the region of normal dispersion are realized only in the case of inversion of the initial population of the levels of the working transition, and represent solitons of Lorentzian shape or $2\pi$ pulses of hyperbolic type. Under arbitrary dispersion conditions, soliton regimes are also possible in the form of periodic trains of $2\pi$ pulses of elliptical and trigonometric types. The solitons of Lorentzian shape, trains of a trigonometric type and special types of isolated solitons with $\theta < 2\pi$ (class 9) are stable, which indicates the possibility of their practical realization (see the estimates in Sec. 1).

We note in conclusion that the soliton regimes of SRS can have great value in the analysis of the detailed temporal structure of the radiation of combination lasers, since the fields arising in such systems have intense fluctuation discharges of short duration. The observation of soliton regimes in "pure form" is advantageously carried out in gases at low pressure ~ 0.1 atm in the case of durations of the initial laser pulses of ~ 1–10 nanosec.

The authors thank S. A. Akhamanov for discussion of the results of the work and useful remains, and also G. M. Makhviladze for discussion of the question of the stability of soliton regimes.

[1)]We note that here and below we neglect the pumping of energy into the antistokes and higher Stokes components, since the intense fields of the exciting and first Stokes radiations are given at the input to the medium, and the fields of the other components are generated from noise nuclei.

[2)]We note that for the special case of the absence of dispersion $\eta_i = \eta_s$, this type of soliton solutions was found also in a recent publication.[8]

[1]I. A. Poluekotov, Yu. M. Popov, and V. S. Roĭtberg, Usp. Fiz. Nauk **114**, 97 (1974) [Soviet Phys. Usp. **17**, 673 (1975)].
[2]S. L. McCall and E. L. Hahn, Phys. Rev. Lett. **18**, 908 (1967); Phys. Rev. **183**, 457 (1969).
[3]É. M. Belenov and I. A. Poluektov, Zh. Eksp. Teor. Fiz. **56**, 1407 (1969) [Sov. Phys. JETP **29**, 754 (1969)].
[4]N. Tan-no, K. Yokoto, and H. Inaba, Phys. Rev. Lett. **39**, 1211 (1972).
[5]T. M. Makhaladze, M. E. Sarychev, and L. A. Shelepin, Preprint, Phys. Inst. Acad. Sci. USSR, No. 18, 1974; Zh. Eksp. Teor. Fiz. **69**, 499 (1975) [Sov. Phys. JETP **42**, 255 (1975)].
[6]S. A. Akhmanov, K. N. Drabovich, A. P. Sukhorukov, and A. S. Chirkin, Zh. Eksp. Teor. Fiz. **59**, 485 (1970) [Sov. Phys. JETP **32**, 266 (1971)].
[7]S. A. Akhmanov, K. N. Drabovich, A. P. Sukhorukov, and A. K. Shehednova, Zh. Eksp. Teor. Fiz. **62**, 525 (1972) [Sov. Phys. JETP **35**, 279 (1972)].
[8]N. Tan-no, T. Shirahata, K. Yokoto, and H. Inaba, Phys. Rev. **A12**, 159 (1975).
[9]P. G. Kryukov and V. S. Letokhov, Usp. Fiz. Nauk **99**, 169 (1969) [Sov. Phys. Usp. **12**, 135 (1969)].
[10]G. I. Barenblatt, Analysis of Dimensions in Self-similar Solutions. Preprint No. **52**, Institute of Semimetals, Moscow, 1975.
[11]S. R. Barone, Lett. Nuovo Cim. **3**, 156 (1972).

Translated by R. T. Beyer

# Crossing of quasistationary levels[1)]

A. Z. Devdariani, V. N. Ostrovskiĭ, and Yu. N. Sebyakin

*Leningrad State University*
(Submitted November 24, 1975)
Zh. Eksp. Teor. Fiz. **71**, 909–918 (September 1976)

The crossing of discrete energy levels, each of which interacts with the continuous spectrum, is discussed. The problem is reduced to the consideration of only two, but quasistationary, levels with suitably modified interaction. A formula for the amplitude of the nonadiabatic transition in this problem is derived for a sufficiently general dependence of the terms on the interatomic distance. The behavior of the populations of such states is investigated, and it is shown that the interaction between the levels through the continuum has an important effect both on the nonadiabatic transition amplitude and on the population of states. It is noted that both the formulation of the problem and the method of solution given by Karas' *et al.* (1974) and by Bazylev and Zhevago (1975) are subject to error.

