Structure of superradiance spectra

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Results of theoretical and experimental studies of the dynamics of the superradiance spectrum and the physical mechanism responsible for the periodic fine structure in the emission of a superradiant laser are presented. It is shown that the fine structure of the spectrum arises from nonstationary diffraction due to time variation of the gain coefficient of the medium. Experimental data on a superradiant dye laser excited by picosecond optical pulses are presented. A quantitative calculation of the magnitude of the fine structure of the spectrum is in good agreement with experiment. The proposed theory can explain the space-frequency structure of the near radiation field.

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INTRODUCTION

Interest in the study of superradiance (SR)[1-10] rose with the appearance of high-gain resonator-free lasers. In SR lasers the emission is formed in one or two traversals of the active region with a definite transverse gain profile. In this connection the problem of the formation of the spatial coherence of the emission arises. Bespalov and Pasmanik[9] analyzed the establishment of spatial coherence for the stationary case in the parabolic approximation and, using the reciprocal operator method, found the asymptotic behavior of the spatial correlation function. The effect of the unsteadiness of the waves due to the difference between the group velocities of the SR and pumping pulses on the formation of the spatial coherence was investigated by Abdulin *et al.*[10] These authors found an asymptotic formula for the correlation-function parameters that characterize the spatial coherence of the beam and calculated these parameters on a computer for finite distances from zero to the diffraction length.

Distinctive features of the formation of pulsed SR laser spectra are the high gain at the diffraction length and the nonstationary character of the interaction of the radiation with matter. Abrosimov[1] and Ishchenko *et al.*[3] have shown experimentally that, within the region of spatial coherence, the SR spectrum consists of equidistant components. A general property of the phenomenon, first noticed by Ishchenko *et al.*,[3] is that it takes place in solid and liquid media under excitation of stimulated Raman scattering,[8] in gas lasers,[1,3,4] and also, as we discovered, in the SR of rhodamin 6G solutions excited by picosecond second harmonic neodymium laser pulses. Studies of the dye SR also showed that during the development of the emission, the total SR spectrum narrows to an extent determined by saturation.

The experimental facts indicate that the formation of the SR spectrum is closely associated with the establishment of spatial coherence and the dynamics of the saturation process. The purpose of the present study is to obtain a consistent solution of such problems. The theoretical results obtained are compared with experimental studies of the SR of a dye excited by an ultrashort second harmonic neodymium laser pulse and with the experimental data of Abrosimov[1] and Ishchenko *et al.*[3]

SATURATION AND SPECTRAL CHARACTERISTICS OF SUPERRADIANCE

The fine structure of the SR spectrum, observed within the limits of a single spatial coherence granule,[1,3] disappears, and the SR spectrum becomes smooth, if radiations from several granules simultaneously enter the spectrometer. Digressing from the fine structure of the spectrum, which is due, as is shown below, to saturation and nonstationary diffraction, we shall investigate the contraction of the spectrum in the geometric optics approximation. Let us consider the change in the "instantaneous" spectrum of a luminescence pulse. Let T be a time interval that is long compared with the correlation time τ of the light pulse but is fairly short compared with the length of the pulse. We define the instantaneous spectrum of the luminescence pulse as follows:

$$E_\omega(t,z)=\frac{1}{2T}\int_{t-T}^{t+T}E(t,z)e^{i\omega t}\,dt. \tag{1}$$

The function $E_\omega(t,z)$ varies slowly in times of the order of the reciprocal width of the luminescence line and satisfies the following equation in the approximation of slowly varying amplitudes:

$$\left(\frac{\partial}{\partial z}+\frac{1}{u}\frac{\partial}{\partial t}\right)E_\omega(t,z)=\frac{2\pi i\omega}{c}P_\omega(t,z)+f_\omega(t,z), \tag{2}$$

where $f_\omega(t,z)$ is the spectral component of the random force that describes the spontaneous transitions, $P_\omega=\chi_\omega E_\omega$,

$$\chi_\omega=\chi_0'+i\left\{\chi_0''-\frac{\alpha_0(\omega,t-z/u_0)}{(4\pi\omega/c)\left(1+\int\varkappa_\omega I_\omega\,d\omega\right)}\right\}, \tag{3}$$

