[6]W. W. Webb and R. T. Warburton, Phys. Rev. Lett. 20, 461 (1968); J. Meyer and G. V. Minnigerode, Phys. Lett. A38, 529 (1972).
[7]D. A. Vincent and B. S. Deaver, Phys. Rev. Lett. 32, 212 (1974).
[8]J. E. Mercereau, Rev. Phys. Appl. 9, 47 (1974).
[9]V. N. Gubankov, V. P. Koshelets, K. K. Likharev, and G. A. Ovsyannikov, Pis'ma Zh. Eksp. Teor. Fiz. 18, 292 (1973) [JETP Lett. 18, 171 (1973)].
[10]R. O. Zaĭtsev, Zh. Eksp. Teor. Fiz. 50, 1055 (1966) [Sov. Phys. JETP 23, 702 (1966)].
[11]I. O. Kulik and A. N. Omel'yanchuk, Zh. Eksp. Teor. Fiz. 68, 2139 (1975) [Sov. Phys. JETP 41, 1071 (1975)].
[12]L. P. Gor'kov and N. B. Kopnin, Zh. Eksp. Teor. Fiz. 65, 396 (1973); 64, 356 (1973) [Sov. Phys. JETP 38, 195 (1974); 37, 183 (1974)].
[13]G. M. Éliashberg, Zh. Eksp. Teor. Fiz. 61, 1254 (1971) [Sov. Phys. JETP 34, 668 (1972)].
[14]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 68, 1915 (1975) [Sov. Phys. JETP 41, 960 (1975)].
[15]A. Schmid and G. Schön, J. Low Temp. Phys. 20, 207 (1975).
[16]L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) [Sov. Phys. JETP 20, 1018 (1965)].
[17]B. I. Ivlev, S. G. Lisitsyn, and G. M. Éliashberg, J. Low Temp. Phys. 10, 449 (1973).

Translated by J. G. Adashko

Coherent effects in superconducting bridges of variable thickness

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(Submitted January 29, 1976)
Zh. Eksp. Teor. Fiz. 71, 348–358 (July 1976)

Coherent microwave phenomena in thin-film superconducting tin junctions of "variable" thickness are investigated experimentally. It is shown that the dependence of the superconducting current on phase difference, which is approximately harmonic, is preserved up to the limiting junction dimensions, which exceed the coherence length by several times. Some features of the coherent phenomena, which manifest themselves when the relative dimensions of the junctions and the conditions for the transition from the resistive model to ordered motion of Abrikosov vortice in the film are varied, are elucidated. The experimental results are compared with current theoretical concepts.

PACS numbers: 74.50.Tk

The possible existence of the Josephson effect in superconductor—constriction— superconductor junctions with small cross sections ($a \ll \xi$) was first demonstrated theoretically by Aslamazov and Larkin. Starting from the Ginzburg–Landau equations, they have shown[1] that the Josephson effect can be due in this case to a redistribution of the current I among the normal and superconducting components without breaking the Cooper pairs. According to Aslamazov and Larkin, near the critical temperature T_c, at currents not greatly exceeding the critical value I_c, the current in the junction region is

$$I=I_n+I_s=V/R+I_c\sin\varphi,\qquad d\varphi/dt=2eV/\hbar. \tag{1}$$

I_n and I_s in (1) are the currents of the normal and superconducting electrons, V is the voltage across the junction, R is the junction resistance in the normal state, and φ is the phase difference of the wave functions of the superconducting electrodes. A relation of the type (1) in superconducting point contacts and in film bridges of small size was confirmed many times in experiment.[1)]

At the same time, phenomena typical of Josephson junctions, for example the appearance of current steps on the current-voltage characteristics following application of microwave radiation, are observed in superconducting point junctions and film bridges having transverse dimensions much larger than ξ.[2,3] These phenomena are attributed to the formation and coherent motion of magnetic-flux quanta (vortices) in a direction perpendicular to the transport current. A detailed analysis of the vortical motion in such junctions was carried out in a number of studies.[4,5] It is impossible to obtain in this case a simple analytic relation similar to (1), although the very concept of coherent vortex motion turned out to be quite productive when it came to explain the properties of large-size junctions.