PACS numbers: 31.10.Bb

1. It is necessary to introduce the concept of quasistationary energy terms when different atomic-collision processes are investigated. In contrast to the usual discrete states, quasistationary terms are characterized not only by the energy $E$ but also by the width $\Gamma$ which describes the possibility of decay, i. e., the possibility of a transition of the system from a given electronic state to the continuum with the emission of an electron or photon. In general, both $E$ and $\Gamma$ are functions of the distance between the colliding particles. The interaction and crossing of such terms play a fundamental role in collisions leading to the formation of vacancies in the inner electron shells of atoms.[2,3] Such states are commonly referred to as the autoionization states. Another example is charge transfer on negative ions. Thus, analysis of experimental data shows[4] that the crossing of quasistationary molecular terms corresponding to different charge-transfer channels must be taken into account in these reactions. Multiple crossing of quasistationary terms is also expected in collision processes involving the participation of atoms in highly excited states, for example, in Penning ionization processes of

the form $A+B^* \to A^*+B+e$. It is interesting to note that similar problems also arise in certain types of waveguide.[5]

The theoretical analysis of the interaction between quasistationary levels was undertaken by Kogan *et al.*[6] and Lisitsa and Yakovlenko.[7] These workers investigated in detail the case of parallel, time-independent terms and their results have extensive applications to atomic processes. Osherov[8] used the example of nonadiabatic transitions in mixtures to consider the crossing of an inclined and a horizontal term.

The aim of the present paper is to investigate the interaction between two electronic states $|\varphi_{1,2}\rangle$ with discrete energies $E_1(t)$ and $E_2(t)$ which cross at $t=0$. Each of these states interacts with the continuum, and this leads to their decay with the emission of electrons or photons. This problem has recently been considered by Karas' *et al.*[9] and Bazylev and Zhevago.[10] However, the formulation and method of solution of the problem given by these workers are subject to error, so that the final result is not correct. This has led us to a more detailed analysis of the various difficulties that have given rise to the misunderstanding.

2. In verifying the initial set of equations, we shall follow the procedure given by one of the present authors in[11]. Assuming that the total energy of the colliding atoms is sufficiently large, so that the quantum-mechanical energy exchange between electronic and nuclear motions can be ignored, we can take the time-dependent Schrödinger equation with the Hamiltonian given by

$$\hat{H}(t)=\hat{H}_0+\hat{V}=E_1(t)|\varphi_1\rangle\langle\varphi_1|+E_2(t)|\varphi_2\rangle\langle\varphi_2|+\int d\omega\,\omega|\omega\rangle\langle\omega|+\hat{V}. \quad (1)$$

The vectors of discrete states $|\varphi_{1,2}\rangle$ and states in the continuum $|\omega\rangle$ form a complete orthonormal set, and we assume that the operator $\hat{V}$ satisfies the relation

$$\langle\varphi_1|\hat{V}|\varphi_1\rangle=\langle\varphi_2|\hat{V}|\varphi_2\rangle=\langle\omega|\hat{V}|\omega\rangle=0.$$

In the ensuing analysis, we shall assume, for simplicity, that $\langle\omega|\hat{V}|\varphi_{1,2}\rangle$ is independent of $\omega$, and that the continuous spectrum extends from $-\infty$ to $\infty$. Physically, this assumption means that the limit of the continuum lies well away from the region of crossing, and the widths are sufficiently slowly varying functions in this region.

We shall take the solution of the Schrödinger equation

$$\hat{H}|\Psi\rangle=i\frac{\partial}{\partial t}|\Psi\rangle$$

in the form

$$|\Psi\rangle=a_1(t)|\varphi_1\rangle+a_2(t)|\varphi_2\rangle+\int d\omega\, b(\omega,t)e^{-i\omega t}|\omega\rangle \quad (2)$$

so that by eliminating $b(\omega,t)$, we obtain the following set of equations for the probability amplitudes:

$$i\frac{d}{dt}\begin{pmatrix}a_1\\a_2\end{pmatrix}=\begin{pmatrix}E_1-{}^1/_2 i\Gamma_1 & V-{}^1/_2 i(\Gamma_1\Gamma_2)^{1/2}\\ V-{}^1/_2 i(\Gamma_1\Gamma_2)^{1/2} & E_2-{}^1/_2 i\Gamma_2\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix}, \quad (3)$$

$$i\frac{d}{dt}b(\omega,t)=(a_1\Gamma_1^{1/2}+a_2\Gamma_2^{1/2})\frac{e^{i\omega t}}{\sqrt{2\pi}}, \quad (4)$$

where $\Gamma_{1,2}$ represents the coupling between the original discrete levels and the continuum:

$$\Gamma_{1,2}=2\pi|\langle\omega|\hat{V}|\varphi_{1,2}\rangle|^2, \quad V=\langle\varphi_1|\hat{V}|\varphi_2\rangle.$$

The problem of the interaction between two discrete arbitrarily time-dependent levels with each other and with the infinite number of states in the continuum has thus been reduced to the interaction between only two but quasistationary levels. The set of equations given by (3) can readily be generalized to the case of a large number of levels and continua.

