of the maximum increments.

The author is grateful to Academician P. L. Kapitza for interest in the work and to L. P. Pitaevskiĭ for suggesting the problem and valuable comments.

[1)]A short exposition of the results for this case is contained in the work of Pitaevskiĭ and the author.[3] We note that a different normalization of the flow velocities is used in this case.

[1]L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Mechanics of Continuous Media, 2nd edition) Gostekhizdat, 1954.

[2]A. V. Gurevich, L. V. Pariiskaya, and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. 54, 891 (1968) [Sov. Phys. JETP 27, 476 (1968)].

[3]A. P. Mescherkin and L. P. Pitaevskiĭ, Proc. XI Intern. Conf. on Phenomena in Ionized Gases, Prague, 1973.

[4]J. Alexeff, K. Estabrook, and M. Widner, Phys. Fluids 14, 2355 (1971).

[5]A. V. Gurevich, Zh. Eksp. Teor. Fiz. 53, 953 (1967) [Sov. Phys. JETP 26, 575 (1968)].

[6]R. A. Morse, Phys. Fluids 8, 308 (1965).

Translated by R. T. Beyer

Frequency of inelastic collisions in plasma

A. V. Vinogradov and V. P. Shevel'ko

P. N. Lebedev Physics Institute, USSR Academy of Sciences, Moscow

(Submitted February 26, 1976)
Zh. Eksp. Teor. Fiz. 71, 1037–1044 (September 1976)

An analysis is given of the influence of polarization plasma effects on the frequency of inelastic atomic transitions induced by the combined electron and ion fields. The inelastic collision frequency W is expressed in terms of the Born matrix element and the longitudinal component of the plasma permittivity at the frequency $\omega_0=\Delta E/h$ of the atomic transition (ΔE is the transition energy). For low-density plasma, the frequency W is equal to $N\langle v\sigma_{01}\rangle$, where σ_{01} is the cross section for the excitation of the particular transition, averaged over the velocity distribution function of the exciting charged particles in the plasma, and N is the density of the particles. The dependence of collision frequency on plasma temperature and density is investigated in detail for optically allowed transitions with a small resonance defect ω_0. It is shown that polarization effects play an important role at temperatures for which the main contribution to the inelastic collision frequency is due to electron-atom collisions. Whilst inelastic transitions in the plasma are largely due to interactions with ions, polarization effects are not appreciable, even at high densities that are close to those of a solid body. Conditions are formulated for the validity of first-order perturbation theory as applied to the frequency of inelastic transitions with a small resonance defect.

PACS numbers: 52.20.Hv

The polarization properties of plasma modify the character of interactions between charged particles. The spectroscopic consequences of this effect have been discussed in connection with the emission of forbidden spectral lines[1] and the theory of broadening of spectral lines in plasma.[2] By polarization properties, we understand two physically clear effects, namely, the screening of Coulomb forces and the interaction between charged particles and plasma oscillations. It is obvious that both these effects should have an important influence on the rate of relaxation of excited states in dense plasma. The present paper is concerned with this question. By dense plasma, we shall understand plasma in which the Langmuir frequency exceeds the frequency of the atomic transition. This condition is realized, for example, for transitions between highly excited atomic states and for a number of transitions between the energy levels of multiply-charged ions in dense laser plasma.

1. FORMULATION OF THE PROBLEM

In low-density plasma, the frequency of inelastic transitions between atomic levels 0 and 1 due to collisions with electrons is given by

$$\nu=N\langle v\sigma_{01}\rangle=\int v\sigma_{01}(v)F(\mathbf{v})\,d\mathbf{v}, \tag{1}$$

where $F(\mathbf{v})$ is the electron velocity distribution function, σ_{01} is the cross section for the excitation of the 0–1 transition, and N is the electron density.

In dense plasma, the interaction between an atom and electrons can no longer be looked upon as the result of successive independent collisions. An incident electron interacts with the ambient electrons and ions, and induces a dipole moment in the plasma. This means that the resultant field acting on the atom is made up of fields produced by the electron and by the dipole moment induced by it in the plasma.

