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Translated by J. G. Adashko

The sound velocity in superconductors

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(Submitted March 25, 1976)
Zh. Eksp. Teor. Fiz. 71, 1172–1179 (September 1976)

We consider the influence of non-linear effects, which arise when sound propagates in a superconductor, on the temperature dependence of the sound speed. We show that the experimentally observed change in this behavior with increasing sound wave amplitude can be explained if we invoke a specific heating of the electron gas. Moreover, we show that the minimum in the temperature dependence of the velocity of transverse sound remains also in the "dirty" limit, in contrast to the result obtained using the two-fluid model of a superconductor.

PACS numbers: 74.20.Ef, 74.50.Nf, 63.20.—e

Recent experiment on sound in superconductors[1] have revealed a non-linear amplitude behavior of the sound velocity. The characteristic dip in the temperature dependence of the velocity below T_c was shifted to the higher temperature region when the sound amplitude increased. It is well known[2] that the presence of such a dip is connected with the appearance of superconducting currents which screen the lattice sound flux and which contribute to the force acting on the lattice. The fields which in that case occur depend on the state of the electrons. We shall in the present paper, as in an earlier one,[3] consider the heating of the electron gas by the sound wave which leads to a non-linear amplitude dependence of the sound velocity.

A survey of papers on the study of the sound speed in superconductors can be found in the monograph by Geĭlikman and Kresin.[4] We note merely that we shall show that in the limit $kl \ll 1$, where l is the mean free path of an electron connected with the scattering by impurities, the temperature dependence of the sound speed in the linear approximation is appreciably different from what follows from the simple two-fluid model of superconductivity used by Ozaki and Mikoshiba.[2]

1. SET OF EQUATIONS FOR A TRANSVERSE SOUND WAVE

We shall consider a sound wave propagating along the z-axis and the displacement vector is u_x. When we change to a comoving system of coordinates in which the lattice is at rest we must add to the electron Hamiltonian a part[5]

$$\mathscr{H}'=ec^{-1}vA_xn_x-iku_xpvn_xn_z+i\omega pn_xu_x, \tag{1}$$

$\mathbf{n}$ is a unit vector directed along the momentum. The second term is the deformation potential, and the third one is caused by the Stewart-Tolman effect.

If we take into account the force exerted by the electrons we can write the equation of motion of the lattice in the form

$$(\omega^2-s_0^2k^2)u_x+f_\omega(k)/Mn=0, \tag{2}$$

where s_0 is the adiabatic sound velocity in a normal metal in agreement with Brovman and Kagan.[6] The volume force density f_ω refers to the laboratory system of coordinates. We shall assume u_x and A_x/c to be independent of the generalized coordinates[7] and the lattice force is then connected with the electron force through the simple relation:

$$f_x=-f_x^{\text{el}}=\frac{\partial\mathscr{H}'}{\partial u_x}+nm\omega^2u_x. \tag{3}$$

The second term on the right is the inertial force which must be taken into account when we change to the laboratory system of coordinates. We can express Eq. (3) in terms of the electron Green function:

$$f_\omega(k)=\frac{ip^3}{\pi^2v}\int\frac{d\varepsilon}{4\pi i}\int\frac{d^3p}{(2\pi)^3}(\omega-kvn_z)n_xG_{\varepsilon,\varepsilon-\omega}(p,p-k)+nm\omega^2u_x, \tag{4}$$

which in the case of a normal metal goes over into the expression obtained by Kontorovich.[8] Using the Maxwell equation $j_\omega = ck^2A_\omega/4\pi$ we can use the Gor'kov and Éliashberg[9] technique to get the current and force densities:

$$\frac{c}{4\pi}k^2A_\omega=\frac{ne^2}{mc}bA_\omega-ien\omega u_\omega(a-b),$$

$$j_\omega=ien\frac{\omega}{c}(a-b)\left(A_\omega+\frac{mc}{e}i\omega u_\omega\right)-\frac{i}{\tau}nmvkdu_\omega+nm\omega^2u_\omega;$$

$$a=\frac{3}{16}\frac{vk}{\omega}\int d\varepsilon\left(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\right)\int_{-1}^{1}x\varphi\,dx, \tag{5}$$

$$b=\frac{3}{16}\int d\varepsilon\left[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\xi_\varepsilon\xi_{\varepsilon-\omega}}-1\right]\int_{-1}^{1}\varphi\,dx,$$

$$d=\frac{3}{16}\int d\varepsilon\left[\frac{\varepsilon(\varepsilon-\omega)-\Delta^2}{\xi_\varepsilon\xi_{\varepsilon-\omega}}-1\right]\int_{-1}^{1}x\varphi\,dx,\quad \varphi=\frac{1-x^2}{\xi_\varepsilon+\xi_{\varepsilon-\omega}+vkx+i/\tau}.$$