Proceeding now to the solution, we note, first of all, that the set of equations given by (3) for quasistationary states differs from the complete system (3) and (4) by the fact that it is not Hermitian. This is why, in the case of the two-level problem, the question of the time-independent transition probability is not trivial in relation to (3). Another important point is that coupling to the continuous spectrum leads not only to the appearance of the level width but also the imaginary term $-\frac{1}{2}i(\Gamma_1\Gamma_2)^{1/2}$. This term describes the interaction between levels through the continuum, which is clearly indicated by the form of the equations given by (3) and (4). This modified interaction between quasistationary levels (as compared with the interaction between the initial discrete states) was not taken into account in[9] or[10] and, therefore, the initial equations that are solved there can be regarded as valid only in the absence of interaction through the continuum. This occurs when the width of one of the terms is zero, or the width of the terms is determined by transitions to states in physically different continua, which is of no interest for applications. The modified interaction for horizontal terms is derived in[7] and, for a horizontal and an inclined term in[8]. It is interesting that the exchange interaction operator between single-nucleon quasistationary states is also proportional to $(\Gamma_1\Gamma_2)^{1/2}$. This has been shown by Dalidchik.[12]

3. We shall write the solution of (3) in the form

$$|\Psi\rangle=a_1|\varphi_1\rangle+a_2|\varphi_2\rangle=\Delta^{-1/2}\left[\frac{A_1H_{12}}{(H_{11}-E_I)^{1/2}}\Phi_1(t,t_0)+A_2(E_{II}-H_{22})^{1/2}\Phi_2(t,t_0)\right]|\varphi_1\rangle+\Delta^{-1/2}\left[A_1(H_{11}-E_I)^{1/2}\Phi_1(t,t_0)-\frac{A_2H_{12}}{(E_{II}-H_{22})^{1/2}}\Phi_2(t,t_0)\right]|\varphi_2\rangle, \quad (5)$$

where

$$H_{11,22}=E_{1,2}-{}^1/_2 i\Gamma_{1,2}, \quad H_{12}=V-{}^1/_2 i(\Gamma_1\Gamma_2)^{1/2},$$

$$\Phi_{1,2}(t,t_0)=\exp\left(-i\int_{t_0}^{t}E_{I,II}dt\right)$$

the energies of the adiabatic terms are

$$E_{I,II}={}^1/_2(H_{11}+H_{22}\mp\Delta),$$

and the splitting is given by

$$\Delta=[(H_{11}-H_{22})^2+4H_{12}^2]^{1/2}. \quad (6)$$

Equation (5) is, at the same time, the definition of the

time-dependent quantities $A_1$ and $A_2$, the significance of which becomes clear if we rewrite the solution (5) in the form

$$|\Psi\rangle = A_1\left[\frac{H_{12}}{\Delta^{1/2}(H_{11}-E_I)^{1/2}}|\varphi_1\rangle + \frac{(H_{11}-E_I)^{1/2}}{\Delta^{1/2}}|\varphi_2\rangle\right]\Phi_1(t,t_0)$$
$$+A_2\left[\frac{(E_{II}-H_{22})^{1/2}}{\Delta^{1/2}}|\varphi_1\rangle - \frac{H_{12}}{\Delta^{1/2}(E_{II}-H_{22})^{1/2}}|\varphi_2\rangle\right]\Phi_2(t,t_0). \quad (7)$$

The expressions in brackets are commonly referred to as the vectors of the adiabatic basis $|\Psi_{I,II}\rangle$.[13] On this basis, the energy matrix of (3) is diagonal with eigenvalues $E_{I,II}$ (in contrast to the basis of the initial states $|\varphi_{1,2}\rangle$), which is called nonadiabatic in collision theory.