A theoretical analysis of the conditions for the change of the model (1) and the transition to the concept of coherent vortex motion when following a change in the weak-coupling geometry has been carried out in the last few years (see, e.g.,[6]). The variable parameters are in this case usually the geometric dimensions of the junction relative to characteristic parameters such as the coherence length ξ or the penetration depth $\delta_\perp$ of the perpendicular magnetic field into the film. One of the most interesting conclusions[6,7] is the existence of "limiting" dimensions; when these are exceeded, a qualitative change takes place in the character of the dynamic processes in the weak-coupling region.

In the experimental studies known to us,[8,9] the conditions under which deviations take place from the dependence of the superconducting current on the phase, similar to (1), have been established numerous times, but were not studied in detail. In this study we have investigated experimentally coherent effects in thin-film superconducting bridges with geometry that is variable relative to ξ and $\delta_\perp$, for the purpose of ascertaining the conditions for the transition from the model of type (1) to coherent vortical motion, and with an aim at comparing the results with contemporary theoretical concepts.

EXPERIMENTAL PROCEDURE

To solve our problem in principle it is very convenient to use thin-film superconducting bridges. Bridges of constant thickness films (Dayem bridges), in which this transition were investigated,[9,10] are not very suitable for this purpose. When the current in such a bridge is increased, the region in which nonlinear effects come into play expands, so that the geometric dimensions do not coincide with the electrodynamic dimensions. In addition, for a study of vortical motion in bridges, it is of considerable interest to use broad and short bridges (with length L smaller than the width W).[3] It is impossible to obtain this case for Dayem bridges because of the strong dependence of the effective length L_e on the width. (By effective length L_e we mean here the distance in the current direction over which $j_c \gtrsim j_{GL}$, where j_c is the critical current density in the bridge and j_{GL} is the density of the pair-breaking current in the conducting film of the banks. $L_e \sim \max\{L, W\}$ for Dayem bridges.) To obtain a distinct spatial localization of the nonlinear effects we chose therefore a bridge with "variable" thickness,[4] with a film thickness much less than that of the bank film.

We have investigated tin bridges of rectangular shape with length $L \approx 0.4$–$8\ \mu$, width $W \approx 0.8$–$30\ \mu$, bridge film thickness $d \approx 100$–1000 Å, and bank film thickness $d_b \approx (2$–$5)\times 10^3$ Å. The samples were prepared by two different procedures.[3,11] The procedure used to prepare the CS series of samples[11] yielded small bridges ($L \sim 0.5\ \mu$, $W \sim 1\ \mu$) with practically identical properties. To obtain bridges with $L > 1\ \mu$, however, it was necessary to use the procedure for the preparation of K-series samples,[3] since the procedure of[11] could not yield bridges that are long enough and are sufficiently homogeneous over their length.

The current-voltage characteristics (CVC) were measured in the temperature interval 4–2 °K. The relative temperature was measured accurate to 10^{-3} °K. To match the bridge impedance to the external electrodynamic system, the sample was connected in the central lead of a two-half-wave coaxial resonator and placed at the midpoint. The microwave-generator frequency was chosen equal to the resonant frequency of the resonator, $f_r = 9.85$ GHz. To decrease the harmful effect of external induced noise, permalloy screens were placed around the helium cryostat, and LC filters were connected in the dc circuit.

The bridge-film parameters were determined in the following manner. The bridge resistance R at room temperature can be calculated from the resistance of the entire sample, putting $R_{\text{sample}} = R + R_{\text{bk}} + R_{\text{leak}}$, where R_{bk} is the resistance of the bank film. The leakage resistance was determined from the expression

$$R_{\text{leak}} = 2\pi^{-1} R_{\square \text{bk}} \ln (2D/W), \tag{2}$$

where D is the smaller of the two quantities, half the substrate thickness or the distance to the contact. In a variable-thickness bridge one adds to this quantity the contribution from the leakage of the current over the thickness of the edge film (from d to d_{bk}). This contribution amounts to $\sim 10^{-1}$ of R_{leak} (2) for a width $W \sim 1\ \mu$ and decreases with width like $1/W \ln (D/W)$.

The bridge resistance R_N at helium temperature was usually determined from CVC that had clearly pronounced sections with constant differential resistance $R_d = \text{const}$ (see Fig. 4 below). In the case when the CVC of the bridge had several sections with constant R_d, the value of R_N was assumed to be equal to the R_d corresponding to the region of small voltages and currents. The correctness of this choice was confirmed by an estimate of R_N from $R(T)$ (see the insert in Fig. 4a). It is possible that the existence of a constant differential resistance not equal to the bridge resistance at high voltages is connected with the start of disintegration of the bank sections adjacent to the bridge.[2)]

The film thicknesses were determined from the resistance $R_\square^{300}$ of a square of film at room temperature and from the residual resistance $R_\square^{4.2}$ at helium temperature, in accordance with the method described by Harper and Tinkham[12] (assuming a phonon resistivity $\rho_{\text{ph}} = 11.5 \times 10^{-6}\ \Omega$-cm).

