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Translated by D. ter Haar

Electromagnetic waves with a discrete spectrum in metallic ferromagnets

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(Submitted December 11, 1975)
Zh. Eksp. Teor. Fiz. 71, 627–634 (August 1976)

The spectrum, attenuation, and polarization of electromagnetic waves with a discrete spectrum in an isotropic metallic ferromagnet are investigated theoretically as functions of the external magnetic field **H**, the wave vector **k**, and the angle between **H** and **k**. Such waves can exist because, in the case of a definite ratio of the wavelength to the cyclotron radius of the conduction electrons, the absorption of the electromagnetic wave energy by the electrons as a result of Landau damping becomes small. The interaction between the electromagnetic wave in the ferromagnet with the magnetic subsystem alters the character of the wave propagation as compared with that in a normal metal. In particular, in a weak external magnetic field, the Landau damping of the wave at points far from the points of the discrete spectrum becomes less than unity. This fact is important for the excitation and experimental observation of electromagnetic waves with a discrete spectrum.

PACS numbers: 75.30.—m

INTRODUCTION

It has long been assumed that electromagnetic waves with a frequency much smaller than the plasma frequency cannot propagate in metals to a depth greater than the skin depth. Konstantinov and Perel'[1] were the first to show that this is not the case under certain circumstances. The existence of electromagnetic oscillations in a metal is due to the presence of a strong magnetic field ($l \gg R$), which localizes the electrons in a region with dimensions of the order of the cyclotron radius R; l is the free path length of the conduction electrons. Localization of the electrons makes the electromagnetic-wave damping connected with dissipative currents small. In addition to this damping, there are resonance mechanisms of absorption of energy of the electromagnetic wave by electrons in the metal. This includes cyclotron absorption and Landau damping. The resonance damping of the wave is determined by the electrons for which the phase relation

$$k_z\bar{v}_z+N\Omega=\omega,\ N=0,\ \pm1,\ \pm2,\dots \tag{1}$$

is satisfied. Here ω is the frequency of the wave, $\Omega = eB/mc$ is the cyclotron frequency, e and m are the values of the charge and the effective mass of the conduction electron in the direction of **B**. The Landau damping turns out to be most important for short waves, whose length is much less than the cyclotron radius.[2] The electron interacts most effectively with the field of such a wave on those portions of its trajectory where it moves almost parallel to the planes of equal phase of the wave. In Fig. 1, this is in the vicinity of the points A and B. The value of the absorption of energy of the wave by the electron will change as a function of the

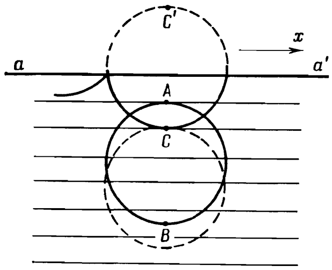

FIG. 1. Trajectories of electrons in a magnetic field close to the surface of the metal. The wave vector of the electromagnetic wave, propagating perpendicular to the surface of the metal aa', makes an angle with the direction of the magnetic field that is close to $\pi/2$. The system of parallel lines represent the planes of equal phase of the wave. The x axis is parallel to the corresponding axis of the system of coordinates (Fig. 2).

phase difference of the wave at these points. Since the electron moves in opposite directions at the points A and B, the absorption will be a maximum if an even number of half-wavelengths fits into the distance AB, and a minimum if the number of half-wavelengths is odd. Thus, the possibility arises of the propagation of short waves ($kR \gg 1$) with a discrete spectrum.

Electromagnetic oscillations with a discrete spectrum in a normal metal were first investigated by Kaner and Skobov.[3,4] In these researches, the spectrum, polarization and damping of the oscillations were obtained. In particular, it was shown that the Landau damping of the wave vanishes in the case

$$kR=\alpha_n\equiv\pi(n+{}^1/_4);\ n=0,\ 1,\ 2,\dots \tag{2}$$

It was shown by us,[5] as a result of a more detailed analysis of the dispersion equation, that two branches of oscillations are present, in a wide range of fields and frequencies, with a discrete spectrum of polarization of opposite sign. The dispersion relation for one of the branches in the case of a small magnetic field transforms into the expression obtained by Kaner and Skobov.[4] Electromagnetic waves with a discrete spectrum were considered in ferromagnetic metals in the work of Chudnovskiĭ,[6] where the dispersion law was obtained for waves in the two simplest cases of the problem of Kaner and Skobov.

