Investigation of the transparency of the wave barrier to electron plasma waves in a magnetoactive plasma

V. V. Lisitchenko, L. I. Romanyuk, and V. V. Ustalov

Nuclear Research Institute, Ukrainian Academy of Sciences
(Submitted December 28, 1975)
Zh. Eksp. Teor. Fiz. 71, 537–546 (August 1976)

Theoretical estimates are presented for the transparency coefficient of the wave barrier to electron plasma waves in a magnetized plasma. Experimental plots of the spatial damping of the wave and of the transparency coefficient of the barrier against the density of the surrounding plasma and the frequency of excited oscillations are obtained for various barrier widths. The experimental plots are compared with the theory. Satisfactory agreement is demonstrated.

PACS numbers: 52.35.–g

The theoretically predicted effect of bleaching of wave barriers[1–3] was observed experimentally and investigated[4–6] for waves with dispersion $\omega=\omega_{pe}\cos\theta$ (ω_{pe} is the plasma electron frequency in the plasma surrounding the barrier and θ is the angle between the wave vector $\mathbf{k}$ and the direction of the magnetic field). The main result of our studies was a qualitative confirmation of the physical picture proposed by Oraevskiĭ and one of us[1] for the effect and consisting in the following. For certain types of waves (e.g., electron Langmuir waves), the plasma regions in which the local plasma frequency is higher than the wave frequency are opaque. A wave incident on such a region is reflected at a point where the refractive index vanishes, penetrating a certain distance into the opacity region. The hydrodynamic theory predicts that the depth of penetration of the field is of the order of the Debye radius of the plasma. However, the wave incident on the opacity region is coupled to a current produced in the pre-barrier plasma, and this current can be subdivided into a current of thermal particles and a current of resonant particles. These currents, penetrating the opacity region, vanish as a result of the loss of phase coherence of the particles, but the depth of their penetration can greatly exceed the Debye radius. Since the current of the resonant particle is practically monoenergetic, its loss of phase is weak and it makes the main contribution at large distances from the boundary. Contributing to the phase loss length is the acceleration of the particle by the potential drop across the boundary. If the opacity region has finite dimensions (wave barrier), then the resonant-particle current penetrating behind the barrier into the plasma excites a wave, thereby increasing effectively the transparency of the barrier. From the fundamental and practical points of view it is very important to establish the character of the dependence of the transparency of the wave barrier on the parameters of the plasma system and on the wave propagating in it. The investigations described in the present article were aimed at solving this problem.

An expression for the coefficient of transmission of the Langmuir waves through a density barrier was obtained in[1] for the one-dimensional case:

$$K=\frac{U_{\rm III}}{U_{\rm I}}\approx\frac{\gamma}{k_\parallel}\exp\left[-\gamma\Delta\Big/\left(1+\frac{e\varphi}{T_e}\frac{v_{Te}^2k_\parallel^2}{\omega^2}\right)^{1/2}\right], \qquad (1)$$

where $U_{\rm III}$ and $U_{\rm I}$ are the amplitudes of the wave "passing" through the barrier and the incident wave, γ, $k_\parallel$, and ω are the spatial damping, the wave vector, and the frequency of the propagating wave, Δ and φ are the width of the barrier and the potential drop across it, while v_{Te} and T_e are the thermal velocity of the electrons and their temperature (in electron volts) in the plasma. Expression (1) was obtained under the assumption that

$$\frac{e\varphi}{T_e}\frac{v_{Te}^2k_\parallel^2}{\omega^2}\leqslant1,\quad \Delta>\frac{v_{Te}}{\omega}\left(1+\frac{e\varphi}{T_e}\right)^{1/2},\ a.$$

(a is the electron Debye radius.) The last condition means that the width of the barrier is sufficient for the already noted vanishing of the thermal-particle current inside the barrier. Thus, expression (1) determines the wave-barrier transparency due to the resonant-electron current.

Under the conditions of the experiments[4–6] there were excited in the plasma waves with dispersion $\omega=\omega_{pe}\cos\theta$, but the criterion for the applicability of the hydrodynamic description was satisfied in a direction transverse to the magnetic field $k_\perp v_{Te}/\omega_{He}\ll1$ (ω_{He} is the cyclotron electron frequency and $k_\perp$ is the wave-vector component perpendicular to the magnetic field). Owing to the latter, the physical picture of the passage of such waves through the barrier has in essence a one-dimensional character. Therefore expression (1) should describe in principle also the transparency of the wave barrier for waves with dispersion $\omega=\omega_{pe}\cos\theta$, if $k_\parallel$ is the component of the wave vector along the magnetic-field direction.

