Ion formation by an electron beam

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Ionization of a gas by an electron beam is considered. In the approximation of a small beam radius in comparison with the mean free path of secondary particles, the Boltzmann equation reduces to a partial differential equation which is solved by the method of characteristics. The distribution function obtained is used to calculate the ion density.

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1. INTRODUCTION

The problem of passage of an electron beam through a gas presents great interest as a result of the possibility of the comparatively simple production of plasma.[1] By solving the kinetic equation and finding the distribution functions of various kinds of particles, it is possible to calculate the current and density of secondary particles. In view of the complexity of the kinetic equation, it is not possible to obtain an exact solution. The most frequently used method of solution is expansion of the distribution function in spherical functions.[2–4] The rate of convergence of the series, however, depends on the conditions in the plasma.

In this article the Boltzmann kinetic equation is solved on the assumption that the ions formed have momentum components along and perpendicular to the beam axis but the azimuthal component is unimportant. A sufficient condition for validity of this assumption is smallness of the beam radius in comparison with the mean free path. Here it is possible to reduce the integro-differential equation to a partial differential equation and solve it by the method of characteristics.

2. THE BASIC EQUATION

In writing the Boltzmann equation for the distribution function of the ions formed, we shall take into account elastic scattering of electrons by ions and ionization of atoms by the electrons of the incident beam. We shall neglect elastic collisions of atoms and ions, excitation of ions by electrons, and excitation of atoms and ions in collision with each other. With these assumptions the Boltzmann equation has the form

$$\frac{\partial f_i}{\partial t}+\mathbf{v}_i\frac{\partial f_i}{\partial \mathbf{r}_i}+\mathbf{F}_i\frac{\partial f_i}{\partial \mathbf{p}_i}=I_{st}, \tag{1}$$

where $\mathbf{F}_i$ is the force acting on an ion and I_{st} is the collision integral, which can be written in the form of the sum

$$I_{st}=I_{st1}+I_{st2}. \tag{2}$$

I_{st1} is due to elastic collisions of ions with beam electrons, and I_{st2} describes the ionization of atoms. We neglect collisions of ions with secondary electrons, assuming that their mean free path is much greater than the beam radius

$$n_e\sigma R \ll 1,$$

where n_e is the concentration of secondary electrons in the region of the beam and R is the beam radius.

The collision integral which describes the inelastic process can be written in the following form[5,6]:

$$I_{st2}=\int d\mathbf{p}_e'\, d\mathbf{p}_e\, d\mathbf{p}_a\, d\mathbf{p}\,(2\pi/\hbar)\,|U_{\text{ion}}|^2$$
$$\times\delta\left(\frac{p_a^2-p_i^2}{2M}+\frac{p_e^2-p_e'^2-p^2}{2m}\right)\delta(\mathbf{p}_a+\mathbf{p}_e-\mathbf{p}_i-\mathbf{p}_e'-\mathbf{p})\,f_a(\mathbf{p}_a)\,f_e(\mathbf{p}), \tag{3}$$

where m and M are the masses of the electron and atom (ion), U_{ion} is the amplitude of ionization of an atom by an electron, $\mathbf{p}_e$ and $\mathbf{p}_a$ are the momenta of the electron and atom before the collision, and $\mathbf{p}_e'$ and $\mathbf{p}$ are the momenta of the scattered electron and secondary (atomic) electron.

In the case when the velocity of the beam electrons is much greater than the velocity of the atomic electrons, U_{ion} can be calculated in the Born approximation. If we set

$$f_e(\mathbf{p}_e)=n_0\delta(\mathbf{p}_e-\mathbf{p}_0),$$

where n_0 and $\mathbf{p}_0$ are the density and momentum of the beam electrons, and if we take into account that elastic scattering of electrons by ions occurs at small angles, we can then obtain the following expression for I_{st1},

$$I_{st1}=8me^4n_0\frac{\partial f_i}{\partial \mathbf{p}_i}\left\{-\pi\frac{\mathbf{p}_0}{p_0^3}\left[\int\frac{dq}{q}+\left(\frac{\langle r_i^2\rangle}{3\hbar^2}-\frac{1}{8p_0^2}\right)\int q\,dq\right.\right.$$
$$\left.\left.-\frac{\langle r_i^2\rangle}{24p_0^2\hbar^2}\int q^3\,dq\right]\right\}\equiv-\varkappa\frac{\partial f_i}{\partial \mathbf{p}_i}\frac{\mathbf{p}_0}{p_0},\quad \varkappa>0. \tag{4}$$

The integrals over q (the momentum transferred to the electron) are taken from $q_{\min}=Jm/p_0$ to $q_{\max}=\hbar/a_0$; here J is the ionization potential of the atom, a_0 is the characteristic dimension of the atom, and $\langle r_i^2\rangle$ is the mean square radius of the ion.

