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Multiregge processes in the Yang-Mills theory

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For nonabelian gauge theories with the Higgs mechanism of mass generation the scattering amplitudes have been calculated in the multiregge kinematics in the leading logarithmic approximation. The reggeization of the vector particle is proved in this approximation.

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1. INTRODUCTION

The hypothesis that all observed hadrons are reggeons turned out to be so fruitful that it gave the possibility of constructing a phenomenological theory of hadron interactions at high energies, based on Gribov's reggeon diagram technique.[1] Testing this theory in the framework of a local field theory presents great interest. As is known, in an abelian gauge theory—quantum electrodynamics (QED)—the spinor particle reggeizes,[2] but the vector particle remains elementary.[3] More realistic models for the strong interactions will probably be based on nonabelian gauge theories with Yang–Mills vector mesons.[4] In such theories the interaction vanishes at small distances, so that in distinction from QED[5] they exhibit approximate scale-invariance.[6]

Moreover, the Higgs mechanism[7] allows the vector mesons in nonabelian gauge theories to acquire a mass without destroying the renormalizability of the theory.[8] In a spontaneously broken theory which arises in this manner the necessary conditions for the reggeization of the vector meson are satisfied.[9] One of the authors has shown by direct calculation of the scattering amplitudes to sixth order of perturbation theory that for the gauge group $SU(2)$ the reggeization of the vector meson does indeed take place.[10] Later this result was generalized to other models.[11] In our preceding short note, based on calculations up to eighth order of perturbation theory, we have shown that the hypothesis of reggeization to any order is self-consistent and have also determined the form of the Pomeranchuk singularity.[12]

In the present paper we give the details of these calculations to eighth order for the group $SU(2)$. Our computation method which is based on the dispersion theory approach, also allows one to obtain the inelastic amplitudes in the multiregge kinematics (Eq. (55)). Making use of the expression (55) for these amplitudes we obtain an equation of the Bethe–Salpeter type for the $2\to 2$ partial wave amplitudes with isospins $T=0$, 1, 2 in the t-channel, Eq. (64). The solution of the equation with $T=1$ is the Regge pole (66). The Appendix contains a generalization of these results to the group $SU(N)$.

2. THE MODEL AND THE RESULTS OF CALCULATIONS OF THE TWO-PARTICLE AMPLITUDES IN LOWEST ORDERS

Following[10], we consider the simplest nonabelian gauge theory whose Lagrangian, after spontaneous symmetry breaking and removal of the unphysical degrees of freedom by means of a gauge transformation has the form

$$L=-\tfrac{1}{4}(\partial_\mu \mathbf{A}_\nu-\partial_\nu \mathbf{A}_\mu-g[\mathbf{A}_\mu \mathbf{A}_\nu])^2+\mathbf{A}_\mu \mathbf{A}^\mu(\tfrac{1}{2}m^2+\tfrac{1}{8}mg\varphi+\tfrac{1}{8}g^2\varphi^2)+\tfrac{1}{2}(\partial_\mu\varphi)^2-\tfrac{1}{2}\mu^2\varphi^2-\tfrac{1}{4}g\mu^2m^{-1}\varphi^3-\tfrac{1}{32}g^2\mu^2m^{-2}\varphi^4+\bar\psi(i\hat\partial-\tfrac{1}{2}g\hat{\mathbf{A}}\boldsymbol{\tau}-M)\psi. \quad (1)$$

As can be seen, this theory contains an isospin triplet of vector fields $\mathbf{A}_\mu$ of mass m, a scalar isosinglet φ of mass μ and the spinor isodoublet ψ of mass M.

In the sequel we shall denote particles by the letters A, B, etc. and the corresponding momenta by p_A, p_B, etc. The isospin indices of the vector (spinor) particles A, B will be denoted by a, b (α, β) and their polarizations by λ_A, λ_B (r_A, r_B). As before,[10] we choose the polarization vectors as:

$$(e_{\lambda_A}p_A)=0,\quad (\mathbf{e}_{\lambda_A}\mathbf{p}_A)|_{\lambda_A=1,2}=0,\quad e_{\lambda_A=3}=e^0\frac{|\mathbf{p}_A|}{m}+\frac{\mathbf{p}_A}{|\mathbf{p}_A|}\frac{p_A^0}{m},$$
$$e^0=(1.0,0.0),\quad e_{\lambda_A}^2=-1. \quad (2)$$

The amplitude of the process $A+B\to A'+B'$, in the Born approximation and the Regge region

$$s=(p_A+p_B)^2\gg m^2,\quad -t=-q^2\sim m^2,\quad q=p_{A'}-p_A \quad (3)$$

can be represented in the form

$$A_{AB}^{A'B'}(s,q)=\Gamma_{AA'}^i\frac{s}{t-m^2}\Gamma_{BB'}^i, \quad (4)$$

where a summation over the isospin index i is understood, and the vertex functions $\Gamma_{AA'}^i$ and $\Gamma_{BB'}^i$ for the vector (V), scalar (S), and spinor (F) particles, in the center of inertia frame $\mathbf{p}_A+\mathbf{p}_B=0$, are

$$\Gamma_{VV'}^i=-2igc_{vv'}^i\delta_{\lambda_V\lambda_{V'}}a_{\lambda_V},\quad \Gamma_{VS}^i=\Gamma_{SV}^i=g\sqrt{2}\delta_v^i b_{\lambda_V}, \quad (5)$$

$$\Gamma_{SS'}^i=0,\quad \Gamma_{FF'}^i=-\frac{g}{\sqrt{2}}(\tau^i)_{\varphi'\varphi}\delta_{r_F r_{F'}},$$
$$b_\lambda=\frac{1}{2}\delta_{\lambda 3},\quad a_\lambda=1-b_\lambda,\quad c_{vv'}^i=\frac{1}{\sqrt{2}}\varepsilon_{ivv'}. \quad (6)$$

Here $c_{vv'}^i$ is the projection operator mapping the state of two vector mesons onto the state with $T=1$. In the absence of fermions these amplitudes have been calculated elsewhere[10]; the amplitudes with fermions present are calculated similarly. We have adopted the spinor normalization $\bar u^r(p)u^{r'}(p)=2M\delta_{rr'}$. As is evident from (4) and (5), in the Born approximation scattering occurs only with isospin $T=1$ in the t-channel, and the relative magnitude of amplitudes with different polarizations follows from the isospin properties of the original massless theory.[10]

We underline the fact that Eqs. (5) and (6) give the vertex functions for definite polarizations in the center-of-inertia frame of the incident particles. In the sequel we shall need the expression for the scattering amplitude in an arbitrary frame. We introduce the notation

$$\{e_V,e_{V'}\}_p=-(e_Ve_{V'})+\frac{(pe_V)(p_Ve_{V'})+(pe_{V'})(p_{V'}e_V)}{(pp_V)}-(m^2-(p_V-p_{V'})^2)\frac{(pe_V)(pe_{V'})}{2(pp_V)^2}. \quad (7)$$

In the frame with $\mathbf{p}_A+\mathbf{p}_B=0$ we have

$$\{e_A,e_{A'}\}_{p_B}=\delta_{\lambda_A\lambda_{A'}}a_{\lambda_A}.$$

We note that $\{e_A,e_{A'}\}_{p_B}$ can be represented in the form

$$e_A^\mu e_{A'}^\nu p_B^\rho\gamma_{\mu\nu\rho}(2p_Ap_B)^{-1}\quad\text{for}\quad (p_A-p_{A'})^2=m^2,$$

where $\gamma_{\mu\nu\rho}$ are the Yang–Mills vertices.[10] In an arbitrary reference frame the amplitude $A_{AB}^{A'B}$ will be given by equation (4) if the vertex functions defined in (5) are subjected to the substitution

$$\delta_{\lambda_A\lambda_{A'}}a_{\lambda_A}\to\{e_A,e_{A'}\}_{p_B},\quad b_{\lambda_A}\to m(p_Be_A)(2p_Ap_B)^{-1},$$
$$\delta_{\lambda_B\lambda_{B'}}a_{\lambda_B}\to\{e_B,e_{B'}\}_{p_A},\quad b_{\lambda_B}\to m(p_Ae_B)(2p_Ap_B)^{-1}, \quad (8)$$
$$\delta_{r_Ar_{A'}}\to\bar u^{r_{A'}}(p_{A'})\hat p_Bu^{r_A}(p_A)(2p_Ap_B)^{-1},\quad \delta_{r_Br_{B'}}\to\bar u^{r_{B'}}(p_{B'})\hat p_Au^{r_B}(p_B)(2p_Ap_B)^{-1}.$$

