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# Hydrogenlike system in crossed electric and magnetic fields

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The problem of a hydrogenlike system in crossed electric and magnetic fields is investigated. A quasiclassical calculation is made of the energy of a state in which the wave function is concentrated at a certain distance from the center of the Coulomb well. The nature of the spectrum is ascertained and a criterion for the validity of the obtained results is established. A discussion is given of the conditions under which observation of the investigated states becomes feasible.

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## 1. INTRODUCTION

A hydrogenlike system (hydrogen atom, exciton) on which crossed electric and magnetic fields act is investigated in this article. The center of inertia is fixed. We shall assume that $\mathscr{E} \ll \mathscr{H}$. This assumption allows us to confine our attention to a nonrelativistic treatment. The posed problem is physically equivalent to the problem of a particle moving with velocity $\mathbf{V}$ in a homogeneous magnetic field $\mathscr{H}$, since an electric field

$$\mathscr{E} = \frac{1}{c}[\mathbf{V} \times \mathscr{H}] \tag{1}$$

appears in the coordinate system attached to the particle.

Choosing the symmetric gauge of the vector potential

$$\mathbf{A} = \tfrac{1}{2}[\mathscr{H} \times \mathbf{r}] \tag{2}$$

($\mathbf{r}$ denotes the distance between the positively charged and negatively charged particles), we obtain the Schrödinger equation for the problem under consideration in the form[1]

$$\left\{ -\frac{\hbar^2}{2\mu}\Delta + \frac{ie\hbar}{2\mu c}\gamma\mathscr{H}[\mathbf{r}\times\nabla] + \frac{e^2}{8\mu c^2}[\mathscr{H}\times\mathbf{r}]^2 + e\mathscr{E}\mathbf{r} - \frac{e^2}{r} \right\}\Psi = E\Psi, \tag{3}$$

where $\varepsilon$ is the dielectric constant ($\varepsilon = 1$ for the hydrogen atom),

$$\mu = \frac{m_+ m_-}{m_+ + m_-}, \qquad \gamma = \frac{m_+ - m_-}{m_+ + m_-}, \tag{4}$$

$m_+$ and $m_-$ denote the masses of the positively and negatively charged particles.

Following Gor'kov and one of the authors,[1] let us set

$$\Psi = \Phi \exp\left\{ i\gamma M \frac{c}{2\hbar\mathscr{H}^2}[\mathscr{H}\times\mathscr{E}]\mathbf{r} \right\}, \tag{5}$$

where $M = m_+ + m_-$. Having directed the $z$ axis along the magnetic field, the $y$ axis along the electric field, denoting the radius vector in the $(x, y)$ plane by $\rho$, and displacing the origin of coordinates along the $y$ axis by the amount

$$y_0 = -Mc^2\mathscr{E}/e\mathscr{H}^2, \tag{6}$$

we obtain the following equation for $\Phi$[1]:

$$\left\{ -\frac{\hbar^2}{2\mu}\Delta_\rho - \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + \frac{ie\hbar\gamma}{2\mu c}[\rho\times\nabla_\rho]\mathscr{H} + \frac{e^2}{8\mu c^2}\mathscr{H}^2\rho^2 \right.$$
$$\left. - \frac{e^2}{\varepsilon[x^2+(y+y_0)^2+z^2]^{\frac{1}{2}}} - \frac{Mc^2\mathscr{E}^2}{2\mathscr{H}^2} \right\}\Phi = E\Phi. \tag{7}$$

If we divide both sides of Eq. (7) by the quantity $e^2$, then three parameters having the dimension of a length are formed in the equation: the Bohr radius $a = \hbar^2/\mu e^2$, the magnetic radius $a_H = (\hbar c/e\mathscr{H})^{1/2}$, and the characteristic length $y_0$. Two, independent, dimensionless parameters can be constructed from these quantities

$$\lambda = y_0/a, \qquad \nu = a_H/a. \tag{8}$$

In order to express $\lambda$ and $\nu$ in terms of the field strength in the most convenient way, let us introduce the charac-

teristic field $\mathscr{H}_0$ for which $a_H = a$. The magnitude of this field is given by

$$\mathscr{H}_0 = c\mu^2 e^3/\hbar^3. \tag{9}$$

(For the hydrogen atom $\mathscr{H}_0 = 3 \times 10^9$ Oe.) Then

$$\nu = \left(\frac{\mathscr{H}_0}{\mathscr{H}}\right)^{1/4}, \quad \lambda = \frac{M}{\mu}\frac{\hbar c}{e^2}\frac{\mathscr{E}}{\mathscr{H}}\frac{\mathscr{H}}{\mathscr{H}_0}. \tag{10}$$

The numerical factor $M\hbar c/\mu e^2$ is approximately equal to $2.4 \times 10^5$ for the hydrogen atom.