The integrals over the frequency must be understood in the analytically continued form:

$$\int d\varepsilon\, f(\xi_\varepsilon\xi_{\varepsilon-\omega})\to\int d\varepsilon\left[\operatorname{th}\left(\frac{\varepsilon-\omega}{2T}\right)f(\xi_\varepsilon^R\xi_{\varepsilon-\omega}^R)\right.$$

$$\left.-\operatorname{th}\left(\frac{\varepsilon}{2T}\right)f(\xi_\varepsilon^A\xi_{\varepsilon-\omega}^A)+\left(\operatorname{th}\frac{\varepsilon}{2T}-\operatorname{th}\frac{\varepsilon-\omega}{2T}\right)f(\xi_\varepsilon^R\xi_{\varepsilon-\omega}^A)\right],$$

$$\xi_\varepsilon^R=-(\xi_\varepsilon^A)^*=\begin{cases}(\varepsilon^2-\Delta^2)^{1/2}\operatorname{sign}\varepsilon, & |\varepsilon|\geqslant\Delta.\\ i(\Delta^2-\varepsilon^2)^{1/2}, & |\varepsilon|<\Delta.\end{cases}$$

In deriving (5) we used the identities

$$\left(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\right)\left(\frac{\xi_\varepsilon+\xi_{\varepsilon-\omega}}{\omega}\right)^{\pm1}=1-\frac{\varepsilon(\varepsilon-\omega)\pm\Delta^2}{\xi_\varepsilon\xi_{\varepsilon-\omega}}.$$

In principle Eqs. (2) and (5) solve the problem of determining the damping and the change in the sound speed. We note that defining the damping as the imaginary part of the force f_ω is equivalent to the definition where the damping is proportional to the time-average of the quantity $-jc^{-1}\dot{A}-f\dot{u}$ and it is equivalent to the definition of the damping as the imaginary part of the polarization operator with, in its vertices, the quantity (1) where A is expressed in terms of u through the Maxwell equations. One can easily show[3] that if we define the damping in this way it is proportional to the energy emitted by the electron system in the form of thermal phonons.

2. TRANSVERSE SOUND VELOCITY IN A "PURE" SUPERCONDUCTOR

We consider the case $kl\ll 1$. The force is then simply

$$f_\omega=enE_\omega, \tag{6}$$

as $a-b\equiv 1$. There is here no part of the force connected with the direct transfer of momentum by the lattice to the electrons, provided the velocity of the latter relative to the lattice does not vanish. This is a consequence of the fact that we assume the electron spectrum to be the vacuum one and that there are no impurities. Similarly the expression for the electron current in the comoving system, which is the same as the total current (the sum of the electron and the ion currents) in the laboratory is

$$j_\omega=\frac{ck^2}{4\pi^2}A_\omega=\frac{ne^2}{mc}bA_\omega-i\omega enu_\omega. \tag{7}$$

Assuming the sound wave to be weakly damped $k\to k+i\varkappa$ and taking the variation of the dispersion Eq. (2) we get[2]

$$\frac{\varkappa}{k}=\frac{1}{2\omega^2Mn}\frac{\operatorname{Im}f}{u},$$

$$\frac{s-s_0}{s_0}=-\frac{1}{2Mn\omega^2}\frac{\operatorname{Re}f}{u}-\frac{1}{2}\left(\frac{1}{2Mn\omega^2}\frac{\operatorname{Im}f}{u}\right)^2. \tag{8}$$

In the pure case we can neglect the square of the imaginary part

$$\frac{s-s_0}{s_0}=-\frac{m}{2M}\operatorname{Re}\frac{1}{h-b},\quad h=\left(\frac{kc}{\omega_p}\right)^2,\quad \omega_p^2=\frac{4\pi ne^2}{m}. \tag{9}$$

The quantity b is connected with the conductivity: $b=i\omega m\sigma/ne^2$.