To find the dependence of the transparency coefficient of the wave barrier on the plasma-system parameters and on the wave propagating in it, it is necessary to determine the corresponding relations for γ and $k_\parallel$ from the dispersion equation for the longitudinal electron waves in a magnetoactive plasma. This equation takes the form (see, e.g.,[7])

$$1+a^2k^2-e^{-\beta}\sum_{n=-\infty}^{\infty}\frac{z_0}{z}I_n(\beta)\left[\varphi(z_n)-i\sqrt{\pi}\,z_n\exp(-z_n^2)\right]=0, \qquad (2)$$

where $\beta=k_\perp^2v_{Te}^2/2\omega_{He}^2$, $v_{Te}=(2T_e/m_e)^{1/2}$, $I_n(\beta)$ is a modified Bessel function,

$$z_n=\frac{\omega-n\omega_{He}}{|k_\parallel|v_{Te}},\quad \varphi(z)=\frac{z}{\pi^{1/2}}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{z-y}dy.$$

Under the experimental conditions[5,6] the inequalities $\beta\ll 1$, $\omega\ll\omega_{He}$, $|z_n|\gg z_0=\omega/|k_\parallel|v_{Te}\gg 1$, and $n\neq 0$ were usually satisfied, so that in (2) we can expand $I_n(\beta)$ and $e^{-\beta}$ in powers of β, and also use the asymptotic expansion for $\varphi(z_n)$. As a result, equation (2) takes the form[7]

$$a^2k^2-\left(\frac{1}{2z_0^2}+\frac{3}{4z_0^4}+\ldots\right)-\beta\left[\frac{z_0}{2}\left(\frac{1}{z_1}+\frac{1}{z_{-1}}+\frac{1}{2z_1^3}+\frac{1}{2z_{-1}^3}\right)-1-\frac{1}{2z_0^2}\right]$$
$$-\beta^2\left[\frac{z_0}{8}\left(\frac{1}{z_2}+\frac{1}{z_{-2}}-\frac{4}{z_1}-\frac{4}{z_{-1}}\right)+\frac{3}{4}\right]+\ldots+i\pi^{1/2}z_0\left[\left(1-\beta+\frac{3}{4}\beta^2\right)\right.$$
$$\times\exp\{-z_0^2\}+\frac{\beta}{2}(1-\beta)(\exp\{-z_1^2\}+\exp\{-z_{-1}^2\})$$
$$\left.+\frac{\beta^2}{8}(\exp\{-z_2^2\}+\exp\{-z_{-2}^2\})+\ldots\right]=0. \tag{3}$$

Retaining the principal terms in the expansion, we arrive at the equation

$$a^2k^2-1/2z_0^2+\beta+i\pi^{1/2}z_0\exp\{-z_0^2\}=0. \tag{4}$$

Under the condition $k_\parallel^2\gg\omega^2k_\perp^2/\omega_{He}^2$, which is also satisfied in the experiment, Eq. (4) can be written in the form

$$\left(1-\frac{\omega_{pe}^2}{\omega^2}\right)k_\parallel^2+k_\perp^2+2i\pi^{1/2}\frac{\omega_{pe}^2}{v_{Te}^2}\frac{\omega}{k_\parallel v_{Te}}\exp\left\{-\frac{\omega^2}{k_\parallel^2v_{Te}^2}\right\}=0. \tag{5}$$

Hence, neglecting the imaginary part of the equation, we obtain an expression for $k_\parallel$:

$$k_\parallel=k_\perp(\omega_{pe}^2/\omega^2-1)^{-1/2}. \tag{6}$$

The expression for the decrement γ (the imaginary part of the wave vector) can be obtained for $\gamma\ll k_\parallel$ by successive approximations. As a result we get

$$\gamma=\omega\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{m_e}{T_e}\right)^{1/2}\frac{\omega_{pe}^2}{k_\perp^2}\exp\left(-\frac{\omega^2}{k_\parallel^2v_{Te}^2}\right). \tag{7}$$