Let us therefore consider a more general formulation of the problem. Suppose that the atom is located in a random field $V(\mathbf{r}, t)$ due to all the charged particles in the plasma. In first-order perturbation theory, the probability of a transition from state 0 to state 1 at time t is then given by the expression[1)]

$$|a(t)|^2=\left|\int_{-\infty}^{t} e^{\gamma t'}V_{10}(t')e^{i\omega_0 t'}dt'\right|^2 \qquad \gamma\to 0, \tag{2}$$

$$V_{10}(t)=\langle 1|V(\mathbf{r},t)|0\rangle=\int \Psi_1^*(\mathbf{r})V(\mathbf{r},t)\Psi_0(\mathbf{r})d\mathbf{r}, \tag{3}$$

where ω_0 is the 0–1 transition energy and $\Psi(\mathbf{r})$ are the atomic wave functions (see[3]).

Substituting (3) in (2), and averaging over all the possible realizations of the random field $V(\mathbf{r}, t)$, we obtain

$$\langle|a(t)|^2\rangle=\int_{-\infty}^{\infty}d\omega\frac{e^{2\gamma t}}{(\omega-\omega_0)^2+\gamma^2}\int d\mathbf{q}|A(\mathbf{q})|^2V^2(\mathbf{q},\omega), \qquad \gamma\to 0, \tag{4}$$

where $A(\mathbf{q})$ is the Born transition matrix element:

$$A(\mathbf{q})=\int\Psi_1^*(\mathbf{r})\Psi_0(\mathbf{r})e^{i\mathbf{q}\mathbf{r}}d\mathbf{r}, \tag{5}$$

and $V^2(\mathbf{q}, \omega)$ is the spectral density of potential fluctuations:

$$\langle V(\mathbf{r},t)V(\mathbf{r}',t')\rangle=\int d\mathbf{q}\,d\omega\exp\{i\mathbf{q}(\mathbf{r}-\mathbf{r}')-i\omega(t-t')\}V^2(\mathbf{q},\omega). \tag{6}$$

The frequency of inelastic 0–1 transitions, i.e., the transition probability per unit time, is obtained by differentiating (4) with respect to t:

$$W=\lim_{\gamma\to 0}\frac{d}{dt}|a(t)|^2=2\pi\int|A(\mathbf{q})|^2V^2(\mathbf{q},\omega_0)d\mathbf{q}. \tag{7}$$

We note that the expression given by (7) can also be obtained by simplifying the kernels of the general kinetic equations for partially ionized plasma (see[4], Chap. 14). In particular, similar expressions were obtained in[2] for the nondiagonal pole-type potential $\zeta(r)=\lambda/R^2$.

Before we consider polarization effects, let us show that, in the case of low-density plasma, the expression given by (7) is equal to the frequency of inelastic collisions given by (1). In fact, in low-density plasma, $V^2(\mathbf{q}, \omega)$ is the spectral density of fluctuations in the electric potential of an ideal electron gas (see, for example, [5], p. 541):

$$V^2(\mathbf{q},\omega)=\frac{2N}{\pi q^4}\int F(\mathbf{v})\delta(\omega-\mathbf{q}\mathbf{v})d\mathbf{v}. \tag{8}$$

Substituting (8) in (7), we obtain

$$W=N\int v\left[\frac{4}{v}\int\frac{d\mathbf{q}}{q^4}|A(\mathbf{q})|^2\delta(\omega_0-\mathbf{q}\mathbf{v})\right]F(\mathbf{v})d\mathbf{v}. \tag{9}$$

It is shown in[6] that the expression in brackets in (9) is the quasiclassical Born approximation to the excitation cross section for the 0–1 transition.[2)] Thus, the expressions given by (1) and (7) yield the same result in the case of low-density plasma, i.e., in the absence of plasma effects. Only the form of these expressions is different: while in (1) the frequency of the inelastic transition is given in the form of a cross section averaged over the Maxwellian electron distribution, in (7) it is presented in the form of an integral over the wave vectors of the spectral density of fluctuations in the plasma potential at frequency ω_0.

