Galvanomagnetic properties of metals with open Fermi surfaces

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The magnetoresistance of pure metals with open Fermi surfaces is considered under conditions when relaxation processes are completely determined by the electron-phonon interaction. "Kirchhoff laws," which yield a complete solution of the problem under the condition that the phonon thermal momentum is small compared with all characteristic distances in momentum space, are formulated for strong magnetic fields. Effects due to finite phonon thermal momentum are discussed. Actual calculations are carried out for a Fermi surface of the "corrugated cylinder" type. The electric conductivity tensor is obtained and its angular and temperature dependences are studied. It is shown, in particular, that the angular dependence of the conductivity is oscillatory in certain temperature ranges. It is demonstrated that in metals for which the Fermi surface contains narrow necks there exists a broad range of intermediate magnetic fields in which the conductivity depends on the field strength in an unconventional manner.

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The present paper, as the preceding one,[1] is devoted to a study of the galvanomagnetic properties of pure metals under conditions when the only relaxation mechanism is the collision of the electrons with the phonons. Principal attention is paid to identification of the singularities connected with the character of the small-angle scattering of the electrons by the phonons. The asymptotic behavior of the electric conductivity as a function of the magnetic field intensity is, naturally, in accord with the general theory of galvanomagnetic phenomena.[2]

For metals with closed Fermi surfaces (FS), an important role is played by umklapp processes; in the absence of these processes, a joint drift of the electrons and phonons takes place, and the transverse conductivity is equal to zero. On the other hand, if the FS is open, then the need for collisions with umklapp is obviated. In the presence of open orbits, the electron goes off to infinity in momentum space (and consequently also in coordinate space), moving along the orbit in a magnetic field. In the case when all the orbits are closed (but the FS is open), the departure to infinity is via diffusion displacement of the electron along a chain of closed orbits. Another nontrivial example of such a situation is the possibility, considered by Pippard[3] of diffusion displacement of an electron near a flat boundary separating the electron and hole orbits.

The most interesting manifestations of the special character of scattering of electrons by phonons appear in those cases when there are layers of open orbits that are small in comparison with the entire FS. We shall investigate below the simplest (but frequently encountered) possibility of such a situation: the open part of the FS is made up of narrow bridges, and the magnetic field is nearly or exactly perpendicular to this direction. (Opening line on the stereographic projection.) A detailed examination of the diffusion relaxation process makes it possible, in particular, to trace the conductivity anisotropy connected with the appearance of open orbits.

We do not deal here at all with effects due to simultaneous action of the phonon and impurity scattering mechanisms.[4] We note that in the presence of narrow layers of open (or extended) orbits the contribution of the impurity scattering is relatively unimportant. Therefore the conditions under which the phonon scattering mechanism prevails over the impurity mechanism are less stringent in slow magnetic fields than in the absence of the field.

1. FUNDAMENTAL EQUATIONS. KIRCHHOFF'S RULES

As in our preceding paper,[1] we start from the diffusion equation

$$-\frac{1}{v}\frac{\partial\chi}{\partial t}+\operatorname{div}\hat{D}(\nabla\chi-\mathbf{a})=-e\mathbf{E}\mathbf{n}. \tag{1}$$

Here $-\chi\partial f_0/\partial\varepsilon$ is the nonequilibrium increment to the electron distribution function, $f_0=[\exp(\varepsilon-\mu/T)+1]^{-1}$, $\mathbf{n}=\mathbf{v}/v$, t is the time of revolution on the orbit in a magnetic field, $\hat{D}$ is the diffusion tensor, and the term $\mathbf{a}$, which is integral in $\nabla\chi$, describes the dragging of the phonons (see[5]). The boundary conditions for this equation are

$$\chi_p=\chi_{p+g},\quad \nabla\chi_p=\nabla\chi_{p+g}, \tag{2}$$

$\mathbf{g}$ is the reciprocal-lattice vector.

Equation (1) is valid under conditions when the thermal momentum of the phonons $q\approx T/s$ (s is the speed of sound) is small in comparison with all the characteristic distances in momentum space. We note that with increasing temperature, besides the diffusion displacement over the FS, an important role can be played by transitions of the electrons between closely-lying sections of this surface (a similar possibility was considered in Sec. 2). To take these such transitions into account it is necessary to add to Eq. (1) a term χ_p containing finite differences of the functions Π_p; this term was already used earlier[1] to describe umklapp processes.

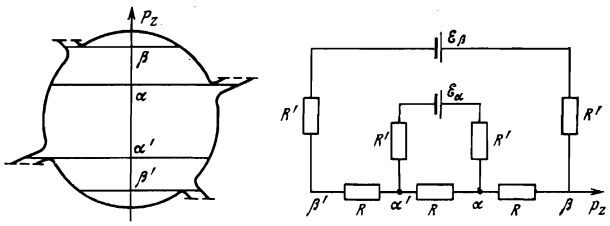

FIG. 1.

