behind the plasma layer.

The increased self-focusing in the case of induced transparency, as compared with the case of transparent plasma ($N_e < N_{crit}$), is probably connected with the appearance in the post-critical plasma illuminated by the incident radiation of a transparency channel of width $d < a$ which, in effect, compresses the beam propagating through it.

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Stabilization of tearing instability and heating of plasma ions by a modulated particle beam

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The tearing instability is considered for nonisothermal plasma penetrated by a low-velocity charged-particle beam. Possible stabilization of this instability due to the transformation of the high-frequency wave energy into the energy of a nonlinear ion-acoustic wave is discussed. This phenomenon is equivalent to the nonlinear absorption of the high-frequency waves. The efficiency of heating of heavy particles (ions) by a charged-particle beam is estimated.

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Tearing (explosive) instability, characterized by a sharp increase in the amplitudes of interacting waves, is known to be possible in nonequilibrium media (see, for example, [1–3]). This instability has been investigated for plasma systems in the case of longitudinal[1,4] and transverse[5,6] waves; efficient generation of microwave radiation in laboratory plasma, previously found under experimental conditions for plasmas[7] and transmission lines,[8] was demonstrated. The main mechanism proposed for limiting the "explosion" was the nonlinear departure from synchronism due to cubic nonlinearity.[9,10,5] In the present paper, we investigate a fundamentally new mechanism for limiting tearing instability. This is connected with the multiwave interaction between hf and lf waves in which the lf waves have positive energy and are attenuated in a linear fashion so that, owing to the presence of a large number of lf branches (or one branch with weak dispersion), nonlinear lf oscillations are generated. The latter situation is equivalent to the nonlinear attenuation of lf waves and leads to the stabilization of the "explosion" in a medium with quadratic nonlinearity. We shall illustrate the analysis by the example of the interaction between a frequency-modulated multivelocity electron beam and the ion-acoustic waves in the main plasma when the modulation frequency is much greater than the Langmuir frequency of the plasma electrons. The higher frequency of the beam wave will then be associated with negative energy, whereas the other beam and ion waves will have positive energies. Linear viscosity-type absorption will be taken into account for the ion sound. It is found that a nonlinear ion-acoustic wave is generated in this system, and this may lead to effective heating of plasma ions as a result of dissipation. The growth of the hf waves is not observed under these conditions, i.e., the tearing instability becomes stabilized by the nonlinear generation of a large number of lf waves, which is equivalent to the nonlinear attenuation of hf waves. The stabilization mechanism for tearing instability is physically interesting in itself, but it can also be used for the effective heating of plasma ions.

1. The basis set of quasihydrodynamic equations is[1)]

$$\frac{\partial E}{\partial x}=4\pi e(\rho_e-\rho_i+\rho_s),$$
$$\frac{\partial v_{e,i}}{\partial t}+v_{e,i}\frac{\partial v_{e,i}}{\partial x}=\frac{e}{m_{e,i}}E-\frac{\varkappa T_{e,i}\partial\rho_{e,i}/\partial x}{m_{e,i}(N_{e,i}+\rho_{e,i})},$$
$$\frac{\partial \rho_{e,i}}{\partial t}+N_{e,i}\frac{\partial v_{e,i}}{\partial x}+v_{e,i}\frac{\partial \rho_{e,i}}{\partial x}=0, \tag{1}$$
$$\frac{\partial v_s}{\partial t}+V_0\frac{\partial v_s}{\partial x}-\frac{e}{m_e}E=-v_s\frac{\partial v_s}{\partial x},$$
$$\frac{\partial \rho_s}{\partial t}+N_s\frac{\partial v_s}{\partial x}+V_0\frac{\partial \rho_s}{\partial x}=0,$$

where $\rho_{e,i,s}$ and $v_{e,i,s}$ are the deviations of the concentrations of the plasma electrons and ions, the beam electrons, and the corresponding velocities from the equilibrium values ($N_e = N_i$; N_s; 0, 0, V_0), and E is the electric field of the wave. In the linear approximation, the above set of equations leads to the following dispersion relation[2)]

$$1-\frac{\omega_0^2}{\omega^2-v_{Te}^2k^2}-\frac{\omega_{0i}^2}{\omega^2}-\frac{\omega_{0s}^2}{(\omega-kV_0)^2}=0, \tag{2}$$

$$\omega_0^2=\frac{4\pi N_e e^2}{m_e};\quad \omega_{0i}^2=\frac{4\pi N_i e^2}{m_i};\quad \omega_{0s}^2=\frac{4\pi N_s e^2}{m_e};\quad v_{Te}^2=\frac{\varkappa T_e}{m_e};$$

where ω and k are the frequency and wave vector, respectively. It is readily seen that, when

$$\omega/k \ll v_{Te},\quad V_0 \gg v_{Te},\quad V_0 \gg C_s,\quad C_s^2=\varkappa T_e/m_i \tag{3}$$

the above dispersion relation describes high-frequency beam waves and ion-acoustic waves in nonisothermal plasma:

