Magnetophonon concentration oscillations in zero-gap semiconductors

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Magnetophonon oscillations of the longitudinal magnetoresistance of HgTe are observed at helium temperatures in a strong electric field (1-10 V·cm^{-1}) and magnetic fields up to 300 kOe. A theory is developed according to which the magnetophonon oscillations are due to resonance singularities in the electron and hole recombination cross sections. Recombination with emission of optical phonons occurs whenever the energy gap between the lowest electron Landau level and one of the hole levels equals the energy of the optical phonon. As a result, the number of excess carriers produced by impact ionization in a strong electric field decreases sharply drops and the magnetoresistance is maximal.

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1. INTRODUCTION

Magnetophonon resonance (MPR), which is connected with inelastic resonant scattering of carriers by optical phonons in a strong magnetic field,[1] was observed in the investigation of the magnetoresistance of semiconductors (see the reviews[2]). At low temperatures ($T < 50\,^\circ$K) magnetophonon oscillations of effects that are linear in the electric field vanish because of the freezing of the optical phonons. However, the magnetophonon oscillations in quantizing magnetic fields can appear in the case of resonant emission of optical phonons by a non-equilibrium system of electrons. The magnetophonon (MP) oscillations of the magnetoresistance when the electrons are heated up by an electric field were observed in n-InSb, n-InAs, n-GaAs, n-InP, and n-CdTe in the temperature region 10–20 °K.[2,3] MP oscillations due to electron heating by light were observed in the photomagnetic effect in n-InSb at helium temperatures.[4] The MP oscillations in the acoustoelectric effect were explained also in the electron-temperature approximation.

In this paper we consider a new type of MP oscillations of the longitudinal magnetoresistance of zero-gap semiconductors of the HgTe type in strong electric and magnetic fields ($\mathbf{E} \parallel \mathbf{H}$). The effect is connected with oscillations of the concentration of the non-equilibrium free carriers, which result from the fact that the rate of the interband recombination with emission of an optical phonon depends nonmonotonically on the magnetic field. The MP oscillations were observed in HgTe at low temperatures in strong magnetic fields, when the width of the energy gap in the magnetic field is comparable with the energy of the optical phonon.

2. MEASUREMENT PROCEDURE

The measurements were performed with pulsed magnetic fields up to $H = 300$ kOe. The measured magnetoresistance signal was registered, with the field and signal as coordinates, by an S8-11 oscilloscope with an additional preamplifier. The current in the sample could vary in a wide range—from 0.1 mA to 3 A. The current pulse, the duration of which exceeded the duration of the magnetic-field pulse, was produced by a generator with large output resistance, thus ensuring stabilization of the current in the sample when the resistance was increased by three orders of magnitude. To separate the oscillating part of the magnetoresistance, we used a procedure wherein the signal linear in H was subtracted and the difference signal was subsequently amplified.

To measure the current-voltage characteristics (CVC) in the magnetic field, a triangular current pulse with front durations 50 μsec was applied to the sample at the instant of the maximum magnetic field. At a duration 2.5 msec of the first half-cycle of H, the change of H during the time of the current pulse was 0.2%. The CVC oscillograms were obtained with the S8-11 oscilloscope, and the exact value of H was measured with a second oscilloscope.

In measurements at large current densities j, the question arose of the degree of the possible Joule heat rise of the sample immersed in the liquid helium. It is known[6] that in stationary nucleate boiling, liquid helium can draw from a surface up to 1 W/cm^2 of heat when the temperature rise relative to the bath is $\Delta T \sim 1\,^\circ$K, while liquid hydrogen can draw up to 10 W/cm^2 at $\Delta T \sim 3\,^\circ$K. In the case of a larger heat release a gas bubble is formed (film boiling), and ΔT increases by one order of magnitude.

To determine the permissible pulsed heat loads, we have performed a control experiment in which a sample immersed in helium at $H = 0$ was heated by a large current pulse. A measure of the heating was the change of the sample resistance (the electric field in the sample was weak). At a heat flux 1.5 W/cm^2, after a very rapid initial heat rise $\Delta T = 1$–$2\,^\circ$K, a constant thermal regime was observed for ~ 1 msec, followed by a heat rise of dozens of degrees. The constant regime was apparently made possible by the nucleate boiling of the helium.

In the measurements of the magnetoresistance $\rho(H)$ and of the CVC, the maximum current density was chosen with allowance for the geometry of the sample

TABLE I.