The energy spectrum and the wave functions of the relative motion are determined by the values of the parameters $\lambda$ and $\nu$.

The case $\lambda \ll 1$, $\nu \ll 1$ was investigated earlier[1] for an exciton. In the present article we shall investigate $\lambda \gg 1$. As to the values of $\nu$, we shall formulate specific conditions below. We note beforehand that the most interesting physical effects arise for values of $\nu$ not exceeding unity by much or smaller than unity.

For large $\lambda$ the length $y_0$ has a simple and intuitive meaning; in order to clarify this meaning let us consider the expression

$$U = \frac{e^2 \mathscr{H}^2}{8\mu c^2}y^2 - \frac{e^2}{\varepsilon|y+y_0|} - \frac{Mc^2\mathscr{E}^2}{2\mathscr{H}^2}. \tag{11}$$

Somewhat arbitrarily it can be called the potential energy along the $y$ axis. The point at which the derivative $dU/dy$ vanishes is found from the cubic equation

$$\left(\frac{y}{y_0}\right)\left(\frac{y}{y_0}+1\right)^2 + \frac{4}{\varepsilon}\frac{\nu^4}{\lambda^3} = 0. \tag{12}$$

In that case, when $\lambda > 3\nu^{4/3}/\varepsilon^{1/3}$, the curve $U(y)$ has the form shown in Fig. 1. If $\lambda$ satisfies the stronger inequality

$$\lambda \gg 4\nu^{4/3}/\varepsilon^{1/3}, \tag{13}$$

then, as is clear from Eq. (12), one can assume that the bottom of the well is located at $y = 0$ without making a large error. Thus, $y_0$ represents (under condition (13)) the distance from the Coulomb center to the bottom of the well.

The figure suggests that states of two types exist in the case under consideration: one type in which the wave function is concentrated in a Coulomb well near $y = -y_0$, and the other type in which it is concentrated in a well near $y = 0$. The possibility of the existence of such states was indicated as long ago as 1969.[2] We shall investigate those states in which the electron is localized in the potential well near $y = 0$.

In that case when the bottom of the band drops down lower than $\hbar^2/2\mu a^2\varepsilon^2$, the ground state of the atom will be of precisely such form. This corresponds, to within small correction terms, to the following relationship between the parameters:

$$\lambda > \nu^2\left(\frac{M}{\mu}\right)^{1/3}\frac{1}{\varepsilon}. \tag{14}$$
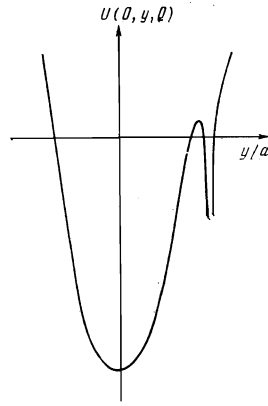


FIG. 1. Form of the potential energy $U(0, y, 0)$ for the hydrogen atom. ($a$ denotes the radius of the Bohr orbit).

Below we shall assume that condition (14) is satisfied. In this connection the hydrogen atom acquires unusual properties. It turns out to have a large dipole moment (in the ground state) and its energy spectrum is radically modified.

## 2. THE ENERGY SPECTRUM

One can utilize the adiabatic approximation[1,3] for the values of the parameters under consideration, i.e., one can represent the wave function $\Phi$ in the form

$$\Phi = g(\rho)f(z), \tag{15}$$

where $g$ denotes the wave function of the motion in only the magnetic field without the Coulomb term. Being interested in the lowest energy states, let us take as $g$ the wave function of the ground state in a magnetic field

$$g = (2\pi)^{-1/2}a_H^{-1}\exp\left(-\rho^2/4a_H^2\right).$$

Substituting (15) into (7), multiplying by $g$ and integrating over $d\rho$, we obtain an equation for $f$

$$\frac{d^2f}{dz^2} + \frac{2\mu}{\hbar^2}(E - E_0 - u)f = 0, \tag{16}$$

$$u = -\frac{e^2}{\varepsilon}\int\frac{g^2 d\rho}{[x^2 + (y+y_0)^2 + z^2]^{1/2}}, \tag{17}$$