The residual electron mean free path in scattering by impurities, Λ_b, was determined from the values of R^{300} and $R^{4.2}$ of the film. Assuming that the Matthiessen rule holds for our films, we have

$$\Lambda_b = \Lambda_{\text{ph}} (R_\square^{300}/R_\square^{4.2} - 1), \tag{3}$$

where $\Lambda_{\text{ph}} = 95$ Å is the electron-phonon mean free path at 300 °K. This estimate is correct when the film thickness greatly exceeds Λ_{ph}, and was used to determine the mean free path in the shores. In the bridge film, the mean free path Λ was estimated[3)] with account taken of the scattering from the boundary, using the formula[13]

$$\Lambda = \Lambda_b \Big/ \left[1 + \frac{3}{8} \frac{\Lambda_b}{d} (1-p) \right], \tag{4}$$

where p is the fraction of the electrons specularly reflected from the boundary. The value of p was estimated from a comparison of the resistivities of the film and of the bulk material in accord with the formulas deduced from Fuchs's theory (see, e.g.,[13]). In our case $p \approx 0.5$ for the films of the bridge and of the banks.

According to Gor'kov's calculations,[14] the scattering of electrons by impurities leads to a change in the value of $\varkappa = \delta/\xi$, which can be expressed in terms of $\varkappa_0$ for the pure material:

TABLE I.

Bridge	L, μ	W, μ	R_N, Ω	$\xi(0)$, Å	$\delta_\perp \Delta T$, μ-°K	T_c, K
CS-26	0.4	0.8	0.1	610	0.27	3.78
CS-8	0.6	1.2	0.06	940	0.14	3.77
CS-13	0.6	1.5	0.065	800	0.17	3.67
CS-31	0.7	0.8	0.7	830	0.87	3.76
K-41	1.5	7	1.5	490	5.8	3.81
K-80	2.5	11	0.4	990	2.2	3.84
K-46	6	28	0.95	570	3.9	3.84

$$\varkappa=\frac{\varkappa_0}{\chi(\rho)},\quad \rho=\frac{\hbar}{8\pi^2 kT_c\sigma\delta_L^2(0)}\approx 0.89\frac{\xi_0}{\Lambda},$$

σ is the conductance of the material in the normal state, $\delta_L(0)$ is the London depth of penetration of the magnetic field at $T=0$, and ξ_0 is the coherence length of the pure material at $T=0$. The function $\chi(\rho)$ tends to unity as $\rho\to 0$ and to $1.17\rho^{-1}$ as $\rho\to\infty$. In the range $\xi_0/\Lambda=0$ to 10 we can approximate $\chi(\rho)$ with accuracy better than 6% by the expression[15]

$$\chi(\rho)=[1+0.76\xi_0/\Lambda]^{-1}. \tag{5}$$

Substituting the proposed approximate formula for $\chi(\rho)$ in the expressions for ξ and $\delta_\perp$, and taking the relation between $\delta_L(0)$ and σ into account, we get

$$\xi(T)=0.74\frac{\xi_0}{(1+0.76\xi_0/\Lambda)^{1/2}}\left(\frac{T_c}{\Delta T}\right)^{1/2}, \tag{6}$$

$$\delta_\perp(T)=0.83(1+1.32\Lambda/\xi_0)R_\square^{1/2}/\Delta T, \tag{7}$$

$\Delta T=T_c-T$. Table I lists the values of $\xi(0)$, $\delta_\perp\Delta T$, and other bridge parameters.

EXPERIMENTAL RESULTS

Figure 1 shows plots of the critical current I_c against the relative temperature $t=T/T_c$ for several typical bridges. The experimental values of I_c were normalized to the theoretical value obtained by Ambegaokar and Baratoff[16] for the critical current at zero temperature, $I_c(0)=\pi\Delta(0)/2eR_n$ (where $\Delta(0)=1.76kT_c$). Such a normalization is convenient because the value of $I_c(t)$ given by Ambegaokar and Baratoff coincides in the region $t\to 1$ with the theoretical $I_c(t)$ obtained by Aslamazov and Larkin[1] in the "dirty" limit[16,1] and from the vortex model,[4] as shown by the solid and dashed lines in Fig. 1, respectively. The plot of $I_c(t)$ for the vortex model[4] is drawn in the region of applicability of this model for the sample K-46.