In dense plasma, when Debye screening and the interaction of the atom with plasma waves must be taken into account, the expression given by (8) must be divided by $|\varepsilon(\mathbf{q}, \omega)|^2$, where $\varepsilon(\mathbf{q}, \omega)$ is the longitudinal permittivity of the plasma. In particular, in plasma with the Maxwellian electron velocity distribution, we have[5] (p. 531):

$$V^2(q,\omega)=\frac{\sqrt{2}N}{\pi^{3/2}q^5v_T^{1/2}}|\varepsilon(\beta,qv_T/\omega)|^{-2}\exp\left(-\frac{\omega^2}{2(qv_T)^2}\right), \tag{10}$$

$$\varepsilon(\beta,x)=1-\beta\Phi(x)+\frac{i\beta}{x}\sqrt{\frac{\pi}{2}}\exp\left(-\frac{1}{2x^2}\right), \tag{11}$$

where

$$x=qv_T/\omega_0, \quad \beta=\omega_L^2/\omega_0^2, \quad v_T=\sqrt{T/m},$$

T is the electron temperature, $\omega_L=(4\pi Ne^2/m)^{1/2}$ is the Langmuir frequency, and $\Phi(x)$ is the plasma dispersion function:

$$\Phi(x)=\begin{cases}1+3x^2+15x^4+\dots, & x\ll 1,\\ x^{-2}+x^{-4}+\dots & x\gg 1.\end{cases} \tag{12}$$

Substituting (10) in (7), and summing $|A(\mathbf{q})|^2$ over the magnetic M_0 and M_1 quantum numbers of the atom, we obtain the following expression for the frequency of inelastic collisions, which takes plasma polarization into account:

$$W=8\sqrt{2\pi}\frac{N}{v_T}\int_0^{\infty}\frac{dq}{q^3}\varphi(q)|\varepsilon(\beta,qv_T/\omega_0)|^{-2}\exp\left(-\frac{\omega_0^2}{2(qv_T)^2}\right), \tag{13}$$

$$\varphi(q)=\frac{1}{2l_0+1}\sum_{M_0,M_1}|A_{M_0M_1}(\mathbf{q})|^2=\frac{1}{2l_0+1}\sum_{M_0,M_1}|\langle M_1|\exp(i\mathbf{q}\mathbf{r})|M_0\rangle|^2. \tag{14}$$

In the case of low-density plasma ($\beta=\omega_L^2/\omega_0^2\ll 1$), we have $\varepsilon(\beta, qv_T/0)\approx 1$, and the formula given by (13) yields the result given by (1). However, even in this case, the form in which (13) is written is more convenient because it enables us to eliminate the integration of the cross section over the velocities, which must usually be carried out numerically.

Thus, allowance for polarization effects reduces to the appearance of the factor $|\varepsilon(\beta, qv_T/\omega_0)|^{-2}$ in front of the atomic formfactor $\varphi(q)$. The problem of inelastic electron-atom collisions is then analogous to the scattering of light by fluctuations in electron density in plasma (see, for example, [5]) and the problem of elastic collisions between charged particles (this follows from the Lennard–Balescu equation[8]).

2. EXCITATION BY ELECTRONS

We now consider the role of polarization effects by examining the example of inelastic optically allowed atomic transitions with low excitation energy ω_0. It is precisely these transitions that are particularly interesting from the point of view of applications because they are the main reason for the broadening of spectral lines and determine the lifetime of metastable states in plasma. It was shown previously in[9] that the Born matrix element for such transitions could be written in the form

$$\varphi(q)=\lambda^2R_0^2q^4K_1^2(qR_0), \tag{15}$$

where K_1 is the Macdonald function, $\lambda=(f/2\omega_0)^{1/2}$, and f is the oscillator strength of the 0–1 transition. The quantity R_0 has a simple physical interpretation. The matrix element (15) corresponds to a nondiagonal dipole-type interaction potential between the atom and an electron:

$$\zeta(R)=\int P_0(r)P_1(r)\frac{r_<}{r_>^2}r^2\,dr=\frac{\lambda R}{(R^2+R_0^2)^{3/2}}, \tag{16}$$

where $P(r)$ is the radial atomic function, and $r_<$ and $r_>$ are, respectively, the smaller and larger of the two quantities r and R.