Номер образца	T, K	σ_0, ом$^{-1}$·см$^{-1}$	$R_0 \cdot 10^{-2}$, см^3·кул$^{-1}$
1	1.6	55	65
	4.2	125	40
	14	365	10
	17	415	8
	20.3	510	5.5
2	1.6	38.5	–
	4.2	77	35
3	1.6	14.3	–
	4.2	30	29

and the duration of the pulses of j and H in such a way that the heat flux through the sample surface did not exceed 1.5 W/cm^2 into the helium and 10 W/cm^2 into the hydrogen. In addition, we monitored the agreement between the curves obtained with increasing and decreasing field (at constant $\rho(H)$ and j) or with increasing and decreasing current (at constant CVC and H).

Thus, the experimental curves presented below correspond to samples heated relative to the bath, the heat rise increasing with increasing H and j, reaching 1–2 °K in helium and 3–5 °K in hydrogen at the maximum values of H and j. We note that temperature of an electron gas with an electron mobility $\mu \geq 10^5$ cm$^2 \cdot$ V^{-1} sec^{-1} in an electric field $E \sim 10$ V/cm is raised several dozen degrees.[7] This gives grounds for connecting the results discussed below with the heating of the electron gas rather than with the heating of the crystal lattice.

3. EXPERIMENTAL RESULTS AND DISCUSSION

The measurements of the resistance in a longitudinal magnetic field were performed on three large-crystal HgTe samples (see Table I) at $1.6 < T < 4.2$ °K, and on sample No. 1 at $T < 20.3$ °K. Figure 1 shows plots of $\rho_H^{\parallel}/\rho_0$ against the field, obtained for sample No. 1 at a bath temperature 1.6 °K and at different current densities in the sample j.

At small j it is possible to separate on the $\rho_H^{\parallel}/\rho_0$ curve two exponential sections: $H < 30$ kOe and $H > 60$ kOe. The first of them is due to the decrease of the concentration of the intrinsic carriers when the gap $E_g(H)$ is formed in the energy spectrum of HgTe in the magnetic field; the second is probably connected with conduction over the impurity states. It is important that for each of the samples the value of $\rho_H^{\parallel}$ in the second section does not depend on T at $1.6 < T < 4.2$ °K. With increasing j, the sample resistance in the magnetic field decreases

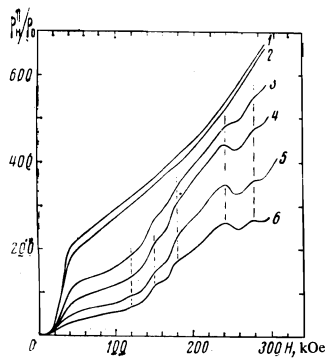


FIG. 1. Longitudinal magnetoresistance of HgTe (sample No. 1, $T = 1.6$ °K) at different densities of the current through the sample: 1—0.067; 2—0.167; 3—0.67; 4—1.0; 5—1.67; 6—2.5 A/cm^2. The electric field at $H = 280$ kOe was respectively 0.7; 1.8; 6.2; 8.1; 10.3; 12.3 V/cm.

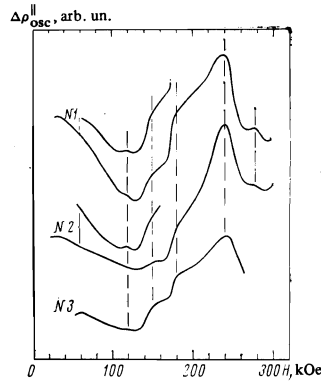


FIG. 2. Oscillating part of the longitudinal magnetoresistance of samples No. 1–3 at $T = 2$–4 °K and at current densities: 1—1.7; 2—5; 3—8 A/cm^2.

and an oscillating component appears on the plot of $\rho_H^{\parallel}/\rho_0$. The positions of the extrema do not depend on the electric field intensity E, and the ratio of the amplitude of the oscillations to the background increases with increasing j. A similar picture was observed on samples No. 2 and 3.

Figure 2 shows the oscillating parts of $\rho^{\parallel}_{osc}$ of the magnetoresistance curves for samples No. 1–3, obtained by subtracting the signal linear in H at a bath temperature 1.6 °K. The curves of Figs. 1 and 2 were used to determine the following values of the magnetic field at which the maxima of $\rho_H^{\parallel}$ were observed: $H_1 = 277$ kOe, $H_2 = 240$ kOe, $H_3 = 180$ kOe, $H_4 = 150$ kOe, and $H_5 = 120$ kOe. The accuracy with which the absolute values of H is determined is ± 5%, and the relative position of the maxima is determined with accuracy ± 2%. The position of the maximum of H_3 is possibly distorted by the presence of an additional maximum in a stronger field (~ 200 kOe).