Let us begin by considering the usual two-level system without attenuation, when $\Gamma_1 = \Gamma_2 = 0$ i.e., $E_{I,II}(t)$ are real for real $t$. The corresponding definitions of the transition probability amplitude $g$ between adiabatic states can be introduced either with the aid of an adiabatic or diabatic basis. In the first case, the only definition is

$$g = \lim_{t_1\to\infty}\frac{A_2(t_1)}{A_1(-t_1)} \quad \text{for} \quad A_2(-t_1)=0, \quad (8)$$

and, in the second,

$$g = \lim_{t_1\to\infty}\frac{a_1(t_1)}{a_1(-t_1)}\exp\left(i\int_{-t_1}^{t_1}E_1\,dt\right) \quad \text{for} \quad a_2(-t_1)=0. \quad (9)$$

It is readily shown that (8) and (9) lead to the same result provided only the quantity $t_0$ in (7) is taken to be the point of closest approach of the diabatic terms, i.e., $H_{11}(t_0) = H_{22}(t_0)$. We note that the choice of $t_0$ affects only the phase and not the modulation of the amplitude $g$.

Bates[16] has shown that, for a linear difference between diabatic terms in the neighborhood of crossing, $H_{11} - H_{22} \approx \Delta F v t$ ($\Delta F$ is the difference between the forces on the terms and $v$ is the relative velocity), the estimated time interval $\Delta t_{ad}$ during which transitions take place between adiabatic terms can be obtained from the condition

$$\int_0^{\Delta t_{ad}} \Delta dt \approx 1$$

and is given by

$$\Delta t_{ad} \approx (4\pi/\Delta F v)^{1/2}.$$

The same estimate is obtained for $\Delta t_{ad}$ in the linear case and from the condition that the exact solutions must have the asymptotic behavior of (7) (see Sec. 5, with $\Gamma_{1,2} = 0$).

In the diabatic basis, the transitions occur in a broader interval $\Delta t_d$ determined from the requirement that one of the coefficients in front of the exponentials in brackets in (5) is much smaller than the other, i.e., $|H_{11} - H_{22}| \gg H_{12}$. This inequality yields $\Delta t_d \ll |H_{12}/\Delta F v|$. Consequently,

$$\Delta t_{ad}/\Delta t_d \approx (\Delta F v)^{1/2}/|H_{12}| \ll 1$$

in the limit of small velocities when, and only when, the concept of an adiabatic term is valid.

Thus, even in this simplest case, the use of the adiabatic basis is to be preferred. In fact, the physical requirement that must be imposed on (8) is $|t_1 - t_0| \gg \Delta t_{ad}$, whereas that corresponding to (9) is $|t_1 - t_0| \gg \Delta t_d$, so that the coefficients $A_{1,2}$ reach the asymptotic region much more rapidly than $a_1$ and $a_2$. This difference is important when the solutions are matched in different regions. As will be shown below, this is particularly important in the case of quasistationary terms.

The traditional approach to quasistationary states is to consider the problem of the dependence of the populations on time. This was done for parallel terms in[6,7]. The equations given by (6) in[10], which are described there as probabilities, in fact determine the populations of two crossing quasistationary terms that do not interact through the continuum.

If we wish to retain the formulation of the problem of finding the time-independent probability, that is traditional for the two-level system, it is convenient to use the definition in the adiabatic basis (8). We note at once that, since the system given by (3) is not Hermitian, the usual requirement $|g| \leq 1$ is not satisfied for this "probability." Nevertheless, this extension of the definition of $g$ is very useful because it enables us to extend the usual formalism of the theory of slow atomic collisions to the case of quasistationary terms.

We also note that, in contrast to the case where $\Gamma_{1,2} = 0$, the choice of $t_0$ affects not only the phase but also the modulus of the amplitude $g$. The condition $H_{11}(t_0) = H_{22}(t_0)$ now gives a complex value for $t_0$ which is somewhat more difficult to interpret but is quite natural in adiabatic theory. On the other hand, any attempt to generalize the definition in the adiabatic basis (9) leads to its own much more substantial difficulties connected with the fact that the decay law for the diabatic states is not exponential (see below for further details).

Finally, we note the third possible formulation of the problem for the system (3), (4), which is: to determine the spectrum of the emitted particles, i.e., $b(\omega, t\to\infty)$, and to elucidate how the crossing of terms affects this spectrum.

4. We must now determine the quantity $g$. For simplicity, we shall assume initially that $E_1 = \alpha_1 t$, $E_2 = -\alpha_2 t$, and that $\Gamma_{1,2}$ is independent of time.[2)] In this approximation, the set of equations given by (3) corresponds to the well-known Landau-Zener problem[13] but now for the crossing of quasistationary levels with constant widths.