The determination of the scattering amplitude in the next orders of perturbation theory is made under the assumption

$$g^2\ln\frac{s}{m^2}\sim 1,\quad g^2\ll 1 \quad (9)$$

retaining only the leading logarithmic terms, and reduces to the determination of the s-channel discontinuities of the amplitude:

$$\mathrm{disc}_s A_{AB}^{A'B'}(s,q)=\sum_N\int d\rho_N A_{AB}^N\overset{*}{A}{}_{A'B'}^N. \quad (10)$$

Here the sum extends over all possible intermediate states N, A_{AB}^N is the amplitude of the transition $A+B\to N$, and $d\rho_N$ is the corresponding density of states for the N particles

$$d\rho_{A_1A_2\ldots A_N}=\int(2\pi)^4\delta(p_A+p_B-p_{A_1}-\ldots-p_{A_N})\prod_{i=N}^{N}\left(\frac{d^3p_{A_i}}{2E_{A_i}(2\pi)^3}\right). \quad (11)$$

The reconstruction of the amplitude in terms of its s-channel discontinuity reduces to carrying out in the expression of the discontinuity the substitution (cf. for details[10])

$$s\ln^n\left(\frac{s}{m^2}\right)\to-\left[s\ln^{n+1}\left(-\frac{s}{m^2}\right)\pm u\ln^{n+1}\left(-\frac{u}{m^2}\right)\right][2\pi(n+1)]^{-1} \quad (12)$$

with the plus sign for positive signature (t-channel isospin $T=0$; 2) and with the minus sign for negative signature ($T=1$). In the order g^4 of perturbation theory only two-particle intermediate states are possible in (10). Utilizing the relation

$$\sum_{A''}\Gamma_{AA''}^i\overset{*}{\Gamma}{}_{A'A''}^j=ig[\Gamma_{AA'}^k c_{ij}^k+\Gamma_{AA'}c_{ij}+\Gamma_{AA'}^{kl}c_{ij}^{kl}], \quad (13)$$

where $\sum_{A''}$ in (13) denotes summation over particle types and over their polarization and isospin states,

$$c_{ij}=\frac{1}{\sqrt{3}}\delta_{ij},\quad c_{ij}^{kl}=\tfrac{1}{2}[\delta_{ki}\delta_{lj}+\delta_{li}\delta_{kj}-\tfrac{2}{3}\delta_{kl}\delta_{ij}] \quad (14)$$

are the projection operators onto states with $T=0$; 2 in the t-channel, and the quantities $\Gamma_{AA'}$, $\Gamma_{AA'}^{kl}$ for vector (V), scalar (S) and spinor (F) particles

$$\Gamma_{SS'}=-ig\frac{\sqrt{3}}{2},\quad \Gamma_{VV'}=-2igc_{vv'}\delta_{\lambda_V\lambda_{V'}}(2a_{\lambda_V}^2+b_{\lambda_V}^2),$$
$$\Gamma_{SV}=\Gamma_{VS}=0,\quad \Gamma_{VV'}^{ij}=-2igc_{vv'}^{ij}\delta_{\lambda_V\lambda_{V'}}(b_{\lambda_V}^2-a_{\lambda_V}^2),$$
$$\Gamma_{SS'}^{ij}=\Gamma_{SV}^{ij}=\Gamma_{VS}^{ij}=\Gamma_{FF'}^{ij}=0, \quad (15)$$
$$\Gamma_{FF'}=-ig\frac{\sqrt{3}}{2}\delta_{\varphi'\varphi}\delta_{r_Fr_{F'}},$$

we obtain the result derived in[10] (in a somewhat different form) for the scattering amplitude to fourth order:

$$A_{AB}^{A'B'}(s,q)=\Gamma_{AA'}^{i}\frac{1}{t-m^2}\Gamma_{BB'}^{i}\left(\frac{s-u}{2}+\frac{\alpha(t)}{1!}\frac{s\ln(-s/m^2)-u\ln(-u/m^2)}{2}\right)$$
$$+(\Gamma_{AA'}\Gamma_{BB'}+\Gamma_{AA'}^{ij}\Gamma_{BB'}^{ij})\beta_2(t)\frac{s\ln(-s/m^2)+u\ln(-u/m^2)}{2}, \quad (16)$$

where

$$\beta_2(t)=g^2(2\pi)^{-3}\int d^2k_\perp[(m^2-k_\perp^2)(m^2-(q-k_\perp)^2)]^{-1},$$
$$\alpha(t)=(t-m^2)\beta_2(t). \quad (17)$$

In order g^6 of perturbation theory both two- and three-particle intermediate states are possible in (10). The contribution of two-particle states is calculated in the same manner as in the preceding order. For the calculation of the three-particle contribution one needs the Born amplitude for the production of three particles. For incident vector particles this amplitude has been obtained in[10]. In the following sections we determine the inelastic amplitudes with arbitrary number of final particles.

3. THE AMPLITUDE FOR THE PRODUCTION OF n PARTICLES IN THE BORN APPROXIMATION

In order to consider the process $A+B\to D_0+D_1+\ldots+D_{n+1}$ we introduce the momentum transfers

$$q_i=q_{i-1}+p_{D_{i-1}},\quad i=1,2,\ldots,n+1,$$
$$q_0=-p_A \quad (18)$$

and their Sudakov expansions

$$q_i=\beta_ip_B-\alpha_ip_A+q_{\perp i}. \quad (19)$$

The main contribution to the unitarity condition (10) to logarithmic accuracy (9) is given by multiregge kinematics, the region in which we will determine the amplitude,

$$1\gg\alpha_1\gg\alpha_2\gg\ldots\gg\alpha_{n+1}\sim m^2/s,$$
$$1\gg\beta_{n+1}\gg\beta_n\gg\ldots\gg\beta_1\sim m^2/s,\quad |q_{\perp i}^2|\sim m^2. \quad (20)$$

Here

$$s\alpha_i\beta_{i+1}=m^2-(q_i-q_{i+1})_\perp^2,\quad q_i^2=q_{i\perp}^2,$$
$$d\rho_{D_0\ldots D_{n+1}}=[(2\pi)^{3n+2}\cdot2^{n+1}\cdot s]^{-1}\prod_{i=1}^{n}\frac{d\alpha_i}{\alpha_i}\prod_{k=1}^{n+1}d^2q_{k\perp} \quad (21)$$

and after integration with respect to α, taking (20) into account,

$$d\rho_{D_0D_1\ldots D_{n+1}}\to\ln^n\left(\frac{s}{m^2}\right)[(2\pi)^{3n+2}\cdot2^{n+1}\cdot s\cdot n!]^{-1}\prod_{k=1}^{n+1}d^2q_{k\perp}. \quad (22)$$

We first consider the amplitude $A_{2\to2+1}$ of the process $A+B\to D_0+D_1+D_2$. The following reasoning differs somewhat from the one in[10], and admits a simple generalization to the general case. We determine the pole part in q_2^2 of the amplitude (we denote this part by $P_{q_2^2}A_{2\to2+1}$). It is obtained from the absorptive part by means of the substitution $\pi\delta(q_2^2-m^2)\to(m^2-q_2^2)^{-1}$. The absorptive part in q_2^2 which is proportional to s, appears only for a vector meson exchange in the q_2^2-channel. After the usual substitution for the spin matrix of that meson

$$-g^{\mu\nu}+q_2^\mu q_2^\nu m^{-2}\approx\frac{2}{s}(p_B^\mu p_A^\nu+p_B^\nu p_A^\mu)$$

we obtain (cf. (4))

$$P_{q_2^2}A_{2\to2+1}=A_{AJ}^{D_0D_1}\frac{s}{q_2^2-m^2}\Gamma_{BD_2}^{i}, \quad (23)$$

where in the frame $\mathbf{p}_A+\mathbf{p}_B=0$ the vertex function Γ_{BD}^i is given by (5), and $A_{AJ}^{D_0D_1}$ is the amplitude of the process $A+J\to D_0+D_1$ where J is a vector particle with momentum q_2, isospin i (which is summed over in (23)) and with the polarization vector replaced by $\sqrt{2}p_Bs^{-1}$. Recalling the discussion following Eq. (6) and taking into account that in the frame where $\mathbf{p}_A+\mathbf{p}_B=0$

$$\{e_{\lambda_A},e_{\lambda_{D_0}}\}_{q_2}=\delta_{\lambda_A\lambda_{D_0}}a_{\lambda_A},\qquad m(q_2e_{\lambda_A})(2p_Aq_2)^{-1}=b_{\lambda_A},$$
$$m(q_2e_{\lambda_{D_0}})(2p_{D_0}q_2)^{-1}=b_{\lambda_{D_0}}, \quad (24)$$

we obtain

$$A_{AJ}^{D_0D_1}=\Gamma_{AD_0}^{k}\frac{s\beta_2}{q_1^2-m^2}\bar\Gamma_{JD_1}^{k}, \quad (25)$$

where $\bar\Gamma_{JD_1}^k$ is obtained from $\Gamma_{JD_1}^k$ by means of the substitution

$$\delta_{\lambda_J\lambda_D}a_{\lambda_D}\to\{e_J,e_D\}_{p_A},\qquad b_{\lambda_J}\to m(p_Ae_J)(2p_Aq_2)^{-1},$$
$$e_J\to\sqrt{2}p_Bs^{-1}. \quad (26)$$