The curves of Fig. 3 indicate that the oscillations vanish in sample No. 1 when the temperature is increased. In the region of liquid hydrogen temperatures, the exponential section of $\rho^{\parallel}(H)$, which is connected with the formation of the gap $E_g(H)$, shifts towards stronger magnetic fields. At 14 and 17 °K a nonmonotonicity of the $\rho^{\parallel}(H)$ curve at large current density is observed in the same magnetic-field region (200–240 kOe) where the

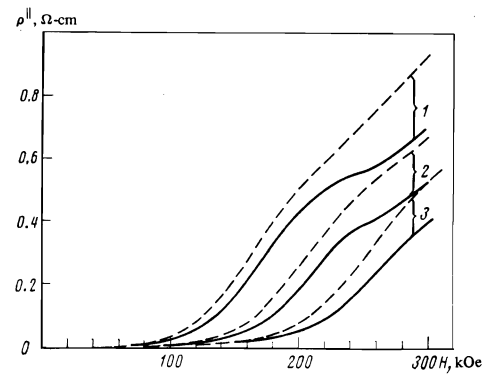
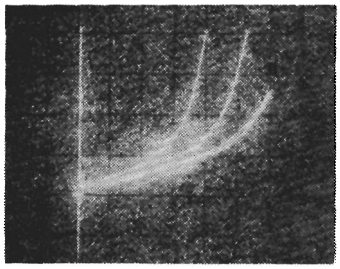
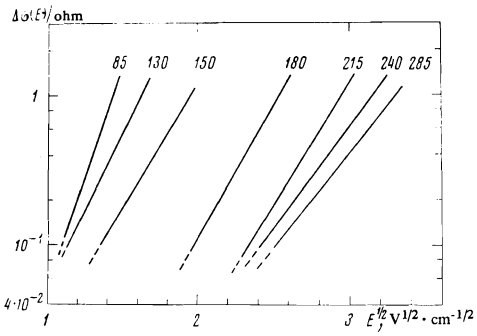


FIG. 3. Resistivity of sample No. 1 in a longitudinal magnetic field at liquid hydrogen temperatures: 1—14; 2—17; 3—20.3 K. Current density: dashed lines: $j \leq 1$ A/cm^2 (ohmic region); solid: 1—12.7; 2—17.6; 3—25.6 A/cm^2. The electric field for the solid curves at $H = 300$ kOe is 1—1.7; 2—5; 3—8 ≈ 10 V/cm.



a



b

FIG. 4. Current-voltage characteristics of sample No. 1 at 4.2 °K in a strong magnetic field. a) CVC oscillograms obtained with three pulses of the magnetic field at (from left to right) $H = 180$, 215, and 285 kOe. The vertical and horizontal scales are 100 mA and 1.15 V per division, respectively. b) Increment of conductivity in the non-ohmic region relative to the conductivity in the same magnetic field H in a weak electric field, calculated from curves of type *a*.

maxima of H_2 and H_3 are located; it was impossible to observe oscillations of $\rho''(H)$ at 20 °K.

To investigate the relations $\sigma(E)|_{H=\text{const}}$ we measured the CVC of sample No. 1 at 4.2 °K. Figure 4a shows the CVC oscillograms, while Fig. 4b shows the CVC in semilog scale

$$\lg\left[\frac{\sigma(E)}{\sigma(0)}\Big|_{H=\text{const}} - 1\right] = f(E^{1/2}).$$

An exponential dependence of the increment of the conductivity is observed:

$$\Delta\sigma(E, H) \sim \exp\left[-(E_0/E)^{1/2}\right].$$

The results of a control experiment (Sec. 2) and the already mentioned independence of the ohmic part of the conductivity $\sigma(0, H)$ on the temperature in the helium region at $H > 100$ kOe allow us to state that the nonlinearity of the CVC is not due to heating of the crystal lattice. As seen from Fig. 4b, the increase of H leads to a decrease of the conductivity in a given electric field E and to a shift in the section of the exponential growth of the conductivity into stronger electric fields.