Thus, R_0 represents the radius of the interaction inducing the inelastic transition. In the case of transitions without a change in the quantum number n, and when hydrogen-like wave functions are employed, the characteristic radius is $R_0=n^2$. In general, however, R_0 must be expressed in terms of integrals of the atomic radial wave functions $P_0(r)$ and $P_1(r)$. Substituting (15) in (13), and assuming that $\varepsilon(q,\omega)=1$, we obtain the following expression for the frequency of inelastic transitions in low-density plasma:

$$W=N\frac{8\sqrt{2\pi}\lambda^2R_0^2}{v_T}\int_0^\infty qK_1^2(qR_0)\exp\left(-\frac{\omega_0^2}{2(qv_T)^2}\right)dq=N\frac{8\sqrt{2\pi}\lambda^2}{\omega_0R_0}f(\alpha), \tag{17}$$

$$f(\alpha)=\frac{1}{\alpha^3}\int_0^\infty\frac{dx}{x^3}e^{-x^2/2}K_1^2\left(\frac{1}{\alpha x}\right),\quad \alpha=\frac{v_T}{\omega_0R_0}. \tag{18}$$

The formula given by (17) leads to good agreement with the results of calculations based on numerical wave functions,[9] and we shall therefore confine our attention to the matrix element $\varphi(q)$ given by (15).

According to (13) and (15), the expression for the rate of inelastic collisions in the general case is of the form[10]

$$W=N\frac{8\sqrt{2\pi}\lambda^2}{\omega_0R_0}f(\alpha,\beta), \tag{19}$$

$$f(\alpha,\beta)=\frac{R_0^2}{\alpha}\int_0^\infty q\,dqK_1^2(qR_0)\left|\varepsilon\left(\beta,\frac{qv_T}{\omega_0}\right)\right|^{-2}\exp\left(-\frac{\omega_0^2}{2(qv_T)^2}\right). \tag{20}$$

The function $f(\alpha,\beta)$ represents the dependence of the frequency of inelastic transitions on the plasma temperature and density.

In low-density plasma ($\beta=0$, $\varepsilon=1$) and when $\alpha\gg1$, the integrand in (20) is localized in the region $\omega_0/v_T<q<1/R_0$, the lower boundary of which is determined by long-wave fluctuations and the upper boundary by the atomic formfactor $\varphi(q)$ in (13), which is proportional to $K_1^2(qR_0)$. When $\alpha\ll1$, the spectral density of fluctuations in the region $q\sim1/R_0$ is exponentially small, and the transition frequency W decreases exponentially with decreasing temperature.

We must now investigate the role of plasma effects and begin by considering the case of moderately dense plasma such that $\beta\lesssim1$. Since the permittivity is different from unity only for $qv_T/\omega_0\leq1$, the influence of plasma effects for $\alpha\gg1$ should be quite small. In fact, calculations yield (see Appendix)

$$W(\alpha,\beta)=N\frac{8\sqrt{2\pi}\lambda^2}{\omega_0R_0\alpha}\left[\ln2\alpha-\frac{1}{2}-C-C(\beta)\right],\quad\alpha\gg1, \tag{21}$$

where $C=0.577$ is Euler's constant and $C(\beta)$ is defined in the Appendix. Hence, it is clear that, at sufficiently low excitation energy, the dependence of the frequency of inelastic transitions on temperature is still logarithmic, just as in low-density plasma. Polarization effects merely modify the value of the constant $C(\beta)$ in this case.

When $\alpha\ll1$, the main contribution to the integral in (20) is provided by the narrow region of transferred momenta in the neighborhood of $q_0\approx\alpha^{-2/3}/2^{1/2}R_0$, in which the permittivity may differ appreciably from unity. In particular, when

$$\beta=\omega_L^2/\omega_0^2\approx1,\quad qv_T/\omega_0\approx[(1-\beta)/3\beta]^{1/2}$$

the spectral density of fluctuations has a sharp peak connected with Langmuir oscillations. Hence, when $\alpha\ll1$, one would expect that polarization effects would exert a substantial influence on the situation. Unfortunately, it has not been possible to express W as a function of temperature and density in an analytic form. We reproduce the result only for the special case when

$$\alpha\ll2[(1-\beta)/3\beta]^{1/2}\ll1,$$

for which the main contribution to the integral in (20) is provided by short-wave oscillations:

$$W=N\frac{8\sqrt{2\pi}\lambda^2}{\omega_0R_0}\left(\frac{\pi^3}{3\cdot2^{7/3}}\right)^{1/2}\alpha^{-4/3}(1-\beta)^{-2}\exp\left(-\frac{3}{(\alpha\sqrt{2})^{2/3}}\right). \tag{22}$$

This differs from the corresponding expression in[9] by the presence of the screening factor $|\varepsilon|^{-2}$, where

$$|\varepsilon|=1-\beta=1-\omega_L^2/\omega_0^2$$

is the plasma permittivity at the frequency of the atomic transition.