After this substitution we obtain for the case when D_1 is a scalar particle

$$P_{q_2^2}A_{2\to2+1}=s\Gamma_{AD_0}^{k}\frac{mg\delta_{ik}}{(q_1^2-m^2)(q_2^2-m^2)}\Gamma_{BD_2}^{i} \quad (27)$$

and when D_1 is a vector particle (taking into account that $(e_{D_1}(q_2-q_1))=0$),

$$P_{q_2^2}A_{2\to2+1}=s[(q_1^2-m^2)(q_2^2-m^2)]^{-1}\Gamma_{AD_0}^{k}ige_{d_1ki}[-(q_1+q_2)_\mu$$
$$-p_{A_\mu}(2(p_Bp_{D_1})(p_Ap_B)^{-1}-(m^2-q_1^2)(p_Ap_{D_1})^{-1})+2p_{B_\mu}(p_Ap_{D_1})(p_Ap_B)^{-1}]e_{D_1}^{\mu}\Gamma_{BD_2}^{i}. \quad (28)$$

The pole part in q_1^2 of the amplitude is determined in exactly the same way. For the case of production of a scalar particle D_1 it coincides with the q_2^2 pole part (27), and for the production of a vector particle

$$P_{q_1^2}A_{2\to2+1}=s[(q_1^2-m^2)(q_2^2-m^2)]^{-1}\Gamma_{AD_0}^{k}ige_{d_1ki}[-(q_1+q_2)_\mu$$
$$+p_{B_\mu}(2(p_Ap_{D_1})(p_Ap_B)^{-1}-(m^2-q_2^2)(p_Bp_{D_1})^{-1})-2p_{A_\mu}(p_Bp_{D_1})(p_Ap_B)^{-1}]e_{D_1}^{\mu}\Gamma_{BD_2}^{i}. \quad (29)$$

The renormalizability of the theory requires that the amplitude decrease in q_1^2 and q_2^2 (cf. [10]), so that Eqs. (27)–(29) imply that the amplitude of the process $A+B\to D_0+D_1+D_2$ has the form

$$A_{2\to 2+1}=s\frac{\Gamma^{i}_{AD_0}\gamma^{D_1}_{ij}(q_1,q_2)\Gamma^{j}_{BD_2}}{(q_1^2-m^2)(q_2^2-m^2)}, \tag{30}$$

where

$$\gamma_{ij}^{D_1}(q_1,q_2)=mg\delta_{ij} \tag{31}$$

for the production of scalar particles, and

$$\begin{gathered}\gamma_{ij}^{D_1}(q_1,q_2)\equiv ig\varepsilon_{d_1ij}\mathscr{P}_\mu(q_1,q_2)e_{D_1}^{\mu}=ig\varepsilon_{d_1ij}[-(q_1+q_2)_\mu\\ -p_{A_\mu}(2(p_Bp_{D_1})(p_Ap_B)^{-1}-(m^2-q_1^2)(p_Ap_{D_1})^{-1})+p_{B_\mu}(2(p_Ap_{D_1})(p_Ap_B)^{-1}\\ -(m^2-q_2^2)(p_Bp_{D_1})^{-1})]e_{D_1}^{\mu}=ig\varepsilon_{d_1ij}[-(q_{1\perp}+q_{2\perp})_\mu\\ -p_{A_\mu}(\alpha_1-2(m^2-q_1^2)(s\beta_2)^{-1})+p_{B_\mu}(\beta_2-2(m^2-q_2^2)(s\alpha_1)^{-1})]e_{D_1}^{\mu},\\ p_{D_1}=q_2-q_1\end{gathered} \tag{32}$$

for the production of vector particles.

The amplitude for the production of a larger number of particles in the Born approximation can be determined in a completely analogous manner. As an example we consider the amplitude $A_{2\to 2+2}$ of the process $A+B\to D_0+D_1+D_2+D_3$ in the case when the particles D_1 and D_2 are vectors. For the q_3^2 pole part we have (cf. Eq. (23))

$$P_{q_3^2}A_{2\to 2+2}=A_{2\to 2+1}\frac{s}{q_3^2-m^2}\Gamma^{k}_{BD_3}, \tag{33}$$

where $A_{2\to 2+1}$ is the amplitude of the process $A+K\to D_0+D_1+D_2$, where K is a vector particle of momentum q_3, isospin index k (over which a summation is carried out in Eq. (33)) and with the polarization vector replaced by $\sqrt{2}p_Bs^{-1}$. In the frame with $\mathbf{p}_A+\mathbf{p}_B=0$ and with the polarization vector of the K particle equal to $e_{\lambda K}$, this amplitude is given by the expression s (30), (32) with the substitution $B\to K$, $p_B\to q_3$. It is clear from (32) that the substitution $p_B\to q_3$ does not change $\gamma^{D_1}_{ij}(q_1,q_2)$; moreover this quantity is written in invariant form. Therefore $A_{2\to 2+1}$ is obtained in the same manner as $A^{D_0D_1}_{AJ}$ in Eq. (23); as a result we obtain

$$\begin{gathered}P_{q_3^2}A_{2\to 2+2}=s[(q_1^2-m^2)(q_2^2-m^2)(q_3^2-m^2)]^{-1}\Gamma^{j}_{AD_0}\gamma^{D_1}_{ji}(q_1,q_2)ig\varepsilon_{d_2ik}\\ \times[-(q_{2\perp}+q_{3\perp})_\mu-p_{A_\mu}(\alpha_2-2(m^2-q_2^2)(s\beta_2)^{-1})+p_{B_\mu}\beta_3]e_{D_2}^{\mu}\Gamma^{k}_{BD_3}.\end{gathered} \tag{34}$$

In exactly the same manner, we obtain

$$\begin{gathered}P_{q_1^2}A_{2\to 2+2}=s[(q_1^2-m^2)(q_2^2-m^2)(q_3^2-m^2)]^{-1}\Gamma^{i}_{AD_0}ig\varepsilon_{d_1ij}[-(q_{1\perp}+q_{2\perp})_\mu\\ -p_{A_\mu}\alpha_1+p_{B_\mu}(\beta_2-2(m^2-q_2^2)(s\alpha_1)^{-1})]e_{D_1}^{\mu}\gamma^{D_2}_{jk}(q_2,q_3)\Gamma^{k}_{BD_3},\end{gathered} \tag{35}$$

and for the q_2^2 pole part we obtain (cf. (23))

$$P_{q_2^2}A_{2\to 2+2}=A^{D_0D_1}_{AJ}\frac{s}{q_2^2-m^2}A^{D_2D_3}_{JB}, \tag{36}$$

where the amplitude $A^{D_0D_1}_{AJ}$ is the same as in Eq. (23), and $A^{D_2D_3}_{JB}$ is obtained from it by the appropriate substitutions, so that

$$\begin{gathered}P_{q_2^2}A_{2\to 2+2}=s[(q_1^2-m^2)(q_2^2-m^2)(q_3^2-m^2)]^{-1}\Gamma^{i}_{AD_0}ig\varepsilon_{d_1ij}[-(q_{1\perp}+q_{2\perp})_\mu\\ -p_{A_\mu}(\alpha_1-2(m^2-q_1^2)(s\beta_2)^{-1})+p_{B_\mu}\beta_2]e_{D_1}^{\mu}ig\varepsilon_{d_2jk}[-(q_{2\perp}+q_{3\perp})_\nu-p_{A_\nu}\alpha_2\\ +p_{B_\nu}(\beta_3-2(m^2-q_3^2)(s\alpha_2)^{-1})]e_{D_2}^{\nu}\Gamma^{k}_{BD_3}.\end{gathered} \tag{37}$$

From Eqs. (34)–(37) and from the circumstance that for renormalizability the amplitude must decrease as a function of q_i^2, we obtain

$$A_{2\to 2+2}=s\frac{\Gamma^{i}_{AD_0}\gamma^{D_1}_{ij}(q_1,q_2)\gamma^{D_2}_{jk}(q_2,q_3)\Gamma^{k}_{BD_3}}{(q_1^2-m^2)(q_2^2-m^2)(q_3^2-m^2)}. \tag{38}$$

The generalization to an arbitrary number of particles is trivial. Thus, the amplitude of the process $A+B\to D_0+D_1+\ldots+D_{n+1}$ in the Born approximation has the form

$$\begin{gathered}A_{2\to 2+n}=s[(q_1^2-m^2)(q_2^2-m^2)\ldots(q_{n+1}^2-m^2)]^{-1}\Gamma^{i_1}_{AD_0}\gamma^{D_1}_{i_1i_2}(q_1,q_2)\\ \times\gamma^{D_2}_{i_2i_3}(q_2,q_3)\ldots\gamma^{D_n}_{i_ni_{n+1}}(q_n,q_{n+1})\Gamma^{i_{n+1}}_{BD_{n+1}},\end{gathered} \tag{39}$$

where the vertex functions $\Gamma^{i}_{AA'}$ are defined in Eq. (5) and the functions $\gamma^{D_1}_{ij}$ are defined in Eqs. (31) and (32).