The electric current density is

$$\mathbf{j}=\frac{2e}{h^3}\int \chi \mathbf{n}\, dS.$$

When calculating the current in the direction perpendicular to the magnetic field, it is convenient to use the relation

$$\mathbf{j}_\perp = e(n_e-n_h)\mathbf{u}_X + \frac{c}{H^2}\sum_{\mathbf{g},\sigma}[\mathbf{g}\times\mathbf{H}]A_{\mathbf{g},\sigma}; \tag{3}$$

$$A_{\mathbf{g},\sigma}=\frac{2}{h^3}\left(\oint \mathbf{Q}\, d\mathbf{l}_{\mathbf{g},\sigma} - e_\sigma \mathbf{S}_{\mathbf{g},\sigma}\mathbf{E}\right), \tag{4}$$

$$\mathbf{Q}=\hat{D}(\nabla\chi-\mathbf{a})+ec^{-1}[\mathbf{H}\times\mathbf{n}]\chi. \tag{5}$$

These formulas can be obtained by multiplying Eq. (1) by $\mathbf{p}\times\mathbf{H}$, integrating over the Fermi surface, and using the identity[5]

$$\int \hat{D}(\nabla\chi-\mathbf{a})\, dS=0.$$

Here $\mathbf{u}_X = cH^{-2}[\mathbf{E}\times\mathbf{H}]$, σ is the number of the energy band, and $\mathbf{g}$ are the reciprocal-lattice vectors corresponding to the faces of the Brillouin zone. The absolute value of the vector $\mathbf{S}_{\mathbf{g},\sigma}$ is equal to the area of the intersection of the FS with the corresponding face and is directed normal to this face; it is assumed that $\mathbf{S}_{\mathbf{g},\sigma}\cdot\mathbf{g}>0$ (the summation in (3) is over all the non-equivalent faces). The integration in (4) is along the line of the intersection of the FS with the face, the vector $d\mathbf{l}_{\mathbf{g},\sigma}$ is perpendicular to this line and is located in a plane tangent to the FS.

In certain zones the FS can be closed, and such zones contribute only to the term $e(n_e-n_h)\mathbf{u}_X$. Zones with open FS contribute to both terms of (3). If the FS is regarded as "electronic" (this manifests itself in the magnitude of the area $S_{\mathbf{g},\sigma}$), then $e_\sigma = e < 0$, while for a hole surface $e_\sigma = -e$. The choice of the treatment is dictated by considerations of convenience and naturally does not affect the result. To verify this statement it is convenient to use the formula

$$\sum_{\mathbf{g}} g_i (\mathbf{S}_{\mathbf{g}})_k = V\delta_{ik},$$

where $S_{\mathbf{g}}$ is the area of the face of the Brillouin zone and V is the volume of the zone.

The first term in (3) is the current connected with the Hall drift. The meaning of this term can be easily understood: the electron trajectory in a plane perpendicular to the magnetic field is a plane from the trajectory in momentum space by rotating through an angle $\pi/2$ and multiplying by c/eH. The quantity $A_{\mathbf{g},\sigma}$ has the meaning of the number of umklapps per unit of time; for each umklapp, the momentum changes by the vector $\mathbf{g}$. (By "umklapp" is meant here the passage of the electron through the boundary of the Brillouin zone.) The umklapps take place both from the lines $l_{\mathbf{g},\sigma}$ ($\mathbf{Q}$ is the flux density of these umklapps) and from the part of the face of the zone which closes the FS. The first term in $\mathbf{Q}$ is the diffusion flux, the second is the flux connected with the rotation of the electrons over the FS under the influence of the magnetic field. (Equation (1) can be written in the form $\operatorname{div}\mathbf{Q} = -e\mathbf{E}\cdot\mathbf{n}$.)

In this and the following sections we shall be interested in the region of strong magnetic fields: $\Omega\tau \gg 1$, Ω is the Larmor frequency, and τ is the characteristic relaxation time, the meaning of which will be explained in Sec. 3. We proceed to solve Eq. (1). We consider separately the case of closed electron orbits in the case when the FS has open intersections with a plane perpendicular to the magnetic field.

1. *There are no open orbits.* The solution of Eq. (1) can be obtained by successive approximations (cf. [2,1]). In first-order approximation

$$\frac{\partial\chi^{(1)}}{\partial t}=e\mathbf{E}\mathbf{v}_\perp, \quad \chi^{(1)}=\mathbf{up}+\psi(p_z), \tag{6}$$

where $\mathbf{v}_\perp$ is the velocity component in the plane perpendicular to the magnetic field; $\mathbf{u}_\perp = \mathbf{u}_X$, $u_z = u_c$. The function $\psi(p_z)$ and the velocity u_c of the co-moving reference frame are determined from the condition that the next-order approximation equation have a solution:

$$\frac{d}{dp_z}J = eE_z\langle v_z\rangle, \quad J(p_z) = -D\frac{d\psi}{dp_z}, \tag{7}$$

$$\frac{2}{h^3}\sum_{\mathbf{g}} g_z J_{\mathbf{g}} = eE_z(n_e-n_h) - E_z\frac{2}{h^3}\sum_{\mathbf{g},\sigma} e_\sigma g_z S_{\mathbf{g},\sigma}. \tag{8}$$

Here $J(p_z)$ is the diffusion current through the given section of the FS, $J_{\mathbf{g}}$ is the value of this current at the boundary of the zone; the quantity $D(p_z)$ and the averaging operations $\langle\ldots\rangle$ were defined in the preceding paper[1]; the z axis was chosen along the direction of the magnetic field.