Formula (1) jointly with (6) and (7) makes it possible to estimate the transparency coefficient of the barrier for waves with dispersion $\omega=\omega_{pe}\cos\theta$ and to determine the theoretical dependence of the transparency on the parameters of the plasma system and of the wave propagating in it. It is easily seen that these parameters, which determine the quantity K, are the following: the density n of the plasma surrounding the barrier, the temperature of the electronic component of the plasma, the width of the barrier, the potential drop across the barrier, the frequency of the wave, and the wave-vector component perpendicular to the magnetic-field direction. All these quantities can be varied in principle in a controlled fashion during the course of the experiment, and consequently it is possible to verify experimentally the dependences of the transparency coefficients of the wave barrier on the parameters listed above as predicted by expressions (1), (6), and (7).

We have verified experimentally the dependences of the transparency coefficient of the wave barrier on the density of the plasma surrounding the barrier, on its width, and on the frequency of the propagating wave.

The dependence of the transparency of the wave barrier on the voltage drop across the barrier was specially investigated earlier,[8] where it was shown that the functional dependence $K(\varphi)$ that follows from (1) is indeed realized in experiment. As to the functions $K(k_\perp)$ and $K(T_e)$, it was difficult to determine them in experiment because of all possible variations of the discharge parameters in the employed apparatus the quantity $k_\perp$ was practically constant at approximately 1.5 cm^{-1},[5] and the electron temperature also varied in a narrow range near $T_e=4$ eV.

The experiments were performed with the setup illustrated schematically in Fig. 1a. To produce the wave barrier we used the symmetrical escape of a plasma along the magnetic field from a Penning discharge produced in chamber II with incandescent cathodes C in vacuum chambers I and III, across an opening of 20 mm diameter in reflectors R_1 and R_2. The discharge anode A and the vacuum chambers I and III were copper cylinders of 100 mm diameter. The discharge was produced in argon at a pressure $p=2\times10^{-4}$ mm Hg (the pressure in the vacuum chamber was lower by one order of magnitude), the discharge voltage was $V_a=100$ V, and the magnetic field intensity was $H=70$ Oe. The electron plasma wave was excited by applying a high-frequency monochromatic signal with amplitude $U_g=1.5$ V on a vibrator probe V located in chamber I.

With the aid of a system of moving flat Langmuir (1, 2) and high-frequency (3, 4) probes we measured directly in the experiment the axial profiles of the plasma density, of the floating-probe potential, and of the HF oscillation intensity. Typical profiles of this type are shown in Fig. 1b (curves 1, 2, and 3, respectively). The radial profile of the density in the plasma surrounding the barrier, as shown by the measurements, is a curve with a broad maximum located on the system axis and with almost flat top, the dimensions of which are 40–50 mm. The wave excited in the pre-barrier plasma constitutes, as a result of reflection from the barrier and the spatial damping, a superposition of a standing wave and a traveling wave. The reflection of the wave from the barrier takes place in the vacuum-cham-

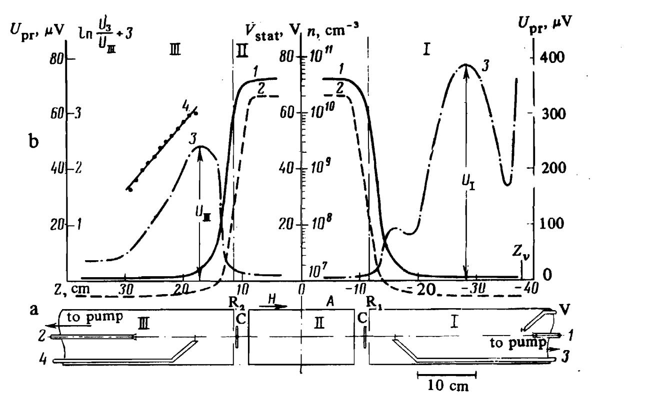

FIG. 1. Diagram of setup (Fig. a): b—axial distributions of the plasma density n (curve 1), of the floating probe potential V_{stat} (curves 2), of the amplitude of the HF oscillations U_{pr} in a linear (curve 3) and logarithmic (line 4) scales; $I_a=0.25$ A, $V_a=90$ V, $f=9$ MHz.

ber region I at a distance ~2–3 cm from the reflector (see Fig. 1). The beam of electrons trapped by the wave in the pre-barrier plasma and collimated by holes in the reflectors[6] passes through the barrier and excites a damped traveling wave in the plasma behind the barrier. The experimental setup, the measurement procedure, the axial profiles of the potential and of the plasma density, as well as the spatial characteristics of the investigated waves are described in greater detail in[5].