Electromagnetic waves with a discrete spectrum were apparently not observed experimentally. This is possibly connected with the difficulty of excitation of such waves.[1)] Electromagnetic waves are usually excited in metals through the skin layer. It is thus difficult to excite in this manner a wave with a discrete spectrum. It was noted above that the Landau damping of a wave can be small only in the mean over a period of the motion of the electron along trajectories in the magnetic field. Near the surface of the sample, in a layer of thickness $2R$, there exist electrons reflected from the surface and therefore moving in the magnetic field along truncated trajectories. This leads to a strong damping of the wave. In the vicinity of the point C (Fig. 1), the energy of the wave is absorbed by the electron whose trajectory is shown in the figure by the dashed line. If this point is located in the interior of the sample, then, simultaneously with absorption of energy by one electron, another electron, which arrives at the point C from above, gives the wave the energy absorbed at the point C'. In the case in which the point C is located near the surface, a reflected electron reaches this point, which brings no energy to the wave. The number of points similar to C on the diameter of the orbit is proportional to n. The damping of the electromagnetic wave at each of these is determined by the maximum damping Γ_{max} of the wave with the discrete spectrum in the interior of the sample.

Thus, in order that excitation of the wave occur, the satisfaction of the following inequality is necessary:

$$n\Gamma_{max}<1. \tag{3}$$

As shown by Kaner and Skobov,[4] the Landau damping of the wave with a discrete spectrum is less than unity only in the vicinity of α_n:

$$\Gamma=(2\pi\alpha_n)^{1/2}\sin^2(kR-\alpha_n), \tag{4}$$

Therefore, the condition (3) is not satisfied for normal metals.

It is shown in the present work that in a metallic ferromagnet, because of the effect of the magnetic subsystem, the damping of the electromagnetic oscillations is decreased, and a spectrum of such oscillations and polarizations are obtained.

DISPERSION EQUATION. SPECTRUM AND POLARIZATION

For a description of the propagation of electromagnetic waves in an unbounded isotropic metallic ferromagnet, we use the set of Maxwell's equations and the Landau–Lifshitz equation:

$$[\mathbf{k}\times\mathbf{h}] = 4\pi c^{-1}\mathbf{j},\ \ [\mathbf{k}\times\mathbf{E}] = i\omega c^{-1}\mathbf{b}, \tag{5}$$

$$-i\omega\mathbf{m}/\gamma=[\mathbf{m}\times\mathbf{H}] + [\mathbf{M}\times\mathbf{h}]\ . \tag{6}$$

Here $\mathbf{E}$ is the alternating electric field, $\mathbf{m}$, $\mathbf{h}$, $\mathbf{b}$ are the high-frequency parts of the magnetization, magnetic field and induction, respectively ($\mathbf{m}, \mathbf{h}, \mathbf{b}, \mathbf{E}\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, $\mathbf{H}$ is the external constant magnetic field, $\mathbf{M}$ the saturation magnetization, $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$, $\mathbf{b}=\mathbf{h}+4\pi\mathbf{m}$, $\mathbf{j}$ is the current density, $\mathbf{k}$ is the wave vector, γ is the gyromagnetic ratio. In the Maxwell equations, we neglect the displacement current, and it follows from (5) that

$$\mathbf{kj}=0, \tag{7}$$

which is similar to the condition of electric quasineutrality of the metal.

We solve the set of equations (5), (6) under the following conditions[4]:

$$\nu\ll\omega\ll k_z v\ll\Omega\ll kv, \tag{8}$$

where ν is the collision frequency of the electron. Here

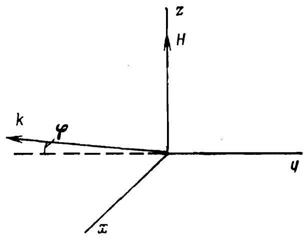

FIG. 2. System of coordinates. The z axis is perpendicular to the plane passing through the magnetic field vector H and the wave vector k. The angle $\varphi \ll 1$.

the first inequality determines ω, the second corresponds to the assumption on strong spatial dispersion in the direction of B. We assume the next inequality in order to "tune out" the cyclotron absorption. Actually, if (8) is valid, then Eq. (1) is satisfied at $N=0$. The next inequality in (8) has already been put in the form $kR \gg 1 (v/\Omega = R)$.