Since all the orbits are closed, the Brillouin zone can always be chosen such that its boundaries cross the FS only along the sections $p_z = \text{const}$ (see Fig. 1). In writing down the quasi-momentum balance equation (8) we have already used this choice of the zone. The boundary conditions (2) then take the form

$$\psi(p_z+g_z)-\psi(p_z) = -ug, \quad J(p_z+g_z)=J(p_z). \tag{9}$$

We note that a given p_z can correspond to several currents and several functions $\psi(p_z)$. At those values of p_z for which there is branching of the sections of the FS, the total current is conserved, and the functions $\psi(p_z)$ are continuous.

In crossed fields $\mathbf{E}$ and $\mathbf{H}$ (i.e., $E_z = 0$), the relations written out above are equivalent to Kirchhoff's laws for a certain electric circuit.[1] By way of example, Fig.

1 shows the FS and the corresponding circuit. R stands for the diffusion resistances of the sections of the large body, and R' are the resistances of the stubs. According to (7), the "potential difference" is

$$\delta\psi = JR, \quad R = \int_{p_{z1}}^{p_{z2}} D^{-1}(p_z)\,dp_z. \tag{10}$$

The sections of the circuit that are joined together through the zone boundaries include sources of the emf $\mathscr{E} = \mathbf{u}\cdot\mathbf{g}$.

At $E_z \neq 0$ we have, in analogy with the preceding paper[1]

$$\psi = \psi_1 + \psi_2, \quad J = J_1 + J_2, \quad J_1 = -D\frac{d\psi_1}{dp_z} = \pm eE_z S(p_z),$$

where the upper and lower signs correspond to the electron and hole FS, $S(p_z)$ is the area of the intersection of the FS with the plane $p_z = \text{const}$, and

$$dS/dp_z = \pm\langle v_z\rangle.$$

The currents J_2 are determined from the same Kirchhoff laws as before, but with an emf

$$\mathscr{E} = \mathbf{ug} + \psi_1(p_z) - \psi_1(p_z + g_z).$$

Thus, Kirchhoff's laws jointly with the quasi-momentum balance equation make it possible to determine all the currents and the velocity u_c of the co-moving system. The electric current density is

$$\mathbf{j}_\perp = e(n_e - n_h)\mathbf{u}_X - \frac{2c}{h^3H^2}\sum_{g,\sigma}[\mathbf{g}\times\mathbf{H}](J_{g,\sigma} + e_\sigma S_{g,\sigma}E_z), \tag{11}$$

$$j_z = \frac{2e}{h^3}\int n_z\psi\,dS + e(n_e - n_h)u_c. \tag{12}$$

In order of magnitude, $J_g \approx \mathbf{u}_X\cdot\mathbf{g}/R$ and the transverse conductivity is

$$\sigma_{xx} \approx (c^2g^2/h^3H^2)R^{-1}.$$

2. *Layers of open orbits are present.* In this case the term $e\mathbf{E}\cdot\mathbf{v}_\perp$ must be relegated to the second-approximation equation. (Otherwise Eq. (6) cannot be solved, since $\langle\mathbf{v}_\perp\rangle \neq 0$.) Thus, $\chi^{(1)} = \psi(p_z) + u_c p_z$. From the condition that the second-approximation equations have a solution, we obtain for open orbits

$$\frac{d}{dp_z}D^\pm\frac{d\psi^\pm}{dp_z} = -e\mathbf{E}\langle\mathbf{v}_\perp\rangle = \mp e\mathbf{E}[\mathbf{g}\times\mathbf{h}], \tag{13}$$

where the ± signs number two layers of open orbits with opposite directions of motion in the magnetic field, $\mathbf{g}$ is the reciprocal-lattice vector corresponding to the given openness direction, and $\mathbf{h} = \mathbf{H}/H$. We assume here for simplicity that $E_z = 0$, since the longitudinal electric conductivity σ_{zz} is not very sensitive to the topology of the electron orbits, and is no longer of interest to us. At the boundaries of the layer $p_{z1} \leq p_z \leq p_{z2}$ of the open orbits, the following conditions are satisfied:

$$\psi^+(1) = \psi^-(1), \quad \psi^+(2) = \psi^-(2), \quad J^+(1) + J^-(1) = J_1,$$

$$J^+(2) + J^-(2) = J_2,$$

where J_1 and J_2 are the diffusion currents entering and leaving the layer.