The transparency coefficient of the wave barrier was determined from the ratio of the maximum amplitudes of the excited HF oscillations in the plasma in front and behind the barrier ($K = U_{III}/U_{I}$, see Fig. 1). The spatial damping of the wave was estimated from the decrease of its intensity along the system axis in the plasma behind the barrier with increasing distance from the barrier. The axial dependences of the oscillation amplitude, obtained in a wide range of the frequencies of the excited waves and densities of the plasma surrounding the barrier, turned out to be close to exponential; the plots of $\ln(U_{pr}/U_{III})$ are well approximated by straight lines (see, e.g., plots 3 and 4 in Fig. 1). From the slopes of these lines we determined the effective spatial damping of the wave.

An investigation of the transparency of the wave barrier was carried out at 5 barrier widths (125, 185, 250, 300, 375 mm) in a propagating-wave frequency range $f = 3$–30 MHz and in a range of the plasma density surrounding the barrier $n = 2\times10^6$–1×10^8 cm^{-3} (the plasma density in the barrier ranged in this case from $n = 1\times10^{10}$ cm^{-3} to $n = 1.2\times10^{11}$ cm^{-3}). The width of the wave barrier and the density of the plasma surrounding it were varied by changing respectively the length of the discharge gap (the length of the anode) and the value of the discharge current ($I_a = 0.05$–0.8 A). The potential drop across the barrier and the electron temperature in the system were changed insignificantly thereby and amounted to $\varphi \approx 80$ V and $T_e \approx 4$ eV.

The wave amplitude U in the plasma ahead of the barrier, at a constant value of the signal applied to the vibrator probe, changed with changing wave frequency and with changing plasma density. However, it always satisfied the condition

$$U \ll 4\pi^2 \frac{m_e}{e} \frac{\omega^2\gamma^2}{k_\parallel^4}\left(1+\frac{k_\parallel^2}{k_\perp^2}\right)^{-2}, \tag{8}$$

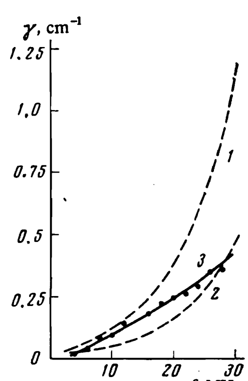

FIG. 2. Dependence of the spatial damping of the wave on its frequency 1—calculation for $n = 3\times10^7$ cm^{-3}; 2—calculation for $n = 4\times10^7$ cm^{-3}; 3—experiment.

which was obtained with allowance for (6) from the criterion for the applicability of the linear theory[9]

$$\gamma_t \gg \frac{k_\parallel}{2\pi}\left(\frac{eU}{m_e}\right)^{1/2}, \tag{9}$$

where γ_t is the time decrement of the wave and $k_\parallel$ and γ are determined by expressions (6) and (7). The criterion was all the more satisfied for waves in the plasma behind the barrier ($U_{III} < U_I$, see Fig. 1). The values of γ obtained from the axial profiles of the wave intensity in this plasma also satisfied relation (8).

It was of interest to compare the experimental dependences of the spatial damping of the wave on its frequency and on the density of the plasma surrounding barrier with the functional relations that follow from (7). Figure 2 shows plots of $\gamma(f)$ calculated in accordance with (7) for $n = 3\times10^7$ cm^{-3} (curve 1) and $n = 4\times10^7$ cm^{-3} (curve 2), and also the experimentally dependence (curve 3) for $n \approx 3\times10^7$ cm^{-3}. It can be seen that the plots are quite similar. At frequencies $f < 15$ MHz, the values of γ calculated for $n = 3\times10^7$ cm^{-3} and those determined from experiment differ from each other by not more than 30%. The difference between them becomes noticeable at $f > 15$ MHz and increases with increasing wave frequency; for $f = 30$ MHz, calculation yields a value of γ which is 2.5 times larger than that observed in experiment.