We choose the set of coordinates xyz as shown in Fig. 2. The x axis is perpendicular to the plane passing through **H** and **k**. Since the ferromagnet is assumed to be isotropic, the direction of the induction vector **B** is identical with that of **H**. It follows from (8) for the angle φ that $\varphi \ll 1$. Eliminating the electric field component (7) longitudinal with respect to **k** with the help of Eq. (7), we reduce Eqs. (5) and (6) by the well-known method[3,4] to a system of linear homogeneous equations for the transverse components of **E**. Setting the determinant of this system equal to zero gives us the dispersion equation of the electromagnetic wave

$$4\pi\frac{\omega^2}{k^2c^2}(\omega^2-\omega_B^2)(\sigma_{xx}\sigma_{zz}+\sigma_{xz}^2)+i\omega(\omega^2-\omega_s^2)\sigma_{xx}$$
$$+i\omega(\omega^2-\omega_B^2)\sigma_{zz}+2\varphi\omega^2\omega_M\sigma_{xz}-\frac{k^2c^2}{4\pi}(\omega^2-\omega_s^2)=0. \qquad (9)$$

Here

$$\omega_s=\gamma(HB)^{1/2}, \quad \omega_B=\gamma B, \quad \omega_M=4\pi\gamma M, \quad \sigma_{\alpha\beta}=\sigma_{\alpha\beta}(\omega, \mathbf{k}, \mathbf{H})$$

is the component of the conductivity tensor ($j_\alpha = \sigma_{\alpha\beta}E_\beta$).

As $\gamma \to 0$, we get from (9) the dispersion equation for the normal metal.[4] If we transform in (9) to the limit $\sigma_{\alpha\beta} \to 0$ (ferromagnetic dielectric), then in the case $k^2 > 0$ we obtain the dispersion law of the magnetostatic wave that is transverse relative to H (see, for example,[7])

$$\omega=\gamma(HB)^{1/2}. \qquad (10)$$

For the components of the conductivity tensor, we use the asymptotic expressions obtained earlier[5]:

$$\sigma_{xx}=\sigma_0\{1-\sin(2kR-\rho^2)$$
$$+i\sqrt{2}[\sin\theta S(\rho^2)-\cos\theta C(\rho^2)]\},$$
$$\sigma_{xz}=-\sigma_{zx}=\sigma_0(\pi kR)^{-1/2}\{\cos(2kR-\pi/4)$$
$$+(2\pi)^{1/2}\rho[\cos\theta S(\rho^2)+\sin\theta C(\rho^2)]$$
$$+i\pi^{1/2}\rho\cos(2kR-\rho^2)\},$$
$$\sigma_{zz}=-\sigma_0 i\rho/(kR)^{1/2}. \qquad (11)$$

Here $S(\rho)^2$ and $C(\rho)^2$ are the Fresnel integrals,[8] $\theta = 2kR - \pi/4 - \rho^2$, $\sigma_0 = 3n_0e^2/2mk^2Rv\varphi$, n_0 is the density of conduction electrons. The dispersion law of the electrons is assumed to be quadratic and isotropic. In Eqs. (11), we have used the notation $\rho = (kR)^{1/2}\omega/k_z v$. It follows from (8) that

$$\rho \ll (kR)^{1/2}. \qquad (12)$$

The real part of σ_{xx} describes the absorption of the wave by the electrons, due to Landau damping.[2)] Equation (2) follows from the condition Re $\sigma_{xx} = 0$, with accuracy to ρ^2/kR. Setting $kR = \alpha_n$, and substituting (11) in (9), we get the following equation after elementary transformations:

$$A_1b^4-A_2b^3-A_3b^2+A_4b-A_5=0, \qquad (13)$$

where

$$A_1=\delta^2-\rho^2, \quad A_2=\mu\delta^2, \quad A_3=2\sqrt{\pi}\alpha_n^{-4}\rho(\delta^2-\rho^2)(\rho+\alpha_n^{1/2}P),$$
$$A_4=2\delta\mu\alpha_n^{-4}\rho[(\pi\alpha_n)^{1/2}\delta P+2\varphi\rho(\cos\beta+\rho\sqrt{\pi}Q)],$$
$$A_5=4\rho^2\alpha_n^{-8}(\delta^2-\rho^2)[(\cos\beta+\rho\pi^{1/2}Q)^2-\pi\alpha_n^{1/2}\rho P],$$
$$b=B/G, \quad \mu=4\pi M/G, \quad G=v(3\pi^{1/2}n_0m)^{1/2},$$
$$\delta=\gamma mc/e\varphi\alpha_n^{1/2}, \quad P=C(\rho^2)-S(\rho^2),$$
$$Q=C(\rho^2)+S(\rho^2), \quad \beta=\pi/4+\rho^2,$$

b and μ are the normalized induction and the magnetization, respectively.

The numerical solution $b(\rho)$ of this equation is shown in Fig. 3 by the solid curves. In the calculations, we used $4\pi M = 22$ kG, $n_0 = 10^{23}$ cm^{-3}, $\gamma = 2\times10^7$ G^{-1}-sec^{-1}, $\varphi = 10^{-2}$. The effective mass of the conduction electron was assumed to be equal to the mass of the free electron. With these values of the physical parameters, the normalized magnetization was $\mu = 0.6\times10^{-2}$ and the dimensionless quantity $\delta = 115/\alpha_n^{1/2} \gg 1$. In the case of a normal metal, the equation that is analogous to (13) is biquadratic. The solution of this equation obtained earlier[5] is

$$b_\pm^2(\rho)=\rho\pi^{1/2}\alpha_n^{-4}[\rho+P\alpha_n^{1/2}\pm(L^2+F^2)^{1/2}], \qquad (14)$$

where $L = \rho - P\alpha_n^{1/2}$, $F = 2(\pi^{-1/2}\cos\beta + \rho Q)$. The dependence of $b_\pm(\rho)$ for $n=5$ and $n=7$ is shown in Fig. 3 by

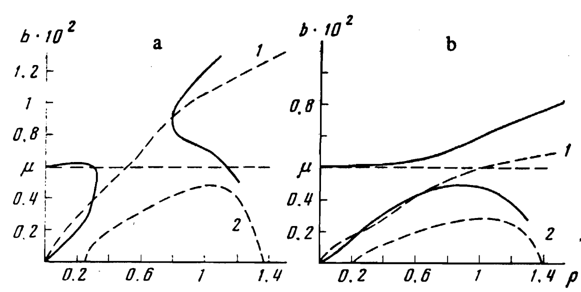

FIG. 3. The solutions $b(\rho)$ of the dispersion equation (13) at fixed α_n are shown by the continuous curves; a) at $n=5$, b) at $n=7$. The dashed lines indicate the spectrum of the electromagnetic waves in a normal metal; the curve 1 corresponds to the upper index in the expression for the spectrum $b_\pm(\rho)$, the curve 2 to the lower index. The dashed line, which intersects the axis of the ordinate at the point μ corresponds to the spectrum b_s of the magnetostatic oscillations in the ferromagnetic dielectric.

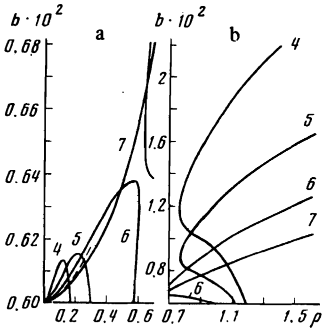

FIG. 4. The spectrum of electromagnetic waves in a metallic ferromagnet. The number of the curve is the same as the value of n for which this curve is constructed. The interval of change of ρ is broken into two parts. On one of them (a), the scale along the ordinate is 20 times greater than on the other (b). The curve shown by the dashed lines is determined by the expression (18).

the dashed lines. The dashed line which nearly coincides with the straight line $b=\mu$ corresponds to Eq. (10), which has the form

$$b_s=\mu\delta^2/(\delta^2-\rho^2). \tag{10a}$$

in the variables ρ and b. As has already been noted, $\delta\gg 1$, and for $\rho\sim 1$, it follows from (10a) that $b_s\approx\mu$.

