOe^2 in the cases $\mathbf{H} \| \langle 110 \rangle$ and $\mathbf{H} \| \langle 111 \rangle$ (Figs. 2a–2c).[6)] The difference between the values of λ_3 for different directions may be due to the anisotropy of the diamagnetic splitting, which was not taken into account in the calculation of the indicated curves. We note that estimates of the coefficients λ_i by formulas (6), using the radii $a_\parallel$ and $a_\perp$ determined by a variational method, yield $\delta = 10^{-4}$ meV/kOe^2 and $\lambda_1 = 28$; $\lambda_2 = 5.7$; $\lambda_3 = 13$; $\lambda_4 = -8.7$; $\lambda_5 = 1.8$; $\lambda_6 = 9.0$; $\lambda_7 = 1.4$; $\lambda_8 = 0.9$ (all in units of δ). According to[16], the experimental value $\lambda_1 = 5.3 \cdot 10^{-4}$ meV/kOe^2, but it is not very reliable, since the data in[16] were reduced without allowance for the diamagnetic splitting. The discrepancy between the indicated experimental values of λ_1, λ_2, and λ_3 and the calculated ones is due to the use of perturbation theory and to the errors in the variational calculation, which underestimates the binding energy E_i and accordingly overestimates $a_\parallel$ and $a_\perp$ (the value $E_i^{theor} = 2.5$ meV calculated by this method is 2.5 meV as against $E_i^{exp} = 4$ meV). However, the ratio of the constants λ_1, λ_2, and λ_3 is close to the experimental one.

We note in conclusion that the results obtain in this paper have demonstrated the essential role of diamagnetic splitting of the excitonic levels in the polarization properties of the emission of free excitons in germanium.

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1) Preliminary results of this investigation were published in[1,2].

2) When account is taken of the mixing of the electronic functions as a result of the spin-orbit interaction, four independent constants exist for the LA phonon, and eight for the TO phonon.

3) The constant that determines the diamagnetic centers was calculated in[8-12] for acceptors and direct excitons in germanium and A_3B_5.

4) Calculation has shown that variation of the parameter λ_2 in the range $1.5 - 2.5$ meV/kOe^2 and of Δ_{cr} in the range $0.8 - 1.0$ meV has little effect on the $P(\mathbf{H})$ plots.

5) The sign of the polarization of the exciton emission corresponds to right-hand rotation. This means that for the lower state of a hole in the magnetic field $j_z = +\frac{3}{2}$ and consequently $g_1 < 0$, whereas for an electron $g_1 > 0$ and for the lower state $s_z = -\frac{1}{2}$.[14]

6) The measured value of the polarization is several times smaller than the theoretical one because of the strong depolarization of the radiation emitted from the sample as a result of multiple reflections from the sample faces.[2,14]

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Magnetostriction and thermal expansion of single crystals of the rare earth gadolinium-dysprosium alloys

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The thermal expansion of single-crystal gadolinium-dysprosium alloys in a magnetic field is measured. The temperature dependence of the magnetostriction contributions to the thermal expansion is compared with that predicted by the theory in which isotropic two-ion exchange interactions and anisotropic one-ion magnetocrystal interactions are taken into account. It is found that the magnetostriction contribution to the thermal expansion along the c axis is mainly due to exchange interaction and, along the a axis, to both exchange and magnetocrystal interactions. It is shown that the derivatives of the exchange parameters with respect to interatomic distances are anisotropic and depend nonlinearly on the dysprosium concentration; an explanation of this may be the deformation of the Fermi surface in $Gd_{1-x}Dy_x$ alloys. The derivatives of the magnetocrystal energy with respect to interatomic distances depend linearly on the dysprosium concentration in accordance with the predictions of the one-ion theory.

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It has been established[1–4] that when heavy rare-earth metals (HREM) become magnetically ordered, giant magnetostriction deformations are produced in them. Nonetheless, the magnetostriction[5,6] and especially thermal expansion[7] of single crystals of HREM alloys with one another have not received much study. Investigations of the thermal expansion as well as of the temperature dependence of the spontaneous striction and its dependence on the magnetization, on the atomic number, and on the concentration of the fused components are needed in order to understand the nature of the giant magnetostriction deformations of REM and their alloys. Only a joint investigation of the magnetostriction induced by an external field and the spontaneous magnetostriction makes it possible in principle to separate the different contributions made to the magnetostriction deformation and to determine all the magnetostriction constants.

In this paper, principal attention is paid to an investigation of the dependence of the magnetostriction component to the thermal expansion on the temperature and on the concentration of the fused components, inasmuch as this contribution is more influenced by exchange interactions than the striction induced by an external field. We have investigated by a tensometric method the thermal expansion of polycrystalline samples of the alloys $Gd_{1-x}Dy_x$, prepared in accordance with a technology described earlier.[5] Measurement of the thermal expansion was carried out in a magnetic field, a procedure needed to separate the magnetostriction contribution to the thermal expansion and to determine the magnetostriction constants. The field was sufficient to destroy the helicoidal structure in the $Gd_{1-x}Dy_x$ alloys at $x > 0.5$ and to realize a single-domain state in the entire temperature region.

Figure 1 shows the thermal expansion of single-crystal $Gd_{1-x}Dy_x$ alloys along different crystallographic directions: in the basal plane (a axis) and for the hexagonal axis c. The a axis is the easy magnetization axis in $Gd_{1-x}Dy_x$ alloys. It is seen from the figure that in all the samples investigated by us magnetic ordering produces anomalies of the thermal-expansion curve with a reversal, in a number of cases, of the sign of the thermal expansion coefficient. These anomalies are attributed to the fact that below the magnetic-ordering temperature there arise giant magnetostriction deformations, which are superimpoed on the phonon part of the thermal expansion. If the thermal expansion is measured at $H = 0$, the value of the magnetostriction contribution depends both on the spontaneous striction and on the domain structure. On cooling below the magnetic-ordering tem-