The temperature of hot photoexcited electrons

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We discuss the competition that arises between electron-electron scattering and emission of optical phonons when hot electrons are excited by light. An energy balance equation is derived, from which the electron temperature can be determined.

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INTRODUCTION

The most popular method of describing disequilibrium in an electron gas in semiconductors is to introduce an electron temperature T_e higher than the lattice temperature T. This description has an exact meaning if the electron energy distribution function is Maxwellian: $f(\varepsilon)\sim e^{-\varepsilon/T_e}$. This distribution is actually established if the collisions between electrons are frequent enough, i.e., when the frequency $1/\tau_{ee}$ of the electron-electron collisions greatly exceeds the frequencies of the collisions with the lattice.

In a number of cases the distribution function can be measured directly by determining the spectral composition of the luminescence. Measurements of this type were made for electrons excited by laser light in GaAs,[1,2] CdS[3] and CdSe.[4] The use of nanosecond excitation pulses and measurement of the time dependence of the luminescence spectrum have made it possible to trace even the temporal evolution of the distribution function.[2]

These experiments indicate that the electron distribution is indeed quite close to Maxwellian. Therefore the interpretation of these experiments was carried out in the spirit of Stratton's theory,[5] and this led to fair agreement between theory and experiment. Nonetheless, this interpretation is incorrect.

An essential point of the theory is the calculation of the electron-energy relaxation rate. Since the lattice temperature is low, $T \ll \hbar\Omega_0$, where $\hbar\Omega_0$ is the energy of the optical phonon, it follows that this relaxation proceeds differently in the active region, where $\varepsilon > \hbar\Omega_0$, than in the passive region, where $\varepsilon < \hbar\Omega_0$. In the active region, emission of optical phonons with a characteristic time $\tau_{PO} \approx 10^{-12}$ sec predominates. In the passive region, the emission of optical phonons is impossible, and their absorption at helium lattice temperature is negligible. The energy relaxation in this region is therefore determined by the quasi-elastic scattering by acoustic phonons with a characteristic time $\tau_A \approx 10^{-9}$ sec. In Stratton's theory[5] it is assumed that the distribution function is Maxwellian both in the passive and in the active region. Yet in order for the electron-electron collisions to dominate in the active region, i.e., if τ_{ee} is to be smaller than τ_{PO}, it is necessary that the electron density n exceed 10^{18} cm^{-3}. The concentrations in all the experiments are known to be lower. There is therefore no doubt that the luminescence experiments yield no information on the form of the distribution in the active region, since the measured spectral range corresponds to recombination of the electrons only from the passive region.

In most experiments $T_e < 80$ °K, so that the number of electrons in the active region is small. Nonetheless, the form of the distribution in this region is very important for a correct calculation of the energy relaxation rate. Levinson and Mazhuolite[6] have shown that the non-Maxwellian character of the distribution in the active region alters qualitatively the mechanism of energy relaxation on the optical phonons in comparison with the Stratton mechanism.[5]

Knowledge of the correct form of the distribution function in the active region is essential also for the calculation of the rate of generation of the nonequilibrium optical phonons[7] observed in a number of experiments.[8,9]

1. SCATTERING MECHANISMS AND THE CHARACTER OF THE DISTRIBUTION FUNCTION

We consider first qualitatively the scattering mechanisms and the form of the distribution function at various energies.

The time of energy relaxation on the acoustic phonons is given by

$$\frac{1}{\tilde{\tau}_A(\varepsilon)} = \frac{2ms^2}{\hbar\Omega_0}\left[\frac{1}{\tau_{PA}y^{1/2}} + \frac{1}{\tau_{DA}}y^{1/2}\right], \quad y = \frac{\varepsilon}{\hbar\Omega_0}. \tag{1}$$

Here m is the electron mass, s is the speed of sound, the two terms in the square brackets correspond to the piezoelectric and deformation scattering by the acoustic phonons, while τ_{PA} and τ_{DA} are the corresponding characteristic scattering times. All the numerical estimates that follow pertain to GaAs, for which the most reliable experimental data are available.[1,2] The numerical parameters for GaAs are $\tau_{DA}^{-1} = 2.2\cdot 10^{11}$ sec^{-1}, $\tau_{PA}^{-1} = 0.9\cdot 10^{11}$ sec^{-1}, $\tau_{PO}^{-1} = 0.7\cdot 10^{13}$ sec^{-1}, $ms^2 = 0.12$ °K, $\hbar\Omega_0 = 420$ K, $m = 0.07\, m_e$, $\varkappa_0 = 12.5$.