In the case $qa_0/\hbar \ll 1$ the integral I_{st2} has the form

$$I_{st2}=\frac{n_an_0}{(2\pi MT)^{3/2}}I_\tau p_{0i}\exp(-p_{0i}^2/2MT), \tag{5}$$
$$I_\tau=\frac{m^3e^4}{3\pi^2M^2\hbar^5}(q_{max}-q_{min})\iint d\tau_1\,d\tau_2\,\Psi_i^*(\tau_1)$$
$$\times\Psi_a^*(\tau_2)\,\Psi_i(\tau_2)\,\Psi_a(\tau_1)\sum_{\alpha=1}^{z}\sum_{\beta=1}^{z}\mathbf{r}_{1\alpha}\mathbf{r}_{2\beta},$$

$$d\tau_i = \prod_{\alpha=1}^{Z} d^3 r_{i\alpha},$$

where Ψ_i, Ψ_a are the wave functions of the ion and the atom, Z is the charge of the nucleus, and p_{0i} is the momentum of the ion at the moment of its formation. In calculation of I_{st2} it was also assumed that the gas is in equilibrium and has a temperature T and density n_a.

If the density of secondary electrons is much less than the density of beam electrons, the force F_i is due to the attraction of the ions to the beam,

$$F_i(r) = eE = \begin{cases} -\mu r, & r<R \\ -\mu R^2/r, & r>R \end{cases}, \quad \mu = 2\pi e n_0. \tag{6}$$

Since there is a distinguished direction $\mathbf{z} \parallel \mathbf{p}_0$, we will look for a solution of the Boltzmann equation in the form

$$f_i = f(r, p_\parallel, p_\perp, t),$$

where $p_\parallel$ and $p_\perp$ are the longitudinal and transverse components of the momentum and r is the distance from the beam axis. With this assumption the basic equation for $r<R$ can be rewritten in the form (dropping the subscript i)

$$\frac{\partial f}{\partial t} + \frac{p_\perp}{m}\frac{\partial f}{\partial r} - \mu r \frac{\partial f}{\partial p_\perp} = -\varkappa \frac{\partial f}{\partial p_\parallel} + I_{st2}. \tag{7}$$

I_{st2} is a time-independent quantity in which, after solution of the equation, $p_{0\parallel}$ and $p_{0\perp}$ must be substituted for the functions depending on r, $p_\parallel$, $p_\perp$, and t in accordance with the equations of the characteristics.

We note that only those ions for which $p_{0\perp}^2 + \mu M r_0^2 \geqslant \mu M r^2$, can leave the beam; here r_0 is the coordinate of the point of formation and $p_{0\perp}$ is the transverse momentum at this point. The depth of the potential well, according to Eq. (6), is

$$U_{max} = \pi e^2 n_0 R^2 = 0.525 \cdot 10^{-2} n_0 R^2 \mathrm{K} \sim 0.5 \cdot 10^8 \mathrm{K}$$

for $R = 1$ cm and $n_0 \sim 10^{10}$ cm^{-3}. The mean kinetic energy of the ions formed is $\varepsilon_T \sim T$. Therefore ions formed in a layer of thickness $\Delta r = R(T/U_{max})^{1/2}$ can cross the beam boundary. Thus, under ordinary conditions ($T \lesssim 10^3$ K), the number of fast ions produced which leave the beam is negligible ($n_{fast}/n_{slow} \sim T/U_{max}$). The case in which $T \sim U_{max}$ is considered in the Appendix.

3. DISTRIBUTION FUNCTION AND DENSITY OF IONS IN THE BEAM

If the momentum of an ion on the axis is $p_{0\perp}^* < (\mu M R^2)^{1/2}$, the ion will execute oscillations inside the beam with a frequency $\omega_0 = (\mu/M)^{1/2}$ and amplitude $R_{max} = p_{0\perp}^*/\omega_0 M$. Here the longitudinal momentum increases in proportion to the time: $p_\parallel = p_{0\parallel} + \varkappa t$. Integrating Eq. (7), we obtain

$$f(r, p_\parallel, p_\perp, t) = \frac{n_a n_0 I_\tau}{(2\pi M T)^{3/2}} [p_\perp^2 + \mu M (r^2 - r_0^2) + (p_\parallel - \varkappa t)^2]^{1/2} \times \exp\{-[p_\perp^2 + \mu M (r^2 - r_0^2) + (p_\parallel - \varkappa t)^2]/2MT\} t, \tag{8}$$

where r_0 is the coordinate of the point of formation.