Let us consider the solution of (3) subject to the initial condition corresponding to the motion of the atoms over the lower adiabatic term $E_I$, i.e.,

$$|\Psi(t\to-\infty)\rangle = \Phi_1(t,t_0)\,|\Psi_I\rangle. \quad (10)$$

The required solution of (3) is

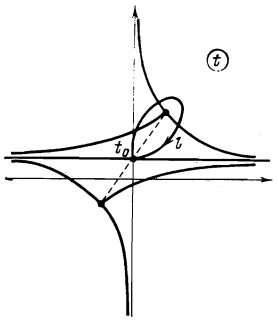


FIG. 1.

$$a_1=C\exp\left[-\frac{i}{2}\int_{t_0}^{t}(H_{11}+H_{22})\,dt\right]D_n\left[-\alpha^{1/2}e^{i\pi/4}\left(t-i\frac{\gamma}{\alpha}\right)\right],$$
$$a_2=-i\sqrt{n}\,C\exp\left[-\frac{i}{2}\int_{t_0}^{t}(H_{11}+H_{22})\,dt\right]D_{n-1}\left[-\alpha^{1/2}e^{i\pi/4}\left(t-i\frac{\gamma}{\alpha}\right)\right], \tag{11}$$

where $\alpha=\alpha_1+\alpha_2$, $\gamma=(\Gamma_1-\Gamma_2)/2$, and $D_n$ is the parabolic cylinder function with

$$n=-\frac{i}{\alpha}\left(V-i\frac{(\Gamma_1\Gamma_2)^{1/2}}{2}\right)^2.$$

Using the well-known asymptotic expressions for $D_n$[14] for $|\alpha^{1/2}t|>\sqrt{2}$, we can verify that the initial condition given by (10) is satisfied. The constant $C$ is uniquely related to the choice of $t_0$.

If we consider the solution (11) for $t>0$, we find that the component of the wave function corresponding to motion over the upper adiabatic term is

$$e^{-i\pi n}\Phi_2(t,t_0)\,|\Psi_{II}\rangle.$$

Consequently, in accordance with (8), the probability amplitude for the nonadiabatic transition is

$$g=e^{-i\pi n},\quad |g|^2=\exp\left[-\frac{\pi}{\alpha}\left(V^2-\frac{\Gamma_1\Gamma_2}{4}\right)\right]. \tag{12}$$

so that, for $V^2<\Gamma_1\Gamma_2/4$, we have $|g|>1$ (see the Appendix).

For an arbitrary function $E_{1,2}(t)$, the transition amplitude can be found by the method of phase integrals assuming that the splitting of the adiabatic terms ($\Delta$) has a pair of complex roots $t_{1,2}$ close to the real axis and sufficiently isolated from other zeroes and singularities in $\Delta$. For $V^2>\Gamma_1\Gamma_2/4$, the Stokes lines on which

$$\mathrm{Im}\int_{t_0}^{t}\Delta\,dt=0,$$

are shown by the solid lines in Fig. 1. To solve the problem, we must specify (10) for sufficiently large $|t|$ on a Stokes line passing through $t_0(t_1+t_2)/2$ ($t_0=-i\gamma/\alpha$ for linear terms).

Beyond this, the solution is constructed as shown in[15] for Hermitian problems on over-barrier reflection or nonadiabatic transitions. We find that

$$\Phi_1(t<-\Delta t_{ad},t_0)\,|\Psi_I\rangle\to g\Phi_2(t>\Delta t_{ad},t_0)\,|\Psi_{II}\rangle, \tag{13}$$
$$g=\exp\left(-\frac{i}{2}\int_{l}\Delta dt\right). \tag{14}$$

The contour $l$ runs around the upper zero in $\Delta$ along the cut and begins at the point $t_0$, as shown in Fig. 1.

When $V^2<\Gamma_1\Gamma_2/4$, the Stokes lines are as shown in Fig. 2. The situation resembles subbarrier transmission with the one difference that the zeros in $\Delta$ no longer lie on the real axis. If we use this analogy, we can readily construct the corresponding approximate solution. The usual approach to subbarrier transmission is to evaluate the phase integral in (13) between the turning points $t_{1,2}$. Correspondingly, the transition amplitude is given by

$$g=\exp\left(-\frac{i}{2}\int_{t_1}^{t_2}\Delta dt\right),$$

which, of course, is equal to the amplitude in which the integral is evaluated over the contour around one of the turning points. To achieve a unified approach, we shall therefore assume that, in both cases, $V\gtrless(\Gamma_1\Gamma_2)^{1/2}/2$, the phase integrals in (13) are taken from the point $t_0$, and that the transition amplitude is given by (14). In the case of linear terms, (14) becomes identical with (12).