$\alpha_0(\omega,t-z/u_0)$ is the spectral profile of the gain coefficient corresponding to the resonance transition, $I_\omega=c|E_\omega|^2/(4\pi)^2$ is the spectral density of the light flux, $\varkappa_\omega$ is the saturation parameter for the component of frequency ω, u and u_0 are the group velocities of the SR and pumping pulses, respectively, and $\chi_0=\chi_0'+i\chi_0''$

is the contribution of the nonresonant transitions to the susceptibility. The saturation model adopted for the formula reflects the basic features of the phenomena under consideration.[6,7]

Let us pass from Eq. (2) to the Fokker–Planck equation for the light flux spectral intensity distribution function $W_\omega(I)$. Here ω enters as a parameter. It is not difficult to see that

$$\left(\frac{\partial}{\partial z}+\frac{1}{u}\frac{\partial}{\partial t}\right)W_\omega=-\frac{\partial}{\partial I}\{\tilde{\alpha}_\omega I W_\omega\}+D_\omega\frac{\partial}{\partial I}\left\{I\frac{\partial}{\partial I}W_\omega\right\}, \tag{4}$$

$$\tilde{\alpha}_\omega=\frac{\alpha_0(\omega, t-z/u_0)}{1+\int\varkappa I_\omega\, d\omega}-C, \tag{5}$$

where $\tilde{\alpha}_\omega$ is the spectral profile of the gain coefficient, $D_\omega(t-z/u_0)$ is the spectral intensity of the noise due to the medium, and $C=(4\pi/c)\omega\chi_0''$ is the nonresonance absorption coefficient. The subscript ω on I_ω indicates that the stochastic value of the spectral intensity corresponding to frequency ω is to be taken.

An idea of the distribution W_ω can be obtained by analyzing the equations satisfied by the first moments of the distribution function. Multiplying both sides of the equation by I and integrating from zero to infinity, we find that the spectral density of the radiation,

$$\bar{I}_\omega=\int_0^\infty IW_\omega(I)\,dI$$

satisfies the equation

$$\left(\frac{\partial}{\partial z}+\frac{1}{u}\frac{\partial}{\partial t}\right)\bar{I}_\omega=\left\{\frac{\alpha_0(\omega, t-z/u_0)}{1+\langle\varkappa I\rangle}-C\right\}\bar{I}_\omega+D_\omega. \tag{6}$$

in which

$$\langle\varkappa I\rangle=\int_0^\infty\varkappa\bar{I}_\omega\,d\omega. \tag{7}$$

The appearance in the denominator of the gain coefficient of the average spectral intensity is quite reasonable, since the intensity fluctuations already contribute little to the saturation on account of the averaging over frequency.

Equation (6) determines the development of the spectrum of the luminescence pulse. Let us integrate it for the case in which the pulses have the same group velocity, $\nu=0$, and the contribution from the intrinsic noise of the medium is small as compared with the intensity of the radiation incident on the boundary of the active medium, $D/\alpha\ll I_0$. This assumption involves no loss of generality in treating the development of the radiation, but merely simplifies the calculation, since the second term on the right in Eq. (6) can be neglected. Let us transform to the variables z, $\xi=t-z/u_0$ in Eq. (6); then the integral

$$I_\omega=I_{0\omega}\exp\left\{\int_0^z\left[\frac{\alpha_0(\omega,\xi)}{1+\langle\varkappa I\rangle}-C\right]dz\right\} \tag{8}$$

gives the luminescence spectrum as a function of z and ξ. Here $I_{0\omega}$ is the spectral distribution of the radiation at $z=0$. As is evident from Eq. (8), in the region of exponential growth the spectrum of the pulse narrows in accordance with the frequency dependence of the gain coefficient. For Lorentz and Doppler line contours, when $\alpha_0(\omega_0)z\gg1$, the spectrum narrows in accordance with an $(\alpha_0(\omega_0)z)^{-1/2}$ law (ω_0 is the central frequency of the active line). As αz increases the narrowing becomes less rapid since the gain coefficient decreases on account of saturation.