4. THE THREE-PARTICLE PRODUCTION AMPLITUDE IN ORDER g^5. GENERALIZATION TO ANY ORDER

A knowledge of the scattering amplitude to the order g^4 of perturbation theory and of the Born amplitude for the production of three particles allows one to obtain the scattering amplitude to the order g^6. In the order g^8 in the unitarity condition (10) there appears the three-particle production amplitude $A+B\to D_0+D_1+D_2$ to order g^5. In order to calculate this amplitude we consider the s-, s_1-; s_2-channel discontinuities of this amplitude ($s=(p_A+p_B)^2\simeq(p_{D_0}+p_{D_2})^2$, $s_1=(p_{D_0}+p_{D_1})^2$, $s_2=(p_{D_1}+p_{D_2})^2$). The s-channel discontinuity (cf. Fig. 1a, c) equals

$$\begin{gathered}\sum_{A',B'}(2\pi)^{-2}\int\left(\frac{d^3p_{A'}}{2E_{A'}}\right)\left(\frac{d^3p_{B'}}{2E_{B'}}\right)\delta(p_A+p_B-p_{A'}-p_{B'})A(A'B'\to AB;s,q)\\ \times A(D_0D_1D_2\to A'B';q_1-q,q_2-q)+\sum_{D_0'D_2'}(2\pi)^{-2}\int\left(\frac{d^3p_{D_0'}}{2E_{D_0'}}\right)\left(\frac{d^3p_{D_2'}}{2E_{D_2'}}\right)\\ \times\delta(p_{D_0'}+p_{D_2'}-p_{D_0}-p_{D_2})A(D_0'D_1D_2'\to AB;q_1-q',q_2-q')\\ \times A(D_0D_2\to D_0'D_2';s,q'),\\ A(AB\to CD;s,q)\equiv A^{AB}_{CD}(s,q).\end{gathered} \tag{40}$$

Here $q_1=p_{D_0}-p_A$, $q_2=p_B-p_{D_2}$, $q=p_{A'}-p_A$, $q'=p_{D_0}-p_{D_0'}$, and $A(D_0D_1D_2\to AB)$ is the three-particle production amplitude. The amplitude $A(D_0D_1D_2\to A'B';(q_1-q),(q_2-q))$ is given by Eqs. (30)–(32) with the substitutions $A\to A'$, $B\to B'$, $q_{1,2}\to q_{1,2}-q$, and in the expression for $\gamma^{D_1}_{ij}$ one may set $p_{A'}=p_A$, $p_{B'}=p_B$. The amplitude $A(D_0D_2\to D_0'D_2';s,q')$ defined by Eqs. (4)–(6) in the center-of-inertia frame of the particles D_0 and D_2 does not change its form when one goes over into the frame where $\mathbf{p}_A+\mathbf{p}_B=0$. The longitudinal components of the vectors q and q' are small and can be neglected in Eq. (40); therefore, taking into account that $\Gamma^{i}_{AA'}=\Gamma^{*i}_{A'A}$, the expression (40) becomes equal to

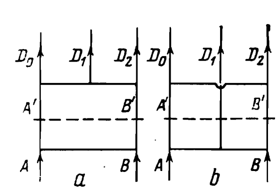

FIG. 1

$$s\left(\sum \Gamma^{i}_{AA'}\dot{\Gamma}^{i'}_{D_0A'}\right)\left(\sum_{B'}\Gamma^{j}_{BB'}\dot{\Gamma}^{j'}_{D_2B'}\right)$$
$$\times(8\pi^2)^{-1}\int d^2q_\perp[(q_\perp{}^2-m^2)((q_1-q)_\perp{}^2-m^2)((q_2-q)_\perp{}^2-m^2)]^{-1}$$
$$\times[\delta_{ij}\gamma^{D_1}_{i'j'}(q_1-q_\perp,q_2-q_\perp)+\delta_{i'j'}\gamma^{D_1}_{ij}(q_1-q_\perp,q_2-q_\perp)]. \quad (41)$$

The summation with respect to A' and B' is effected by means of Eq. (13). We need the three-particle production amplitude to leading logarithmic accuracy, so that there is no need here to distinguish between $\ln(s/m^2)$ and $\ln(-s/m^2)$; therefore in (13) one may retain just the first term (with $T=1$), since for $T=0$ and 2 there occurs a cancellation of the contributions of the s- and u-channels. Thus, the discontinuity (40) determines a contribution to the amplitude equal to

$$2s\ln(s/m^2)\Gamma^{k}_{AD_0}\Gamma^{k'}_{BD_2}c^{k}_{ii'}c^{k'}_{jj'}\delta_{ij}g^2(2\pi)^{-3}\int d^2q_\perp$$
$$\times[(q_\perp{}^2-m^2)((q_1-q_\perp)^2-m^2)((q_2-q_\perp)^2-m^2)]^{-1}\gamma^{D_1}_{i'j'}(q_1-q_\perp,q_2-q_\perp). \quad (42)$$

Here the factor 2 is due to the contribution of the u-channel. The calculation of the discontinuity of the amplitude in the s_1-channel requires a more detailed consideration. This discontinuity (cf. Fig. 2a) equals

$$\sum_{D_0'D_1'}(2\pi)^{-2}\int d^3p_{D_0'}(2E_{D_0'})^{-1}d^3p_{D_1'}(2E_{D_1'})^{-1}\delta(p_{D_0'}+p_{D_1'}-p_{D_0}-p_{D_1})$$
$$\times A(D_0'D_1'D_2\to AB;\ q_1',q_2)A(D_0,D_1\to D_0'D_1';\ s_1,(q_1-q_1')), \quad (43)$$

where $q_1=p_{D_0}-p_A$, $q_1'=p_{D_0'}-p_A$, $q_2=p_B-p_{D_2}$; the parameters of the Sudakov expansion $q_i=\beta_i p_B-\alpha_i p_A+q_i$ satisfy the conditions

$$1\gg\alpha_1\gg\alpha_2\sim\frac{m^2}{s},\quad 1\gg\beta_2\gg\beta_1\sim\frac{m^2}{s},\quad 1\gg\alpha_1'\gg\alpha_2\sim\frac{m^2}{s},$$
$$1\gg\beta_2\gg\beta_1'\sim\frac{m^2}{s},\quad q_i^2=q_\perp^2,\quad s\alpha_1\beta_2=m^2-(q_1-q_2)_\perp{}^2,$$
$$s\alpha_1'\beta_2=m^2-(q_1'-q_2)_\perp{}^2. \quad (44)$$

The amplitude $A(D_0'D_1'D_2'\to AB;\ q_1',q_2)$ is given by Eqs. (30)–(32) with the substitution $D_0\to D_0'$, $D_1\to D_1'$, $q_1\to q_1'$. The amplitude $A(D_0D_1\to D_0'D_1')$ in the frame with $\mathbf{p}_A+\mathbf{p}_B=0$ can be written in the form

$$A(D_0D_1\to D_0'D_1':s_1,q_1-q_1')=\Gamma^{i}_{D_0'D_0}\frac{s_1}{(q_1-q_1')^2-m^2}\Gamma^{i}_{D_1'D_1}, \quad (45)$$

where $\Gamma^k_{D_0'D_0}$ is given by Eq. (5) and $\overline{\Gamma}^k_{D_1'D_2}$ is obtained from it by means of the substitution

$$\delta_{\lambda_{D'}\lambda_D}\to\{e_{D'},e_D\}_{p_A},\quad b_{\lambda_D}\to m(p_Ae_D)s_1^{-1},$$
$$b_{\lambda_{D'}}\to m(p_Ae_{D'})s_1^{-1}. \quad (46)$$