$$\omega-kV_0\approx\pm\omega_{0s}, \tag{4}$$

$$\Omega=qC_s(1+q^2C_s^2/\omega_{0i}^2)^{-1/2}. \tag{5}$$

For waves for which the dispersion relations given by (4) and (5) are valid, we have the synchronism condition

$$\omega_2=\omega_1+\Omega,\quad k_2=k_1+q, \tag{6}$$

so that, if we use this in conjunction with (4) and (5), we can readily show that

$$\Omega\approx 2\omega_{0s}C_sV_0^{-1} \tag{7}$$

(we recall that $V_0 \gg C_s$, so that the usual ion-acoustic instability need not be taken into account) and

$$k_1\sim k_2\sim\omega_{1,2}/V_0. \tag{8}$$

Beats produced by the high-frequency beam waves will generate the ion-acoustic wave which, because of weak dispersion ($\Omega \ll \omega_{0i}$), will excite a number of ion-sound harmonics (for simplicity, we shall confine our attention to three harmonics because, as will be shown below, this is sufficient for the correct description of the above interaction). Using standard procedures (see, for example, [11,12]), we obtain the following equation for the complex amplitudes of the interacting waves:

$$\begin{aligned}
&\frac{\partial a_2}{\partial t}+V_0\frac{\partial a_2}{\partial x}=\sigma_2a_1b_1,\qquad \frac{\partial a_1}{\partial t}+V_0\frac{\partial a_1}{\partial x}=\sigma_1a_1b_1,\\
&\frac{\partial b_1}{\partial t}+C_s\frac{\partial b_1}{\partial x}=\sigma a_2a_1^*+\delta(2b_2^*b_3+3b_2b_1^*)-\eta b_1,\\
&\frac{\partial b_2}{\partial t}+C_s\frac{\partial b_2}{\partial x}=\frac{2}{3}\delta(-3b_1^2+2b_3b_1^*)-4\eta b_2,\\
&\frac{\partial b_3}{\partial t}+C_s\frac{\partial b_3}{\partial x}=-\frac{9}{2}\delta b_2b_1-9\eta b_3,
\end{aligned} \tag{9}$$

where

$$\sigma_1\sim\sigma_2\sim-\frac{e}{4m_eV_0}N_sN_e^{-1}(\Omega\omega)^2(\omega_0\omega_{0i})^{-2},$$

$$\sigma\sim-\frac{1}{8}\frac{e}{m_iC_s}N_sN_e^{-1}\left(\frac{\omega_0}{\omega}\right)^4\left(\frac{\omega_{0s}\omega_{0i}}{\omega\Omega}\right)^2,$$

$\delta\sim e/3C_sm_i$; η is the viscosity, and $a_{1,2}$ and $b_{1,2,3}$ are the complex amplitudes of electric fields in the hf beam and lf ion-acoustic waves.

For simplicity, we shall now confine our attention to the propagation of waves for $x>0$ in the time-independent approximation ($\partial/\partial t=0$). It is readily shown that, for reasonable parameters of the beam-plasma system, $\sigma \gg \sigma_{1,2}$ and, moreover, for $V_0 \gg C_s$, the ion sound will grow rapidly and will subsequently "follow" the changes in $a_{1,2}$. Under these conditions, the expression given by (9) yields the following equilibrium distribution for the generated ion-sound field in the system:

$$\begin{aligned}
&b_1^0=\sigma a_2a_1^*\eta_{\text{eff}}^{-1}(|b_1^0|^2),\qquad b_2^0=-3\delta b_1^2(6\eta^2+|b_1^0|^2\delta^2)^{-1}\\
&b_3^0=3b_1^3\delta^2(6\eta^2+|b_1^0|^2\delta^2)^{-2},
\end{aligned} \tag{10}$$

$$\eta_{\text{eff}}=\eta\left[1+\frac{3}{2}\delta^2|b_1^0|^2(6\eta^2+\delta^2|b_1^0|^2)^{-1}+\frac{3}{2}\delta^4|b_1^0|^4(6\eta^2+|b_1^0|^2\delta^2)^{-2}\right],$$

where the function $\eta_{\text{eff}}(|b_1^0|^2)$ characterizes the nonlinear attenuation of ion sound due to the transfer of energy to higher harmonics. Qualitatively speaking, η_{eff} increases with η for small $|b_1^0|$ and reaches the saturation value $\sim 4\eta$ for large $|b_1^0|$. To determine the nature of the function $a_{1,2}(x)$, we must substitute for b_1^0 from (10) in (9) with the time derivative equal to zero. However, this does not yield any useful expressions for the variation in the amplitude of the hf waves, and further analysis was therefore carried out numerically on a computer. It was found that $a_{1,2}(x)$ did not undergo substantial variation, i.e., the ion sound grew in nearly the same way as in a given field of hf beam waves and reached the values given by (10). Further dissipation of the nonlinear ion-acoustic wave results in the transformation of the ion-sound energy into heat, i.e., the heavy plasma particles (ions) become heated. Unlimited growth of the lf and hf waves, i.e., the tearing instability described by the first three terms in (9), without including $b_{2,3}$, does not occur. Effective nonlinear attenuation of the ion-sound harmonics may therefore result in the dynamic stabilization of the "explosion."