The essential region in which the energy ε varies significantly extends from the thermal energies $\varepsilon = T(y \approx 10^{-2}$ at helium temperatures) to the photoproduced-electron energies $\varepsilon = \varepsilon_0 (y \approx 10$ in the existing experiments). In the entire region we have $\tilde{\tau}_A \approx 10^{-9}$ sec.

The frequency of energy exchange between the electrons in ee scattering is

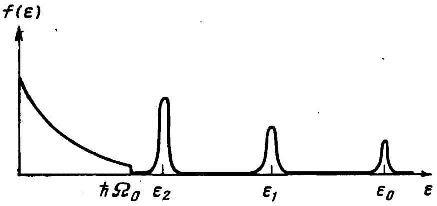

FIG. 1. The electron distribution function.

$$\frac{1}{\tau_{ee}(\varepsilon)}=\frac{8\pi e^4 n}{\varkappa_0^2 m^2 v^3}\ln\frac{T_e}{\varepsilon\chi^2_{min}}, \tag{2}$$

where $\varkappa_0$ is the static dielectric constant, v is the velocity of an electron of energy ε, and $\chi_{\min}$ is the minimum Coulomb-scattering angle. The function under the logarithm sign is the ratio of the energy transfer to the electron temperature; this expression is derived below. Substituting in (2) the numerical values for GaAs and assuming the logarithmic factor to be equal to unity, we get

$$\frac{1}{\tau_{ee}(\varepsilon)}=\frac{1}{\tau_{ee}}y^{-3/2}, \quad \frac{1}{\tau_{ee}}=10^9\left(\frac{n}{10^{14}}\right). \tag{2a}$$

In scattering by optical phonons, the polarization mechanism prevails in all the investigated crystals. In this case the time of spontaneous emission of an optical phonon is given by

$$\frac{1}{\tau_{PO}(\varepsilon)}=\frac{1}{\tau_{PO}y}\operatorname{Arch}\sqrt{y}, \quad y>1. \quad \frac{1}{\tau_{PO}}=2\alpha\Omega_0, \tag{3}$$

where α is the Fröhlich coupling constant. The induced emission and absorption are proportional to $\exp\{-\hbar\Omega_0/T_e\}$, and can be neglected at helium temperatures. In the photoproduction region (at $y \gg 1$) we can introduce the energy relaxation time

$$\frac{1}{\tilde\tau_{PO}(\varepsilon)}=\frac{1}{\tau_{PO}(\varepsilon)}\frac{\hbar\Omega_0}{\varepsilon}=\frac{1}{\tau_{PO}y}\ln 2\sqrt{y}. \tag{4}$$

For order-of-magnitude estimates, the last formula is suitable also at $y \approx 1$. It is seen from it that $\tilde\tau_{PO} \approx 10^{-11}$ sec in the entire energy range.

If we compare (2) with (4), omitting the logarithmic factors, then

$$\tilde\tau_{PO}(\varepsilon)/\tau_{ee}(\varepsilon)=n/n_c, \tag{5}$$

where n_c is a certain critical concentration. For GaAs we have $n_c \approx 0.7\times10^{18}$ cm^{-3}. Therefore *ee* scattering cannot prevail at $n < 10^{18}$ cm^{-3} over *PO* scattering in the active region $\varepsilon > \hbar\Omega_0$, and the distribution in this region is not Maxwellian. The *A*-scattering is also of little significance in this region compared with *PO* scattering. Thus, the dominant scattering process in the active region is the cascade emission of optical phonons.