Assuming that the width $\Delta\omega$ of the SR spectrum is much smaller than the width $\overline{\Delta\omega}$ of the active line and that the second derivative $\alpha_0''(\omega_0,\xi)$ is equal in order of magnitude to $2\alpha_0(\omega_0,\xi)/(\overline{\Delta\omega})^2$, we obtain, using Eq. (8), the expression

$$\frac{\Delta\omega}{\overline{\Delta\omega}}\approx\left[\alpha_0(\omega_0,\xi)\int_0^z\frac{dz}{1+\langle\varkappa I\rangle}\right]^{-1/2}, \tag{9}$$

which describes the z dependence of the width $\Delta\omega$ of the spectrum. On account of saturation, the contraction of the spectrum continues only to a certain limiting width $\Delta\omega_\infty$, which is equal in order of magnitude to

$$\overline{\Delta\omega}[\ln(I_{sat}/I_0)]^{-1/2},$$

where $I_{sat}=1/\varkappa(\omega_0)$.

SPATIAL COHERENCE AND TRANSVERSE MODES OF AN SR LASER

To elucidate the physical phenomena associated with diffraction it is sufficient to consider a two-dimensional model, assuming that the gain coefficient is a function of just one coordinate x and that the group velocities of the pumping and SR pulses are equal. The spatial coherence of the SR is determined by the correlation function

$$\Gamma(x,x'|z,\xi)\equiv\langle E(x',z;\xi)E(x,z;\xi)\rangle, \tag{10}$$

where $E(x,z;\xi)$ is the slowly varying amplitude of the radiation field, which, in the parabolic approximation, satisfies the equation

$$\left(\frac{\partial}{\partial z}+\frac{1}{2ik}\frac{\partial^2}{\partial x^2}\right)E(x,z;\xi)=\frac{\alpha(x,\xi)}{2}E(x,z;\xi). \tag{11}$$

The angle brackets indicate an average over realizations, and $\alpha(x,\xi)$ is the gain coefficient profile.

Using the Green's function, the solution of (11) can be expressed in terms of the field at the input end of the SR laser. The latter is found analytically in the paraxial approximation, in which

$$\alpha(x,\xi)=\alpha_0(\xi)(1-2x^2/a^2), \tag{12}$$

and has the form[11]

$$G(x,x'|z,\xi)=F(z,\xi)\times\exp\left\{\frac{\alpha_0 z}{2}+\frac{i\varepsilon}{\sin(2\varepsilon z/k)}\left[(x^2+x'^2)\cos\left(\frac{2\varepsilon z}{k}\right)-2xx'\right]\right\},$$

$$F(z,\xi)=\left[\frac{\varepsilon}{\pi i\sin(2\varepsilon z/k)}\right]^{1/2},\quad \varepsilon(\xi)=\left[-\frac{ik\alpha_0(\xi)}{2a^2}\right]^{1/2}. \tag{13}$$

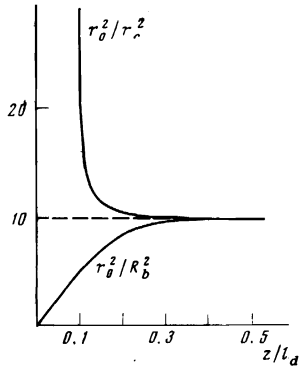

FIG. 1. Distance dependences of the beam radius and coherence range.

The root in the last of Eqs. (13), which defines ε, is chosen so that $\operatorname{Im}\varepsilon<0$.

With the aid of formula (13) one can find an expression for the correlation function for a δ-correlated signal at the input:

$$\Gamma(x,x'|z,\xi)=\Phi(z,\xi)\exp\left\{-\frac{R^2}{2R_b^2}-\frac{\rho^2}{2r_c^2}-iD\rho R\right\}. \tag{14}$$