Finally, when $\beta=1$ ($\omega_L^2=\omega_0^2$), we have

$$W=N\frac{8\sqrt{2\pi}\lambda^2}{\omega_0R_0}\frac{2^{1/6}}{3}\left(\frac{\pi}{3}\right)^{1/2}\alpha^{-2/3}\exp\left(-\frac{3}{(\alpha\sqrt{2})^{2/3}}\right). \tag{23}$$

Comparison of (23) with (22) shows that, when the electron Langmuir frequency is in exact resonance with the frequency ω_0 of the atomic transition, there is a qualitative change in the dependence of the frequency of inelastic collisions on the parameter α, i.e., the electron temperature.

We have not so far taken into account the ion part of the plasma permittivity. When $\beta\lesssim1$, the presence of ions has an important effect on the spectral density of fluctuations in the region

$$q\leqslant(\omega_L/v_T)\sqrt{m/M},$$

where M is the ion mass. However, this region provides a negligible contribution to the integral in (20) except for the case of very low temperatures [$\alpha<(m/M)^{3/2}$],

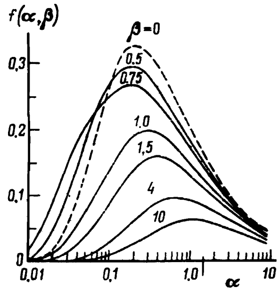

FIG. 1. Dependence of the function $f(\alpha,\beta)$ in (20) on the parameters $\alpha = v_T/(\omega_0 R_0)$ and $\beta = \omega_L^2/\omega_0^2$.

when the excitation rate is exponentially small. The influence of ion-acoustic and ion-Langmuir oscillations must, in general, be taken into account for $\beta > M/m$ when the frequency of the corresponding oscillations is close to the frequency of the atomic transition. For real transitions, such values of β correspond to the density of a solid body ($N > 2\times10^{23}$ cm^{-3} for $\hbar\omega_0 \sim 0.3$ eV), and we shall not consider them here.

To summarize the results of this section, we may say that, when $\beta\sim1$, the region in which polarization effects have an appreciable influence is determined by the inequality $\alpha \lesssim 1$, i.e., the Langmuir plasma frequency should be comparable with the frequency of the atomic transition ($\omega_L \sim \omega_0$) and the Debye length should be smaller than the characteristic range of the interaction, i.e., $r_D \lesssim R_0$, where $r_D = (T/4\pi N e^2)^{1/2}$.

3. RESULTS OF CALCULATIONS. DISCUSSION

The rates of excitation of optically allowed transitions can be obtained from (19) and from the figure which gives the results of numerical calculations of the function $f(\alpha, \beta)$ based on (20). This can be done for arbitrary plasma temperature and density. In the most interesting region, where $\alpha \gtrsim 1$, the influence of polarization effects leads to a reduction in the excitation rate. When $\alpha < 1$, the function $W(\beta)$ becomes nonmonotonic: as the density increases, the quantity $W(\beta)$ at first increases and them, for $\beta \geqslant 1$, it rapidly decreases and is given by $W(\beta)\sim\beta^{-2}$ [see (22)].

Let us now consider the validity of the first-order perturbation theory used in the derivation of (7) for the frequency of inelastic collisions. The condition for the validity of the Born approximation in calculations of cross sections for inelastic transitions is[3)]

$$\frac{\lambda\omega_0}{v^2}\frac{\lambda}{R_0 v}\ll 1, \tag{24}$$

where v is the relative velocity of the colliding particles.

It is quite clear that a similar criterion for the collision frequency should contain the electron density because the mean field acting on the atom decreases with increasing mean distance between the particles. This criterion can readily be obtained from the condition

$$W\tau\ll1, \tag{25}$$

where τ is the correlation time for the potential $V_{10}(t)$ in (2).

Substituting $\tau\sim R_0/v_T$ in (25), which corresponds to the potential given by (16), we obtain the condition for the validity of the Born approximation in the following form:

$$\begin{aligned} &N\lambda^2/\omega_0 v_T \ll 1, \quad \alpha\sim1,\\ &N(\lambda/v_T)^2 R_0 \ll 1, \quad \alpha\gg1. \end{aligned} \tag{26}$$

Finally, let us briefly consider the influence of polarization on inelastic atomic transitions due to collisions with ions. In this case, the main contribution to the frequency of inelastic collisions is provided by the region of wave vectors $q \gtrsim \omega_0/v_{Ti}$ (where $v_{Ti} = \sqrt{T/M}$). At the same time, Debye screening ensures that the permittivity differs from unity only for $q \lesssim \omega_0/v_T$ so that polarization effects are unimportant in this case.