Adding Eqs. (13), we verify that $J^+(p_z) + J^-(p_z) = J = \text{const}$, i.e., the layer of open orbits is not a source of current. This layer, however, is a source of a potential difference:

$$\psi(2) - \psi(1) = J\mathscr{R} + \mathscr{E}; \tag{14}$$

$$\mathscr{R}^{-1} = \frac{1}{R^+} + \frac{1}{R^-}, \quad \mathscr{E} = \frac{e\mathbf{E}[\mathbf{g}\times\mathbf{h}]}{R^+ + R^-}(R^+K^- - R^-K^+);$$

$$R^\pm = \int_1^2 \frac{dp_z}{D^\pm}, \quad K^\pm = \int_1^2 \frac{p_z\,dp_z}{D^\pm}.$$

The density of the electric current connected with one layer of open orbits is

$$\mathbf{j}_\perp = [\mathbf{g}\times\mathbf{h}]\frac{2e}{h^3}\int_1^2 (\psi^+ - \psi^-)\,dp_z = [\mathbf{g}\times\mathbf{h}]\frac{2e}{h^3} \tag{15}$$

$$\times\left\{e(\mathbf{E},[\mathbf{g}\times\mathbf{h}])\int_1^2 \frac{(p_z - p_{z0})^2\,dp_z}{D^\cdot} + J\frac{R^+K^- - R^-K^+}{R^+ + R^-}\right\},$$

$$p_{z0} = \frac{K^+ + K^-}{R^+ + R^-}, \quad D^\cdot = \left(\frac{1}{D^+} + \frac{1}{D^-}\right)^{-1}.$$

The two terms in (15) are in general of the same order of magnitude. The first is determined only by the layer of open orbits. The current J in the second term is obtained from Kirchhoff's laws for a circuit in which each open layer corresponds according to (14) to an emf $\mathscr{E}$ and a resistance $\mathscr{R}$. The presence of open orbits does not influence the condition (8) for the balance of the longitudinal momentum.

If all the open-orbit layers are narrow ($d \ll g$, where d is the layer thickness), then the second term in (15) is small in comparison with the first, and the dependence of $D^\pm$ on p_z can be neglected. Then each layer of open orbits makes the following contribution to σ_{xx}:

$$\sigma_{xx} = \frac{1}{6}\frac{e^2g_y^2d^3}{h^3D^\cdot}. \tag{16}$$

Comparing this expression with the known formulas for the electric conductivity of the open orbits $\sigma_{xx} = n^{\text{eff}}e^2\tau^{\text{eff}}/m$, we have $n^{\text{eff}} \approx (d/g)(g/h)^3$ is the number of the electrons in the layer, $\tau^{\text{eff}} \approx \tau_F(d/g)^2$ is the time of the diffusion of the electron out of the layer, $\tau_F \approx mg^2D^{-1}$ is the time of diffusion through the entire FS. This result ($\sigma_{xx} \propto d^3T^{-5}$) was obtained earlier by Kaganov, Kadigrobov, and Slutskin.[6]

2. STRONG FIELDS. ANISOTROPY AND TEMPERATURE DEPENDENCE OF THE CONDUCTIVITY

In this section we consider the behavior of the electric conductivity tensor when "opening sets in," i.e., at magnetic-field directions close to those for which layers of open orbits appear. (We recall that we are dealing with open directions that are mapped by lines on the stereographic projection.)

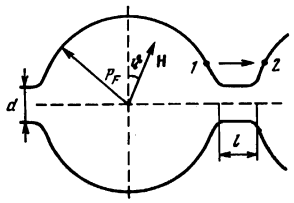

FIG. 2.

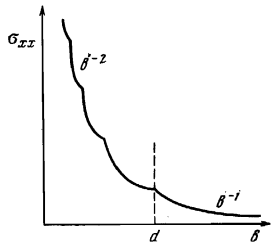

FIG. 4.

The actual calculations will be carried out for the model shown in Fig. 2. The principal part of the FS is a sphere of radius p_F; the bridges are assumed to be narrow and short; the diameter of the bridge is $d \ll p_F$ and its length is $l \lesssim d$. (The FS of noble metals are similar in shape.) The magnetic field is perpendicular (or almost perpendicular) to a certain opening direction. It is no less important that there are also additional opening directions[1)] (not shown in Fig. 2). We shall explain later on the behavior of the conductivity for a somewhat more general model of the FS.

The topology of the electron orbits depends on the misorientation angle ϑ between the magnetic field and the direction corresponding to the open orbits. We introduce the symbol $b = g\vartheta$. At $b > d$ there are two types of orbits: those consisting of one circle and those consisting of two circles joined by a narrow constriction—"figure-8 formations." At $b < d$, orbits with large numbers of loops appear. As indicated in Sec. 1, it is convenient to choose a Brillouin zone with boundaries that do not disturb the closed orbits, e.g., as shown in Fig. 3. (It is assumed that $b < d$ and the intersection of the FS with the boundaries of the zone is marked dashed.)

1. We consider first the case when the thermal momentum of the phonons is small in comparison with all the characteristic distances in momentum space ($q \approx T/s \ll b, d, l$), and therefore the diffusion approximation can be used. Elementary calculations based on formulas (9)–(11) lead to the result (according to (8), $u_c g_z \ll u_x g_y$)

$$\sigma_{xx} = \frac{2c^2 g^2}{h^3 H^2} R^{-1},$$
$$R^{-1} = 2\pi D_0 \frac{g}{b}\left(1+\frac{d}{b}\right)\eta\left(\frac{d}{b}\right),$$
$$\frac{1}{\eta(x)} = 1 + \frac{\Delta(x)(1-\Delta(x))}{(A(x)+1)(A(x)+2)}. \qquad (17)$$

Here $A(x)$ is the integer part of x and $\Delta(x) = x - A(x)$; the diffusion tensor is assumed to be a constant scalar $D_{ik} = D_0\delta_{ik}$. The function $\eta(x) \approx 1$ oscillates with a unity period and has kinks at $x = n$. The angular dependence of σ_{xx} is shown in Fig. 4. We note that the period of the oscillations is $\delta(1/b) = 1/d$, and the oscillation amplitude is independent of b.