The summation over D_0' in (43) is carried out by means of Eq. (13); owing to the cancellation of logarithms in terms with positive signature in (13) one must retain only terms with $T=1$, as before. Thus, we obtain from (43)

$$\frac{s}{8\pi^2}\Gamma^{k}_{AD_0}\Gamma^{j}_{BD_2}igc^{k}_{ii'}\sum_{D_1'}\int\frac{d^2q_{1\perp}'\gamma^{D_1'}_{ij}(q_1',q_2)\Gamma^{i'}_{D_1'D_1}}{(q_2^2-m^2)(q_1'^2-m^2)((q_1-q_1')^2-m^2)}. \quad (47)$$

In the summation over the polarization vectors of the particles in (47) we make use of the fact following from the definition of $\mathscr{P}_\mu$ (32) that $p^\mu_{D_1'}\mathscr{P}_\mu(q_1',q_2)=0$, to obtain the following result

$$\sum_{\lambda_{D_1'}}\{e_{D_1'},e_{D_1}\}_{p_A}(e_{D_1'}\mathscr{P}(q_1',q_2))=-\{e_{D_1},\mathscr{P}(q_1',q_2)\}_{p_A}$$
$$=e_{D_1}{}^{\mu}\left[\mathscr{P}_\mu(q_1,q_2)-\frac{m^2-q_2^2}{m^2-(q_2-q_1')_\perp{}^2}\mathscr{P}_\mu(q_1-q_{1\perp}',q_2-q_{1\perp}')+\frac{m^2}{s_1}p_{A_\mu}\right]. \quad (48)$$

Therefore, in the case when D_1 is a vector particle, we obtain

$$\sum_{D_1'}igc^{k}_{ii'}\gamma^{D_1'}_{ij}(q_1',q_2)\Gamma^{i'}_{D_1'D_1}$$
$$=ig^3\varepsilon_{d_1jk}\left[-\frac{m^2}{s_1}(p_Ae_{D_1})+\sum_{\lambda_{D_1'}}\{e_{D_1'},e_{D_1}\}_{p_A}(e_{D_1'}\mathscr{P}(q_1',q_2))\right]$$
$$=-g^2\left[\gamma^{D_1}_{kj}(q_1,q_2)-\frac{m^2-q_2^2}{m^2-(q_2-q_1')_\perp{}^2}\gamma^{D_1}_{kj}(q_1-q_{1\perp}',q_2-q_{1\perp}')\right]. \quad (49)$$

In the case when D_1 is a scalar particle,

$$ig\sum_{D_1'}c^{k}_{ii'}\gamma^{D_1'}_{ij}(q_1',q_2)\Gamma^{i'}_{D_1'D_1}=2g^3\delta_{kj}\sum_{\lambda_{D_1'}}\frac{m(p_Ae_{D_1'})}{s_1}(e_{D_1'}\mathscr{P}(q_1',q_2))$$
$$=-mg^3\delta_{kj}\left(1-2\frac{m^2-q_2^2}{m^2-(q_2-q_1')_\perp{}^2}\right). \quad (50)$$

Both cases can be written as a single formula:

$$\sum_{D_1'}igc^{k}_{ii'}\gamma^{D_1'}_{ij}(q_1',q_2)\Gamma^{i'}_{D_1'D_1}$$
$$=-g^2\left[\gamma^{D_1}_{kj}(q_1,q_2)-\frac{2(m^2-q_2^2)}{m^2-(q_2-q_1')_\perp{}^2}c^{k}_{ii'}c^{j}_{i'j'}\gamma^{D_1}_{i'j'}(q_1-q_{1\perp}',q_2-q_{1\perp}')\right]. \quad (51)$$

Thus, the contribution of the s_1-channel discontinuity to the amplitude turns out to be equal to

$$s\ln\left(\frac{s_1}{m^2}\right)\Gamma^{k}_{AD_0}\Gamma^{j}_{BD_2}g^2(2\pi)^{-3}\int d^2q_\perp[(q_2^2-m^2)(q_\perp{}^2-m^2)((q_1-q_\perp)^2-m^2)]^{-1}$$
$$\times[\gamma^{D_1}_{kj}(q_1,q_2)-2(m^2-q_2^2)(m^2-(q_2-q)_\perp{}^2)^{-1}c^{k}_{ii'}c^{j}_{i'j'}\gamma^{D_1}_{i'j'}(q_1-q_\perp,q_2-q_\perp)]. \quad (52)$$

The contribution of the s_2-channel discontinuity is calculated in exactly the same manner to yield

$$s\ln\left(\frac{s_2}{m^2}\right)\Gamma^{k}_{AD_0}\Gamma^{j}_{BD_2}g^2(2\pi)^{-3}\int d^2q_\perp[(q_1^2-m^2)(q_\perp{}^2-m^2)((q_2-q)_\perp{}^2-m^2)]^{-1}$$
$$\times[\gamma^{D_1}_{kj}(q_1,q_2)-2(m^2-q_1^2)(m^2-(q_1-q)_\perp{}^2)^{-1}c^{k}_{ii'}c^{j}_{ij'}\gamma^{D_1}_{i'j'}(q_1-q_\perp,q_2-q_\perp)]. \quad (53)$$

Adding (42), (52), (53) and (30), taking into account that in our approximation $\ln(s/m^2)=\ln(s_1/m^2)+\ln(s_2/m^2)$, we obtain the three-particle production amplitude in the g^5-approximation:

$$A_{2\to2+1}=s\frac{\Gamma^{i}_{AD_0}\gamma^{D_1}_{ij}(q_1,q_2)\Gamma^{j}_{BD_2}}{(q_1^2-m^2)(q_2^2-m^2)}\left[1+\alpha(q_1^2)\ln\left(\frac{s_1}{m^2}\right)+\alpha(q_2^2)\ln\left(\frac{s_2}{m^2}\right)\right]. \quad (54)$$

The contribution (42) which destroys the Born structure of the amplitude cancels completely, and as a re-

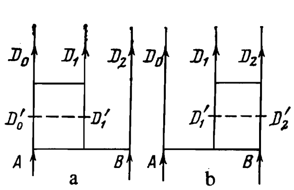

FIG. 2

sult of this the amplitude takes on a simple multi-regge form. This allows us to assume that such a form will remain valid also in higher orders of perturbation t theory, and not only for the amplitude A_{2-2+1}, but also for A_{2-2+n}.

Thus we make the conjecture that to all orders of perturbation theory, in the leading logarithmic approximation, the amplitude for the process $A+B\to D_0+D_1+\ldots+D_{n+1}$ has the form (cf. (39))

$$A_{2\to2+n}=s\Gamma^{i_1}_{AD_0}\frac{(s_1/m^2)^{\alpha(q_1^2)}}{q_1^2-m^2}\gamma^{D_1}_{i_1i_2}(q_1,q_2)\frac{(s_2/m^2)^{\alpha(q_2^2)}}{q_2^2-m^2}\cdots\frac{(s_{n+1}/m^2)^{\alpha(q^2_{n+1})}}{q^2_{n+1}-m^2}\Gamma^{i_{n+1}}_{BD_{n+1}},\qquad s_k=(p_{D_{k-1}}+p_{D_k})^2. \tag{55}$$

The distinction from the Born amplitude consists in the replacement of the ordinary vector meson by a reggeon with the trajectory $j=1+\alpha(t)$.