It is necessary to average the distribution function over r_0, taking into account the fact that the beam is homogeneous and the density of ions formed at any moment of time does not depend on r_0. Averaging leads to the following result:

$$f = \frac{n_a n_0 I_\tau t}{\pi^{1/2}(\varepsilon_\perp + \varepsilon_r)}\left[\gamma\left(3/2; \frac{\varepsilon_\perp + \varepsilon_r + \varepsilon_T}{\varepsilon_T}\right) - \gamma\left(3/2; \frac{\varepsilon_\parallel}{\varepsilon_T}\right)\right],$$
$$\varepsilon_\perp = p_\perp^2, \quad \varepsilon_\parallel = (p_\parallel - \varkappa t)^2, \quad \varepsilon_r = \mu M r^2, \tag{9}$$
$$\varepsilon_T = 2MT, \quad \gamma(p, x) = \int_0^x t^{p-1} e^{-t}\, dt.$$

The density of ions is

$$n_i = \frac{16}{3} n_a n_0 I_\tau t (MT)^{1/2} \left\{ \frac{2}{3}(4-\xi)(1-\xi)^{1/2} + \ln \xi + 2^{3/2} \ln[1 + 2(1-\xi)^{1/2}/(1-(1-\xi)^{1/2})] \right\}, \quad \xi = \varepsilon_T/\varepsilon_r. \tag{10}$$

For $\varepsilon_r \gg \varepsilon_T$, we can obtain the following expression for the density:

$$n_i = (8/3\pi) n_a n_0 I_\tau t (MT)^{1/2} \ln(R/r). \tag{11}$$

Thus, the ion density increases with increasing beam density and with the passage of time. This leads to a situation in which, beginning at some moment of time, the inverse effect of the ions on the beam becomes important. In addition, with passage of time the effect of the ions on the secondary electrons continues to increase: The compensation of the beam field has the result that electrons which left the beam in the initial stage of plasma formation begin to return to it. These effects can be discussed in taking into account the self-consistency of the problem posed.

APPENDIX

In the case of high temperatures ($T \sim U_{max}$) the gas is ionized to a certain degree. Therefore we must add to the solution (8) of Eq. (7) a distribution function for the primary ions in the field of the beam which does not vanish at $t = 0$:

$$\frac{N_i}{(2\pi M T)^{3/2}} \exp\left(-\frac{\varepsilon_\parallel}{2MT}\right) \exp\left(-\frac{\varepsilon_\perp}{2MT}\right), \tag{A.1}$$

where $\varepsilon_\perp = p_\perp^2 + \mu M r^2$, $\varepsilon_\parallel = (p_\parallel - \varkappa t)^2$. For primary ions outside the beam we must make the following substitution in Eq. (A.1):

$$\varepsilon_\perp = p_\perp^2 + \mu M R^2 + 2\mu M R^2 \ln(r/R)$$

(in accordance with Eq. (6)),

$$\varepsilon_\parallel = (p_\parallel - \varkappa t_0)^2.$$

The quantity t_0 is the time spent by the ion inside the beam. It can be determined by noting that the ion crosses the beam in a direction perpendicular to the beam axis in a time $\arcsin(\mu M R/p_{0\perp}^*)/\omega_0$, while the period spent by the ion outside the beam is

$$T = \frac{2^{1/2}}{\omega_0}[\bar{p}_{0\perp}/(2\mu M R^2)^{1/2} + \gamma(3/2; \bar{p}_{0\perp}^2/2\mu M R^2)], \tag{A.2}$$

where $\bar{p}_{0\perp}$ is the momentum at the beam boundary and $\gamma(p, x)$ is the incomplete gamma function:

$$\gamma(p,x)=\int_0^x e^{-t}t^{p-1}\,dt.$$

The expression for the distribution function of secondary ions (that is, ions produced by the beam) is only slightly changed. For slow ions Eq. (9) remains valid. For fast ions we must substitute $p_\perp^2+\mu M r^2$ for μMR^2 and t for t_0 in accordance with Eq. (A.2). The density of slow ions can be calculated by integration of their distribution function over $p_\perp$ up to $\mu M(R^2-r^2)$, and the density of fast ions can be calculated with this same integral taken to infinity. Outside the beam $\varkappa=0$ and the longitudinal momentum does not change. Since for $r>R$ we have $I_{st2}=0$, the value of the distribution function outside the beam is determined by its value at the boundary

$$f=\frac{n_a n_0 I_\tau t_0}{\pi^{3/2}\varepsilon_R}\left[\gamma\left(3/2;\frac{\varepsilon_\perp+\varepsilon_\parallel+\varepsilon_r}{\varepsilon_T}\right)-\gamma\left(3/2;\frac{\varepsilon_\perp+\varepsilon_\parallel+\varepsilon_r-\varepsilon_R}{\varepsilon_T}\right)\right],$$