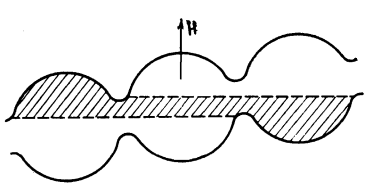

FIG. 3.

The physical mechanism that leads to transverse conductivity consists in the fact that on going from certain trajectories to others the electron is displaced along the p_y axis in momentum space and accordingly along the x axis in coordinate space. The kinks in the oscillations are connected to the jumplike appearance of orbits having a number of loops greater by unity. The smooth part of the angular dependence of $\sigma_{xx}(b)$ can be interpreted as the result of random walks of the electron along a chain consisting of layers of extended orbits:

$$\sigma_{xx} \approx \frac{n^{\text{eff}} e^2}{\varepsilon_F} \frac{(\Delta x)^2}{\tau^{\text{eff}}}. \qquad (18)$$

Here $\Delta x \approx cg/eH$ is the step of the walk, $\tau^{\text{eff}} \approx \tau_F (b/p_F)^2 \propto T^{-5}$ is the time of the diffusion displacement of the electron through a layer of thickness b, $n^{\text{eff}} \approx n(b+d)g^{-1}$ is the effective number of electrons, and $n = (8\pi/3)(p_F/h)^3$, ε_F is the Fermi energy. We note that the angular dependence $\sigma_{xx} \approx b^{-2}$ at $b \ll d$, obtained by Lifshitz and Peschanskiĭ[7] in the relaxation-time approximation, is of different origin: τ does not depend on b, but the step is $\Delta x \approx (cg/eH)\, d/b$.

Under these conditions, the conductivity σ_{xx} is determined completely by a narrow layer of electron orbits that are close to infinite. The presence of additional opening directions (see footnote 1) does not manifest itself here in explicit form: in view of the smallness of the resistance (17), the diffusion current flowing through the layer is $J_z \cong \mathscr{E}/R$, i.e., this section of the "circuit" is regarded as isolated.

The situation is different in the calculation of the conductivity σ_{xx}, which is determined by the behavior of the function $\psi(p_z)$ within the limit of the entire FS (see the first term of (12)). At $b \ll d$, the principal part of the FS consists of two "caps," which are widely separated in reciprocal space (Fig. 3). As a result, a large emf $\mathscr{E} \approx \mathbf{u}_X \cdot \mathbf{g}(1 + d/b)\eta(d/b)$ appears on the sections S_z corresponding to the "additional" opening directions, with a corresponding potential drop $\psi(p_z)$ within the limits of the caps.

According to (12), we have

$$\sigma_{xx} \approx \frac{nec}{H}\left(1+\frac{d}{b}\right)\eta\left(\frac{d}{b}\right). \qquad (19)$$

(As shown by calculations, $u_c \approx u_X(1 + d/b)$, and therefore both terms in (12) are of the same order.) We note that the result (19) remains valid in order of magnitude also at $n_e = n_h$. The numerical coefficients in (19) depend, naturally, on the orientation of the additional openness directions.

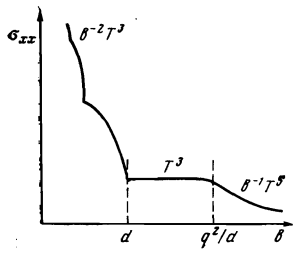

FIG. 5.

It is easy to see how the results are altered if we forgo certain assumptions connected with the FS model. If the neck is not narrow ($d \propto g$), then the dependence $\sigma_{xx} \propto b^{-1}$ may not manifest itself, but at $b \ll d$ the law $\sigma_{xx} \propto b^{-2}\eta(d/b)$ remains in force. If the neck is long enough ($l \gg d$), orbits with more than one loop vanish at $b \gtrsim dg/l$, and consequently the relation $\sigma_{xx} \propto b^{-1}$ is violated. (In this region, the $\sigma_{xx}(b)$ dependence may be determined by the anisotropy of the resistance of the neck.) At smaller b, however, the results remain in force if we assume in them $d = d(b)$, where $d(b)$ is the thickness of the layer of the "through" orbits that pass through the neck.

2. We proceed now to consider effects connected with the finite character of the thermal momentum of the phonons $q \approx T/s$. We assume first that the momentum q is small in comparison with the characteristic dimensions of the neck, but exceeds the width of one layer of extended orbits: $b < q \ll d, l$. It is easy to understand the result of the violation of the diffusion approximation in this case: a random walk along a chain of orbits takes place, as before, although the electron can jump in one step over several layers of orbits. Therefore all the preceding results remain in force, but the oscillations at $q > b$ become smeared out. (It is curious that with increasing temperature, at $q > d$, the oscillations, as we shall see, appear again.)

