tion the terms proportional to Ω_A^2 in those equations which describe the rotation of the magnetization. As a result we have $\beta = ht'$, $\dot{\alpha} = \dot{\Phi} = \dot{M} = 0$.

To obtain the next approximation, it is necessary to substitute the values $M = 1$ and $\Phi = 0$ into the terms proportional to Ω_A^2 of the right-sides of the aforementioned equations, after which it is easily seen that these equations have an integral that is obtained from (54) by expansion in terms of $\Omega_A^2/4h$, and for $\beta_{\max}$ we get

$$\pi - \beta_{max} = \Omega_A{}^2/4h.$$

For the experimental curves of[3], plotted at $T/T_c = 0.754$ and $T/T_c = 0.854$ we have in accordance with (56) $180° - \beta_{\max} \approx 20°$. It must be borne in mind that for these curves $\Omega_A^2/4h \approx 0.4$, so that the employed approximation is not very accurate and the result should be regarded as an estimate. For $T/T_c = 0.930$ we get $180° - \beta_{\max} \approx 10°$, and for $T/T_c = 0.952$ we have $180° - \beta_{\max} \approx 4°$. In this case $\Omega_A^2/4h$ is equal to 0.2 and 0.06, respectively, and the approximation should give good results.

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Critical magnetic fields in layered thin-film vanadium–carbon structures

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The critical magnetic fields $H_{c\perp}$ and $H_{c\parallel}$ of superconducting layered thin-film C-(V-C-V)-C samples, in which a nonmonotonic dependence of the critical temperature T_c and resistivity ρ on the thickness of the intermediate carbon layer was previously observed, are investigated. It is found that $|dH_{c\perp}/dT|$ and $|dH_{c\parallel}^2/dT|$ also depend near T nonmonotonically on the thickness of the layer. Estimates of the electron density of the states derived on the basis of data on $|dH_{c\perp}/dT|$ and data on the residual resistivity agree qualitatively with the assumption that the nonmonotonic dependence of T_c and ρ on the thickness of the carbon layer is due to variation of the electron density of the states.

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An oscillatory dependence of the critical temperature of superconducting films of thickness $d < 300$ Å on the thickness d_c of the dielectric coating (in the range of d_c from 0 to ~30 Å) was observed in[1–4]. The objects of these investigations were superconducting films of Mo–C,[1,2] Tc,[2] V,[3] and Al,[4] the dielectric being carbon[1–3] and silicon oxide.[4] Similar oscillations were observed in a film system with two vanadium layers, $C-(V-C-V)-C$,[3] as functions of the thickness of the intermediate dielectric C layer.

Golyanov *et al.*[2,3] have observed also correlations between T_c and the resistivity in films having different dielectric-coating thicknesses and in sandwiches with different intermediate dielectric-filler layers, viz., the positions of the maxima of T_c corresponded to the minima of the resistivity. It was indicated in this connection[3] that this correlation can be explained by assuming that the oscillations are the result of a change in the electron density of states.

The easiest way to estimate experimentally the electron density of states N in film samples is to calculate N from data on the perpendicular magnetic fields $H_{c\perp}$ and on the residual resistivity ρ_n.[5–8] This is the method that will be used here to determine N. The investigations were carried out on two-layer vanadium-carbon C–(V–C–V)–C sandwiches. Samples of this type were chosen because they exhibited most distinctly the correlation of T_c and ρ_n.[3] Besides $H_{c\perp}$ and ρ_n we measured also the parallel critical magnetic fields $H_{c\parallel}$.

SAMPLES AND MEASUREMENT PROCEDURE

The C–(V–C–V)–C sandwiches were prepared by ion sputtering in an ultrahigh-vacuum installation.[3] The vanadium layers were each 100 Å thick and the thickness of the central carbon filler was varied from 0 to 10 Å. To protect the samples against external agents, a carbon sublayer and a carbon coating were applied, of thickness 15 Å. The type of crystal lattice and the parameter of the vanadium lattice in the sandwiches were the same as for bulky samples and remained unchanged when the carbon liner thickness was varied. No noticeable change of the grain dimension (~400–500 Å) was observed in this case. The critical magnetic fields of the samples were measured by a resistive method in a superconducting solenoid. The procedure of these measurements was described in detail in earlier papers.[7,8]

TABLE I.