5. EQUATIONS FOR THE PARTIAL WAVE AMPLITUDES

The expression for the amplitude $2\to2+n$, (55), together with the unitarity relation (10) allow one to obtain the contribution to the $2\to2$ amplitude corresponding to the $(n+2)$-particle intermediate state in the t-channel. In the sequel it will be convenient to consider separately the different isospin states in the t-channel:

$$A^{A'B'}_{AB}(s,q)=\Gamma_{AA'}A^{(0)}(s,q)\Gamma_{BB'}+\Gamma^i_{AA'}A^{(1)}(s,q)\Gamma^i_{BB'}+\Gamma^{ij}_{AA'}A^{(2)}(s,q)\Gamma^{ij}_{BB'}. \tag{56}$$

We expand the bilinear combination of the γ^D_{ij} in (31), (32) in terms of the projection operators onto states with definite isospin value, (6), (14):

$$\sum_{D_l}\gamma^{D}_{ii'}(q_l,q_{l+1})\gamma^{*D_l}_{jj'}(q_l-q,q_{l+1}-q)$$
$$=2g^2K^{(0)}_{l,l+1}c_{ij}c_{i'j'}+2g^2K^{(1)}_{l,l+1}c^k_{ij}c^k_{i'j'}+2g^2K^{(2)}_{l,l+1}c^{kk'}_{ij}c^{kk'}_{i'j'}, \tag{57}$$

where

$$K^{(T)}_{l,l+1}\equiv K^{(T)}(q_l,q_{l+1},q)=A_T(q^2)-c_T\frac{(l)(\widetilde{l+1})+(\tilde{l})(l+1)}{(l,l+1)}$$
$$A_T(q^2)={}^1/_2m^2+c_T(q^2-{}^3/_2m^2),\quad c_T=2-{}^1/_2T(T+1),$$
$$(l)=q_l^2-m^2,\quad(\tilde{l})=(q_l-q)^2-m^2,\quad(l,l+1)=(q_l-q_{l+1})^2-m^2. \tag{58}$$

Here and in the sequel q_l and q stand for $q_{l\perp}$ and $q_\perp$. Substituting (55) into the unitarity condition (10), and making use of the relations (13) and (57) for the summation over the particles $D_0, D_1, \ldots, D_{n+1}$ and the expression (21) for $d\rho_N$, we obtain

$$\mathrm{Im}_s A^{(T)}(s,q)$$
$$=-\frac{1}{2}\pi s\sum_{n=0}^{\infty}\left(\frac{g^2}{(2\pi)^3}\right)^{n+1}\int\prod_{i=1}^{n}\frac{d\alpha_i}{\alpha_i}\prod_{l}^{n+1}\frac{d^2q_l}{(l)(\tilde{l})}\left(\frac{s_l}{m^2}\right)^{\alpha(q_l^2)+\alpha((q_l-q)^2)}$$
$$\times K^{(T)}_{1,2}\cdot K^{(T)}_{2,3}\ldots K^{(T)}_{n,n+1}. \tag{59}$$

We go over to the j-representation of the amplitudes $A^{(T)}(s,q)$:

$$A^{(T)}(s,q)=\delta_{T,1}\frac{s}{q^2-m^2}+\frac{s}{4i}\int_{\delta-i\infty}^{\delta+i\infty}d\omega\left(\frac{s}{m^2}\right)^{\omega}\frac{e^{-i\pi\omega}-(-1)^T}{\sin\pi\omega}F^{(T)}_\omega(q^2), \tag{60}$$

where $\omega=j-1$,

$$F^{(T)}_\omega(q^2)=-\frac{2}{\pi}\int_1^\infty d\left(\frac{s}{m^2}\right)\left(\frac{s}{m^2}\right)^{-\omega-1}s^{-1}\,\mathrm{Im}_s A^{(T)}(s,q)$$
$$=\sum_{n=0}^{\infty}\left(\frac{g^2}{(2\pi)^3}\right)^{n+1}\int K^{(T)}_{1,2}\ldots K^{(T)}_{n,n+1}\prod_{l=1}^{n+1}\frac{d^2q_l[(l)(\tilde{l})]^{-1}}{\omega-\alpha(q_l^2)-\alpha((q-q_l)^2)}. \tag{61}$$

When taking the integral in (61) we made use of the fact that to the desired accuracy

$$\int_1^\infty d\left(\frac{s}{m^2}\right)\left(\frac{s}{m^2}\right)^{-\omega-1}\int_{m^2/s}^{1}\frac{d\alpha_1}{\alpha_1}\int_{m^2/s}^{\alpha_1}\frac{d\alpha_2}{\alpha_2}\cdots\int_{m^2/s}^{\alpha_{n-1}}\frac{d\alpha_n}{\alpha_n}\prod_{l=1}^{n+1}\left(\frac{s_l}{m^2}\right)^{\alpha(q_l^2)+\alpha((q_l-q)^2)}$$
$$=\prod_{l=1}^{n+1}\int_1^\infty d\left(\frac{s_l}{m^2}\right)\left(\frac{s_l}{m^2}\right)^{-\omega-1+\alpha(q_l^2)+\alpha((q_l-q)^2)}. \tag{62}$$

Representing (61) in the form

$$F^{(T)}_\omega(q^2)=\frac{1}{\omega}\frac{g^2}{(2\pi)^3}\int\frac{d^2k}{(k^2-m^2)((k-q)^2-m^2)}f^{(T)}_\omega(k,q-k)A_T(q^2), \tag{63}$$

we obtain for $f^{(T)}_\omega(k,q-k)$ the equation

$$[\omega-\alpha(k^2)-\alpha((q-k)^2)]f^{(T)}_\omega(k,q-k)=\frac{\omega}{A_T(q^2)}$$
$$+\frac{g^2}{(2\pi)^3}\int\frac{d^2q_1}{(1)(\tilde{1})}K^{(T)}(k,q_1,q)f^{(T)}_\omega(q_1,q-q_1), \tag{64}$$

where

$$f^{(T)}_\omega(k,q-k)\big|_{k^2=(q-k)^2=m^2}=\frac{1}{A_T(q^2)}+F^{(T)}_\omega(q^2), \tag{65}$$

For the case $T=1$ it is easy to see that the solution of (64) is

$$f^{(T)}_\omega(k,q-k)=\frac{\omega A_T^{-1}(q^2)}{\omega-\alpha(q^2)},$$
$$F^{(1)}_\omega(q^2)=\frac{\alpha(q^2)}{(q^2-m^2)(\omega-\alpha(q^2))} \tag{66}$$

which proves the reggeization of the vector meson and the self-consistency of our assumptions on the amplitude (55).

The graphic representation of Eq. (64) is given in Fig. 3, where the block in the left-hand side represents $f^{(T)}_\omega(k,q-k)$; momentum conservation is satisfied at each vertex; to each internal line with momentum q_i one associates the factor $(q_i^2-m^2)^{-1}$, to each closed loop with loop momentum q_i one associates $g^2(2\pi)^{-3}\int d^2q_i$; the boldface dot in a four- (three-) particle vertex is represented by the factor $A_T(q^2)((q_i^2-m^2))$, where q_i is the momentum flowing through this point from right to left.

If one iterates the equation of Fig. 3, the last four terms will cancel for $T=1$, so that $f^{(T)}_\omega(k,q-k)$ will be represented by the series of Fig. 4a, which shows that for isospin 1 there are only two-particle intermediate states in the t-channel.

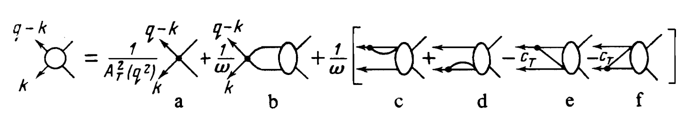

FIG. 3

As to the isospins $T=0$ and 2, the iteration of the equation given by Fig. 3 leads from them to diagrams with an arbitrary number of particles in the intermediate state in the t-channel. In order g^{2n} of perturbation theory (we recall that g^2 is contained in the vertex functions in (56)) there appear in the expression for $F_\omega^{(T)}(q^2)$ diagrams of the type represented in Fig. 4b with n lines and with coefficient $\omega^{-n+1}R_n^{(T)}$:

$$R_2^{(T)}=1,\qquad R_3^{(T)}=2(1-c_T),\qquad R_n^{(T)}=2(1-c_T)^2(-c_T)^{n-4},\qquad n\geqslant 4. \tag{67}$$

The contribution of such diagrams to $A^{(T)}(s,q)$ for $T=0$; 2 equals

$$-{}^1/_2\pi i s\ln^{n-2}(s/m^2)R_n^{(T)}\beta_n(q^2)\,[(n-2)!]^{-1}. \tag{68}$$

where $\beta_n(q^2)$ corresponds to the contracted diagram (Fig. 4b) with n lines.

The presence of diagrams with arbitrary number of particles in the t-channel has an essential influence on the character of singularities of partial-wave amplitudes with $T=0$; 2 in the j-plane.