At higher temperatures, when $q \gtrsim d$ and $q \gtrsim l$, the violation of the diffusion approximation again becomes significant. To avoid umklapp process, i.e., transitions that bypass the neck (of the type 1 – 2 in Fig. 2), we shall first assume the neck to be long and that $l \gg q \gg d$. (Then the umklapp probability is exponentially small.) The transverse conductivity mechanism is then the following: during the time of one collision with the phonon $\tau^{eff} \approx \tau_F (p_F/q)^2 \propto T^{-3}$, the electron located in the layer of extended orbits has an overwhelming probability of leaving the layer and falling in the region of circular orbits—the cap. The electron is then displaced in reciprocal space by an amount $\Delta p_y \approx (d/b+1)g$ and, accordingly, in coordinate space by an amount $\Delta x \approx (cg/eH)(d/b+1)$. To estimate the conductivity we can use formula (18).

A more accurate analysis, based on the solution of the kinetic equation and on the assumption that the function $\psi(p_z)$ is constant within the limits of individual layers of extended orbits, yields

$$\sigma_{xx} = \frac{4\pi}{h^3} p_F \frac{c^2}{H^2} \frac{D_0}{q^2} \int_{-b/2}^{b/2} dp_z \sum_k [\Delta p_y(k)]^2.$$

Here $\Delta p_y(k) = g_y k$ is the change of the momentum on going from the k-th loop of the extended orbit to the cap; within the limits of the interval $b/2 \geq p_z \geq -b/2$, there are three layers of extended orbits, differing by unity in the number of loops.

The results of simple calculations yield

$$\sigma_{xx} \cong \frac{3}{2} n \left(\frac{c}{H}\right)^2 \frac{D_0}{q^2} d\lambda\left(\frac{d}{b}\right), \qquad (20)$$

$$\lambda(x) = x^2 + 3x + 2 + x^{-1}\Delta(x)[1-\Delta(x)][3x+4-2\Delta(x)].$$

It is seen from this formula that, in contrast to the case $q \ll d$, the conductivity is $\sigma_{xx} \propto T^3$ and is independent of b if $b > d$. Incidentally, with further increase of b we return to the result of the diffusion approximation: $\sigma_{xx} \propto b^{-1}T^5$. The point is that at sufficiently large b, when only single and figure-8 orbits remain, the diffusion time between the figure-8 orbits over the layer of the single orbit may turn out to be appreciable. Indeed, a diffusion displacement through a distance b is executed after $(b/q)^2$ Brownian steps; on the other hand, the electron lands in a small region (in comparison with the step q) of width d, as is well known, after $(q/d)^2$ steps. Therefore at $b \gg q^2/d$ the conductivity is determined by the diffusion resistance, i.e., the result (17) is valid. The angular dependence of the conductivity at $l \gg q \gg d$ is shown in Fig. 5.

At $q > l$, umklapp processes come into play. This mechanism, which ensures transverse conductivity of metals with closed FS, was considered earlier.[1] In the temperature region of interest to us, the contribution of the umklapp processes to the electric conductivity is given by[2)]

$$\sigma_{xx} \approx \frac{ne^2}{m\Omega^2\tau_F} \frac{b+(p_F q)^{1/2}}{p_F} \left[\left(\frac{b}{p_F}\right)^2 + \left(\frac{q}{p_F}\right)^{3/2}\right]^{-1}. \qquad (21)$$

Comparing (20) with (21) we easily verify that the umklapp mechanism is the fundamental one at $b > d(d/q)^{1/2}$ and is immaterial at smaller b. Figure 6 shows the conductivity σ_{xx} for ratio of d to l at $q \gg d$ and $q \gg l$.

It is easy to verify that, just as in the preceding subsection, the conductivity σ_{xz} exhibits an appreciable anisotropy:

$$\sigma_{xz} \approx \frac{nec}{H}\left[\frac{\sigma_{xx}(b)}{\sigma_{xx}(p_F)} \frac{b}{p_F} + \varphi\right]. \qquad (22)$$

Here $\varphi(b) \lesssim 1$ is a smooth function of the angles and changes significantly over distances on the order of p_F. In the symmetrical directions (alignment of H with a

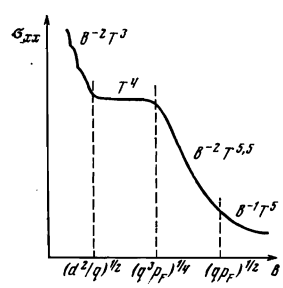

FIG. 6.

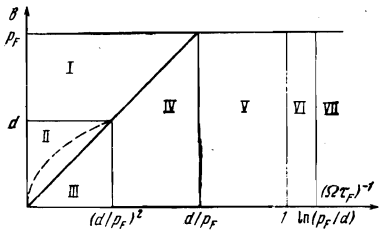

FIG. 7. Regions of different behavior of the transverse conductivity: I—$\sigma \propto H^{-2}b^{-1}$, II—$\sigma \propto H^{-2}b^{-2}$, III—$\sigma \propto H^0$, IV—$\sigma \propto H^{-1}$, V—$\sigma \propto H^{-2}(\ln H)^{-1}$, VI—$\sigma \propto H^{-2}$, VII—$\sigma \propto H^0$.

twofold axis is sufficient) we have $\varphi = 0$. We note that relation (22) is valid in all the cases considered in this section.