The energy distribution of the photoproduced electrons is described by the function

$$G(\varepsilon)=G_0\frac{\Phi(\varepsilon-\varepsilon_0)}{g(\varepsilon)}, \tag{6}$$

where G_0 is the total number of electrons produced in 1 cm^3 per second, $g(\varepsilon)\,d\varepsilon = d^3p/h^3$ is the density of states, and $\Phi(\varepsilon-\varepsilon_0)$ is a form factor localized near $\varepsilon=\varepsilon_0$ in an interval $\Delta\varepsilon_0$, with $\varepsilon_0 \gg \hbar\Omega_0$ and $\Delta\varepsilon_0 \ll \hbar\Omega_0$. The form factor is normalized by the relation

$$\int d\varepsilon\,\Phi(\varepsilon-\varepsilon_0)=1. \tag{7}$$

In cascade emission of optical phonons, the distribution is "multiplied" near the points $\varepsilon_k=\varepsilon_0-k\hbar\Omega_0$, where k is an integer, with $\varepsilon_k > \hbar\Omega_0$. In other words, the electron distribution in the active region is of the form

$$g(\varepsilon)f(\varepsilon)=\sum_{k=0}^{k_0} n_k\Phi(\varepsilon-\varepsilon_k), \quad \hbar\Omega_0<\varepsilon_k\leqslant\varepsilon_0, \tag{8}$$

where n_k is the number of electrons with energies near ε_k. This distribution $f(\varepsilon)$ is shown in Fig. 1.

It follows from (3) that at energies close to $\hbar\Omega_0$ the optical-phonon spontaneous-emission time increases

$$\frac{1}{\tau_{PO}(\varepsilon)}=\frac{1}{\tau_{PO}}\sqrt{y-1}\to 0, \tag{9}$$

i.e., the *PO* scattering ceases to predominate.

If, as will be assumed, ε_0 is not a multiple of $\hbar\Omega_0$, then the electrons can land in this region only through slow processes—electron-electron or acoustic. The number of electrons in this energy region is therefore small.

In the passive region $\varepsilon < \hbar\Omega_0$, the most effective *PO* scattering does not take place, so that *ee* scattering may predominate, in which case the distribution in the passive region is Maxwellian:

$$f(\varepsilon)=\hat n\left(\frac{2\pi m T_e}{h^2}\right)^{-3/2}e^{-\varepsilon/T_e}. \tag{10}$$

This is evidenced quite definitely by the experimental data on luminescence. If $T_e \ll \hbar\Omega_0$, as is the case in all the experiments, then $\hat n$ is equal to the electron concentration in the passive region. The criterion for the existence of the distribution (10) is $\tau_{ee}(\varepsilon) \ll \tilde\tau_A(\varepsilon)$ at $\varepsilon \approx T_e$. Even at the highest temperature observed in GaAs, $T_e \approx 80$ °K, this criterion is satisfied already at $n > 10^{17}$ cm^{-3}.

2. KINETIC EQUATION IN ACTIVE REGION

In the active region, at energies not close to $\hbar\Omega_0$, we can use a kinetic equation that takes into account only the *PO* scattering:

$$g(\varepsilon)G(\varepsilon)-\frac{g(\varepsilon)}{\tau_{PO}(\varepsilon)}f(\varepsilon)+\frac{g(\varepsilon+\hbar\Omega_0)}{\tau_{PO}(\varepsilon+\hbar\Omega_0)}f(\varepsilon+\hbar\Omega_0)=0. \tag{11}$$

Substituting here (8) and integrating over the vicinity of ε_k, we obtain n_k:

$$n_0=G_0\tau_{PO}, \quad n_k=G_0\tau_{PO}(\varepsilon_k). \tag{12}$$

For the distribution function we obtain ultimately the expression

$$g(\varepsilon)f(\varepsilon)=G_0\sum_{k=0}^{k_0}\tau_{PO}(\varepsilon_k)\Phi(\varepsilon-\varepsilon_k). \tag{13}$$