Bazylev and Zhevago[10] have noted, in connection with the reappraisal of the work of Karas' *et al.*,[9] that the Stueckelberg method, i.e., the method of phase integrals, is not valid in the case of a complex $\Delta$. A derivation of (14) shows that this is not so. However, the results in[9] and of our own work are quite different. In fact, it follows from (12) and (14) that, in the presence of interaction through the continuum (which is essentially the case in[9]), the transition amplitude is $e^{-\pi V^2/\alpha}$ according to the Landau-Zener theory, whereas the formula given by (4) in[9] gives a much more complicated expression. The reason for the discrepancy is that the phase integrals in[9] are evaluated between 0 and $t$ along the real axis which, even in the absence of interaction through the continuum ($H_{12}$ real), is not a Stokes line. This integration gives rise to factors on the left and right of (13), whose moduli are not equal. The inclusion of these factors (and they were not allowed for in[9]) leads to the correct expression for the amplitude, i.e., $e^{-\pi V^2/\alpha}$.

5. We must now consider the time dependence of the populations in the problem with linear terms. For diabatic states, the result is given by (11) for any $t$. Using the asymptotic behavior of $D_n$ and omitting the unimportant common normalizing constant, we have for $t<-(2/\alpha)^{1/2}$

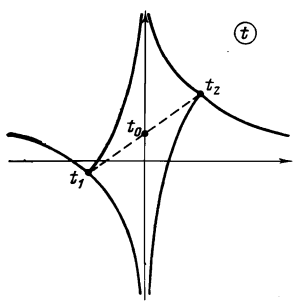


FIG. 2.

$$a_1(t)=(-\alpha^{1/2}t)^n e^{(1)},\quad a_2(t)=-in^{1/2}e^{i\pi/4}(-\alpha^{1/2}t)^{n-1}e^{(2)}, \tag{15}$$

and for $t>(2/\alpha)^{1/2}$

$$a_1(t)=e^{-i\pi n}(\alpha^{1/2}t)^n e^{(1)}+\frac{\sqrt{2\pi}}{\Gamma(-n)}\exp\left(-i\frac{\pi}{2}n-i\frac{\gamma^2}{2\alpha}-i\frac{\pi}{4}\right)(\alpha^{1/2}t)^{-n-1}e^{(2)},$$
$$a_2(t)=-in^{1/2}\left[-\exp\left(-i\pi n-i\frac{\pi}{4}\right)(\alpha^{1/2}t)^{n-1}e^{(1)}\right. \tag{16}$$
$$\left.+\frac{\sqrt{2\pi}}{\Gamma(1-n)}\exp\left(-i\frac{\pi}{2}n-i\frac{\gamma^2}{\alpha}\right)(\alpha^{1/2}t)^{-n}e^{(2)}\right],$$

where

$$e^{(1)}=\exp\left(-\frac{i}{2}\alpha_1 t^2-\frac{\Gamma_1 t}{2}\right),\qquad e^{(2)}=\exp\left(\frac{i}{2}\alpha_2 t^2-\frac{\Gamma_2 t}{2}\right).$$

It is clear from these expressions that, in general, the attenuation has a complicated nonexponential character for each diabatic state.

Let us now consider in greater detail the important case of small widths $V(\Gamma_1\Gamma_2)^{1/2}/\alpha=-\mathrm{Re}n\ll 1$ when the power factors in (15) and (16) need not be taken into account. For $\Gamma_1\ll\Gamma_2$, we find from (16) that the attenuation of both states is determined by the width of the initial populated state. For sufficiently large positive times we have $|a_1/a_2|\gg 1$, i.e., the second state is not, in fact, populated as a result of crossing. When $\Gamma_2\ll\Gamma_1$, on the other hand, the population of the second state is much greater than the population of the initial first state after the crossing when $\alpha^{1/2}t\gg 1$. Finally, when $\Gamma_1\approx\Gamma_2$, both exponents must be taken into account in (16).