3. REGION OF INTERMEDIATE MAGNETIC FIELDS

As we have seen, in asymptotically strong magnetic fields at $b \ll p_F$ the conductivity is determined by a relatively short relaxation time τ^∞, which is either the time required for the electron to go out of the layer of extended orbits, or the time required to diffuse over a distance b. In the absence of a magnetic field, the conductivity is determined by the much longer (at $d \ll p_F$) time τ_F of diffusion through the entire FS. Therefore the region of strong magnetic fields ($\Omega\tau^\infty \gg 1$) is not in contact with the region of weak fields ($\Omega\tau_F \ll 1$) and there appears a wide region of intermediate fields: $H_{max}/H_{min} \approx (p_F/d)^2$. We note that in the relaxation-time approximation, owing to the smallness of the contributions to the conductivity of the narrow layers of extended orbits, there also arises a region of intermediate magnetic fields. However, the width of this trivial intermediate region $H_{max}/H_{min} \approx (p_F/d)^{1/2}$ is small in comparison with the case when small-angle processes predominate.

In this section we shall not present the calculations and confine ourselves to formulation of the fundamental qualitative results for FS with narrow and short bridges ($l \lesssim d \ll p_F$) under conditions when the diffusion approximations are valid ($q \ll d, l$).

The diagram of Fig. 7, in which the variables are b and $(\Omega\tau_F)^{-1}$, shows schematically the regions in which the conductivity behaves in a qualitatively different manner, and we indicate the asymptotic dependences of σ_{xx} on H and b. The conductivity in the regions I–VII is given respectively by

$$\sigma_{xx}\frac{m}{ne^2} \approx \frac{1}{\Omega^2\tau_F}\frac{p_F}{b},\ \frac{1}{\Omega^2\tau_F}\frac{p_F d}{b^2},\ \tau_F\left(\frac{d}{p_F}\right)^3,\ \frac{1}{\Omega}\frac{d}{p_F},\ \frac{1}{\Omega^2\tau_F}\left[\ln\left(\frac{p_F}{\Omega\tau_F d}\right)\right]^{-1},\ \frac{1}{\Omega^2\tau_F\ln(p_F/d)},\ \tau_F\ln\frac{p_F}{d}. \tag{23}$$

The dashed curve corresponds to the relation $p_F(\Omega^*\tau_F)^{-1/2} = b$, where $\Omega^* \approx \Omega b/d$ is the frequency of revolution of an extended orbit ($b \ll d$). This curve divides region II into two parts, with the conductivity undergoing oscillations in the upper part (see Sec. 2) and no oscillations in the lowest part. All the remaining lines in Fig. 7 are straight lines the positions of which are clear from the designations on the coordinate axes.

Let us explain the physical meaning of the results. Strong magnetic fields corresponded to region I, and strictly speaking only to the upper part of region II. The layers of orbits that have unequal numbers of loops have widths on the order of b. In the lowest part of region II, the diffusion displacement of the electron during the period of motion on the extended orbit is $p_F(\Omega^*\tau_F)^{-1/2} > b$ and there are therefore no oscillations.

In region III, time $1/\Omega^*$ is sufficient for the electron to leave the layer of extended orbits via diffusion: $p_F(\Omega^*\tau_F)^{-1/2} \gg d$. Under these conditions the electron does not manage to sense the difference between an extended orbit and an open orbit, and the electric conductivity is $\sigma_{xx} \approx n^* e^2\tau/m$, where $n^* \approx nd/p_F$ and $\tau \approx \tau_F(d/p_F)^2$.

In the region IV, the electron manages to leave the layer of extended orbit within a time $1/\Omega$: $p_F(\Omega\tau_F)^{-1/2} \gg d$. The electron emerging from a certain neck is almost certain not to land in an equivalent neck. The electric conductivity determined by formula (18), in which the random-walk step is $\Delta x \approx v_F\Omega^{-1}$ and the corresponding time is $\tau^{eff} \approx \Omega^{-1}$. It must be borne in mind, however, that in the same interval of magnetic fields at $p_F(\Omega^*\tau_F)^{-1/2} < d$ (region I) the time $\tau_F(b/p_F)^2$ of diffusion between the layers of figure-8 orbits exceeds the time $\Omega^{-1}(p_F(\Omega\tau_F)^{-1/2}d^{-1})^2$ within which the electron lands in a bridge (see the analogous reasoning in connection with the limits of applicability of formula (20) in Sec. 2; in this case the length of the Brownian step is $p_F(\Omega\tau_F)^{-1/2}$). Under these conditions the conductivity is determined by the diffusion time $\tau_F(b/p_F)^2$, just as in the entire region I.

In region V, the specific mechanism considered earlier[1] comes into play. The diffusion displacement is $\delta(t) \approx p_F(t/\tau_F)^{1/2}$ within a time t, and therefore on small segments of the path the electron velocity $\delta \propto \delta^{-1/2}$ exceeds its velocity along the orbit in the magnetic field $p_F\Omega$. The corresponding critical distance $\delta_0 \approx p_F(\Omega\tau_F)^{-1}$, if it is larger than d, plays the role of the effective width of the neck. In region V we have $\delta_0 \gg d$, and therefore σ_{xx} can be obtained from the corresponding expression for region IV, by replacing d by δ_0 (accurate to a factor $\ln(\delta_0/d)$, which characterizes the probability of falling into a neck from a distance δ_0).