We can indicate at least two causes of the above-noted difference between the calculated and experimental values of γ. First, the expression for the spatial wave decrement (7) was obtained under the assumption that $\gamma/k_\parallel \ll 1$. For $n = 3\times10^7$ cm^{-3}, calculation in accordance with (7) yields values of γ that must be regarded as approximate, since they satisfy only the condition $\gamma/k_\parallel < 1$, with the ratio $\gamma/k_\parallel$ increasing with increasing frequency and becoming close to unity at $f = 30$ MHz. Second, in the plasma surrounding the barrier there are small density gradients directed towards the barrier. At low frequencies the wave in the plasma behind the barrier is registered over the entire length of this plasma. With increasing frequency, the damping increases and the region in which a wave with noticeable amplitude exists becomes shorter and contracts towards the barrier. Thus, the average plasma density, which determines the value of γ, increases somewhat with increasing wave frequency. It is therefore probable that in the frequency region $f < 15$ MHz the experimental points (curve 3) agree better with the results of the calculation for $n = 3\times10^7$ cm^{-3} (curve 1), and at $f > 15$ MHz they agree better with the results of calculation for $n = 4\times10^7$ cm^{-3} (curve 2).

Figure 3 shows results of the calculation (curve 1) and of the experiment (curve 2) for the $\gamma(n)$ dependence at $f = 9$ MHz. Here too, good qualitative agreement is observed between the plots, and the difference between the values of γ does not exceed those indicated above and is largest in the region of the maximum of $\gamma(n)$, where $\gamma/k_\parallel \approx 0.5$–$0.75$. Attention is called also to the fact that in the density region $n > 4\times10^7$ cm^{-3}, i.e., at resonant-electron velocities $v_e^* = \omega/k_\parallel > 2.4\times10^8$ cm/sec, the experimental values of γ are larger than the calcu-

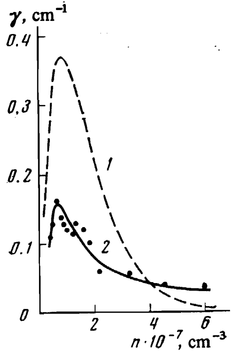

FIG. 3. Dependence of the spatial damping of the wave on the density of the plasma surrounding the barrier: 1—calculation; 2—experiment.

lated ones. The possible cause is the deviation of the electron velocity distribution function from Maxwellian in this range of velocities. Investigations of the distribution function of the electrons with respect to the velocity component longitudinal relative to the magnetic field in the plasma of a Penning discharge with an incandescent cathode[10] have shown that such a deviation does take place.

When account was taken of the possible errors in the calculation and in the experiment, the aggregate of the described facts, namely: the correctness of criterion (8) under the experimental conditions, the exponential decrease of the amplitude of the wave in the plasma behind the barrier, the similar character of the experimental functions $\gamma(f)$ and $\gamma(n)$ and those calculated from (7), as well as the proximity of the experimentally realized values of γ to the calculated ones—all allow us to conclude that the observed spatial damping of the wave is predominantly collisionless and is satisfactorily described by expression (7). The contribution made to the effective damping of the wave by other factors (e.g., the transverse energy outflow) is apparently insignificant. Indeed, there is little likelihood that the spatial decrease of the wave amplitude due to other factors can be described also by an exponential law, with the functional dependences of the argument of the exponential on f and n the same as obtained for collisionless damping.

The wave-barrier widths obtained in the present experiments exceeded by more than 1000 times the electron Debye radius, with the following relation always satisfied:

$$\Delta > \frac{v_{Te}}{\omega}\left(1+\frac{e\varphi}{T_e}\right)^{1/2}.$$

Thus, all the conditions imposed by the theory on the applicability of expression (1) for the transparency coefficient of a wave barrier, with $k_\parallel$ and γ in the form (6) and (7), respectively, were satisfied in the experiment.

Figure 4a shows the dependences of the transparency coefficient of the wave barrier on the density of the plasma surrounding it, calculated from (1), for three frequencies of the propagating wave. Attention is called to the fact that the theory predicts the following: a) the presence of two maxima on the $K(n)$ plot, b) an abrupt decrease of the transparency at $\omega \gtrsim \omega_{pe}$, c) a smooth decrease of the transparency coefficient past the aforementioned maxima with further increase of the plasma density, d) a displacement of the maxima towards lower plasma densities with decreasing wave frequency.