The attenuation of the populations of adiabatic states is much simpler. In the problem involving linear terms, the expressions given by (13)–(16) for $|t|>\Delta t_{ad}$ yield the following expressions for the population of the state $|\Psi_I\rangle$:

$$\exp\left(-i\int_{t_0}^{t}E_1 dt\right),\qquad t<-\Delta t_{ad};$$
$$f\exp\left(-i\int_{t_0}^{t}E_1 dt\right),\qquad t>\Delta t_{ad} \tag{17}$$

and the population of the state $|\Psi_{II}\rangle$:

$$0,\quad t<-\Delta t_{ad}; \tag{18}$$
$$g\exp\left(-i\int_{t_0}^{t}E_1 dt\right),\qquad t>\Delta t_{ad}.$$

The transition amplitude $g$ was found above, and the quantity

$$f=-\frac{i(2\pi n)^{1/2}}{\Gamma(1-n)}\exp\left[-i\frac{\pi}{2}n+n-n\ln(in)\right] \tag{19}$$

can be interpreted as the probability amplitude for remaining on the initial adiabatic term after passing through the nonadiabatic region. When $\Gamma_1=\Gamma_2=0$, the expression given by (19) becomes identical with the usual Landau-Zener result.[13] In general, however, $f$ and $g$ are independent because the relation $|f|^2+|g|^2=1$ is no longer satisfied. Both $g$ and $f$ must be known, for example, when we wish to calculate the transition probabilities and populations in the case of a double transit through the region of nonadiabatic term interaction. In this case, the solution obtained within the framework of the linear model must be matched to the solutions corresponding to the motion over real terms outside the nonadiabatic region. In (17) and (18), the matching of the solution is achieved simply by replacing the model dependence $E_{I,II}(t)$ by the real dependence. Finally, we note that, in the case of the general function $E_{I,II}(t)$, the use of the method of phase integrals again leads to (17) and (18) but with $f=1$. This is a reflection of the well-known fact that the method of adiabatic integration of (3) does not take into account the change in the amplitude for the initial state due to the nonadiabatic transition.[13]

The formulas given by (17) and (18) enable us to elucidate once again the significance and usefulness of the concept of transition probability in the non-Hermitian problem which we have considered. It follows from these expressions that this probability takes into account the effect of the interaction in the transition region on the population of states.

As in the case of discrete terms, the transition probability amplitude can be interpreted as the factor in front of the corresponding exponential in the dependence of the populations of diabatic terms on time (16). The definition of the probability in the diabatic basis (9) rests on the possibility of isolating this coefficient in the limit as $|t|\to\infty$. However, as we have seen, this separation cannot, in general, be achieved by this method for quasistationary terms, even in the case of a linear time dependence. On the other hand, when this separation is possible, the approach to asymptotic behavior may be found to be slower as compared with the case of discrete terms, which gives the impression of an expansion of the region of interaction between the quasistationary terms.[10] Here, we must note that this "expansion" is only a manifestation of decay effects in the relatively large region of interaction between the diabatic terms, $-\Delta t_d<t<\Delta t_d$. The region of interaction between the adiabatic terms, which has a direct physical significance, remains unaltered.

6. We note in conclusion that it follows from the form of $H_{12}=V-i(\Gamma_1\Gamma_2)^{1/2}$ that the effects of the usual interaction and the interaction through the continuum are, in general, of equal importance. For example, for many processes involving the emission of an electron, $\Gamma\sim 1$ eV, which is even greater than the usual magnitude of $V$. It is therefore difficult to agree with the main conclusion in[6] that the attenuation of states has no effect on the transition probability.

The authors are indebted to Prof. N. P. Penkin for his constant interest and support, to Prof. Yu. N. Demkov, Prof. G. F. Drukarev, and V. S. Rudakov for useful discussions, and to S. S. Moiseev for suggestions that have led to an improvement in our presentation.

## APPENDIX

It follows from[12] that $|g|>1$ when $V^2<\Gamma_1\Gamma_2/4$. To elucidate the physical reason for the behavior of the

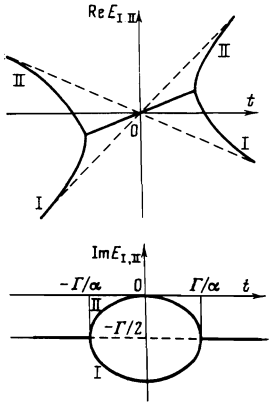


Fig. 3.

"probability," let us consider the limiting case when the terms interact only through the continuum ($V = 0$) and, for simplicity, let us suppose that $\Gamma_1 = \Gamma_2 = \Gamma$. The power factors $t^n$ in (15) and (16) have moduli equal to unity, so that the difficulties associated with the application of the diabatic basis are of lesser significance.

The solid lines in Fig. 3 represent the time dependence of the real and imaginary parts of the energy of the adiabatic terms in this case, and the broken lines represent the same quantities for diabatic terms. It is clear that, when $-\Gamma/\alpha < t < \Gamma/\alpha$, the effective width of the term to which the transition takes place is less than $\Gamma$.