Here the function $\Phi(z,\xi)$ is independent of the transverse coordinate, $R=(x+x')/2$, $\rho=(x-x')/2$,

$$r_c^{-2}(z,\xi)=-4[(\operatorname{Re}A)^2+(\operatorname{Im}B)^2]/\operatorname{Re}A,$$
$$R_b^{-2}(z,\xi)=-4[(\operatorname{Re}A)^2-(\operatorname{Re}B)^2]/\operatorname{Re}A,$$
$$D(z,\xi)=2[2\operatorname{Im}A\operatorname{Re}A-(\operatorname{Im}B)^2]/\operatorname{Re}A;$$

r_c is a characteristic dimension of the region of spatial coherence, R_b is a characteristic dimension of the beam, and D is the phase of the correlation function. The functions A and B are defined as follows:

$$A(z,\xi)=i\varepsilon(\xi)\operatorname{ctg}\left(\frac{2\varepsilon(\xi)z}{k}\right),\qquad B(z,\xi)=i\varepsilon(\xi)\operatorname{cosec}\left(\frac{2\varepsilon(\xi)z}{k}\right).$$

Figure 1 shows the z dependences of r_0^2/r_c^2 and r_0^2/R_b^2 normalized at $r_0^2=a^2/4$ for $G_d=\alpha_0 l_d=100$ ($l_d=ka^2/4$ is the diffraction length, $r_c(0)=0$, and $R_b^{-1}(0)=0$). It is evident from the figure that the size of the coherence region increases monotonically with increasing z, while the radius of the beam decreases, tending to the single value $D_c/2=a/2(G_d)^{1/4}$ for $z\gg l_d/\sqrt{G_d}$. The distribution of the field on the cross section does not change for $z\gg l_d/\sqrt{G_d}$ and is given by the formula

$$E(x,z;\xi)=E_0(z,\xi)\exp(-\varepsilon x^2), \tag{15}$$

in which $E_0(z,\xi)$ depends only on z and ξ.

Expression (15) is an example of a field configuration (mode) that retains its form when propagating in a medium with the gain coefficient $\alpha(x,\xi)$. A complete set of transverse modes of the active channel is defined by the eigenfunctions of the operator

$$\hat{R}=-\frac{1}{2ik}\frac{\partial^2}{\partial x^2}-\frac{\alpha_0 x^2}{a^2}. \tag{16}$$

and can be expressed in terms Hermite polynomials as follows:

$$E_n(x)=C_nH_n(\sqrt{2\varepsilon}x)\exp(-\varepsilon x^2), \tag{17}$$

where C_n is a normalizing factor. The eigenvalues corresponding to (17) are

$$\lambda_n=-i\frac{\varepsilon}{k}(2n+1),\qquad n=0,1,2,\ldots \tag{18}$$

Functions (17) are not orthogonal, and they form a complete set. In other words, an arbitrary solution of Eq. (11) can be expressed as a linear combination of functions (17) with coefficients independent of x. Let us expand an arbitrary field of the SR laser in the eigenfunctions (17). Then the solution of Eq. (11) takes the form

$$E(x,z;\xi)=\sum_{n=0}^{\infty}C_n(0)E_n(x)\exp\left[\left(\frac{\alpha_0}{2}+\lambda_n\right)z\right]. \tag{19}$$

where the $C_n(0)$ are constant coefficients determined by the field $E(x,0;\xi)$ at the boundary of the active medium. Owing to the difference between the gain coefficients for two successive modes, the increase in the amplitude of each higher mode on propagating through the distance $z=2l_d/\sqrt{G_d}$ is less by a factor e than the corresponding increase in the amplitude of the next lower mode. Thus, the relative weights of the higher modes in expansion (19) decrease as a result of the higher diffraction losses. Asymptotically, only the zeroth mode remains when $z\gg l_d/\sqrt{G_d}$, and spatial coherence is fully established.