Figure 4b shows the experimentally obtained plots of $K(n)$ for the same frequencies. The comparison of the data presented in Figs. 4a and 4b show that all the singularities noted above of the $K(n)$ dependence, which follow from the theoretical estimates, are indeed realized in the experiment. It can also be seen that the measured and calculated values of K are close to each other (a substantial difference between the values of K is observed only in the region of the minimum of $K(n)$).

It follows from Fig. 4a that in the plasma-density range $n=(1-4)\times 10^7$ cm^{-3} the barrier transparency should increase substantially with decreasing frequency of the propagating waves (the relation is reversed in denser plasmas). The experimental data shown in Fig. 4b for the three frequencies agree with this conclusion. The results of a more detailed investigation of the function $K(f)$ can be illustrated, by way of example, by the data shown in Fig. 5. It is seen that curve (1) calculated for $n=3\times 10^7$ cm^{-3} is quite close to the experimental curve (2) both with respect to the behavior of the function $K(f)$ and with respect to the absolute values of K. The deviation of the experimental curve from the calculated one at $f>15$ MHz is due, as already noted, to the approximate character of the calculation and to the displacement of the zone of the oscillation excitation in the plasma behind the barrier towards higher densities.

It follows from (1) that, other conditions being equal, the transparency coefficient of the wave barrier should increase with decreasing barrier widths. To check this assumption we measured the functions $K(n)$ at the indicated five widths of the barrier for several frequencies

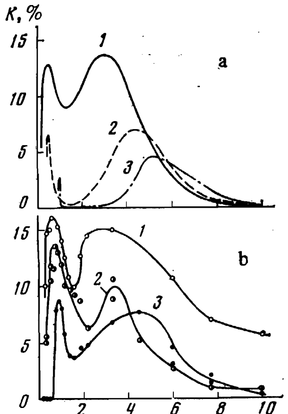

FIG. 4. Dependence of the wave-barrier transparency on the density of the surrounding plasma: a—calculation, b—experiment. Curves 1, 2, and 3—f=9, 18, and 27 MHz.

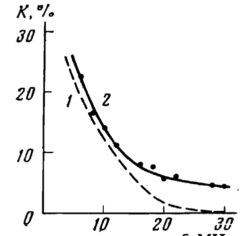

FIG. 5. Dependence of the wave-barrier transparency on the wave frequency, $n=3\times 10^7$ cm^{-3}; 1—calculation, 2—experiment.

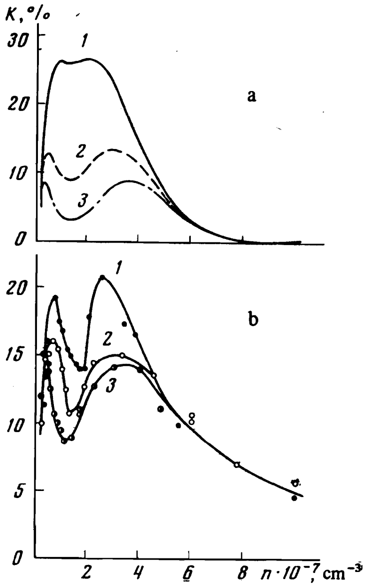

FIG. 6. Dependence of the transparency of wave barriers with different thicknesses on the density of the surrounding plasma, $f=9$ MHz; a—calculation, b—experiment. Curves 1, 2, and 3 are for $\Delta=125$, 250, and 375 mm.

of the exciting waves and used (1), (6), and (7) to calculate $K(n)$ for the same conditions. By way of illustration, Fig. 6 shows the calculated (a) and experimental (b) dependences of $K(n)$ for a wave of frequency $f=9$ MHz for three barrier widths $\Delta=125$, 250, and 375 mm. The main features of the theoretical curves are the following: a) the weak dependence of the transparency of the wave barrier on the width in the region of "large" densities of the plasma surrounding the barrier, when γ are very small (see Fig. 3), b) the growth of the barrier transparency with decreasing barrier width in less dense plasma, and c) the coming together of the maxima with decreasing Δ. It is easily seen that these singularities are possessed also by the experimental curves, the measured transparency coefficients of the wave barrier being close in magnitude to the calculated ones. Some difference between them is observed in the region of the minima of $K(n)$, namely, the depth of the minimum for $f=9$ MHz greatly exceeds the calculated value.