A. We consider first of all the case of relatively low frequencies:

$$\omega\ll\Delta(s/v)^2. \tag{10}$$

As the sound wave leads to a heating of the electrons we must, as was shown by Éliashberg,[10] make in Eqs. (5) for the kinetic coefficients the substitution: $\tanh(\varepsilon/2T)\to 1-2n_F(\varepsilon)-2n'_\varepsilon$, where n'_ε is the non-equilibrium correction to the distribution function which must be determined from the kinetic equation.[3] Bearing this in mind

$$b=-\frac{7\zeta(3)}{4\pi^2}\left(\frac{\Delta}{T}\right)^2+\frac{3\pi i}{4}\frac{\omega}{vk}$$

$$-\frac{1}{2}\int_\Delta^\infty\frac{\varepsilon\,d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\frac{\partial n'}{\partial\varepsilon}+\frac{3}{2}\omega\int_\Delta^\infty\frac{\varepsilon^2\,d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\frac{\partial n'}{\partial\varepsilon}\int_{-1}^{1}\frac{(1-x^2)\,dx}{vkx(\varepsilon^2-\Delta^2)^{1/2}+\omega\varepsilon}, \tag{11}$$

the third term causes a static correction to the Meissner penetration depth in the non-equilibrium case.

At this stage it is necessary to choose a concrete form of the non-equilibrium distribution function. We shall assume that the characteristic scale ε_0 of the spread in n'_ε in energy in the low-frequency limit (10) satisfies $\Delta(s/v)^2\ll\varepsilon_0\ll\Delta$ and then[3]

$$n_\varepsilon'=\frac{\varepsilon_0}{T}f\left(\frac{\varepsilon-\Delta}{\varepsilon_0}\right),\quad \varepsilon_0=\Delta\left(\frac{\alpha\omega^2}{\Delta^2}\right)^{2/5},$$

$$\alpha=\frac{2}{9\pi}\frac{(kvpu_\omega)^2}{\gamma vk}\left[1+\left(\frac{7\zeta(3)}{3\pi^2}\frac{\Delta^2}{T^2}\frac{kv}{\omega}\right)^2\right]^{-1}.$$

The quantity γ is the reciprocal of the inelastic collision time T^3/ω_D^2. We then get

$$b=-\frac{7\zeta(3)}{4\pi^2}\left(\frac{\Delta}{T}\right)^2-\frac{3}{4}\frac{\Delta}{T}\delta+\frac{3}{4}\pi i\frac{\omega}{vk}\left(1-\frac{\Delta}{T}\beta\right). \tag{12}$$

We neglected here a small equilibrium superconducting correction to the imaginary part.

Under the above-mentioned limits large x are important in the integrals so that we can estimate them:

$$\delta=c_1\left(\frac{\varepsilon_0}{\Delta}\right)^{1/2},$$

$$\beta=c_2\ln\left[\frac{\varepsilon_0}{\Delta}\left(\frac{vk}{\omega}\right)^2\right],\quad c_1\sim c_2\sim 1.$$

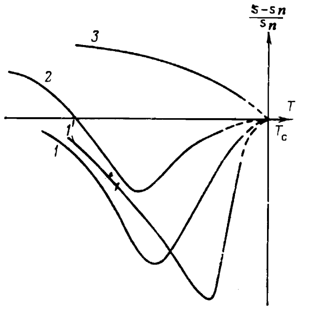

FIG. 1.

Using (8) we get the change in the sound speed:

$$\frac{s-s_n}{s_n}=-\frac{2}{3\pi}\frac{v}{s}\frac{m}{M}\left(1+\beta\frac{\Delta}{T}\right)\frac{B}{1+B^2},$$
$$B=\frac{vk}{\pi\omega}\left[\frac{7\zeta(3)}{3\pi^2}\left(\frac{\Delta}{T}\right)^2+\frac{\Delta}{T}\delta\right], \tag{13}$$

s_n is the sound speed in the normal metal. At equilibrium $\beta=c_1=0$ and we get the well known behavior (curve 1 in Fig. 1).[2]