The observed phenomena are due apparently to heating of the electrons and not the holes, in view of the difference between the effective masses ($m_h/m_e \gg 1$). As a result of the degeneracy of the conduction band and of the valence band of HgTe and as a result of the high electron mobility, the hot electrons can produce a nonequilibrium concentration of electron-hole pairs at easily attainable electric fields $E \sim 1$ V/cm. The gap $E_g(H)$ produced in the magnetic field between the conduction and valence bands determines the energy threshold for impact ionization. The increment of the conductivity in a strong electric field $\Delta\sigma(E, H)$ is determined by the balance between the processes of impact ionization and recombination of the nonequilibrium carriers.

In the absence of a magnetic field, there are two possible channels for direct recombination transitions: Auger processes and transitions with phonon emission. In the purest zero-gap semiconductors at low temperatures, the principal role is played by Auger transitions, since the energies of the electrons and holes are small in comparison with the energy of the optical phonon ($\hbar\omega_0 (T < E_F < \hbar\omega_0$, where E_F is the Fermi energy). When the gap $E_g(H)$ that opens up in the magnetic field H approaches in magnitude the energy of the optical phonon, the principal role is assumed by the process with emission of an optical phonon. The reason is that the Auger processes take place with participation of three particles, for example two electrons and a hole, so that their probability is proportional to n^2p (n and p are the electron and hole densities), whereas the probability of processes with emission of optical phonons is proportional to np. The electron and hole densities decrease exponentially with increasing magnetic field. In addition, the probability of the Auger processes contains an additional small quantity proportional to $[E_B/E_g(H)]^{1/2}$, where E_B is the exciton energy in the magnetic field if $E_B < E_g(H)$. The recombination connected with the emission of optical phonons has a sharp maximum whenever the energy gap between the lowest Landau level of the electrons and one of the hole levels coincides with the energy of the optical phonon. As a result, the number of excess carriers produced by impact ionization in a strong electric field decreases sharply, and the magnetoresistance has a maximum at

$$\hbar\omega_0 = E_e(H) + E_{hl}(H), \tag{1}$$

where $E_e(H)$ and $E_{hl}(H)$ are the absolute values of the energy of the lowest Landau level of the electron and of the l-th hole level, reckoned from the point of tangency of the band at $H = 0$.

From the positions of the maxima of the magnetoresistance we can determine the parameters of the electron and hole bands in a zero-gap semiconductor if we know the limiting frequency of the optical phonons.

Preliminary experimental results show that oscillations are observed in the same magnetic-field region also in the transverse magnetoresistance of HgTe ($\mathbf{H} \perp \mathbf{j}$).

4. THEORY OF MP OSCILLATIONS OF THE CONCENTRATION IN A ZERO-GAP SENICONDUCTOR

We consider a zero-gap semiconductor of the α-Sn or HgTe type with an inverted band scheme[8] in a strong electric field and in a quantizing magnetic field parallel to it (along the z axis). In the magnetic field, a gap of

the order of $\hbar eH/(4m_ec)$ is produced (where m_e and e are the effective mass and charge of the electron and c is the speed of light), if the mass of the hole is much larger than the mass of the electron. At low temperatures, the free electrons are for the most part at the lowest Landau level. The excess of free carriers is produced as a result of impact ionization by the hot electrons. The heavy holes have practically the same temperature as the lattice. The electron-density balance equation can be written in the form

$$\int \alpha(p_z) f_e(p_z)\, dp_z = R_{opt} + R_{nr}, \tag{2}$$

where $f_e(p_z)$ is the electron distribution function at the lower Landau level, $\alpha(p_z)$ is the coefficient of impact ionization of an electron with longitudinal momentum p_z, R_{opt} is the rate of recombination with emission of an optical phonon, and R_{nr} is the rate of the remaining nonresonant recombination processes (for example, recombination on impurities or Auger recombination). In the balance equation, the only oscillating quantity is R_{opt}. It is just the singularities of this quantity which determine the position of the oscillation minima of the concentration and accordingly the maxima of the resistance.

We consider now the recombination of electrons and holes with emission of an optical phonon. The probability of emission of an optical phonon with wave vector $\mathbf{q}$ depends on the number l of the Landau level on which the hole was situated prior to the recombination

$$W_l(p_z, \mathbf{q}) = \frac{2\pi}{\hbar} M_l(p_z, \mathbf{q}) |C_{\mathbf{q}}|^2 \delta[E_e(H) + E_{hl}(H) - \hbar\omega_0 + \varepsilon_c(p_z) + \varepsilon_{hl}(p_z + q_z\hbar)],$$
$$M_l(p_z, \mathbf{q}) = \int |\langle e, \mathbf{p}| e^{i\mathbf{qr}} |h, l, \mathbf{p}+\hbar\mathbf{q}\rangle|^2 \, dp_x\, dp_y, \tag{3}$$