It should be noted that the discrepancy between the experimental and calculated curves, which is usually observed in the region of the minimum of the $K(n)$ dependence (see Figs. 4 and 6) is apparently connected with the already noted approximate character of the calculation of γ. Indeed, in this plasma-density region, calculation of γ and $k_\parallel$ from formulas (7) and (6), obtained in the approximation $\gamma \ll k_\parallel$, yields values of γ and $k_\parallel$ that satisfy only the condition $\gamma < k_\parallel$. The difference between the calculated and experimental values of the transparency coefficient of the wave barrier in the region of large plasma densities, where the longitudinal phase velocity of the wave is large, turned out to be correlated with the difference between the calculated and experimental values of γ at these densities (see Fig. 3). It is probable that it is precisely the increased values of the spatial damping of the wave, which is realized in the plasma surrounding the barrier, which ensure larger values of the transparency coefficient than expected from the theory.

On the basis of the foregoing we arrive at the following conclusion. The bleaching of the wave barrier, for a wave with dispersion $\omega=\omega_{pe}\cos\theta$ at a fixed value of ω, takes place in practice only in a definite range of densities of the surrounding plasma, bounded from below by the condition $\omega=\omega_{pe}$ and from above by the condition $\gamma \to 0$. The maximum wave-barrier transparency is reached at two values of the plasma density, located inside this range. An increase in the frequency of the wave incident on the barrier leads to a decrease of the maximum values of the transparency coefficient of the wave barrier and to an increase of the densities at which they are realized. In the plasma-density region where the transparency coefficient of the wave barrier is extremal, the decrease of the width of the barrier increases its transparency. At large densities of the plasma surrounding the barrier, the influence of the barrier width on this transparency is very small.

These conclusions follow both from theoretical estimates and from the experimental results, which are in perfectly satisfactory agreement with one another. The aggregate of the experimental results previously obtained[4–6, 8] and described in the present paper indicate that the previously developed theory[1] of the bleaching of a wave barrier for electron Langmuir waves describes satisfactorily also the bleaching of the wave vector for electron plasma waves in a magnetoactive plasma. This pertains both to the mechanism whereby the perturbation is transported through the barrier and to the order of magnitude and the functional dependences of the transparency of the barrier on the parameters of the plasma system and of the wave propagating in it.

In conclusion, the authors are grateful to L. L. Pasechnik, A. A. Gurin, N. E. Svavil'nyĭ, and S. N. Reznik for interest in the work, for useful discussions, and for help with the computer calculations.

[1]V. V. Lisitchenko and V. N. Oraevskiĭ, Dokl. Akad. Nauk SSSR 201, 1319 (1971).

[2]A. A. Vodyanitskij, N. S. Erokhin, V. V. Lisitchenko, S. S. Moiseev, and V. N. Oraevskij, Nucl. Fusion 14, 267 (1974).

[3]N. S. Erokhin and S. S. Moiseev, in: Voprosy teorii plazmy (Problems of Plasma Theory), ed. M. A. Leontovich, Atomizdat, 7, 192 (1973).

[4]V. N. Oraevskiĭ, L. I. Romanyuk, N. E. Svavil'nyĭ, and V. V. Ustalov, Pis'ma Zh. Eksp. Teor. Fiz. 17, 288 (1973) [JETP Lett. 17, 207 (1973)].

[5]L. I. Romanyuk, N. E. Svavil'nyĭ, and V. V. Ustalov, Zh. Eksp. Teor. Fiz. 67, 579 (1974) [Sov. Phys. JETP 40, 286 (1975)].

[6]L. I. Romanyuk and V. V. Ustalov, Fiz. Plazmy 1, 504 (1975) [Sov. J. Plasma Phys. 1, 281 (1975)].

[7]A. G. Sitenko, Élektromagnitnye fluktuatsii v plazme (Electromagnetic Fluctuations in c Plasma), KhGu, 1965, p. 78.

[8]L. I. Romanyuk, N. E. Svavilnyi, and V. V. Ustalov, Twelfth Intern. Conf. Phenomena Ioniz. Gases. Eindhoven, 1975, Contrib. pap., North Holland, 1975, p. 308.

[9]V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in Plasma), Nauka, 1967, p. 124. [Pergamon, 1971].

[10]L.I. Romanyuk and N.E. Svavil'nyĭ, Ukr. Fiz. Zh. 21, 981 (1976).

Translated by J. G. Adashko