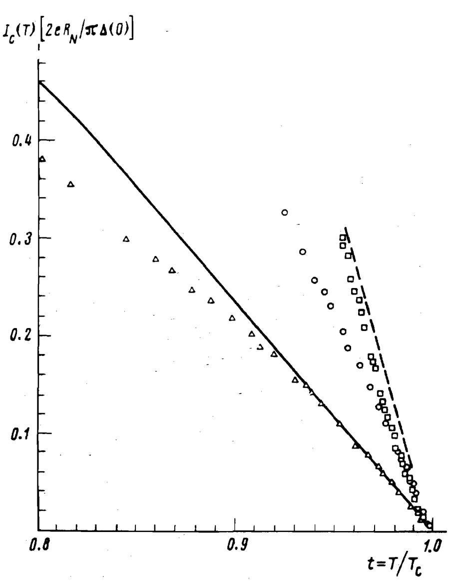

FIG. 1. Temperature dependences of the critical current for three samples: □—K-46, ○—K-80, Δ—CS-13. Solid line—theoretical plot for the resistive model,[1] dashed—for the vortex-motion model.[4]

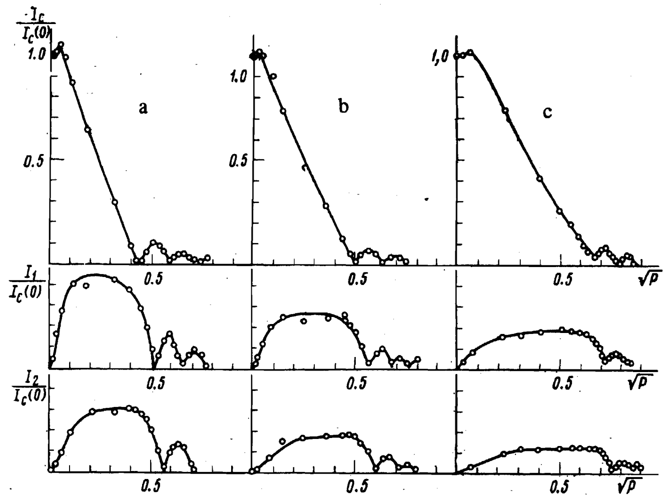

FIG. 2. Plots of the critical current and of the amplitudes of the first two steps, for the sample CS-13 at three different temperatures, vs the level of the incident microwave power. The critical current I_c, the relative temperature t, and the ratio of the voltage $\overline{V}_f$ corresponding to the microwave frequency to the characteristic voltage $V_0=I_cR_N$ are given by: a) $I_c=0.88$ mA, $t=0.975$, $\overline{V}_f/V_0=0.34$; b) $I_c=2.04$ mA, $t=0.910$, $\overline{V}_f/V_0=0.14$; c) $I_c=3.9$ mA, $t=0.872$, $\overline{V}_f/V_0=0.08$.

A change in the slope of $I_c(t)$ is observed for the bridge CS-13 at $t<0.9$. The dependences of the critical current and of the heights $I_{1,2}$ of the first two current steps of the CVC on the power level P of the incident microwaves, are shown for the same bridge in Fig. 2. It is seen that lowering the temperature causes a gradual change in the form of the curves, and in particular a vanishing of the zero-voltage minima of the oscillations.

Figure 3 shows analogous $I_{c,1}(P^{1/2})$ plots for a K-80 bridge at two different temperatures. Attention is called to the qualitative difference between these curves; below a certain threshold temperature, the oscillations of the critical current and of the steps vanish abruptly in the microwave field. The change of the character of the function $I_{c,1}(P^{1/2})$ of the K-80 bridge is not accompanied by a noticeable change in the slope of the plot of the critical current against the temperature.

Figure 4 shows a family of CVC of the CS-13 bridge as functions of the temperature. This family is typical of the small bridges investigated by us.