where $C_{\mathbf{q}}$ is the electron-phonon interaction constant, $|e, \mathbf{p}\rangle$ is the wave function of the electron, $\varepsilon_e(p_z)$ is the energy of the electron at the lowest Landau level ($\varepsilon_e(0) = 0$), $\varepsilon_{hl}(p_z)$ is the energy of the hole at the l-th Landau level, and $|h, l, \mathbf{p}\rangle$ is the wave function of the hole. At a given l there are two hole bands[9] marked by the index a or b. The recombination rate R_{opt} is connected in the following manner with the probability W_l:

$$R_{opt} = \frac{1}{(2\pi)^3\hbar} \int dp_z\, d^3q \sum_{i=a,b} \sum_l W_{l,i}(p_z, \mathbf{q}) f_e(p_z) f_{hl\,i}(p_z + \hbar q_z), \tag{4}$$

where $f_{hl}(p_z)$ is the distribution function of the hole on the l-th Landau level.

We shall calculate the quantities $W_l(p_z, \mathbf{q})$ within the framework of a model that takes into account only the s-p interaction of the valence band and the conduction band with the nearest band having s-symmetry at the center of the Brillouin zone (the point Γ). The spectrum of the electrons in the magnetic field is particularly simple in this case[10] and for the lowest Landau it takes the form

$$E_c(H) = \hbar eH/4m_cc, \quad \varepsilon_c(p_z) = p_z^2/2m_c. \tag{5}$$

The energy spectrum of the holes can be obtained by taking into account the interaction with the other bands by perturbation theory. The wave functions of the electrons and holes have been written out in explicit form in[10,11]. In a quantizing magnetic field, when $\hbar\omega_h = \hbar eH/m_hc \gg T$ and $\hbar\omega_e = \hbar eH/m_cc \gg T_e$ (T_e is the temperature of the hole electrons), the characteristic quantities $p_z \sim (m_eT_e)^{1/2}$ and $\hbar q_z \sim (m_hT)^{1/2}$ in (4) are small in comparison with $\hbar q_\perp = \hbar(q_x^2 + q_y^2)^{1/2} \sim hs = (heH/c)^{1/2}$. The matrix elements $M_l(p_z, \mathbf{q})$ in (3) can be calculated in the lowest approximation in p_z and q_z

$$M_{0a}(q_\perp) = (q_\perp^2/2s^2)\exp(-q_\perp^2/2s^2),$$
$$M_{la}(q_\perp) = \frac{3(l-1)}{l!(4l-3)}\left(l - \frac{q_\perp^2}{2s^2}\right)^2 \left(\frac{q_\perp^2}{2s^2}\right)^{l-1} \exp\left(-\frac{q_\perp^2}{2s^2}\right), \quad l \geqslant 2,$$
$$M_{lb}(p_z, q_\perp) = \frac{6l}{(4l-1)(l-1)!s^2\hbar^2}\left[|p_z|(l-1)^{1/2} - \left(l - 1 - \frac{q_\perp^2}{2s^2}\right)|p_z + \hbar q_z|\right]^2 \left(\frac{q_\perp^2}{2s^2}\right)^{l-2} \exp\left(-\frac{q_\perp^2}{2s^2}\right), \quad l \geqslant 2. \tag{6}$$

It follows from (6) that the transitions into the series a have a larger probability than the transitions into the series b, since they contain one extra power of $(p_z/\hbar s)^2$. The transition to the hole level of series b with $l = 0$ is forbidden if the problem is solved in the spherical approximation. Allowance for the anisotropy of the equal-energy surfaces leads to a mixing of the wave functions obtained under the assumption of the equality $\gamma_2 = \gamma_3$, where γ_2 and γ_3 are the Luttinger parameters.[9] Therefore the amplitude of the oscillations connected with the transition to the level b_0 will be smaller than that of the oscillations in the transition to the level of the series a.