where $E_0$ denotes the boundary of the continuous spectrum:

$$E_0 = e\hbar\mathscr{H}/2\mu c - Mc^2\mathscr{E}^2/2\mathscr{H}^2. \tag{18}$$

Since $y_0 \gg a_H$, one can take the square-root outside the integral sign at the point $x = y = 0$. Then

$$u = -\frac{e^2}{\varepsilon(z^2+y_0^2)^{1/2}}. \tag{19}$$

Furthermore, for the parameters of interest to us the motion along the $z$ axis is quasiclassical, as one can easily verify. We obtain the following result for the determination of the energy levels:

$$\int_0^{\bar{z}}\left\{2\mu\left[E - E_0 + \frac{e^2}{\varepsilon(z^2+y_0^2)^{1/2}}\right]\right\}^{1/2}dz = \frac{\pi\hbar}{2}\left(n+\frac{1}{2}\right), \tag{20}$$

where $\bar{z}$ is the classical turning point.

$\bar{z} \ll |y_0|$ in the ground state and in the lower excited states, as a simple estimate shows. Therefore, one can expand $1/\varepsilon(z^2+y_0^2)^{1/2}$ in a series in powers of $z^2$, confining our attention to the term $\sim z^2$. Then Eq. (20) reduces to

$$\int_0^{\bar{z}} \left[ 2\mu \left( E-E_0 + \frac{e^2}{\varepsilon y_0} - \frac{e^2}{2\varepsilon y_0} z^2 \right) \right]^{1/2} dz = \frac{\pi \hbar}{2} \left( n + \frac{1}{2} \right), \qquad (21)$$

from which

$$E - E_0 = -\frac{e^2}{\varepsilon y_0} + \frac{e^2}{\varepsilon y_0} \left( \frac{a}{y_0} \right)^{1/2} \left( n + \frac{1}{2} \right),$$

or, having divided by $e^2/a = 27.5$ eV,

$$\frac{E-E_0}{e^2/a} = -\frac{1}{\varepsilon\lambda} + \frac{1}{\varepsilon\lambda^{1/2}} \left( n + \frac{1}{2} \right). \qquad (22)$$

This expression is valid up to $n \lesssim \lambda^{1/2}$. On the other hand, for $n \gg \lambda^{1/2}$ the turning point lies far beyond $|y_0|$, and the tail of the potential $-e^2/\varepsilon|z|$ plays a major role, which leads to the appearance of Coulomb condensation of the levels.

## 3. DISCUSSION

Let us clarify the conditions under which one can expect noticeable physical effects associated with the appearance of the states investigated above. Let us consider magnetic fields $\mathscr{H} \sim 10^6$ Oe, the maximum attainable under laboratory conditions at the present time. For such fields $\nu \sim 50$ for the hydrogen atom, and the minimum value of $\lambda$, permissible by condition (14), amounts to $\sim 10^5$. In this connection the binding energy $\sim 2.5\times10^{-4}$ eV, and the distance between levels adjacent to the ground state $\sim 10^{-6}$ eV. Such a weakly bound system cannot exist in practice for any appreciable length of time under laboratory conditions in a gas consisting of atomic hydrogen. However, the magnetic fields near certain astronomical objects are sometimes substan-

tially greater than $10^6$ Oe (for example, $\mathscr{H} \sim 10^{12}$ Oe on the surface of a pulsar). In this case the binding energy and the distances between the levels will be substantially larger. Thus, for example, for $\mathscr{H} = \mathscr{H}_0$, which corresponds to $\nu = 1$ and $\lambda = 50$, the binding energy will be $\sim 0.55$ eV, and the distance between levels will be $\sim 0.08$ eV.

Systems having such binding energies may quite possibly exist for long periods of time in interstellar space. In this connection, the states pertaining to Landau levels with different values of the magnetic quantum number should be taken into consideration together with the ground state. (We note that lines corresponding to transitions between levels in hydrogen atoms with quantum number $n = 100$ are observed in radio spectra. Such a value of $n$ corresponds to a binding energy $10^{-3}$ eV, and a separation of $10^{-15}$ eV between neighboring levels.) If a cloud of atomic hydrogen exists near a pulsar, it will absorb electromagnetic radiation in the range determined by formula (22).

As for excitons, observable effects can be obtained for them only in the case of a very small effective mass. Thus, if $\mu \sim 10^{-2} m_e$, the characteristic field $\mathscr{H}_0$ will be $3\times10^5$ Oe, which is quite attainable under laboratory conditions.

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