When $t \ll -\Delta t_d$, the decay of the diabatic state $|\varphi_1\rangle$ occurs in accordance with the formula $\exp(-\Gamma t)$ and, for $t \gg \Delta t_d$, it occurs in accordance with the law $|g|^2 e^{-\Gamma t}$ with $|g| > 1$. This behavior of the probability of transition between quasistationary states is connected with the fact that the interaction through the continuum leads to a slowing down of the decay of the populated state in the region of term interaction. This is clear from Fig. 3. Outside the interaction region, where the terms decay with constant width, this slowing down leads to the appearance of the factor $|g| > 1$ in the formula for the population.

[1)] A brief report of this research was presented to the First All-Union Seminar on Autoionization Phenomena, Moscow State University, and to the Ninth International Conference on the Physics of Electronic and Atomic Collisions, Seattle, USA, 1975.[1]

[2)] The same approximations were used in[9,10].


[1] A. Z. Devdariani, V. N. Ostrovsky, and Yu. N. Sebyakin, Abstracts of Papers read at the Ninth ICPEAC, Seattle, 1975, p. 220; A. Z. Devdariani, V. N. Ostrovskiĭ, and Yu. N. Sebyakin, Sbornik trudov pervogo vsesoyuznogo seminara po avtoionizatsionnym yavleniyam (Collection of Papers given at the First All-Union Seminar on Autoionization Phenomena), Izd. MGU, 1976, p. 102.

[2] V. V. Afrosimov, Yu. S. Gordeev, A. N. Zinov'ev, D. Kh. Rasulov, and A. P. Shergin, Sbornik trudov pervogo Vsesoyunogo seminara po avtionizatsionnym yavleniyam (Collection of Papers given at the First All-Union Seminar on Autoionization Phenomena), Izd. MGU, 1976, p. 25.

[3] V. V. Afrosimov, G. A. Leĭko, and M. N. Popov, Sbornik trudov pervogo Vsesoyunogo seminara po avtoionizatsionnym yavleniyam (Collection of Papers given at the First All-Union Seminar on Autoionization Phenomena), Izd. MGU, 1976, p. 37.

[4] R. F. Mathis and W. R. Snow, J. Chem. Phys. **61**, 4274 (1974).

[5] A. V. Lazuta, S. Yu. Slavyanov, and I. B. Tampel', Sbornik trudov V Vsesoyuznogo simpoziuma po difraktsii i rasprostraneniyu voln (Collection of Papers given at the Fifth All-Union Symposium on Diffraction and Propagation of Waves), Nauka, Leningrad, 1971, p. 125.

[6] V. I. Kogan, V. S. Lisitsa, and A. D. Selidovkin, Zh. Eksp. Teor. Fiz. **65**, 152 (1973) [Sov. Phys. JETP **38**, 75 (1974)].

[7] V. S. Lisitsa and S. I. Yakovlenko, Zh. Eksp. Teor. Fiz. **66**, 1981 (1974) [Sov. Phys. JETP **39**, 975 (1974)].

[8] V. I. Osherov, Fiz. Tverd. Tela (Leningrad) **10**, 30 (1968) [Sov. Phys. Solid State **10**, 21 (1968)].

[9] V. I. Karas', S. S. Moiseev, and V. E. Novikov, Zh. Eksp. Teor. Fiz. **67**, 1702 (1974) [Sov. Phys. JETP **40**, 847 (1975)].

[10] V. A. Bazylev and N. K. Zhevago, Zh. Eksp. Teor. Fiz. **69**, 853 (1975) [Sov. Phys. JETP **42**, 436 (1975)].

[11] V. N. Ostrovskiĭ, Vestn. Leningr. Univ. Ser. Fiz. Khim. **16**, 31 (1972).

[12] F. I. Dalidchik, Yad. Fiz. **21**, 51 (1975) [Sov. J. Nucl. Phys.]

[13] E. E. Nikitin, Teoriya élementarnykh atomno-molekulyarnykh reaktsiĭ (Theory of Elementary Atom-Molecule Reactions), Part 1, NGU, Novosibirsk, 1965.

[14] A. Kratzner and W. Franz, Transcendental Functions (Russ. Transl. IIL, 1963).

[15] G. M. Zaslavskiĭ, Lektsii po primeneniyu metoda VKB v fizike (Lectures on the Application of the WKB Method in Physics), NGU, Novosibirsk, 1965.

[16] D. R. Bates, Proc. R. Soc. Ser. A **257**, 22 (1960).


Translated by S. Chomet