In connection with the diagrams of Fig. 4b the following remarks are in order. These diagrams are obtained by contracting Feynman graphs of the eikonal type (Fig. 5a). It is claimed[13] that in QED each of the diagrams of eikonal type does not yield more than one power of $\ln(s/m^2)$. In our case, for the fermion scattering amplitude in the $T=0$ channel, the contribution from each diagram of Fig. 5a is obtained by multiplying the QED graph with photon mass m by the factor

$$\frac{1}{2}\,\mathrm{Tr}\left(\frac{\tau^{i_1}}{2}\frac{\tau^{i_2}}{2}\cdots\frac{\tau^{i_n}}{2}\right)\cdot\frac{1}{2}\,\mathrm{Tr}\left(\frac{\tau_{i_{k_1}}}{2}\frac{\tau_{i_{k_2}}}{2}\cdots\frac{\tau_{i_{k_n}}}{2}\right). \tag{69}$$

We make use here of the Feynman gauge for the vector mesons. The use of the previously adopted "unitary" gauge, in which the Lagrangian contains only physical particles, does not change the result, but complicates the consideration, since in addition to the graphs of Fig. 5a, one must consider other graphs for the compensation of the divergences which arise from the term $k^\mu k^\nu m^{-2}$ in the vector meson propagator.

It is clear from (69) that the graphs for which the vertices on the line BB' of Fig. 5a are obtained from one another by cyclic permutation enter with the same factor. We shall say that such diagrams form a class. A direct calculation of the diagrams of Fig. 5a shows that if, introducing the usual parametrization $q_i=\alpha_i p_B+\beta_i p_A+q_{i\perp}$, we do the integrations with respect to the α_i by means of residues, then in the region where all the denominators of the fermion propagators on the line

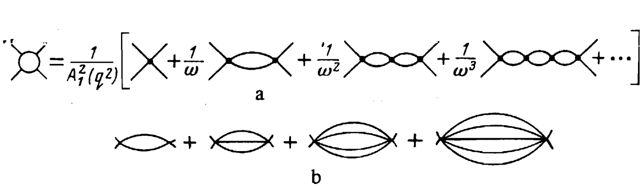

FIG. 4

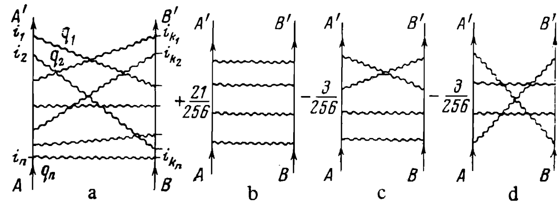

FIG. 5

BB' are much larger than $m^2(\beta_{i_{k_1}}\gg m^2/s,\ \beta_{i_{k_1}}+\beta_{i_{k_2}}\gg m^2/s$, etc.) there occurs a complete cancellation of the diagrams of a given class. However, if in a propagator one takes the residue in the integration with respect to one of the β_i, a cancellation does not occur and the integration over the remaining $\beta_{k\neq i}$ yields large logarithms. In this manner there occurs a growth of the degree of the logarithm with n; the assertion in[13] that graphs of the eikonal type can only yield on $\ln(s/m^2)$ is incorrect.

We have carried out a calculation of the contribution of eikonal diagrams to the fermion scattering amplitude with $T=0$ in the t-channel up to $n=4$. For $n=2$ there is only one class of diagrams, so that the result differs from QED only by the isospin factor and coincides with (68). For $n=3$ there are two classes of diagrams and the results for each class agree with the calculations of Chester[14]; the sum of these multiplied with the appropriate factors coincides with Eq. (68). For $n=4$ there are six classes of diagrams such that pairs of classes for which the vertices on the line BB' have reciprocal orders enter with the same isospin factor. One of the diagrams from each pair of classes is illustrated in Fig. 5b, c, d, together with its isospin factor (69). The contribution of each pair of classes to $A^{(0)}(s,q)$ (i.e., after the separation of $\Gamma_{AA'}$, $\Gamma_{BB'}$) equals

$$\begin{aligned}A_b^{(0)}(s,q)&=-\frac{7}{16}\pi i\beta_4(q^2)\cdot s\ln^2\left(\frac{s}{m^2}\right);\\ A_c^{(0)}&=\frac{\pi i}{16}\beta_4(q^2)\cdot s\ln^2\left(\frac{s}{m^2}\right),\\ A_d^{(0)}(s,q)&=-\frac{2\pi i}{16}\beta_4(q^2)s\ln^2\left(\frac{s}{m^2}\right).\end{aligned} \tag{70}$$

Their total contribution coincides with (68).

We note that if one sums the contributions without the isospin factors they cancel.

CONCLUSION

Our assumption that the $2\to 2+n$ amplitude has a multiregge form (55) is based on the form of the Born approximation amplitude (39) in the multiregge kinematic region and the corrections to the $2\to 3$ amplitude in the order g^5, (54). The fact that the form (55) is valid can be tested in higher orders of g^2 making use of the method of reconstructing an amplitude in terms of its discontinuities in overlapping channels.[16] With the help of the amplitude $2\to 2+n$ (55) we further obtain an equation for the partial-wave amplitudes $2\to 2$, (64) for isospins $T=0$; 1; 2 in the t-channel. Its solution for $T=1$ is the Regge pole (66), which is an argument in favor of the assumption (55) about the form of the $2\to 2+n$ amplitude. An analysis of the equation (64) for $T=0$; 2

shows that in the t-channel there are intermediate states with arbitrarily large particle number, in distinction from the case $T=1$, where there are only two-particle states. As will be shown in another place, this circumstance is related to the appearance for $T=0$ of a fixed singularity in the j-plane.

The reggeizability of a particle is one of the criteria for its n nonelementary character. Another related definition[17] is the vanishing of the wave-function renormalization constant Z. It follows from our results that the Yang–Mills vector meson is a composite object and one can expect that its renormalization constant Z vanishes. One should also expect a decrease of its form-factor with the increase of the momentum transfer, as for any composite system.

APPENDIX

The results derived by us for the group $SU(2)$ allow a simple generalization to the group $SU(N)$. In order that the spontaneous symmetry breakdown should lead to the acquisition of masses by all vector mesons V^a and the global $SU(N)$ symmetry be conserved[15] we select the original Lagrangian in the form

$$L=-\frac{1}{4}\mathrm{Tr}\left[\left(\partial_\mu V_\nu-\partial_\nu V_\mu+\frac{ig}{\sqrt{2}}[V_\mu, V_\nu]\right)^2\right]+\frac{1}{2}\mathrm{Tr}\left[(D_\mu M)^+(D_\mu M)\right]+\mathscr{P}(M), \quad \text{(A.1)}$$

where

$$V_\mu=\lambda_a V_\mu^a\cdot 2^{-1/2}, \quad M=\lambda_a(S^a+i\pi^a)\cdot 2^{-1/2}+I(\varphi'+i\sigma)N^{-1/2}, \quad D_\mu M=\partial_\mu M+igV_\mu\frac{M}{\sqrt{2}},$$

S^a, π^a, φ', σ are hermitean scalar fields, $\mathscr{P}(M)$ is the part of the Lagrangian which contains the mass terms and the self-interaction terms of these fields; λ_a are the matrices of the fundamental (adjoint) representation of the group $SU(N)$

$$\mathrm{Tr}\,\lambda_a\lambda_b=2\delta_{ab}, \quad [\lambda_a, \lambda_b]=2if_{abc}\lambda_c, \quad \{\lambda_a, \lambda_b\}=2d_{abc}\lambda_c+4N^{-1}I\delta_{ab}. \quad \text{(A.2)}$$

The indices a, b, c, ... take on the values from 1 to N^2-1 and a summation over repeated indices is understood, as usual. The following relations will be useful:

$$\begin{gathered} f_{iac}f_{jbc}-f_{jac}f_{ibc}=f_{ijk}f_{kab}, \quad f_{iac}d_{jbc}-f_{icb}d_{jac}=f_{jik}d_{kab}, \\ d_{iac}d_{jbc}-d_{jac}d_{ibc}+\frac{2}{N}(\delta_{ia}\delta_{jb}-\delta_{ja}\delta_{ib})=f_{ijk}f_{kab}, \\ f_{aij}f_{bij}=N\delta_{ab}, \quad d_{aij}d_{bij}=\frac{N^2-4}{N}\delta_{ab}, \quad d_{aii}=0. \end{gathered} \quad \text{(A.3)}$$

As a result of spontaneous symmetry breaking ($\varphi'=\varphi+\langle\varphi'\rangle_0$) and the removal of the fields π^a by means of a gauge transformation the Lagrangian takes on the form

$$\begin{aligned} L=&-{}^1/_4(\partial_\mu V_\nu^a-\partial_\nu V_\mu^a)^2+{}^1/_2 m^2 V_\mu^a V_\mu^a+{}^1/_2(\partial_\mu S^a)^2+{}^1/_2(\partial_\mu\varphi)^2+{}^1/_2(\partial_\mu\sigma)^2 \\ &+gf_{abc}\partial_\mu V_\nu^a V_\mu^b V_\nu^c+{}^1/_2 mgV_\mu^a V_\mu^b\left(S^c d_{abc}+\varphi\sqrt{\frac{2}{N}}\delta_{ab}\right) \\ &-\frac{g}{2}f_{abc}(S^a\partial_\mu S^b)V_\mu^c-\frac{g}{2}\sqrt{\frac{2}{N}}V_\mu^a(\sigma\partial_\mu S^a-S^a\partial_\mu\sigma)+\dots \end{aligned} \quad \text{(A.4)}$$

Missing from (A.4) are the terms involving the masses of the scalar fields, the self-interaction terms and all terms with four-particle interactions. It suffices to know that these terms guarantee the renormalizability of the theory; their explicit form is immaterial.