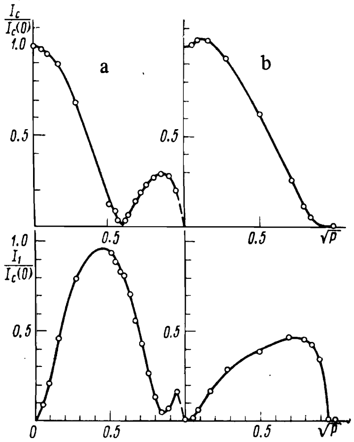

FIG. 3. Plots of the critical current and of the amplitude of the first step vs the level of the incident microwave power for the sample K-80. The dashed lines show the section on which the critical current and the jump steps vanish. a) $I_c = 63$ μA, $t = 0.994$, $\overline{V}_f/V_0 = 0.76$; b) $I_c = 292$ μA, $t = 0.975$, $\overline{V}_f/V_0 = 0.16$.

DISCUSSION OF EXPERIMENTAL RESULTS

We note first that to determine the conditions under which the Josephson effect appears in bridges, it is important to determine not the absolute values of their geometrical dimensions, but the relative ones, as compared with the characteristic parameters. In our experiments we varied $\xi(T)$ and $\delta_\perp(T)$ of samples with different dimensions by changing the temperature.

Let us discuss first the results obtained for the bridge CS-13. As seen from Fig. 1, in the region $t \gtrsim 0.95$, when the bridge dimensions are $L \lesssim \xi(t)$ and $W \leqslant \xi(t)$, the linear character and the slope of the experimental $I_c(t)$ plot agree with the theoretical ones.4) [1] In the same temperature region, we observe oscillatory dependences of I_c and of the current steps on the microwave power level (Fig. 2); although the agreement between the curves calculated earlier[17] and the experimental $I_{c,1,2\ldots}(P^{1/2})$ curves is not complete, the zero minima of the oscillations of $I_{c,1,2}$ indicate that the higher harmonics (of the $\sin m\varphi$ type, where $m > 1$ is an integer) make a small contribution to the function $I_s(\varphi)$.5) The ratio of the oscillation periods of $I_{c,1,2}$ in this temperature range are also in good agreement with the calculation.

With decreasing t, the $I_c(t)$ plot of the CS-13 bridge deviates from a straight line. This deviation is most likely due to the non-uniformity of the current distribution over the bridge cross section. Calculation[18] has shown that a non-uniform distribution of the current over the width changes the slope of $I_c(t)$ by about 10% at $W = \pi\delta_J$, where δ_J is a parameter that depends on the electrodynamic properties of the bridge and of the banks. In the bridges investigated by us $\delta_J \approx (\delta_\perp L)^{1/2}$.[18] For the CS-13 bridge a 19% change in the slope of $I_c(t)$ occurs in the region $t = 0.89$, where $W/\delta_J \approx 3$, in good agreement with the theory.

When the current is not uniformly distributed over the bridge width, it is necessary to take into account the appearance of an additional phase difference in $I_s(\varphi)$, due to the currents flowing from the center to the edges of the bridge. A qualitative analysis of the results of[18] shows that this is equivalent to increasing the influence of the higher harmonics in the integral dependence of superconducting current on the phase, at least if the non-uniformity is small. In turn, the influence of the higher harmonics on $I_s(\varphi)$ eliminates the zero-voltage minima of the oscillations of the critical current and of the steps as functions of the microwave power.[17] This deformation of the functions $I_{c,1,2}(P^{1/2})$ has been observed in experiment; it is clearly seen on Fig. 2. The temperature at which non-zero voltage minima appear corresponds to the temperature at which the change of the slope of $I_c(t)$ begins.

The appearance of higher harmonics $\sin m\varphi$ in the function $I_s(\varphi)$ may be due6) also to an increase of the relative length L/ξ. Estimates[17,19] show, however, that the deviation of the first minimum of the oscillatory function $I_c(P^{1/2})$ from zero does not exceed 3% even at $L/\xi \approx 3$.

In contrast to CS-13, an abrupt vanishing of the oscillations is observed in the K-80 bridge when a threshold temperature $0.97 < t_{\rm thr} < 0.99$ is reached. This indicates a strong qualitative change of the form of the $I_s(\varphi)$ dependence (see Fig. 3). In this case $3 < L/\xi < 4$.