FINE STRUCTURE OF SR-LASER SPECTRA

Experimental studies of the spectra of the radiation from regions of spatial coherence (granules) at the face of an SR laser have shown that the SR spectrum has a quasiperiodic fine structure.[1,3,4] The period of the fine structure corresponds to the coherence time of the SR laser radiation.[1]

The real parts of the eigenvalues of the operator $\hat{R}$ determine the gain coefficients for the corresponding modes ($\alpha_n=\alpha_0+2\operatorname{Re}\lambda_n$, where $\operatorname{Re}\lambda_n<0$) and describe the diffraction losses, which increase with increasing mode-number n. The imaginary parts give the phase shifts, which depend on the distance:

$$\Delta\varphi_n=-\operatorname{Im}\lambda_n z=\frac{z}{2l_b}\sqrt{G_b}(n+1/2). \tag{20}$$

The phase shift of the field of the n-th mode follows from the Huygens–Fresnel principle. The vectors in the complex plane that describe the contributions of the secondary waves from elementary areas (Fresnel zones) turn through angles that depend on the optical path and the mode number. The resultant vector, which determines the resultant field of the n-th mode, turns through the angle (20). For a pulsed SR laser, the phase of each mode changes with time, and the n-th mode suffers a frequency shift proportional to n. The power of the SR emission beats at the difference frequencies. Because of the saturation effect, these beats are shifted according to the gain coefficient and phase shift of each of the modes.

Let us examine this process within the limitations of

perturbation theory. We expand the field in a functional series in the transverse modes of the active wave guide and introduce the correction term due to saturation,

$$U(x,z)E=-{}^1/_2\alpha_0(\xi)(1-2x^2/a^2)\varkappa IE, \tag{21}$$

on the right in Eq. (11). Using the properties of the Hermite polynomials, it is not difficult to obtain the following set of coupled equations for the coefficients of the expansion functions:

$$\frac{dC_n(z)}{dz}=\left(\frac{\alpha_0}{2}+\lambda_n\right)C_n(z)+\sum_{l,m,k}U_{n;\,l\bar{m}k}C_l(z)C_m^*(z)C_k(z), \tag{22}$$

where the $U_{m;\,l\bar{m}k}$ are the matrix elements of the perturbation operator (21).

It is evident from Eq. (22) that the nonlinearity leads to two effects: 1) a change in the increments and phase shifts of the complex amplitudes of the individual modes; and 2) a redistribution of energy among the modes. Let us consider the case in which the nonlinearity becomes appreciable at distances $z\gtrsim l_d/\sqrt{G_d}$. By that time the field is represented mainly by the zeroth mode. Terms proportional to the matrix elements $U_{0,0\bar{k}l}=U_{0,l\bar{k}0}$ contribute to the change in the increment of the zeroth mode. The transfer of energy into this mode is determined by the matrix elements $U_{0,1\bar{1}2}$, $U_{0,1\bar{2}1}$, etc., i.e., by terms of the next higher order in the perturbation parameter. The change in the amplitude $C_1(z)$ takes place in a similar manner, and only for $C_2(z)$ does the source proportional to $U_{2,0\bar{0}0}$ turn out to be significant.

An elementary estimate of the contribution of saturation to the change in the increment of the zeroth mode shows that the diffraction losses become comparable with the correction to the gain coefficient when $\varkappa I\sim 1/\sqrt{G_b}$. For the higher modes ($n\geq 2$) the matrix elements are so small that the change in the increments due to saturation can be neglected. Thus, when the intensity $I_{sat}/\sqrt{G_b}$ is reached, the losses in the higher modes are comparable with those in the lower ones and the rate of formation of spatial coherence falls.

The further development of the radiation consists in transfer of energy from the zeroth mode to the second mode. Assuming that $C_n=R_ne^{i\varphi_n}$, let us find the contribution to the amplitude R_2 due to energy transfer up to the time when saturation just begins to play a part:

$$R_2(z)=\frac{\mathrm{Re}[U_{2,0\bar{0}0}]}{\alpha_0-2\,\mathrm{Re}\,\lambda_0}R_0^3(z). \tag{23}$$

From Eq. (23) it is evident that the induced part of R_2 becomes comparable with the value due to the boundary conditions at the input face of the SR laser when $\varkappa I\sim 0.1$. Subsequently, R_2 continues to grow until it becomes of the order of R_0. Then energy exchange between even-numbered modes, accompanied by an increase in the angular spread of the radiation, becomes important. The phenomenon under discussion has been observed experimentally by Ishchenko *et al.*[3]