The authors are indebted to B. Ya. Zel'dovich, Yu. L. Klimontovich, V. V. Pustovalov, I. I. Sobel'man, V. N. Tsytovich, and E. A. Yukov for useful suggestions.

APPENDIX

We must now consider the properties of the integral $f(\alpha, \beta)$ in (20):

$$f(\alpha,\beta)=\frac{1}{\alpha}\int_0^\infty\frac{dt}{t}\left[\frac{t}{\alpha}K_1\left(\frac{t}{\alpha}\right)\right]^2|\varepsilon(\beta,t)|^{-2}\exp\left(-\frac{1}{2t^2}\right). \tag{A.1}$$

Since

$$\frac{1}{x}[xK_1(x)]^2=-\frac{d}{dx}\left\{\frac{x^2}{2}[K_0(x)K_2(x)-K_1^2(x)]\right\}$$

and integrating by parts, we obtain

$$f(\alpha,\beta)=\frac{1}{\alpha}\int_0^\infty dt\left\{\frac{1}{2}\frac{t}{\alpha}\left[K_0\left(\frac{t}{\alpha}\right)K_2\left(\frac{t}{\alpha}\right)-K_1^2\left(\frac{t}{\alpha}\right)\right]\right\}\left[|\varepsilon(\beta,t)|^{-2}\exp\left(-\frac{1}{2t^2}\right)\right]'. \tag{A.2}$$

If we expand the expression in the braces into a series for $t/\alpha\ll1$, and retain only the nonvanishing terms, we obtain

$$\frac{t}{2\alpha}\left[K_0\left(\frac{t}{\alpha}\right)K_2\left(\frac{t}{\alpha}\right)-K_1^2\left(\frac{t}{\alpha}\right)\right]\approx\ln\frac{\alpha}{t}-\frac{1}{2}-\ln2+C, \tag{A.3}$$

where C is the Euler constant. Substituting (A.3) in (A.2), and using (11), we obtain

$$f(\alpha,\beta)=\frac{1}{\alpha}\left[\ln2\alpha-\frac{1}{2}-C-C(\beta)\right],\quad \alpha\to0, \tag{A.4}$$

$$C(\beta)=\int_0^\infty\left[\left([1-\beta\Phi(y)]^2+\frac{\pi\beta^2}{2y^6}\exp\left(-\frac{1}{y^2}\right)\right)^{-1}\exp\left(-\frac{1}{2y^2}\right)\right]_y'\ln y\,dy. \tag{A.5}$$

In particular, in the case of low-density plasma ($\beta=0$), we have

$$C(0)=(C-\ln2)/2.$$

When $\alpha\ll1$, we can use the asymptotic properties of the MacDonald function for large values of the argument. The integrand in (A.1) will then contain the factor $\exp\{-2t/\alpha - 1/2t^2\}$ so that we use the saddle point method which yields

$$f(\alpha,\beta)=\frac{\pi^{3/2}}{3^{1/2}\cdot 2^{7/6}}\alpha^{-4/3}\left|\varepsilon\left[\beta,\left(\frac{\alpha}{2}\right)^{1/3}\right]\right|^{-2}\exp\left(-\frac{3}{(\alpha\sqrt{2})^{2/3}}\right). \quad \text{(A.6)}$$

Using (11) and (12), we have

$$\left|\varepsilon\left[\beta,\left(\frac{\alpha}{2}\right)^{1/3}\right]\right|^2_{\alpha\to 0}=\begin{cases}(1-\beta)^2, & \beta\neq 1,\\ 9(\alpha/2)^{4/3}, & \beta=1,\end{cases} \quad \text{(A.7)}$$

so that the final expression for $\alpha \ll 1$ becomes

$$f(\alpha,\beta)=\begin{cases}\dfrac{\pi^{3/2}}{3^{1/2}\cdot 2^{7/6}}\alpha^{-4/3}(1-\beta)^{-2}\exp\left(-\dfrac{3}{(\alpha\sqrt{2})^{2/3}}\right), & \beta\neq 1\\ \dfrac{2^{1/6}}{3}\left(\dfrac{\pi}{3}\right)^{3/2}\alpha^{-8/3}\exp\left(-\dfrac{3}{(\alpha\sqrt{2})^{2/3}}\right), & \beta=1.\end{cases} \quad \text{(A.8)}$$

[1]We are using the atomic system of units.