The total number of electrons in the active region is

$$n^{\bullet}=\sum_k n_k=G_0\tau_{PO}^{\bullet}, \tag{14}$$

where

$$\tau_{PO}^{\bullet}=\sum_k \tau_{PO}(\varepsilon_k)=\int\limits_{\hbar\Omega_0}\frac{d\varepsilon}{\hbar\Omega_0}\tau_{PO}(\varepsilon)=\frac{2}{3}\tau_{PO}\left(\frac{\varepsilon}{\hbar\Omega_0}\right)^{3/2} \tag{14a}$$

is the total number that the electron stays in the active region.

At $\varepsilon\gg\hbar\Omega_0$ it is convenient now to introduce the averaged distribution function[7]

$$g(\varepsilon)\bar{f}(\varepsilon)=\int\limits_{\varepsilon}^{\varepsilon+\hbar\Omega_0}\frac{d\varepsilon}{\hbar\Omega_0}g(\varepsilon)f(\varepsilon)=\frac{G_0}{\hbar\Omega_0}\tau_{PO}(\varepsilon). \tag{15}$$

Owing to the condition $\Delta\varepsilon_0\ll\hbar\Omega_0$, the averaged distribution $\bar{f}(\varepsilon)$ does not depend on the type of the form factor $\Phi(\varepsilon)$.

3. ENERGY BALANCE IN PASSIVE REGION

We turn now to the passive region, where the kinetic equation, with allowance for all the scattering and recombination mechanisms, can be written in the form

$$S_{PO}(f|\varepsilon)+S_A(f|\varepsilon)+C_{ee}(f,f|\varepsilon)+R(f|\varepsilon)=0. \tag{16}$$

The individual terms in the left hand side describe the polarization scattering by the optical phonons (*PO*), the scattering by the acoustic phonons (*A*), the electron-electron scattering (*ee*), and the last term describes the recombination.

To obtain the balance equation for the determination of the electron temperature in the passive region, we multiply this equation by $\varepsilon g(\varepsilon)$ and integrate with respect to ε from $\varepsilon=0$ to $\varepsilon=\hbar\Omega_0$. We assume also that the recombination proceeds from the region of small ε and makes no contribution to the energy balance. We then have

$$Q_{PO}+Q_A(T_e)+Q_{ee}(T_e)=0. \tag{17}$$

Here

$$Q_{PO}=-(\varepsilon_{k0}-\hbar\Omega_0)\frac{n_{k0}}{\tau_{PO}(\varepsilon_{k0})}=-(\varepsilon_{k0}-\hbar\Omega_0)G_0 \tag{18}$$

is the rate of increment of the energy of the passive region as a result of the fact that all the photoproduced electrons fall into this region after the termination of the cascade. The term $Q_A(T_e)$ describes the relaxation of the energy of the passive region following acoustic scattering. Since this scattering is quasi-elastic, it follows that $S_A(f|\varepsilon)$ at $\varepsilon<\hbar\Omega_0$ does not contain the $f(\varepsilon)$ distribution at $\varepsilon>\hbar\Omega_0$, and is therefore determined completely by the distribution (10). Since $T_e\lesssim\hbar\Omega_0$, it is possible to extend the integration to $\varepsilon=\infty$ and assume that the distribution is Maxwellian at all ε. We can then readily calculate

$$Q_A(T_e)=\hat{n}\frac{2}{\pi^{1/2}}\frac{2ms^2}{\hbar\Omega_0}(T_e-T)\left[\frac{1}{\tau_{PA}}y_e^{-1/2}+\frac{1}{\tau_{DA}}y_e^{1/2}\right],\quad y_e=\frac{T_e}{\hbar\Omega_0}. \tag{19}$$