An important new factor characteristic of SR under saturation conditions is the establishment of definite phase relations among the even modes. As a result there arises a regular amplitude–phase modulation. Let us illustrate this by a calculation of the correction to φ_0 proportional to the product of R_0 and expression (23):

$$\delta\varphi_0=q^2\sin\{[\mathrm{Im}\,(\lambda_2-\lambda_0)+\varepsilon(\alpha_0-2\,\mathrm{Re}\,\lambda_0)]z+\delta\}, \tag{24}$$

where ε is a numerical factor approximately equal to 0.0178. The modulation depth q^2 and phase shift δ are found from the equation

$$q^2e^{i\delta}=\frac{2U_{0,2\bar{2}0}-U^*_{0,0\bar{2}0}}{2(\alpha_0+2\,\mathrm{Re}\,\lambda_0)+i\,\mathrm{Im}\,(\lambda_2-\lambda_0)}R_2(z)R_0(z).$$

The SR pulse changes the inversion in a time of the order of τ. Then the phase (20) changes with a frequency of the order of

$$\Delta\omega=2z[\mathrm{Im}\,(\lambda_2-\lambda_0)]/\tau=8z/\tau kD_c^2, \tag{25}$$

if $\alpha z\lesssim 100$. An estimate of the modulation frequencies according to the formula is in good quantitative agreement with the experimental results of[1,3]. Estimating the size of the spatial coherence region with the formulas of the preceding section, we find that under Abrosimov's experimental conditions[1] the theoretical value is $\Delta\omega=0.003$[1)]; The experimental value is 0.003. The corresponding figures for the work of Ishchenko *et al.*[3] are 0.054 and 0.058, respectively. The experimental results for the dye SR are presented below.

EXPERIMENTAL STUDY OF A PULSED SR DYE LASER EXCITED BY AN ISOLATED PICOSECOND PULSE

For an experimental test of the theoretical conclusions we investigated the SR excited in a dye solution by a traveling ultrashort pulse (USP). The picosecond (5 psec) second harmonic pulse produced a traveling inversion pulse in a rhodamin 6G solution. With pumping throughout the entire length ($l=10$ cm) of the cell containing the dye solution at a concentration of 10^{16} cm^{-3}, the product of the gain coefficient by the length reached values of 10–20. The traveling inversion pulse formed an SR pulse with a small angular spread ($(3-5)\cdot 10^{-3}$ rad) and a broad (300 cm^{-1}) frequency spectrum at a frequency of 17 850 cm^{-1}. The conversion factor for USP energy into SR radiation reached 5%. The cross-section area of the SR beam was 10^{-3} cm^2. Measurements with an ultrafast optical gate[5] of the duration of the SR pulse gave a value of 10–15 psec and made it

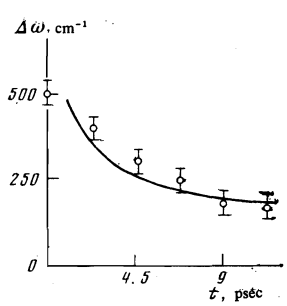

FIG. 2. Width of the SR spectrum at the exit from the cell vs time reckoned from the pumping pulse.

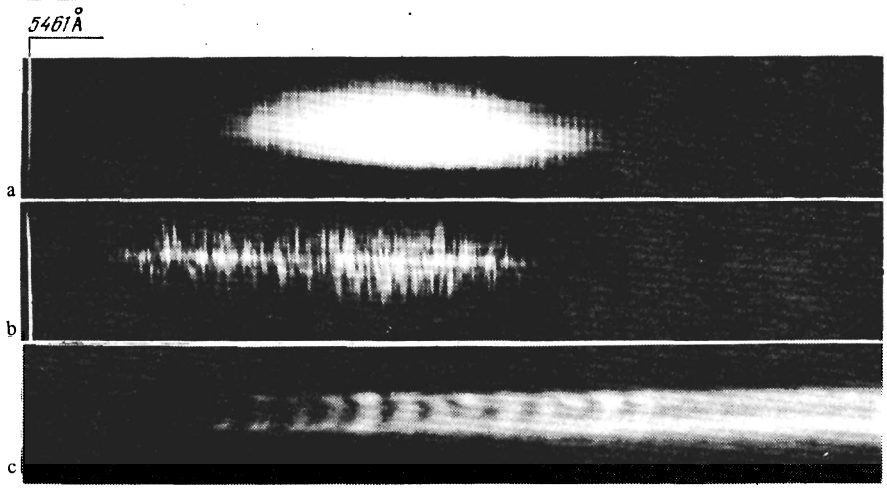

FIG. 3. Fine structure of the SR spectrum of rhodamin 6G: a) recorded with a smooth Gaussian pumping-beam profile, b) recorded with an irregular pumping-beam profile, c) space-frequency structure of the spectrum.