It is clear from (13) that the minimum in the temperature behavior of the sound speed becomes deeper and is shifted into the region of higher temperatures. The magnitude of this shift is given by the condition

$$\frac{28}{3\pi^2}\zeta(3)\left(\frac{\Delta}{T}\right)^2+c_1\left(\frac{\Delta}{T}\right)^{3/5}\left(\frac{\omega}{T}\right)^{2/5}\alpha^{1/5}=\pi\frac{\omega}{vk}. \tag{14}$$

For not too large sound intensities the second term on the left-hand side plays the role of a correction. On the other hand, in our case the intensity must not be too small in order that the excitations are pushed out in energy by an amount $\varepsilon_0\gg\Delta(s/v)^2$. Combining the two conditions we find that when

$$\left(\frac{T}{\omega}\right)^2\left(\frac{s}{v}\right)^6\ll\alpha\ll\left(\frac{T}{\omega}\right)^2\left(\frac{s}{v}\right)^{7/2} \tag{15}$$

the shift in the minimum in the temperature dependence of the sound speed is

$$\frac{\delta T}{T}\sim\alpha^{1/5}\left(\frac{\omega}{T}\right)^{2/5}\left(\frac{s}{v}\right)^{3/10},$$

while the relative increase in the minimum itself is

$$\left(\frac{s}{v}\right)^{1/2}\ln\left[\alpha\left(\frac{\omega}{T}\right)^2\left(\frac{v}{s}\right)^6\right]$$

(curve 1′ in the figure). The non-linearity parameter α can be expressed in terms of the sound energy flux density W which is incident upon the sample; $\alpha\approx\varepsilon_F^2W/\gamma vknMs^3$. For a sound frequency $\omega\sim10^7\ \mathrm{s}^{-1}$ used in the experiments[1] our considerations are valid if α lies within the range (15), $10^{-2}\ll\alpha\ll10^3$. For a flux density of the order of $10^{-2}\ W/\mathrm{cm}^2$ the parameter $\alpha\sim10^2$ and the relative shift of the minimum $\delta T/T\sim10^{-2}$ and the relative increase in its minimum is of the order of 10^{-1}. This picture agrees qualitatively with the experimental data.[1]

Let now the sound wave amplitude be rather small $\alpha\ll s/v$ so that the non-linearity is weak and the correction to the distribution function is quadratic in the sound wave amplitude:[3]

$$n_\varepsilon'=\frac{\alpha\omega}{4T}\frac{\xi_\varepsilon}{\varepsilon}(\psi_{-\omega}-\psi_\omega),$$
$$\psi_\omega=\left[\frac{\varepsilon(\varepsilon+\omega)+\Delta^2}{\xi_\varepsilon\xi_{\varepsilon+\omega}}+1\right]\left[1-\left(\frac{\xi_\varepsilon-\xi_{\varepsilon+\omega}}{vk}\right)^2\right]\theta\left[1-\left(\frac{\xi_\varepsilon-\xi_{\varepsilon+\omega}}{vk}\right)^2\right], \tag{13a}$$

the θ-function arises in the low-frequency limit $\omega\ll\Delta(s/v)^2$ due to the Landau condition $\mathbf{v}\cdot\mathbf{k}=\xi_\varepsilon-\xi_{\varepsilon+\omega}$.

When the sound speed changes Eq. (13) retains its form with the parameters

$$\delta=\frac{10}{9}\alpha\left(\frac{\omega}{\Delta}\right)^2\left(\frac{v}{s}\right)^4,\quad\beta=\frac{16}{105}\alpha\frac{\omega}{\Delta}\left(\frac{v}{s}\right)^2. \tag{13b}$$

Hence it is clear that for small sound amplitudes $\alpha\ll s/v$ the effect is insignificant.

B. In the other limiting case as far as the frequencies are concerned, $\omega\ll\Delta$ and $T\ll vk$, at equilibrium with $\Delta\ll T$ the function b which is proportional to the conductivity equals

$$b=-\frac{3\pi^2}{8}\frac{\Delta^2}{Tvk}+\frac{3\pi i}{4}\frac{\omega}{vk}\left(1+\frac{\Delta}{2T}\ln\frac{8\Delta}{e\omega}\right)-16\left(\frac{\omega}{vk}\right)^2$$

and in agreement with Ozaki and Mikoshiba[2] the relative change in the sound speed as compared with the normal state has the form

$$\frac{s-s_n}{s_n}=-\frac{2}{3\pi}\frac{m}{M}\frac{v}{s}\left[\frac{h'+B_0}{(h'+B_0)^2+1}-\frac{h'}{h'^2+1}\right]$$
$$h'=\frac{4}{3\pi}\frac{v}{s}\left[h+3\left(\frac{\omega}{vk}\right)^2\right]. \tag{16}$$