The distribution function of the heavy holes having the lattice temperature is of the Boltzmann type. Assuming that the distribution function of the electrons in the ultraquantum limit is also of the Boltzmann type with a temperature $T_e \gg T$, we obtain from (4), (3), and (6) an expression for the recombination rate $T_{opt}(l)$ near resonance, when

$$\Delta_l = \hbar\omega_0 - E_e(H) - E_{hl}(H) \ll \hbar\omega_0$$
$$R_{opt}(l) = \frac{e^2\omega_0 n}{\hbar(\pi T_e)^{1/2}}\left(\frac{1}{\varkappa_\infty} - \frac{1}{\varkappa_0}\right)\exp\left\{-\frac{\Delta_l}{T_e} + \frac{1}{T}[E_F - E_{hl}(H)]\right\} F_l\left(\Delta_l, \frac{1}{T} - \frac{1}{T_e}\right),$$
$$F_{l,a}(\Delta, \gamma) = \frac{3}{|4l-3|}\int_0^\Delta \frac{d\varepsilon}{(\Delta-\varepsilon)^{1/2}}\left(\frac{dp_z}{d\varepsilon}\right)_{la} e^{-\gamma\varepsilon}, \tag{7a}$$

$$F_{2,b}(\Delta, \gamma) = -\frac{6}{7\hbar^2s^2}\int_0^\Delta \frac{d\varepsilon}{(\Delta-\varepsilon)^{1/2}} p_z^2\left(\frac{dp_z}{d\varepsilon}\right)_{2,b} e^{-\gamma\varepsilon}\ln\left(\frac{p_z^2}{2\hbar^2s^2}e^{C+1}\right),$$
$$F_{l,b}(\Delta, \gamma) = \frac{3l}{(4l-1)(l-2)\hbar^2s^2}\int_0^\Delta \frac{d\varepsilon}{(\Delta-\varepsilon)^{1/2}} p_z^2\left(\frac{dp_z}{d\varepsilon}\right)_{l,b} e^{-\gamma\varepsilon}, \quad l \geqslant 3, \tag{7b}$$

where $\varkappa_0$ and $\varkappa_\infty$ are the low-frequency and high-frequency dielectric constants and C is the Euler constant. Formula (7b) was obtained the assumption that $m_e/m_h \ll T/T_e$. Since $\hbar\omega_h > T$, the chemical potential of the holes E_F is connected with the hole density p by the relation

$$p = \frac{eH}{4\pi^2c\hbar^2}\int dp_z \exp\left\{\frac{1}{T}[E_F - E_{h,m}(H) - \varepsilon_{h,m}(p_z)]\right\}, \tag{8}$$

where $E_{h,m}$ is the energy at the top of the valence band, and $\varepsilon_{h,m}(p_z)$ is the energy of a hole with momentum p_z on the ground level m, reckoned from the top of the valence band. The function F multiplied by $\exp(-\Delta/T_e)$, in (7) describes the contour of the resonance lines. The contour is asymmetrical, since it is necessary to have

$\Delta > 0$. For example, in the case of a parabolic hole dispersion, when $\varepsilon_h(p_z) = p_z^2/2m_h$, we have

$$F_{l,a}\left(\Delta, \frac{1}{T}-\frac{1}{T_e}\right) = \frac{3}{|4l-3|}\left(\frac{m_h}{2}\right)^{1/2}\int_0^1 \frac{dx}{[x(1-x)]^{1/2}}\exp\left[-x\Delta\left(\frac{1}{T}-\frac{1}{T_e}\right)\right]$$
$$= \frac{3\pi}{|4l-3|}\left(\frac{m_h}{2}\right)^{1/2}\exp\left[-\frac{\Delta}{2}\left(\frac{1}{T}-\frac{1}{T_e}\right)\right] I_0\left[\frac{\Delta}{2}\left(\frac{1}{T}-\frac{1}{T_e}\right)\right], \quad (9)$$

where $I_0(x)$ is a Bessel function of imaginary argument. It follows from (9) that at $\Delta/T \gg 1$, at the edge of the line, F behaves like

$$\frac{3}{|4l-3|}\left(\frac{\pi T m_h}{2\Delta}\right)^{1/2}.$$

At those values of H at which (1) is satisfied n and p experience a jump of finite magnitude, whereas the derivative with respect to H has a discontinuity.

We calculate the electron impact-ionization coefficient in the approximation of an infinitely heavy hole. In this case the threshold energy, starting with which impact ionization by fast electrons is possible, is equal to the width of the forbidden band in the magnetic field $E_g(H)$. At the threshold value of the energy, the electron gives up its entire momentum to the hole produced as a result of impact ionization. Since $E_g(H) \gg T_e$, it follows that in the calculation of the impact-ionization cross sections we can expand all the quantities that depend on the momentum near the threshold momentum $p_t = [2m_e E_g(H)]^{1/2}$. The initial expression for $\alpha(p_z)$ takes the form