There are two types of solutions (Fig. 3a and b) of Eq. (13), depending on how close the maximum of the curve b_- is to b_s. Curves of the type shown in Fig. 3a are obtained for a wave with $n<7$; in the case $n\geq 7$, the situation shown in Fig. 3b is characteristic. In both cases, the changes in the spectrum of the electromagnetic wave in the ferromagnetic metal in comparison with the normal metal are most significant near the intersection of b_+ and b_s. In an isotropic ferromagnet, the value of the induction $B<4\pi M$ has no physical meaning; therefore, of all the solutions of Eq. (13), we must choose only those satisfying the inequality $b\geq\mu$. It then turns out that, depending on n, the spectrum of oscillations consists of one or two branches. Figure 4 also shows the solutions for $n=4-7$. At $\rho\leq 0.7$, (Fig. 4a), the scale along the b axis is increased by a factor of 20 for visualization.

We now make clear the character of the polarization of the electromagnetic field in waves with a discrete spectrum. From Maxwell's equations, we get the connection between E_x and E_z:

$$E_z=-iDE_x. \tag{15}$$

The coefficient of polarization D here can be obtained only in the form of a complicated function of ρ and b; therefore, the analytic expression for D will not be written down. The dependence of D on ρ is shown in Fig. 5. It is seen that in the case in which the spectrum of the electromagnetic oscillations consists of two branches, one of which has alternating polarization. This branch, as is seen from Fig. 3a, tends toward the curve of b_+ at values of $b>\mu$, for which the sign of the coefficient D is positive, and at $b\approx\mu$, it tends toward the curve b_-, which has the polarization that is opposite in sign to b_+.[5] Upon decrease of ρ the quantity D increases rapidly. In a normal metal, the polarization of the wave at small ρ is close to circular.[4]

ADDITIONAL CONDITION. SPECTRUM, DAMPING

If we take in place of (12) the condition $\rho\ll 1$, which does not follow from (8), then the expression (11) for the conductivity is simplified. As a result of the expansion of $\sigma_{\alpha\beta}$ in the vicinity of the point $\rho=0$ the results of Kaner and Skobov are obtained for the conductivity.[4] In this case, the dispersion equation (9) is brought to the simple form

$$\begin{gathered}[\cos^2\psi+a(\chi\cos\psi+(\pi/4kR)^{1/2})]\rho^2+i(\pi/4)^{1/2}a\chi\lambda\rho-a\chi/4kR=0,\\ \chi=H/B,\ \ \lambda=1-\sin(2kR),\ \ \psi=2kR-\pi/4,\ \ a=(k/R)^{9/2}b^2.\end{gathered} \tag{16}$$

In the case $kR=\alpha_n$, the quantity $\lambda=0$, and it is easy to obtain the following dispersion law for the electromagnetic wave from (16):

$$\omega_n=2^{-1/2}\Omega\varphi a_n\chi^{1/2}[1+a_n(\chi\sqrt{2}+(\pi/\alpha_n)^{1/2})]^{-1/2}, \tag{17}$$

where $a_n\equiv a$ at $kR=\alpha_n$. If $a_n\ll 1$, then ω_n is identical with the expression obtained by Chudnovskiĭ[6] in the case $\omega\ll\nu$, i.e., in a ferromagnet as also in a normal metal, the spectrum of such a wave does not depend on the relation between ω and ν.[4]

In the more interesting case $a_n\gg 1$ (strong field b) at $\chi\ll\alpha_n^{-1/2}$, the expression (18) becomes

$$\omega_n=(4\pi)^{-1/4}\alpha_n^{5/2}\varphi b\frac{e}{mc}\sqrt{HB}. \tag{18}$$

This expression does not obey the Chudnovskiĭ formula[6] $\omega_n=\omega_{0n}\sqrt{\chi}$, where ω_{0n} is one of the expressions for the spectrum of the wave in the normal metal. The dependence $b(\rho)$ at $n=7$, which follows from (18), is shown in Fig. 4a by the dashed lines. In the case of a large external field $H(\chi\gg\alpha_n^{-1/2})$ the expression (17) goes over