Sample	$d_{\overline{\text{fil}}}$ A	ρ_n, μΩ-cm	T_c, K	$\lvert dH_{c\perp}/dT\rvert$, kOe²/°K	$\lvert dH^2_{c\parallel}/dT\rvert$, kOe/°K	N, $\frac{\text{states}}{\text{eV-atom}}$
A 1a	0	7.93	4.42	4.57	647	1.66
B 1a	0	12.1	4.07	5.40	1110	1.33
A 2a	3	11.8	3,83	5,47	601	1,37
B 5a	4	15.3	3.63	6.25	708	1,25
B 6a	4.5	12.5	3,78	5.96	632	1.44
A 3a	5	7.74	4,20	4.82	367	1.83
B 8a	5.5	11.7	3,81	5.46	558	1.39
A 4a	7	11.5	3,61	5.78	604	1.52
A 4b	7	11.1	3,86	5,15	532	1.39
B 2a	8	22.2	3,13	5.46	1190	0.75
B 3a	9.5	13.7	3,51	5.62	602	1.25
A 5a	10	12,3	3.61	5.49	550	1.35
A 5b	10	15.9	3.49	5.67	572	1.08

*The first letter designates the evaporation date: A–October 29, 1974; B–May 7, 1975.

RESULTS AND DISCUSSION

The critical fields $H_{c\perp}$ and $H_{c\parallel}$ were determined from the $R(H)$ transition curves plotted at constant temperature. $H_{c\perp}$ (or $H_{c\parallel}$) was assumed to be the field at which $R=0.5R_n$ (R_n is the sample resistance after the transition to the normal state). Near T_c, the $H_c(T)$ dependence is linear. The width and shape of the $R(H_\perp)$ transition curves for a given sample remain practically unchanged with changing temperature near T_c (the transition width changes from sample to sample in the range from ~0.3 to ~0.5 kOe). Thus, the derivative $|dH_{c\perp}/dT|$, which is used to calculate the electron density of states, does not depend on the transition-curve point from which $H_{c\perp}$ is determined.

In parallel fields the widths of the transition curves vary with temperature. $H_{c\parallel}$ varies with temperature near T_c like $(T_c-T)^{1/2}$. If we determine $H_{c\parallel}$ from the start or the end of the transition curve, then we obtain $H_{c\parallel}(T)$ curves that are shifted relative to each other by a constant value ΔT, which is $\lesssim 0.1$ °K for the different samples. A decrease of the slope of $H_{c\parallel}(T)$ with decreasing temperature leads to a decrease of the transition curves $R(H_\parallel)$.

The obtained values of $|dH_{c\perp}/dT|\equiv h_\perp$ and $|dH^2_{c\parallel}/dT|\equiv h^2_\parallel$ near T_c are listed in Table I. To compare the results obtained for samples from different evaporation series, Figs. 1a and b show the dependences of the same quantities on the thickness of the carbon filler in a relative form, i.e., $h_\perp(d_{\rm fil})/h_\perp(0)$ and $h^2_\parallel(d_{\rm fil})/h^2_\parallel(0)$, where $h_\perp(d_{\rm fil})$ or $h^2_\parallel(d_{\rm fil})$ pertain to the sample with carbon filler thickness $d_{\rm fil}$, while $h_\perp(0)$ or $h^2_\parallel(0)$ pertain to the sample of the same evaporation series with zero filler thickness. The figure shows also $\rho_n(d_{\rm fil})/\rho_n(0)$ and $T_c(d_{\rm fil})/T_c(0)$ for the same samples. It is seen from the figure that $h_\perp$ and $h^2_\parallel$ oscillate with changing filler thickness and that the position of the extrema coincides with the positions of the extrema for ρ_n, $h_\perp$, and $h^2_\parallel$, correspond to the minima (maxima) of T_c.

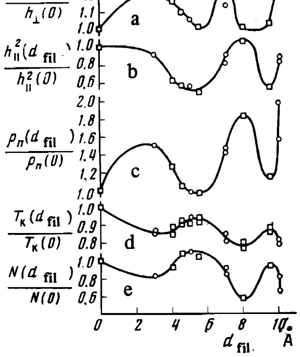

FIG. 1. Dependence of various characteristics of C–(V–C–V)–C sandwiches on the thickness of the carbon filler; the circles and squares pertain to different sputtering series (see the note to the table); ○—series A, □—series B.

The discussion of the behavior of the electron density of states, to which we now proceed, is also qualitative in character, since the calculation of N is based on formulas that are valid, strictly speaking, to homogeneous samples, whereas we are dealing with a film system that is inhomogeneous in thickness. To calculate the electron density of states we use the same system for the analysis of the obtained data on $h_\perp$ and ρ_n as in[8]. The values of N were calculated from the formula1) (which is approximately valid for ranges shorter than half the coherence length $\xi_0=0.18\,\hbar v_F/kT_c$)

$$N=[h_\perp/2.57\ kec\,\eta-(13.2/\hbar e^2)T_c/v_F^2]/\rho_n, \qquad (1)$$

where k is Boltzmann's constant, c is the speed of light in vacuum, $\hbar$ is Planck's constant, e is the electron charge, v_F is its Fermi velocity, and η is a coefficient that takes into account the strong-coupling effect. We shall assume that for C–V–C–V–C sandwiches the coefficient η does not differ from the value of η for vanadium films. In the latter case, $\eta=1.2$.[8] In the calculation of ρ_n we used the assumption that two layers of vanadium up to 100 Å thick can be regarded as connected in parallel. This assumption was confirmed by experiment.