As shown in[6,19], bridges with variable thickness should have a critical length L_c that depends in the general case on W (in bridges of sufficient width ($W/\xi \gtrsim 10$) we have $L_c \approx 3.49\xi(T)$), and Abrikosov vortices can be produced in the bridge when this critical length is exceeded.[6] This is accompanied by an abrupt transition from an unambiguous relation such as $I_s = I_c \sin\varphi$, and consequently from oscillations of $I_{c,1,2}(P^{1/2})$, to a multiply valued $I_s(\varphi)$ function that is therefore complicated by virtue of its nonlocal character and is accompanied by a non-oscillating variation of the current steps in the microwave field, a variation typical of coherent vortical motion.[3] In this transition there should be no noticeable change in the slope of $I_c(t)$,[4,19] and the $I_c(t)$ plot itself should lie between the theoretical curves of Aslamazov and Larkin[1] and of Likharev.[4] This is confirmed by the experimental results for the K-80 bridge (see Fig. 1), for which, according to the esti-

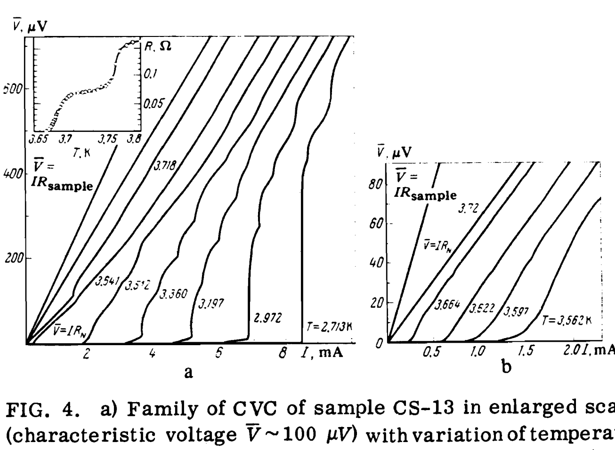

FIG. 4. a) Family of CVC of sample CS-13 in enlarged scale (characteristic voltage $\overline{V} \sim 100$ μV) with variation of temperature. The insert shows the change of the bridge resistance $R(T)$ as a function of the temperature at a measuring current $I = 5$ μA; b) CVC family in small scale (characteristic voltage $\overline{V} \sim 20$ μV).

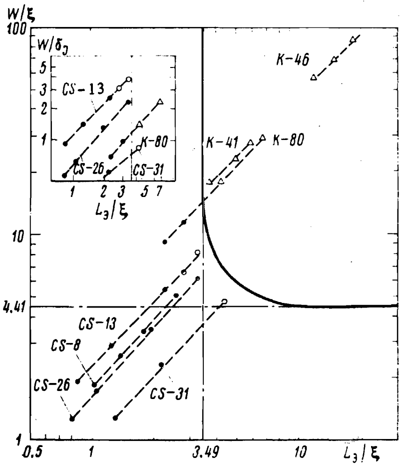

FIG. 5. Variation of the relative bridge dimensions with changing temperature: ●—relative dimensions at which the oscillations of $I_{c,1,2}(P^{1/2})$ have zero-voltage minima; ○—there are no zero-voltage minima of the $I_{c,1,2}(P^{1/2})$ oscillations, with $I_c(P^{1/2})/I_c(0)) \gtrsim 0.03$; Δ—current steps are present, but there are no oscillations of $I_{c,1,2}(P^{1/2})$. The dashed lines correspond to the change of the parameters of one and the same sample with changing temperature. The solid line shows the boundary of the region of existence of Abrikosov vortices in the bridge according to[6]. The insert shows the region of applicability of the relative bridge parameters when the width is normalized $\delta_J = (\delta_\perp L)^{1/2}$.

mates, the current region of the transition from the resistive to the vortex model occupies the temperature interval $0.84 < t < 0.998$. At the same time, the temperature dependence of $I_c(t)$ of the K-46 bridge, as seen from Fig. 1, is in good agreement with the theoretical[4] slope of $I_c(t)$ obtained when the conditions $\xi \ll L \ll W \ll \delta_\perp$ for the vortical model are satisfied. These conditions are satisfied for the K-46 bridge at $t < 0.99$.

The general situation observed when the relative dimensions of the bridges are changed is illustrated in Fig. 5. It shows the experimental results of the temperature variations of the bridge parameters and at the same time their behavior in a microwave field. Since the length and width are normalized in the diagrams of Fig. 5 to the values of ξ and δ_J, which have identical temperature dependences, all the trajectories of the bridge parameters are parallel to the diagonals of the diagrams. Figure 5 shows also the boundary of the region, calculated in[6], in which a solution of the two-dimensional problem of the current and field distribution in the form of an Abrikosov vortex exists for a variable-thickness bridge. According to the calculation,[6] the vortices can exist in the bridge only when the parameters lie above this limit. It is seen that the temperature region where the oscillations of $I_{c,1}(P^{1/2})$ vanish for the K-80 bridge correspond exactly to passage through this boundary. For the bridges K-41 and K-46, the parameters of which lie in the region above the boundary, there are no oscillations of $I_{c,1}(P^{1/2})$.