The calculation of

$$Q_{ee}(T_e)=\int\limits_0^{\hbar\Omega_0}d\varepsilon\, g(\varepsilon)\varepsilon C_{ee}(f,f|\varepsilon) \tag{20}$$

differs substantially from the calculation of $Q_A(T_e)$. The point is that if we assume the distribution in (20) to be Maxwellian for all ε and extend the integration to $\varepsilon=\infty$ (as was done in the calculation of $Q_A(T_e)$, then we obtain $Q_{ee}(T_e)=0$, since the total energy of the electron system is conserved in *ee* scattering. Therefore the integral (20) calls for a more careful examination. The collision term can be represented in the form of the divergence

$$C_{ee}(f,f|\varepsilon)=-\frac{1}{g(\varepsilon)}\frac{\partial}{\partial\varepsilon}(g(\varepsilon)J_{ee}(\varepsilon)), \tag{21}$$

$$g(\varepsilon)J_{ee}(\varepsilon)=\int\limits_0^{\varepsilon}d\varepsilon'\,g(\varepsilon')\int\limits_{\varepsilon}^{\infty}d\varepsilon''g(\varepsilon'')[f(\varepsilon')W_{ee}(\varepsilon',\varepsilon'')-f(\varepsilon'')W_{ee}(\varepsilon'',\varepsilon')]. \tag{22}$$

Here $J_{ee}(\varepsilon)$ is the electron flux along the energy axis as the result of *ee* scattering with probability $W_{ee}(\varepsilon,\varepsilon')$, which itself depends on $f(\varepsilon)$. However, independently of the form of $f(\varepsilon)$, the probability $W_{ee}(\varepsilon,\varepsilon')$ has a singularity $|\varepsilon-\varepsilon'|^{-3}$, which means predominance of scattering acts with small energy changes. For scattering by a Maxwellian distribution, the probability was calculated by one of us and Mazhuolite[10]:

$$W_{ee}^{M}(\varepsilon,\varepsilon')=\frac{ne^4}{\varkappa_0^2}\frac{1}{4m^2}\frac{T_e}{(\varepsilon\varepsilon')^{1/2}}\frac{1}{|\varepsilon-\varepsilon'|^3}\exp\left\{-\frac{\varepsilon'-\varepsilon}{2T_e}\right\}. \tag{23}$$

At a low energy transfer we can transform the integral expression (22) for the current into a differential one. In the case of a smooth function $f(\varepsilon)$, the current takes the form[10,11]

$$J_{ee}(\varepsilon)=-\left[Q_{ee}(\varepsilon)f(\varepsilon)+D_{ee}(\varepsilon)\frac{\partial f}{\partial\varepsilon}\right]. \tag{24}$$

Explicit expressions for $Q_{ee}(\varepsilon)$ and $D_{ee}(\varepsilon)$ in the form of integrals of $f(\varepsilon)$ can be obtained in the review of Ginzburg and Gurevich.[11] Substitution of the Maxwellian distribution and the use of the probability (23) lead to the expression

$$Q_{ee}(\varepsilon)=\frac{D_{ee}(\varepsilon)}{T_e}=\frac{2\pi ne^4}{\varkappa_0^2(2m\varepsilon)^{1/2}}\ln\frac{T_e}{\varepsilon\chi_{min}^2}=\frac{\varepsilon}{\tau_{ee}(\varepsilon)}, \tag{25}$$

where $\tau_{ee}(\varepsilon)$ is the time of energy exchange between electrons, introduced earlier in (2).

In the present case, when $f(\varepsilon)$ has a discontinuity at $\varepsilon=\hbar\Omega_0$, expression (24) cannot be used, since it does not yield the correct current at energies close to $\hbar\Omega_0$. Yet it follows from (20) and (21) that

$$Q_{ee}(T_e)=\hbar\Omega_0\{g(\varepsilon)J_{ee}(\varepsilon)\}_{\varepsilon\to\hbar\Omega_0-0}, \tag{26}$$

i.e., it is just these values of ε that matter in the calculation of $Q_{ee}(T_e)$.