$$\alpha(p_z) = \frac{4\pi e^2}{(2\pi)^3 \varkappa_0^2}\int \frac{d^3q}{q^4}\int d^3p_1\, dp_x\, dp_y |\langle e, \mathbf{p}|e^{-i\mathbf{qr}}|e, \mathbf{p}+\hbar\mathbf{q}\rangle|^2$$
$$\times |\langle e, \mathbf{p}_1|e^{-i\mathbf{qr}}|h, m, \mathbf{p}_1+\hbar\mathbf{q}\rangle|^2 \delta[\varepsilon_e(p_z)-\varepsilon_e(p_z+\hbar q_z)$$
$$-\varepsilon_e(p_{1z})-\varepsilon_{h,m}(p_{1z}+\hbar q_z)-E_g(H)]. \quad (10)$$

As a result of the impact ionization, holes are produced at the uppermost Landau level m with energy $\varepsilon_{h,m}(p_{1z}+\hbar q_z)$. In the case of bands with a parabolic dispersion law (the relation between the Luttinger parameters being $\gamma_1 - \bar{\gamma} - k = 0$) the uppermost band will be b_0. The matrix elements will be determined at the threshold values of the momenta

$$\int dp_x\, dp_y |\langle e, \mathbf{p}|e^{-i\mathbf{qr}}|h, a_0, \mathbf{p}+\mathbf{q}\rangle|^2 = \frac{3q_\perp^2}{2s^2}\left(3+\frac{2p_t^2}{\hbar^2 s^2}\right)^{-1}\exp\left(-\frac{q_\perp^2}{2s^2}\right),$$
$$\int dp_x\, dp_y |\langle e, \mathbf{p}|e^{-i\mathbf{qr}}|e, \mathbf{p}+\mathbf{q}\rangle|^2 = \left(1+\frac{2p_t^2}{\hbar^2 s^2}\right)^{-1}\exp\left(-\frac{q_\perp^2}{2s^2}\right). \quad (11)$$

Expanding $p_z = p_t + p_z'$ and $\hbar q_z = -p_t + \hbar q_z'$, near the threshold, we obtain

$$\int dp_{1z}\, dq_z'\, \delta\left\{p_z'(2E_g m_e^{-1})^{1/2} - \frac{1}{2m_e}[p_{1z}^2 + \hbar^2 (q_z')^2]\right\} = 2\pi m_e. \quad (12)$$

Substituting (11) and (12) in (10), we obtain the final expression for in the ultraquantum limit

$$\alpha = \frac{3m_e e^4}{8\hbar^3 \varkappa_0^2}\left[\frac{3}{2}\int_0^\infty \frac{dx\, e^{-x}}{x+1/2} - 1\right] \approx 0{,}14\frac{m_e e^4}{\varkappa_0^2 \hbar^3}. \quad (13)$$

Allowance for the processes in which the electrons in the final states exchange momenta leads to the appearance of the factor $\frac{2}{3}$ in (13). It follows from (13) that the impact ionization coefficient near the threshold does not depend on the electron momentum. The number of pairs produced per second according to (2) and (13) is equal to (we take into account the fact that there are two threshold values $\pm p_t$)

$$\int \alpha(p_z) f_e(p_z)\, dp_z = n\alpha\left(\frac{T_e}{\pi E_g(H)}\right)^{1/2}\exp\left[-\frac{E_g(H)+\hbar\omega_h}{T_e}\right]. \quad (14)$$

In the case of an intrinsic zero-gap semiconductor with a parabolic dispersion for the Landau hole bands, when $n = p$, it follows from (14), (7), and (2) that the electron and hole density near the resonance is equal to

$$p = n = \frac{\alpha e H T_e |4l-3|}{6\pi^{3/2} c\omega_0 e^2 \hbar}\left(\frac{T}{E_g(H)}\right)^{1/2}\left(\frac{1}{\varkappa_\infty}-\frac{1}{\varkappa_0}\right)^{-1}\exp\left\{-\frac{E_g(H)}{T_e}\right.$$
$$\left.+\hbar\omega_h\left(\frac{l}{T}-\frac{1}{T_e}\right)+\frac{\Delta_l}{2}\left(\frac{1}{T}+\frac{1}{T_e}\right)\right\}\left\{I_0\left[\frac{\Delta_l}{2}\left(\frac{1}{T}-\frac{1}{T_e}\right)\right]\right\}^{-1}. \quad (15)$$

Formula (15) determines the dependence of p on H near the resonance ($\hbar\omega_0 = E_e(H) + (l-1)\hbar\omega_h$). It takes into account only the resonant part of the recombination with emission of optical phonons, as a result of which p increases exponentially far from resonance. Allowance for the nonresonant recombination leads to an essentially much weaker dependence of p on H far from the resonance. The temperature T_e of the hot electrons is determined by the energy-balance equation. In the case of a doped semiconductor, the unknown n, p, and T_e can be obtained from the equations for the neutrality, the energy balance, and the electron-density balance (2).