2. In conclusion, let us estimate the efficiency of heating of the plasma ions. We note that the inclusion of only three harmonics of the lf wave is entirely sufficient, since dispersion and the attenuation of sound ($\sim q^2$) is such that departure from synchronism is important for the higher harmonics and, therefore, their excitation is of minor importance in this system. Let us now suppose that the parameters of the beam-plasma system are as follows: $N_sN_e^{-1}\sim 10^{-2}$, $\omega_0\sim 2\times 10^{10}$ sec^{-1}, $\Omega\sim 8\times 10^7$ sec^{-1} (wavelength of ion-acoustic wave ~ 0.5 mm), $C_s\sim 10^6$ cm·sec^{-1}, $V_0\approx 10^8$ cm·sec^{-1}, beam current ~ 20 mA/cm^2, power ~ 10 mW/cm^2, beam modulation frequency $\sim 5\times 10^{10}$ sec^{-1}, and pump amplitude $|a|/(mV_0^2N_s/2)^{1/2}\sim 10^{-1}$($|a_{1,2}|\sim 2$ V/cm). A nonlinear ion-acoustic wave with the following amplitudes of harmonics is generated under these conditions in the beam-plasma system: $b_1^0/E_{\max}\sim 0.2$; $E_{\max}=(mV_0^2N_s/2)^{1/2}$; $b_2^0/E_{\max}\sim 0.08$; $b_3^0/E_{\max}\sim 0.05$; in the case of dissipation, this corresponds to heating at a rate of 50 deg/sec in a layer with a cross section of the order of 10 cm^2 and length ~ 50 cm.

[1)]It is assumed that the ion temperature $T_i \ll T_e$ (T_e is the electron temperature). We put henceforth for simplicity $T_i\approx 0$.

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Decay of the initial density discontinuity in the hydrodynamics of a collisionless plasma

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The problem is considered of the decay of the initial density and velocity discontinuity in a plasma with cold ions and hot electrons. It is shown that the equations of two-stream hydrodynamics should be used to describe the evolution of such a discontinuity. The region of initial values of the plasma parameters, $u_- - u_+$ and N_-/N_+ is found in which the self-similar solutions obtained are applicable. The stability of the resultant solutions is investigated.

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1. INTRODUCTION

In recent years, great attention has been paid in plasma physics to the description of collisionless shock waves—nonlinear waves that arise in a collisionless plasma and transform the plasma from one stationary state to another. The analog of such a problem in gas dynamics is the Riemann problem[1]—the determination of the asymptotic (as $t\to\infty$) motion of the gas, in which the gas on the right halfspace ($x>0$) is maintained (up to the initial moment) at a pressure p_+, and has a velocity u_+; in the left half space ($x<0$) these quantities have the values p_- and u_-, respectively.

The solution of the Riemann problem for a plasma in the case in which $T_e\sim T_i$ is given in the work of Gurevich *et al.* [2] with the use of the kinetic equation, with a self-consistent field for the ion distribution function. In the case in which the ions are cold ($T_i\ll T_e$) their velocity distribution function degenerates into a δ function, and in the description of the motion of the ions, a transition from the kinetic equation to equations of the hydrodynamic type becomes possible. These equations in the dimensionless variables $u_\alpha=v_\alpha(T_e/M)^{-1/2}$, $\varphi=e\psi/T_e$, $\xi=x(M/T_e)^{-1/2}$, have the form

$$\frac{\partial u_\alpha}{\partial t}+u_\alpha\frac{\partial u_\alpha}{\partial \xi}=-\frac{\partial\varphi}{\partial\xi},\tag{1a}$$

$$\frac{\partial N_\alpha}{\partial t}+\frac{\partial}{\partial\xi}(N_\alpha u_\alpha)=0,\tag{1b}$$

$$N=N_1+N_2,\quad \alpha=1,2.\tag{1c}$$

Here N_1, N_2, v_1, v_2 are the concentrations and velocities of the two ion fluxes, ψ is the electric field potential, M is the mass of the ion, T_e is the temperature of the electrons. The system (1) is written under the assumption that all the variables depend only on the single coordinate and the time t.

The introduction of the index α corresponds to the fact that, in the absence of collisions, ions can be found at each point in space, arriving both from the right and from the left; "two-layer hydrodynamics" is required for the solution of such a system of particles. Equations (1a) express the conservation of momentum of each of the components of the ionic part of the plasma, and Eqs. (1b) express the conservation of the number of particles of these components.

The system (1) should be satisfied by the Poisson relation

$$-\frac{d^2\psi}{dx^2}=4\pi e(N_i-N_e),\tag{2}$$

where ψ, N_1, N_e are functions of the coordinate and the time.