It is interesting to note that the CS-31 bridge reveals a similar gradual deformation of the oscillatory variations of $I_{c,1,2}(P^{1/2})$ as in the CS-13 bridge. In contrast to the CS-13 bridge, however, the uneven distribution of the current over the cross section is much less for the CS-31 bridge, since the condition $W/\delta_J < 1$ is satisfied for this bridge in the entire investigated temperature region (see the insert in Fig. 5). In all probability, the change in the $I_{c,1,2}(P^{1/2})$ dependences is connected in this case to a considerable degree with the change of the bridge length. As shown in[7,19], if the current is uniformly distributed over the cross section, an increase of the bridge length in the region $L \gtrsim 3.5\xi(T)$ leads to Ginzburg–Landau pair breaking and to a monotonic decrease of the amplitude of the Josephson generation in proportion to $1/L$, owing to the appearance of an ambiguity in the function $I_s(\varphi)$, which should be accompanied by a gradual distortion of $I_{c,1,2}(P^{1/2})$.

Thus, the previously obtained theoretical results[1,4,6,7,19] are confirmed by experiments. We note that the calculations in[7,19] were made on the assumption that the order parameter in the banks remains constant and equal to its equilibrium value. This assumption is satisfied if the effect of destruction of the superconductivity of the banks is small enough when the critical current of the bridge is reached, i.e., under the condition $\Delta L = \frac{1}{2}(L_e - L) \ll \xi, L$; ΔL was determined from the formula $\Delta L \approx (\delta_{bk}/\delta)^2(\xi_{bk}/L)d$, which is valid if $\Delta L \ll \min\{\delta_{bk}, d_{bk}\}$; in the opposite case we have $\Delta L \approx (\delta_{bk}/\delta)^2(d/d_{bk})(\xi_{bk}/L)W$, where ξ_{bk}, δ_{bk}, and d_{bk} are the corresponding parameters for the shore film. For all the bridges reported here, the condition $\Delta L/\xi < 0.1$ was satisfied, so that a comparison of the results with the calculations of[7,19] is quite correct.

Let us examine in conclusion the temperature dependences of the CVC of small bridges (Fig. 4). Without discussing the details of the CVC, we note the following features: 1) in contrast to formula (1), the CVC of the bridge become straight lines at $I \gg I_c$, and these lines are shifted relative to the CVC for the normal state; 2) in the region of small $\bar{V}$, when the temperature is lowered, hysteresis jumps appear in the voltage; 3) at large $\bar{V}$, on the order of several hundred microvolts, $d\bar{V}/dI$ undergo strong changes that shift towards larger $\bar{V}$ with decreasing temperature, like $(\Delta T)^{1/2}$.

The existence of the first singularity was observed by us earlier[20] and was explained[21] by taking into account, in the model of Aslamazov and Larkin,[1] the finite relaxation time of the order parameter in the weak-binding region; this however, does not affect the periods and the existence of zero minima of the $I_{c,1,2}(P^{1/2})$ oscillations at currents that do not exceed greatly the critical value. Our previous conclusions remain therefore in force. The singularities of the second type were observed in bridges also by other investigators. These voltage jumps in the region of small $\bar{V}$ are attributed in[9] to vortex motion, and the region of existence of the vortices is revealed by the appearance of a voltage jump with decreasing temperature. No estimate was made in[9], however, of the local heating of the bridges because of the imperfect cooling, nor was account taken of the possible appearance of voltage jumps on account of thermal processes (these effects were investigated in bridges, e.g., by Tinkham and coworkers[22]). We therefore regard as more correct the estimate of the point at which vortices appear from the results of microwave experiments. The nature of the singularities of the last type is not quite clear at present; investigation of their causes are being continued.

The authors thank K. K. Likharev and V. K. Semenov for a discussion of the results and for useful remarks.

1)We note that the resistive model (1) is used in most calculations of the practical devices based on the Josephson effect.

2)In this case it is no longer possible to estimate the leakage resistance by means of formula (2).

3)This estimate of Λ was made for CS-series of small samples, in which the films of the bridge and of the banks were evaporated simultaneously.[11] The value of Λ for bridges of series K, which were obtained by evaporation in two stages,[3] was determined from measurements of $R_{\square}^{300}/R_{\square}^{4.2}$ of a control bridge film.