When $h\ll s/v$ the behavior reminds one of (13) (curve 1 in the figure)

$$\frac{s-s_n}{s_n}=-\frac{2}{3\pi}\frac{m}{M}\frac{v}{s}\frac{B_0}{1+B_0^2},\quad B_0=\frac{\pi}{2}\frac{\Delta^2}{T\omega}, \tag{17}$$

while for $s/v\ll h$ (curve 3 in the figure) we have

$$\frac{s-s_n}{s_n}=\frac{1}{2}\frac{m}{M}\frac{1}{h}\frac{B_0}{(B_0+4vh/3\pi s)}. \tag{18}$$

Curve 2 in the figure is constructed for wavelengths of the order of the skin depth when the frequency $\omega\sim10^9$ to $10^{10}\ \mathrm{s}^{-1}$.

The electron current which tends to screen the lattice current flows in the opposite direction to it and by virtue of the specific dispersion in the current kernel which is diamagnetic in character there occurs an electric field proportional to u. The sign of that field is such that it diminishes the elastic force acting on the lattice and this leads to some softening as compared to the adiabatic situation in a normal metal. In the superconducting state the dispersion in the current response to the vector potential is appreciable and this leads to a large renormalization of the sound speed in (13) and (17): $|(s-s_n)/s_n|\sim s/v$. When the temperature is lowered the diamagnetism of the electron system is strengthened and the magnitude of the screened field decreases. Far from T_c the sound speed thereby reaches its adiabatic value

$$\frac{s-s_n}{s_n}=\frac{1}{2}\frac{m}{M}\frac{1}{h}$$

for a normal metal. For a comparison we point out that the adiabatic correction to the elastic modulus in a superconductor causes a change in the sound speed $(s - s_n)/s_n \sim (\Delta/\varepsilon_F)^2$, [11] which gives a considerably smaller contribution than the effects considered above.

Taking the heating of the electron gas into account

$$b=-\frac{3\pi^2}{8}\frac{\Delta^2}{Tvk}-3\left(\frac{\omega}{vk}\right)^2+\frac{3\pi}{2vk}\int_{\Delta}^{\Delta+\omega}\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{(\varepsilon^2-\Delta^2)^{1/2}[\Delta^2-(\varepsilon-\omega)^2]^{1/2}}n_\varepsilon'\,d\varepsilon$$

$$+\frac{3\pi i}{4}\frac{\omega}{vk}\left(1+\frac{\Delta}{2T}\ln\frac{8\Delta}{e\omega}\right)-\frac{3\pi i}{2vk}\int_{\Delta+\omega}^{\infty}\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{(\varepsilon^2-\Delta^2)^{1/2}[(\varepsilon-\omega)^2-\Delta^2]^{1/2}}(n_\varepsilon'-n_{\varepsilon-\omega}')\,d\varepsilon. \tag{19}$$

In the approximation linear in the intensity $(\alpha \ll (\omega/\Delta)^{1/2})$[12] we have

$$n_\varepsilon'=\frac{\alpha\omega}{4T\varepsilon}\left\{\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{[(\varepsilon-\omega)^2-\Delta^2]^{1/2}}\theta(\varepsilon-\omega-\Delta)-\frac{\varepsilon(\varepsilon+\omega)+\Delta^2}{[(\varepsilon+\omega)^2-\Delta^2]^{1/2}}\theta(\varepsilon-\Delta)\right\},$$
$$\alpha=\frac{2}{9\pi}\frac{(vkpu_\omega)^2}{\gamma vk}\left[1+\left(\frac{\pi}{2}\frac{\Delta}{T\omega}\right)^2\right]^{-1}. \tag{20}$$