The Born approximation amplitudes for the process $A+B\to A'+B'$ are determined in the same manner as in[10] in terms of the t-channel imaginary part, making use of the renormalizability of the theory. They have the form (4), where the summation with respect to i runs from 1 to N^2-1, and the nonvanishing vertex functions are

$$\begin{gathered} \Gamma^i_{VV'}=-ig\sqrt{2}f_{ivv'}\delta_{\lambda_V\lambda_{V'}}a_{\lambda_V}, \quad \Gamma^i_{VS}=\Gamma^i_{SV}=g\sqrt{2}d_{ivs}b_{\lambda_V}, \\ \Gamma_{FV}{}^i=\Gamma_{VF}{}^i=g\frac{2}{\sqrt{N}}\delta_{iv}b_{\lambda_V}, \quad \Gamma^i_{SS'}=\frac{-ig}{\sqrt{2}}f_{iss'}, \quad -\Gamma_{ZS}{}^i=\Gamma_{SZ}{}^i=ig\frac{\delta_{is}}{\sqrt{N}}. \end{gathered} \quad \text{(A.5)}$$

Here and in the sequel F and Z denote the particles φ and σ.

The relation (13) was essential for the calculation of the higher-order amplitudes for the group $SU(2)$, and in this expression the antisymmetric part, since only for this part do the contributions of the s- and u-channels cancel. For the case of the group $SU(N)$, making use of Eqs. (A.3) and (A.5), we obtain

$$\sum_{A''}\Gamma^i_{AA''}\Gamma^j_{A'A''}=ig\sqrt{\frac{N}{2}}\left(\Gamma^k_{AA'}\frac{f_{kij}}{\sqrt{N}}+\Gamma_{AA'}\frac{\delta_{ij}}{\sqrt{N^2-1}}+\dots\right), \quad \text{(A.6)}$$

where the omitted terms are symmetric with respect to i,j. We have explicitly separated from the symmetric part the term with δ_{ij}, i.e., the state with the quantum number of the vacuum in the t-channel. The nonvanishing $\Gamma_{AA'}$ are

$$\begin{gathered} \Gamma_{VV'}=-ig\sqrt{2N}\frac{\delta_{vv'}}{\sqrt{N^2-1}}\delta_{\lambda_V\lambda_{V'}}\left(2a^2_{\lambda_V}+\frac{2(N^2-2)}{N^2}b^2_{\lambda_V}\right) \\ \Gamma_{SS'}=-ig\sqrt{2N}\frac{\sqrt{N^2-1}}{N^2}\delta_{ss'}, \quad \Gamma_{FF'}=\Gamma_{ZZ'}=-ig\sqrt{2N}\frac{\sqrt{N^2-1}}{N^2}. \end{gathered} \quad \text{(A.7)}$$

With the vertex function (A.5) and (A.7) the contribution to the amplitude from states with the vacuum quantum numbers and the vector meson in the t-channel, to fourth order, has the form (16) with the substitution $\alpha(t)\to\alpha_N(t)$, $\beta_2(t)\to\beta_{2(N)}(t)$ where $\alpha_N(t)$ and $\beta_{2(N)}(t)$ are obtained from $\alpha(t)$ and $\beta_2(t)$ by means of the substitution $g^2\to g^2_N=g^2N/2$.

The n-particle production amplitudes in the Born approximation are derived in the same manner as in Sec. 3. For them Eq. (39) is valid with the vertex functions (A.5) and $\gamma^{D_n}_{ij}$ equal to $mg(2/N)^{1/2}\delta_{ij}$ for the production of φ particles, equal to mgd_{sij} for the production of S-particles, and equal to $igf_{vij}(e_V\mathscr{P}(q_n, q_{n+1}))$ for the production of V-particles; $\mathscr{P}(q_1, q_2)$ is given by Eq. (32).

As in the case of $SU(2)$, the three-particle production amplitude calculated to order g^5 has the form (54) with the substitution $\alpha\to\alpha_N$; it is natural to assume that to any order the $2\to 2+n$ amplitude has the form (55) with the corresponding Γ^i_{AA} and γ^D_{ij}, and $\alpha(q^2)\to\alpha_N(q^2_i)$. Reconstructing the $2\to 2$ amplitude by means of analyticity and unitarity for the partial waves with the quantum number of the vacuum ($F^{(0)}_\omega(q^2)$) and of the vector meson ($F^{(1)}_\omega(q^2)$ in the t-channel, defined according to (60), we obtain the expression (61) with the substitution $g^2\to g^2_N$,

$\alpha(q^2) \to \alpha_N(q^2)$; $K^{(1)}_{l,l+1}$ is as before given by Eq. (58), and in $K^{(0)}_{l,l+1}$ one must set $A_0(q^2) = 2q^2 - 2(N^2+1)N^{-2}m^2$.

The reggeization of the vector meson is proved in the same manner as for $SU(2)$, since Eqs. (63)–(66) remain valid with the indicated substitutions.

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Properties of deep-lying levels in a strong electrostatic field

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The properties of deep levels (real, virtual, and quasistationary) lying near the boundary of the lower continuum are investigated. The effective range expansion is generalized to the case of the Dirac equation. With its aid the motion of the levels near the boundary $\epsilon = -mc^2$ is investigated for different values of the angular momenta j and l. The formulas become simpler in the limiting cases $R < \hbar/mc$ and $R > \hbar/mc$, where R denotes the range of the forces. In particular, the case of a wide potential well $R > \hbar/mc$ reduces to a determination of the spectrum of the bound and quasistationary states in the Schrödinger equation with a power-law potential. The asymptotic behavior of the critical nuclear charge Z_{cr} is found in the region $R_N > \hbar/mc$ (R_N denotes the nuclear radius), and Z_{cr} is calculated for the muon for various distributions of electric charge inside the nucleus. Differences in the behavior of the levels near $\epsilon = -mc^2$ for scalar and spinor particles and the inapplicability of the single-particle Klein-Gordon equation for $Z \geq Z_{cr}$ are discussed.

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1. INTRODUCTION

Deep levels, lying near the boundary of the lower continuum of solutions to the Dirac equation, are investigated in the present article. The appearance of levels near the boundary $\varepsilon = -mc^2$ is of interest for quantum electrodynamics (the critical nuclear charge Z_{cr} and the spontaneous production of positrons from the vacuum for $Z > Z_{cr}$ or associated with the approach of two heavy nuclei to within a distance $R < R_{cr}$ [1–8]) and for nuclear physics (the formation of a π-meson condensate and a phase transition in nuclei, the possible existence of superheavy nuclei with charge $Z \sim (137)^{3/2}$ [9,10]).

The motion of the levels near the boundary of the lower continuum has been considered by a number of authors[2,11–15] in connection with problems of quantum electrodynamics in strong external fields. In this connection the Dirac and Klein–Gordon equations were solved analytically or numerically for potentials of the following special forms: rectangular well, Coulomb potential, and Hulthén potential. We shall consider the question of the motion of the levels and the analytic properties of the S-matrix for an arbitrary potential, using a generalization of the effective range method to the relativistic case. In this connection one is able to express the energy levels and the expansion parameters of the S-matrix for $\varepsilon \to -mc^2$ in terms of the wave function at the critical point $V = V_{cr}$, i.e., at the moment when the level intersects the boundary of the lower continuum.

Let us describe the contents of this article. A generalization of the effective range expansion to the case of the Dirac equation is presented in Sec. 2. The limiting cases $R \ll \lambda_c = \hbar/mc$ and $R \gg \lambda_c$ (R denotes the characteristic range of action of the potential and m is the particle's mass) are considered in Secs. 3 and 4. By using the effective potential method,[2,3] it is shown in Sec. 4 that investigation of the levels near $\varepsilon = -mc^2$ in the case $R \gg \lambda_c$ reduces to the solution of the Schrödinger equation. In Sec. 5 the asymptotic form of Z_{cr} is found in the region $R_N \gg \lambda_c$ which has not been pre-