In region VI, the electron diffuses during the time of one revolution through the entire FS, but does not manage to fall into a neck: $\tau_F < \Omega^{-1} < \tau^{(0)} \approx \tau_F\ln(p_F/d)$. Accordingly $\sigma_{xx} \propto (\Omega^2\tau^{(0)})^{-1}$. Finally, VII is a region of weak magnetic fields, in which $\Omega\tau^{(0)} \ll 1$ and $\sigma_{xx} \propto \tau^{(0)}$.

Let us consider also the conductivity σ_{xz}. Calculations show that

$$\sigma_{xz} = \sigma_{xz}^a + \sigma_{xz}^s, \tag{24}$$

where the term σ_{xz}^a, which is odd in the magnetic field, is connected with the conductivity σ_{xx} by the relation (22), while the even term takes the form

$$\sigma_{xz}^s \approx \frac{en}{g}\frac{\mathscr{E}}{E_x}(1+\Omega^2\tau'^2)^{-1},\quad \tau' \approx \tau_F\frac{db}{p_F^2}. \tag{25}$$

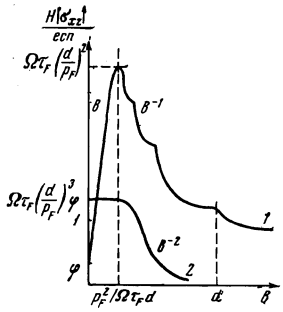

FIG. 8.

The potential difference $\mathscr{E}$ which is produced here at the edges of the layer of the open orbits is determined by expression (14). In the case of a short neck ($l \lesssim d \ll p_F$) we have

$$\mathscr{E}/E_z \approx (eg/m)\tau_F (d/p_F)^3 \varphi,$$

where $\varphi \lesssim 1$ is a smooth function of the angles and is of the same type as in (22).

It is easy to see that at $\Omega \gg \tau_F^{-1}(p_F/d)^2$ the quantity σ_{xz}^a is strongly anisotropic and has a sharp maximum at $b \approx p_F^2(\Omega\tau_F d)^{-1}$. Figure 8 shows for this case plots of $|\sigma_{xz}^a|$ (curve 1) and $|\sigma_{xz}^s|$ (curve 2) as functions of the angle variable b.

Let us write down in conclusion the resistance tensor $\hat{\rho} = \hat{\sigma}^{-1}$ for arbitrary magnetic fields, including weak ones ($\Omega\tau^{(0)} \gg 1$). We use the fact that the components $\sigma_{yy} \approx ne^2/m\Omega^2\tau_F$, $\Omega_{yz} \approx ne^2/m\Omega$, $\sigma_{zz} \approx ne^2\tau_F/m$ are not sensitive to the onset of open orbits. We have, accurate to numerical factors of the order of unity, the following: at $n_e \neq n_h$

$$\rho_{yy} \cong \frac{\sigma_{xx}}{\sigma_{xy}^2}, \quad \rho_{yz}^a \cong \frac{\sigma_{xz}^a}{\sigma_{zz}\sigma_{xy}}, \quad \rho_{yz}^s \approx \frac{\sigma_{xx}}{\Omega\tau_F\sigma_{xy}^2}, \quad \sigma_{xy} = \frac{ec(n_e - n_h)}{H}; \tag{26}$$

at $n_e = n_h$

$$\rho_{xx} \cong \frac{1}{\sigma_{xx}}, \quad \rho_{xy} \approx \frac{m}{ne^2}\left[\Omega^2\tau_F \frac{b}{p_F}\eta\left(\frac{d}{b+b_1}\right) + \Omega\right],$$
$$\rho_{xz} \approx \frac{m}{ne^2}\left[\Omega\frac{b}{p_F}\eta\left(\frac{d}{b+b_1}\right) + \frac{1}{\tau_F}\right], \quad b_1 = \left(\frac{p_F^2 d}{\Omega\tau_F}\right)^{1/3}. \tag{27}$$

The remaining components of the resistance tensor do not contain any singularities with respect to b and depend on H in the usual manner. (These components can be obtained in the relaxation-time approximation if this time is taken to be τ_F.)

[1)]Under phonon-dragging conditions, in the presence of only one opening direction, the situation is special: a joint drift of the electrons and phonons along the magnetic field is produced, such that the resultant velocity $\mathbf{u} = \mathbf{u}_X - \mathbf{h}(\mathbf{u}_X \cdot \mathbf{g})(\mathbf{g} \cdot \mathbf{h})^{-1}$ turns out to be perpendicular to the vector $\mathbf{g}$. In this case $\mathbf{j} \equiv e(n_e - n_h)\mathbf{u}$ and $\sigma_{xx} \equiv \sigma_{yy} = 0$. (A similar situation was considered earlier,[1] p. 2315 [p. 1148 of the translation].) In the presence of additional opening directions, such a drift is obviously impossible.

[2)]We point out an error in[1]: in the region $p_F(q/p_F)^{3/4} < b < (p_F q)^{1/2}$, the quantity σ_{xx} is proportional not to T^5, but to $T^{5.5}$, as follows from (21) and from formula (25) of[1].

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