The line shape near resonance was obtained in explicit form for the simplest model, in which the hole spectrum was assumed to be parabolic and the electrons were assumed to have a Boltzmann distribution with an effective temperature T_e. This model, however, reflects the main features of the phenomenon. The oscillations are connected with the fact that the probability of the departure of the particles upon recombination oscillates as a function of the magnetic field and is proportional to the product np, whereas the probability of impact ionization by the hot electrons is proportional to n and depends monotonically on H. The position of the oscillation minima of the concentration for a real semiconductor is determined by formula (1), which can be used to determine the parameters of the energy spectrum of the carriers in a zero-gap semiconductor.

We note that the problem of the resonance line shape was solved for hot electrons. The production of electron-hole pairs following absorption of a thermal optical phonon was not taken into account, since the oscillations were observed at helium temperatures, when the number of thermal phonons was exponentially small.

5. CONCLUSION

The band parameters of HgTe, determined from the oscillations of the reflection and absorption coefficients in a magnetic field,[12-15] are subject to a large scatter. To determine the parameters from the positions of the maxima of the MP oscillations of the longitudinal magnetoresistance, it is necessary to know in greater de-

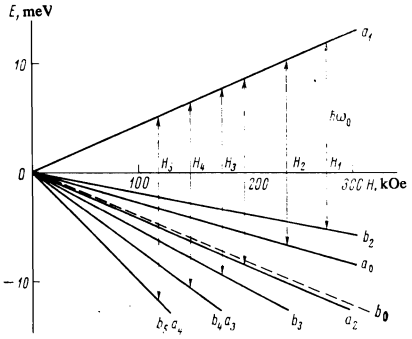


FIG. 5. Scheme of resonant interband transitions in recombination in HgTe. The lower electron level was constructed without allowance for the non-parabolicity[9]: $E_{a1}=\frac{3}{2}(\gamma_1-\gamma)-\frac{1}{2}k$ $=4.2$ (in units of $\hbar eH/m_0c$) at the parameter values taken from[15]. The Landau levels of the holes were drawn according to the experimental results H_1-H_5 at a distance $\hbar\omega_0=17.1$ meV below $E_{a1}(H_N)$.

tail the dependence of the hole energy on the wave vector along the magnetic field. The reason is that in a magnetic field the energy maxima of the series-a Landau bands are located at $p_z\neq 0$.[14,16] For this reason, the MP oscillations are the result of indirect transitions between the conduction band and the valence band, in contrast to optical transitions. The channel of recombination with emission of an optical phonon is turned on at the instant when the width of the indirect gap is equal to the energy of the optical phonon ($\hbar\omega_0$ $=17.1$ meV[17]). Preliminary estimates show that of the three published possible series of parameters, those obtained in[15] agree best with the positions of the maxima of the MP oscillations.

With this set of parameters, the observed maxima of the longitudinal magnetoresistance should correspond to resonance recombination transitions between the lowest electron level a_1 and the lowest hole Landau levels: $H_1\to b_2$; $H_2\to a_0$; H_3 and $H\approx 200$ kOe $\to a_2$ and b_3; $H_4\to a_3$ and b_4; $H_5\to a_4$ and b_5 (Fig. 5). The transition to the level b_0 is forbidden in the spherical approximation. At large l we have $E_a(l)\approx E_b(l+1)\approx|\gamma_1-2\bar{\gamma}|\,l$. Since the lowest hole Landau level is b_2, the obtained data are used to determine the width of the gap in the energy spectrum of HgTe in a magnetic field at 4.2 °K

$$\frac{E_g(H)}{H}=\frac{\hbar\omega_0}{H_1}=5.35\frac{\hbar e}{m_0c}=6.2\cdot10^{-2}\ \text{meV}\cdot\text{kOe}^{-1}.$$

The observed new type of magnetophonon oscillations—oscillations of the rate of interband recombination—adds to the possibilities of using the method of magnetophonon spectroscopy for zero-gap semiconductors such as HgTe and α-Sn. By this method, if the frequencies of the optical phonons are known, one can determine the band parameters and measure directly the width of the energy gap in a magnetic field.

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