It is therefore convenient to use the following procedure to calculate $Q_{ee}(T_e)$: We put $W_{ee}=\hat{W}_{ee}+W_{ee}^*$, where the first and second terms represent the probabilities of scattering by passive and active electrons with distributions $\hat{f}(\epsilon)$ and $f^*(\varepsilon)$, respectively. Accordingly,

$$C_{ee}(f,\ f|\varepsilon)=C_{ee}(\hat{f},\ f|\varepsilon)+C_{ee}(f^*,\ f|\varepsilon), \tag{27a}$$
$$J_{ec}(\varepsilon)=\hat{J}_{ee}(\varepsilon)+J_{ee}^*(\varepsilon), \tag{27b}$$
$$Q_{ee}(T_e)=\hat{Q}_{ee}(T_e)+Q_{ee}^*(T_e). \tag{27c}$$

We consider initially the first term of (27a). It contains the distribution $f(\varepsilon)$, in general, for all ε. However, since the dominant scattering events in *ee* scattering are those at small angle of the order of the cutoff angle $\chi_{\min}$, it follows that the effective energy transfer

$$(\Delta\varepsilon)_{ee}=vp(1-\cos\chi)\approx\varepsilon\chi_{min}^2$$

is also small. This means that transfer to the passive region is possible only from a small part of the active region, where $\varepsilon-\hbar\Omega_0\lesssim\hbar\Omega_0\chi_{\min}^2$. If the photoexcitation region does not contain energies that are multiples of $\hbar\Omega_0$, then according to (13) the photoexcited electrons, emitting optical phonons, do not enter that part of the active region which is of interest to us, so that it can be approximately assumed that $f(\varepsilon)=0$ at $\varepsilon\geq\hbar\Omega_0$. The collision term then becomes simpler:

$$C_{ee}(\hat{f},\ f|\varepsilon)=C_{ee}(\hat{f},\ \hat{f}|\varepsilon). \tag{28}$$

In this form, it describes the Maxwellization of the distribution $\hat{f}$ concentrated in the passive region. It is obvious that following this Maxwellization a fraction of the electrons will go over into the active region, to which it will carry part of the energy. It is therefore clear from physical considerations that $\hat{Q}_{ee}(T_e)>0$, i.e., this term describes the outflow of energy from the passive region as a result of collisions of the passive electron. The probability $\hat{W}_{ee}$, which we need to calculate $\hat{Q}_{ee}(T_e)$, can be calculated by replacing the truncated Maxwellian distribution $\hat{f}$ by the full Maxwellian distribution f_M; This replacement leads only to a small change of the transition probability. This means that we use W_{ee}^M in place of $\hat{W}_{ee}$ in accord with (23). Knowing $\hat{W}_{ee}$, we can use (22) to calculate $\hat{J}_{ee}$. Now, however, it is necessary to take into account explicitly the presence of a discontinuity in the distribution $\hat{f}(\varepsilon)$ that enters in (22). We then obtain[6]

$$g(\varepsilon)\hat{J}_{ee}(\varepsilon)=\frac{4\pi^2m\hat{n}e^4}{\varkappa_0^2(2\pi mT_e)^{3/2}}\exp\left\{-\frac{\hbar\Omega_0}{T_e}\right\}\left(\frac{\varepsilon-\hbar\Omega_0}{T_e}\right)^{-1}. \tag{29}$$

It is seen from the last formula that the current $\hat{J}_{ee}$ diverges at the boundary of the passive region. This divergence is due to the Coulomb singularity in (23) and must therefore be cut off at the minimal energy transfer $(\Delta\varepsilon)_{ee}$ at $\varepsilon=\hbar\Omega_0=\hbar\Omega_0\chi_{\min}^2$. We then obtain from (26), with $\hat{J}_{ee}$ in place of J_{ee},

$$\hat{Q}_{ee}(T_e)=\frac{4\pi^2m\hat{n}^2e^4}{\varkappa_0^2(2\pi mT_e)^{3/2}}T_e\exp\left\{-\frac{\hbar\Omega_0}{T_e}\right\}\chi_{min}^{-2}. \tag{30}$$