$$\varepsilon_\perp=p_\perp^2,\quad \varepsilon_\parallel=(p_\parallel-\varkappa t_0)^2,\quad \varepsilon_r=\varepsilon_R+2\mu MR^2\ln(r/R),$$

where $\varepsilon_R=\mu MR^2$. The density of fast ions is

$$n_i=\frac{4(2MT)^{1/2}n_a n_0 I_\tau t_0}{\xi_1}\left[\frac{3}{2}\xi_1\rho^{3/2}-\frac{2^{5/2}}{3}\xi_1\xi_2^{3/2}\right.$$
$$\left.+\frac{1}{3}(\tau^{3/2}-2\xi_1^{3/2}-\sigma_1^{3/2}-\sigma_2^{3/2})-\frac{2}{15}(\tau^{5/2}-2\xi_1^{5/2}+\sigma_2^{5/2})\right],$$
$$\xi_1=\varepsilon_R/\varepsilon_T,\quad \xi_2=\varepsilon_r/\varepsilon_T,\quad \rho=1/2+\xi_2,\quad \tau=1/2+\xi_1-\xi_2,\quad \sigma_{1,2}=1/2-\xi_{1,2}.$$

[1] S. V. Antipov, M. V. Nezlin, E. N. Snezhkin, and A. S. Trubnikov, Zh. Eksp. Teor. Fiz. **65**, 1866 (1973) [Sov. Phys. JETP **38**, 931 (1974)].

[2] B. I. Davydov, Zh. Eksp. Teor. Fiz. **6**, 471 (1936).

[3] L. M. Kovrizhnykh, Zh. Eksp. Teor. Fiz. **37**, 490 (1959) [Sov. Phys. JETP **10**, 347 (1960)].

[4] V. L. Ginzburg and A. V. Gurevich, Usp. Fiz. Nauk **70**, 201 (1960) [Sov. Phys. Usp. **3**, 115 (1960)].

[5] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, 1974, Chapters 17 and 18. English transl. (earlier edition), Quantum Mechanics, Pergamon, London, 1959.

[6] V. P. Silin, Vvedenie v kinesticheskuyu teoriyu gazov (Introduction to the Kinetic Theory of Gases), Nauka, 1971, Chapter I.

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Stationary flows of a plasma through a magnetic barrier

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The possibility of plasma flow through a region in which strong magnetic fields perpendicular to the direction of motion are produced by external currents is discussed. It is shown that stationary flow of a cold plasma is possible even if the (dynamic) plasma pressure is much lower than the pressure of the external magnetic field, i.e. $\beta\ll 1$. The passage of the plasma through the barrier is caused by the fact that the plasma generates a diamagnetic current that destroys the external field, and the plasma moves in fact in a very weak magnetic field. The field in the plasma is concentrated in a very thin skin layer with a thickness of the order of $\delta\approx c/\omega_0$ where ω_0 is the plasma electron frequency and c the velocity of light in vacuum. The value of β required for the existence of a stationary flow with practically constant velocity depends on the shape of the barrier and may be very small for a smoothly shaped barrier: $\beta\approx(\delta/\Delta)^2$ where Δ is the width of the transition region in which the external field drops to zero, $\delta\ll\Delta$. An exact solution is obtained for the case of the most difficult passage of singular external currents. The distributions of the fields, currents, charges, plasma density, and velocity are calculated. The feasibility of establishment of such stationary flows is discussed.

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1. INTRODUCTION

The possibility of the motion of a plasma stream across a strong[1)] magnetic field was observed experimentally more than ten years ago.[1] The investigation of the features of plasma injection into a strong-field region has since been pursued quite intensively, as is evidenced by the large number of experimental papers.[2–6] In the interpretation of the extensive experimental material, the passage of the plasma into the region of the strong field is attributed to the presence of an E_y component of the electric field and the subsequent drift of the plasma in the crossed fields E and B. The main cause of the appearance of the E_y component is assumed to be polarization of the plasmoid boundaries as the plasma enters into the magnetic-field region. It appears that the presence of E_y is necessary for the initial stage of the plasma motion in the region of the strong magnetic field.

We present an extremely simplified scheme of the interaction of the plasma stream with a magnetic barrier, in the following form: The plasma stream moves along the x axis from the region $x\to-\infty$ into the region $x\to+\infty$. The magnetic barrier is chosen for simplicity in the form of a rectangle of width $2a$ and field intensity B_0. The field is directed along the positive z axis (Fig. 1a).