4)We note that although the expression for $V_0 = I_c R_N$, with allowance for the value of $\chi(\rho)$, takes in the range $\xi_0/\Lambda \approx 0-10$ the form $V_0[\mu V] \approx 630\ 1+1.32\Lambda/\xi_0\ ^{-1},\ \Delta T[K]$, the experimental values of V_0 of all the small samples agree much better with those calculated for the dirty limit.

5)This conclusion is confirmed also by data for other similar bridges and with experiments on Josephson generation.[11]

6)The effect due to the increase of the relative length can be separated, in principle from the influence of the increase of $R_{\square}^{4.2}$ on the width, which leads to an increase of δ_J.

[1] L. G. Aslamazov and A. N. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 150 (1969) [JETP Lett. **9**, 87 (1969)].

[2] A. Dayem and J. Wiegand, Phys. Rev. **155**, 419 (1967).

[3] V. N. Gubankov, V. P. Koshelets, K. K. Likharev, and G. A. Ovsyannikov, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 292 (1973) [JETP Lett. **18**, 171 (1973)].

[4] K. K. Likharev, Zh. Eksp. Teor. Fiz. **61**, 1700 (1971) [Sov. Phys. JETP **34**, 906 (1972)].

[5] L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **68**, 766 (1975) [Sov. Phys. JETP **41**, 381 (1975)].

[6] M. Yu. Kupriyanov, K. K. Likharev, and L. A. Maslova, Proc. of LT-14 **4**, Helsinki, 1975, p. 104; K. K. Likharev, Fiz. Tverd. Tela (Leningrad) **17**, 2787 (1975) [Sov. Phys. Solid State **17**, 1856 (1975)].

[7] I. O. Kulik and A. N. Omel'yanchuk, Zh. Eksp. Teor. Fiz. **68**, 2139 (1975) [Sov. Phys. JETP **41**, 1071 (1975)].

[8] L. D. Jackel, J. M. Warlon, J. C. Brown, T. D. Clark, R. A. Buhrman, and M. T. Levinson, Proc. of LT-14 **4**, Helsinki, 1975, p. 148.

[9] C. Guthmann, J. Mawer, J. Bok, and A. Libchaber, Proc. of LT-14, **4**, Helsinki, 1975, p. 60; C. Guthmann, J. Mawer, M. Belin, J. Bok, and A. Libchaber, Phys. Rev. **B11**, 1909 (1975).

[10] P. E. Gregers-Hansen and M. T. Levinsen, Phys. Rev. Lett. **27**, 847 (1971).

[11] V. N. Gubankov, V. P. Koshelets, and G. A. Ovsyannikov, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 489 (1975) [JETP Lett. **21**, 226 (1975)].

[12] F. E. Harper and M. Tinkham, Phys. Rev. **172**, 441 (1970).

[13] K. L. Chopra, Thin Film Phenomena, McGraw, 1969 (Russ. transl., Mir, 1972).

[14] L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **37**, 1407 (1959) [Sov. Phys. JETP **10**, 998 (1960)].

[15] B. B. Goodman, IBM J. Res. Dev. **6**, 63 (1962).

[16] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963); **11**, 104 (1963).

[17] A. N. Vystavkin, V. N. Gubankov, L. S. Kuzmin, K. K. Likharev, V. V. Migulin, and V. K. Semenov, Rev. Phys. Appl. **9**, 79 (1974).

[18] G. M. Lapir, K. K. Likharev, K. A. Maslova, and V. K. Semenov, Fiz. Nizkikh Temp. **1**, 1235 (1975) [Sov. J. Low Temp. Phys. **1**, 590 (1975)].

[19] K. K. Likharev and L. A. Yakobson, Zh. Tekh. Fiz. **45**, 1503 (1975) [Sov. Phys. Tech. Phys. **20**, 950 (1975)].

[20] V. N. Gubankov, V. P. Koshelets, K. K. Likharev, and G. A. Ovsyannikov, Tezisy (Abstracts) NT-18, Kiev, 1974, p. 228.

[21] K. K. Likharev and L. A. Yakobson, Zh. Eksp. Teor. Fiz. **68**, 1150 (1975) [Sov. Phys. JETP **41**, 570 (1975)].

[22] W. J. Skocpol, M. R. Beasley, and M. Tinkham, J. Low Temp. Phys. **16**, 145 (1974).

Translated by J. G. Adashko