Substituting (19) and (20) into (9) we get the change in the sound speed:

$$\frac{s-s_n}{s_n}=-\frac{2}{3\pi}\frac{m}{M}\frac{v}{s}\left(1+\alpha\left(\frac{\Delta}{2\omega}\right)^{1/2}\frac{\Delta}{T}\ln\frac{\omega}{\Delta\alpha^2}\right)\frac{B}{1+B^2},$$
$$B=\frac{\pi}{2}\frac{\Delta}{T\omega}\left[1+\frac{\Gamma^2(1/4)}{2\pi^{3/2}}\alpha\left(\frac{\omega}{\Delta}\right)^{1/2}+\alpha\left(\frac{\Delta}{2\omega}\right)^{1/2}\frac{\Delta}{T}\ln\frac{\omega}{\Delta\alpha^2}\right]. \tag{21}$$

Here, as in (13), the non-linear change in the London penetration depth leads to a shift of the minimum in the temperature dependence of the sound speed to the region of higher temperatures. The change in the real part of the conductivity entails a deepening of the minimum.

We neglect the change in the order parameter under the action of the sound which also can serve as a source for non-linearity:

$$1=g\int_{\Delta}^{\omega_D}\frac{1-2n(\varepsilon)}{(\varepsilon^2-\Delta^2)^{1/2}}d\varepsilon.$$

We have shown earlier[3] that this non-linearity is important in a narrower temperature range near T_c:

$$(T_c-T)/T\sim(\omega/T)^2.$$

3. TRANSVERSE SOUND VELOCITY IN AN "IMPURE" SUPERCONDUCTOR

We consider the "impure" limit $\omega \ll \Delta \ll T \ll 1/\tau$, $kl \ll 1$. If in a pure metal the force acting upon the lattice was entirely electromagnetic in origin, in the impure limit due to the scattering by impurities there appears additionally a force connected with the direct transfer of lattice momentum to the electrons which does not vanish for $E = 0$. To evaluate this force we must again use Eqs. (5) while it is advisable to split the force into its electromagnetic f^e and its deformation f^d parts:

$$f_\omega^e=nm\omega^2u\frac{(a-b)^2}{h-b};$$
$$f_\omega^d=nm\omega^2u_\omega(1+b-a)-\frac{i}{\tau}nmkvdu_\omega. \tag{22}$$

In the impure case calculations give from Eqs. (5)

$$a=\frac{(kl)^2}{5},\quad b=-\frac{\pi}{2}\frac{\Delta^2\tau}{T}+i\omega\tau,$$
$$d=\frac{i\pi}{160}\frac{\omega vk}{\Delta T}(\omega\tau)^2-\frac{1}{5}kl\omega\tau\left(1-\frac{\Delta}{2T}\right).$$

For the sound speed we have correspondingly

$$\frac{s-s_n}{s_n}=-\frac{m}{4M}\frac{\pi\Delta^2\tau}{T}\left[\frac{(kl)^2}{5}-h\right]^2\left[(\omega\tau)^2-h^2-h\frac{\pi\Delta^2\tau}{2T}\right]$$
$$\times\left\{\left[\left(h+\frac{\pi}{2}\frac{\Delta^2\tau}{T}\right)^2+(\omega\tau)^2\right][h^2+(\omega\tau)^2]\right\}^{-1}-\frac{\pi}{320}\frac{m}{M}\frac{v}{s}kl\frac{\omega^2}{\Delta T}. \tag{23}$$

The last term is the deformation contribution to the change in the sound speed.

In contrast to the case of absorption the electromagnetic mechanism for the change in the velocity dominates in all regions of practical interest. For $h \ll \Delta^2\tau/T \sim \omega\tau$ we have

$$\frac{s-s_n}{s_n}=-\frac{m}{M}\frac{v}{s}\frac{(kl)^3}{50}\frac{B_0}{1+B_0^2}, \tag{24}$$

which agrees qualitatively with the pure case (17) (curve 1 in the figure) except for a decrease in the magnitude of the effect by a factor $3\pi(kl)^3/100$ which makes it impossible to observe the effect for sufficiently small kl. However, the results obtained show that as the impurity of the sample is increased (or the frequency decreased) the minimum in the temperature dependence of the sound speed below T_c does not vanish up to the limit of the resolution of the experiment. The calculations by Ozaki and Mikoshiba[2] indicated the vanishing of the minimum for $(s - s_n)/s_n \sim 10^{-5}$ but they were based upon the simple two-fluid model of superconductivity which in the impure limit gives an incorrect result.