[2]This result could be foreseen because the plasma field $V(\mathbf{r},t)$ in (2) is assumed to be classical, which, in collision language, corresponds to the description of the relative motion of colliding particles in terms of classical trajectories. Strictly speaking, this approach is valid for cross sections only when $\hbar\omega_0 < E$, where E is the energy of the relative motion of the colliding particles. However, a slight modification enables us to extend this approach to the region where $\hbar\omega_0 \sim E$, at least for optically allowed transitions.[7]

[3]The presence of the factor $\lambda\omega_0/v^2$ on the left of (24) is specific precisely for transitions with small resonance defect ω_0.[9]

[1]I. I. Sobel'man and E. L. Feĭnberg, Zh. Eksp. Teor. Fiz. **34**, 494 (1958) [Sov. Phys. JETP 7, 339 (1958)].

[2]É. A. Asmaryan and Yu. L. Klimontovich, Vestn. Mosk. Univ. Fiz. **3**, 273 (1974).

[3]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Nauka, 1974.

[4]Yu. L. Klimontovich, Kineticheskaya teoriya neideal'nogo gaza i neideal'noĭ plazmy (Kinetic Theory of a Nonideal Gas and Nonideal Plasma), Nauka, 1975.

[5]Sb. Élektrodinamika plazmy (Collection: Plasma Electrodynamics), ed. by A. I. Akhiezer, Nauka, 1974.

[6]A. V. Vinogradov, Opt. Spektrosk. 22, 663 (1967) [Opt. Spectrosc. (USSR) 22, 361 (1967)].

[7]L. A. Vainstein and A. V. Vinogradov, J. Phys. B 3, 1090 (1970).

[8]R. Balescu, Phys. Fluids 3, 52 (1960).

[9]V. P. Shevel'ko, A. M. Urnov, and A. V. Vinogradov, J. Phys. B 9 (1976).

[10]A. V. Vinogradov and V. P. Shevel'ko, Abstracts of Papers of Twelfth ICPIG, Part 1, Eindhoven, Holland, 1975 (North-Holland, American Elsevier), p. 261.

Translated by S. Chomet

Collisionless emission of radiation by an inhomogeneous plasma

B. É. Meĭerovich

Institute of Physics Problems, USSR Academy of Sciences, Physics Laboratory

(Submitted March 31, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1045–1058 (September 1976)

Collisionless radiation by an inhomogeneous plasma, due to the finite motion of charges in the field of external forces and collective interaction forces, is studied. The radiation intensity is inversely proportional to the square of the transverse dimensions of the plasma. It apparently makes the main contribution to the radiation from a vacuum spark and other relativistic beams compressed to a small size by collective interaction forces. The intensity of the collisionless radiation is calculated with account taken of the Fermi statistics of the electrons. The spectral radiance in the low frequency range increases with frequency, reaches a maximum at the frequency of the finite motion of the emitters, and then decreases. Measurement of the collisionless radiation by a plasma compressed to a small size by the pinch effect is a natural way of diagnosing the plasma.

PACS numbers: 52.25.Ps

1. INTRODUCTION

A widely used method of plasma diagnostics is the determination of its parameters from the experimentally measured radiation. The electrons and ions of the plasma are under the influence of external forces and of the collective-interaction forces. These forces accelerate the charges and cause them to radiate.

If the plasma is a gas, i.e., if the time between collisions of the particles is large in comparison with the duration of the collision, then the charges are subjected for the greater part of the time to smooth acceleration under the influence of the forces of the external field and of the average collective interaction. In the case of short-range collisions, the charges are appreciably accelerated within short time intervals (on the order of the duration of the collision). The radiation produced by a gaseous fully ionized plasma is of two types, the radiation accompanying the short-range pair collisions, and the radiation due to the external forces and the average collective-interaction forces. The latter type has no bearing on the particle collisions and can be naturally called collisionless.

If the plasma is uniform and is not situated in an external field, then there is no collisionless radiation. The radiation connected with the Coulomb collisions of