possible to estimate the total SR power density as 10^8 W/cm^2 at the exit face of the laser. The SR intensity at which saturation of rhodamin 6G sets in is 10^6 W/cm^2. Hence the SR pulse was formed under nonlinear conditions.

Using a spectrum-time analyzer consisting of an ultrafast optical gate and a spectrograph,[5] we investigated the dynamics of the SR spectrum. The spectrum-time development of the SR pulse for each separate shot was recorded on an RF-3 photographic film. The results on the spectrum-time behavior of the SR are presented in Fig. 2, which shows the contraction of the instantaneous SR spectrum obtained by measuring the spectrum-time recording with a photometer. The experimental points give the width of the SR spectrum at different times with respect to the second harmonic pulse exciting the dye. As is evident from the figure, the greatest contraction of the spectrum is reached 9 psec after arrival of the pumping pulse. The time T_{rel} = 9 psec corresponds to the vibrational relaxation time of rhodamin 6G.[12] The theoretical curve was calculated with formula (9), it being assumed that $\alpha_0(\xi)$ was proportional to $1 - \exp(-\xi/T_{rel})$.

To investigate the fine structure of the pulsed SR laser spectrum, the radiation was focused onto the slit of an ISP-51 spectrograph equipped with a UF-90 camera, using a long focus (F = 1 m) lens mounted at the focal distance from the slit. Typical experimental spectra corresponding to different SR intensities and different pumping beam profiles are shown in Fig. 3a and b. The spectrum of Fig. 3a was taken with a beam having a smooth Gaussian profile. Fine structure with a minimum period of 2 cm^{-1} can be seen. Formula (25) gives the same result for the period of the fine structure. The spectrum of Fig. 3b was excited by pumping with a beam having an irregular profile and is therefore scattered. We note that a similar situation is observed in gaseous lasers[3] pumped with a traveling transverse discharge. We also investigated the spatial–frequency structure of the SR spectrum in the near field (Fig. 3c). According to the theory developed here, in addition to the modulation there should be a frequency shift of the radiation on moving away from the axis of the beam, described by the exponential in formula (17). As a result, a spectrum line in the near field should be curved. The observed curvature of the lines in Fig. 3c is in qualitative agreement with the theory.

CONCLUSION

The theory of SR lasers presented above makes it possible to explain the formation of the spatial coherence of the radiation, the dynamics of the contraction of the spectrum, the effect of saturation on the angular structure of the beam, and the quasiperiodic fine structure of the emission spectrum. The explanation of the fine structure of the spectrum by nonstationary diffraction is applicable to any pulsed laser with a large total gain (product of the gain coefficient by the length) and a limited profile of the active region. The agreement between the theoretical and experimental results for such diverse media[1,3,8] and excitation mechanisms and such different ratios of relaxation times to pumping pulse length permits us to assert that the fine structure of the SR spectrum and stimulated Raman scattering is due to nonstationary diffraction.

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[1)]Frequencies are given in reciprocal centimeters.

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Effect of a strong electromagnetic wave on the radiation emitted by weakly excited electrons moving in a magnetic field

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A quantum-mechanical treatment is given of the spontaneous emission of radiation by electrons moving in a constant uniform magnetic field and the field of a circularly polarized plane wave propagating in the direction of the magnetic field. Electrons occupying low-lying levels $n = 0, 1$ are considered. An analysis of the emission probability is presented. The behavior of spin during the emission process is considered and it is shown that the $n = 1$ state with the electron spin lying along the magnetic field is stable against a transition to the $n = 0$ state.