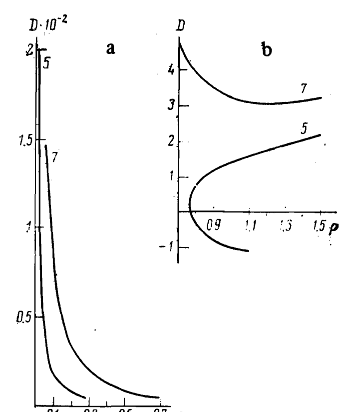

FIG. 5. The dependence on ρ of the coefficient of polarization of the electric field in the electromagnetic wave. The number of the curve is the same as the value of n for which this curve is constructed. At $\rho<0.7$(a) and at $\rho\geq 0.7$ (b), the scales along the ordinate axes is chosen differently.

into the expression for the spectrum of the wave in a normal metal.

At $kR \neq \alpha_n$ the electromagnetic wave is damped. For a wave with the dispersion law (18), we get from the solution of Eq. (16), assuming the relative damping $\Gamma = -\mathrm{Im}\,\rho/\mathrm{Re}\,\rho$ to be small,

$$\Gamma = (\pi/4)^{1/4}(kR)^{-3/2}b^{-1}\lambda\chi^{1/2}. \tag{19}$$

It is seen that at small χ, i.e., $H \ll B$, the damping can be small at any kR. Substituting the value $\Gamma = \Gamma_{max}$ corresponding to the maximum value of λ in Eq. (3), we estimate the magnetic field at which excitation of an electromagnetic wave with a discrete spectrum through the skin layer becomes possible. It follows from (3) that

$$\chi < \alpha_n^3 b^2 / 2\pi^{1/2} n^2. \tag{20}$$

For a wave with $n = 7$, we get $H < 50$ G.

CONCLUSION

Thus, in a metallic ferromagnet, there exists an electromagnetic wave the maximum damping of which is less than unity at arbitrary kR. The spectrum of such a wave is discrete upon satisfaction of the condition $1 > \Gamma_{max} \gg \Gamma'$, where Γ' is the damping not connected with the Landau mechanism. At $1 \gg \Gamma_{max} \approx \Gamma'$. the spectrum of the wave becomes complicated.

The decrease in the attenuation of the electromagnetic wave at small H is due to its polarizations. The absorption of energy of the wave by an electron, connected with the Landau damping, takes place only in the case of motion of the electron in the x direction. If the electric field in the wave in this direction is small, then the absorption of energy of the wave becomes small. It follows from Fig. 5a that at $\rho \ll 1$ the polarization coefficient D can reach values of several orders of magnitude. This means that the transverse (relative to $\mathbf{k}$) part of the electric field of the wave is basically directed along the z axis. The results obtained for the polarization are easily understood if we turn to the expression (18). We can draw the conclusion from the dependence $\omega_n(H)$ that at small H ($\rho \ll 1$), the electromagnetic wave is close in its character to the magnetostatic one. Since the magnetic field $\mathbf{h}$ in the magnetostatic wave is directed along the x axis, then we can assert that the magnetic field is polarized in the direction of the x axis and in the electromagnetic wave (18). Consequently, the electric field in the wave is principally directed along the z axis.

We have identified throughout the oscillations considered in the work as electromagnetic. Such a conclusion follows from Eq. (13). The gyromagnetic ratio enters into this equation only through the parameter δ. As already noted, $\delta \gg 1$; therefore, Eq. (13) can be divided by δ with great accuracy and γ can be eliminated from the dispersion equation at the same time.

[1)] É. A. Kaner called the attention of the authors to this fact.
[2)] However, we must now keep it in mind that there are two mechanisms that lead to Landau damping—ordinary (motion in an electric field) and "magnetic" (motion in a magnetic field).

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