The dependence of the relative electron density of states $N(d_{\rm fil})/N(0)$, calculated using formula (1), on the thickness of the filler, is shown in Fig. 1e. The value of N oscillates "in phase" with T_c. This result agrees qualitatively with the assumption that for superconducting films with dielectric coating of low thickness and for sandwiches with thin dielectric fillers the nonmonotonic dependences of T_c and ρ on the dielectric layer is due to the change in the density of the electronic states, although for a quantitative explanation of the changes of T_c the obtained amplitudes of the oscillations of the electron density of states is too high.

It should be noted that $h_\perp$ is inversely proportional to the product of the mean free path of the electrons by the Fermi velocity lv_F, so that oscillations of $h_\perp$ mean that lv_F also oscillates with changing thickness of the dielectric filler. This is possibly due to the nonmonotonic change of coefficient of electron penetration through the barrier corresponding to the dielectric filler, which should lead to oscillations of the effective mean free path of the electrons.

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[1)] This formula (at $\eta=1$) follows (see[18]) from Gor'kov's theory.[9] The coefficient η has been introduced to take into account possible tight-binding effects.

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Turbulent mixing at contact surface accelerated by shock waves

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Turbulent mixing at a contact surface between two gases of different density which is accelerated by plane stationary normally-incident shock waves is observed experimentally. A system of equations describing the nonstationary turbulent flow of the compressed gases is proposed. Results are presented of numerical calculations of the experimental data on turbulent mixing of the contact surface between gases of different density following multiple passage of shock waves through the surface. Satisfactory agreement between the calculations and experiments can be attained by suitable choice of the empirical constants.

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It is known that the boundary between gases having different densities is unstable to small perturbations if the pressure and density gradients on the boundary are directed oppositely (the so-called gravitational instability), or else if a shock wave passes through the boundary. The development of small perturbations can lead to the onset of turbulence in the vicinity of the boundary, and accordingly to turbulent mixing of the gases.[1,2]

1. MODEL OF TURBULENT MIXING

To describe gasdynamic flow with allowance for the possible onset of turbulence and the ensuing turbulent mixing, we used the following system of equations:

$$d\rho/dt=-\rho\,\mathrm{div}\,\mathbf{v}, \tag{1}$$

$$\frac{d\mathbf{v}}{dt}=-\frac{1}{\rho}\nabla(P+P_t)+\frac{1}{\rho}\frac{\partial}{\partial x_k}[(\nu+\nu_t)\rho\sigma_{ik}], \tag{2}$$

$$\frac{dE}{dt}=\frac{P}{\rho^2}\frac{d\rho}{dt}-\frac{1}{\rho}\mathrm{div}\,\mathbf{q}_t+\nu\sigma_{ik}\frac{\partial v_i}{\partial x_k}-D_t\left(\varkappa\nabla P-\frac{\nabla\rho}{\rho}\right)\frac{\nabla P}{\rho}$$
$$+k_1\left(1+k_2\frac{\nu}{\nu_t}\right)\frac{\varepsilon_t^2}{\nu_t}, \tag{3}$$

$$\frac{dc_i}{dt}=-\frac{1}{\rho}\mathrm{div}\,\mathbf{j}_{ti}, \tag{4}$$

$$\frac{d\varepsilon_t}{dt}=\frac{1}{\rho}\mathrm{div}\,\rho D_t\nabla\varepsilon_t-\frac{1}{\rho}P_t\,\mathrm{div}\,\mathbf{v}+\nu_t\sigma_{ik}\frac{\partial v_i}{\partial x_k}$$
$$+D_t\left(\varkappa\nabla P-\frac{\nabla\rho}{\rho}\right)\frac{\nabla P}{\rho}-k_1\left(1+k_2\frac{\nu}{\nu_t}\right)\frac{\varepsilon_t^2}{\nu_t}. \tag{5}$$

Here

$$\frac{d}{dt}=\frac{\partial}{\partial t}+(\mathbf{v}\nabla),\qquad \sigma_{ik}=\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\,\mathrm{div}\,\mathbf{v},$$

ν is the coefficient of kinematic viscosity, ν_t is the coefficient of turbulent viscosity, D_t is the coefficient of turbulent diffusion,

$$\mathbf{q}_t=-\rho D_t\nabla\left(E+\frac{P}{\rho}\right)$$

is the turbulent heat flux, c_i is the concentration of the i-th component, $\mathbf{j}_{ti}=-\rho D_t\nabla c_i$ is the turbulent mass flux of the i-th component, ε_t is the kinetic energy of the turbulence, $P_t=\frac{2}{3}\rho\varepsilon_t$ is the pressure increment due to ε_t,