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There has been undoubted recent interest in the interaction between a strong radiation field and electrons moving in a magnetic field. Spontaneous emission by electrons in fields of this configuration is of particular interest since such problems arise in calculations involving lasers, electron excitation, behavior of electron spin, and so on. In many respects, these problems are analogous to those arising in connection with the interaction between radiation and systems of atoms or molecules. The latter subject was reviewed in[1a]. The spontaneous emission of electrons moving in a constant uniform magnetic field and the field of a plane wave propagating in the direction of the magnetic field can be investigated in the greatest detail. This is so because the Dirac equation for an electron in a field of this kind has been solved exactly.[2]

Individual problems connected with the properties of radiation emitted by a relativistic electron in fields of this kind have been treated by the methods of classical electrodynamics in[1b,3]. In particular, Bagrov and Khalilov[3] have derived the combination spectrum of frequencies emitted by an electron, and have obtained expressions for the spectral and angular distribution of the emitted radiation, and for the total radiated power. They also investigated the polarization of the emitted radiation. An analysis of the total cross section for the scattering of a strong wave by plasma electrons in a magnetic field, taking into account deceleration by radiation, is given in[1b]. Some aspects of this problem were considered quantum-mechanically in[4–11], in which equations were obtained for the radiation frequencies, and the problem of scattering of a weak wave by a relativistic electron in a magnetic field was investigated. In addition to the relativistic case, there is considerable interest in the interaction between a strong electromagnetic wave and electrons occupying low-lying energy states in a magnetic field. This interaction and the associated electromagnetic emission by electrons are investigated in the present paper. In particular, we report an analysis of the properties of the radiation emitted when an electron undergoes transitions between the $n = 0, 1$ levels. The effect of the field due to a strong wave on the behavior of the electron spin in this process is also discussed.

Consider a charge e moving in a uniform magnetic field H parallel to the z axis and the field of a circularly polarized plane wave propagating along the z axis with frequency $\omega_0 = c\varkappa_0$ and electric-field amplitude E_0 ($g = 1$ and $g = -1$ will correspond to right-handed and left-handed polarizations, respectively). The wave function that is the exact solution of the Dirac equation is known[2] and can be written in the following form in terms of the two-dimensional Pauli matrices σ:

$$\Psi = NL^{-1}\exp(-iS)\begin{pmatrix} G(1) \\ \varepsilon g G(-1)\end{pmatrix} v, \quad \varepsilon = \frac{eg}{|e|},$$

$$S = c\lambda t - kx - \frac{(\gamma y + \varepsilon g k)k_0\gamma_0\cos\varkappa_0\xi}{\lambda\varkappa_0(1+\varepsilon\delta)} + \frac{\beta\xi}{1-\beta} - \frac{\varepsilon\delta k_0^2\gamma_0^2\sin 2\varkappa_0\xi}{4\varkappa_0\lambda(1+\varepsilon\delta)^2},$$

$$G(s=\pm 1) = [(\lambda A^*\sigma_1 + \lambda + sk_0)U_{n-1}(a) + (2\gamma n)^{1/2}\sigma_1 U_n(a)](1-\varepsilon g\sigma_3) - s[(\lambda A\sigma_1 - \lambda - sk_0)U_n(a) + (2\gamma n)^{1/2}\sigma_1 U_{n-1}(a)](1+\varepsilon g\sigma_3), \quad (1)$$

$$a = \sqrt{\gamma}\, y + \frac{egk}{\gamma^{1/2}} + \frac{\varepsilon\sqrt{\gamma}\,k_0\gamma_0\sin\varkappa_0\xi}{\lambda\varkappa_0(1+\varepsilon\delta)}, \quad \xi = ct - z,$$

$$k_0 = \frac{mc}{\hbar}, \quad \gamma = \frac{|e|H}{c\hbar}, \quad \gamma_0 = \frac{|e|E_0}{mc\omega_0}, \quad \delta = \frac{\gamma}{\lambda\varkappa_0}, \quad A = \frac{\varepsilon k_0\gamma_0\exp(i\varepsilon\varkappa_0\xi)}{i\lambda(1+\varepsilon\delta)}.$$