In the normal metal the force (5) acting on the lattice can be written in the form[8]

$$f=en(E-j/\sigma),\quad \sigma=ne^2\tau/m, \tag{25}$$

where j is the current of the electrons relative to the lattice, i.e., in the comoving system of coordinates. In a superconductor the expression for the force can be reduced to a form resembling (25) with the normal current instead of j. The heating effects in an impure superconductor which arise when a sound wave passes through it are small for the reason that the absorption is then basically due to the deformation while the scalar potential causes a smaller change in the distribution function due to the scalar coherence factor. Moreover, the effect itself decreases by a factor $kl \ll 1$.

We make a remark about the longitudinal sound velocity. In that case we must introduce two potentials—a scalar and a longitudinal vector one—which are determined from the electrical neutrality condition. In the pure case the force acting on the lattice is, as before,

$$j = enE = (-c^{-1}\dot{A} - \nabla\varphi)en.$$

For the high-frequency limit $vk \gg T$ one can show that the role of the vector potential is unimportant and the corrections to the sound speed are given by the usual polarization operator:

$$\frac{s-s_n}{s_n} = \frac{\pi}{12}\frac{m}{M}\frac{v}{s}\int_{\eta}^{\Delta+\omega}\frac{\varepsilon(\varepsilon-\omega)-\Delta^2}{(\varepsilon^2-\Delta^2)^{1/2}[\Delta^2-(\varepsilon-\omega)^2]^{1/2}}\frac{\varepsilon}{2T}\frac{d\varepsilon}{\omega},$$
$$\eta = \Delta + (\omega-2\Delta)\theta(\omega-2\Delta); \qquad (26)$$

in the limiting cases

$$\frac{s-s_n}{s_n} = -\frac{\pi^2}{48}\frac{m}{M}\frac{v}{s}\frac{\Delta}{T}\begin{cases}{}^1/_8(\omega/\Delta)^2, & \omega\ll\Delta\\ \Delta/\omega, & \Delta\ll\omega\end{cases}. \qquad (27)$$

Hence it is clear that in the high-frequency limit $vk \gg T$ the temperature dependence of the longitudinal sound speed has a minimum for $(T_c - T)/T_c \sim (\omega/T)^2$ with a relative depth of the order $s\omega/vT$.

We are grateful to G. M. Éliashberg for discussions of the problems mentioned here.

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Translated by D. ter Haar

Energy absorption and the size effect in solid helium

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(Submitted March 26, 1976)
Zh. Eksp. Teor. Fiz. 71, 1180–1191 (September 1976)

Owing to the quantum nature of the diffusion of point defects (defectons) in quantum crystals such as solid helium, energy is dissipated at the substitution defects even in the case of spatially homogeneous deformation. The absorption is due to diffusion flow of the defectons in momentum space in the absence of particle fluxes in coordinate space. Internal friction is connected with the deformation potential at low frequencies and with the inertial and activation components of the energy spectrum at high frequencies. The collision integral within the crystal is determined by quasielastic defecton–phonon scattering. It is found that the law of interaction between the vacancions and the crystal surface may be determined at low temperatures. The dissipation accompanying interaction between the defectons and the surface is determined. Diffusion-viscous flow in quantum crystals is discussed.

PACS numbers: 67.80.Mg

1. INTRODUCTION

Internal friction is one of the characteristic manifestations of the diffusion properties of point defects in crystals. It is customarily assumed that in the case of spatially-homogeneous deformation of the crystal there is no energy dissipation at substitution defects, e.g., vacancies. The reason is that in the case of uniform deformation all the crystal lattice sites remain equivalent and no particle diffusion fluxes that lead to the absorption of energy are produced (the force acting on a point defect is proportional to the gradient of the deformation).

This reasoning is not applicable to quantum crystals of the type of solid helium with large zero-point vibration amplitudes. In such crystals[1,2] the point defects are localized and are transformed into quasiparticles—defectons—which move practically freely through the crystal. The correct quantum numbers for defectons are the values of the quasimomenta and not of the coordinates. When the crystal is deformed, the energy spectrum of the defectons is altered. As a result, even in the case of uniform deformation, diffusion fluxes of particles are produced in momentum space. Such a diffusion leads to dissipation of the energy also in the absence of particle fluxes in configuration space.

For this reason, it would be of great interest to investigate experimentally the internal friction at low-frequency compression deformations (at high frequencies the spatial inhomogeneity of the vibrations, which is connected with the finite speed of sound, is apprecia-