In contrast to the usual situations, the cutoff parameter is not under a logarithm sign here; in other words, the resultant effective *ee*-scattering frequency is much larger than (2). This is due to the discontinuity of $f(\varepsilon)$ at $\varepsilon=\hbar\Omega_0$. For a smooth function, the electron flow from the passive region to the active and the reverse flow, i.e., both terms of (22), practically cancel each other, whereas in the presence of a discontinuity there is no such cancellation. Formally, the discontinuity causes the appearance, in the transformation of the integral equation for the current into a differential equation, of not only the first and second moments of the transition probability, but also of the quantity

$$\int_0^{\hbar\Omega_0}d\varepsilon'\,g(\varepsilon')\int_{\hbar\Omega_0}^{\infty}d\varepsilon''\,g(\varepsilon'')W_{ee}(\varepsilon',\varepsilon''),$$

which has a stronger divergence than the first and second moments.

We turn now to the second term of (27a), which can be written in the form

$$C_{ee}(f^*,\ f|\varepsilon)=C_{ee}(f^*,\ \hat{f}|\varepsilon)+C_{ee}(f^*,\ f^*|\varepsilon). \tag{31}$$

The second term of (31), which corresponds at $\varepsilon<\hbar\Omega_0$ to scattering of active electrons by active electrons with a transfer to the passive region, is small because the concentration of the active electrons is small. The first term of (31) describes the scattering of passive electrons by active ones, at which the energy of the passive electrons increases. It is therefore clear that $Q_{ee}^*(T_e)<0$, i.e., this term describes the influx of energy into the passive region. To calculate $Q_{ee}^*(T_e)$ it is convenient to rewrite it by using the energy conservation in *ee* scattering, in the form

$$Q_{ee}^*(T_e)=\int_0^{\hbar\Omega_0}d\varepsilon\,g(\varepsilon)\varepsilon C_{ee}(f^*,\hat{f}|\varepsilon)=-\int_{\hbar\Omega_0}^{\infty}d\varepsilon\,g(\varepsilon)\varepsilon C_{ee}(\hat{f},f^*|\varepsilon). \tag{32}$$

In this form, $Q_{ee}^*(T_e)$ denotes the change of the energy of the active electrons when they are scattered by the passive ones. The main contribution to the last integral is determined by the region $\varepsilon\approx\varepsilon_0\gg\hbar\Omega_0$. To calculate it we can therefore use for f^* the averaged distribution (15) rather than the detailed distribution (15), and use the ordinary form of the current (24), where $Q_{ee}(\varepsilon)$ and $D_{ee}(\varepsilon)$ are calculated with the aid of the probability $\hat{W}_{ee}$. In the calculation of $\hat{W}_{ee}$ it is possible, as before, to replace $\hat{f}$ by f_M. It is then easy to verify that if $\varepsilon\gg\hbar\Omega_0$ then the second term in (24) can be neglected, and $\hat{Q}_{ee}(\varepsilon)$ can be calculated from (25) with n replaced by $\hat{n}$. Integrating by parts, we get

$$Q_{ee}^*(T_e)=-\int_{\hbar\Omega_0}^{\infty}d\varepsilon\,g(\varepsilon)\bar{f}(\varepsilon)\hat{Q}_{ee}(\varepsilon). \tag{33}$$

We substitute here (15) and (25) and use (2) and (5). We get then

$$Q_{ee}^{*}(T_e)=-G_0\hat{n}\varepsilon_0/n_c. \tag{34}$$

This quantity is independent of T_e because in the case of scattering of active electrons of energy $\varepsilon \gg \hbar\Omega_0$ by passive electrons of energy $T_e \lesssim \hbar\Omega_0$ the passive electrons can be regarded as immobile, i.e., $T_e=0$.

Summarizing the results, we can write down the energy balance equation of the passive energy, for the determination of the electron temperature, in the form

$$Q_A(\hat{n},\ T_e)+\hat{Q}_{ee}(\hat{n},\ T_e)=G_0[(\varepsilon_{k_0}-\hbar\Omega_0)+\varepsilon_0\hat{n}/n_c]. \tag{35}$$

We have displayed here also the explicit dependence of the relaxation terms in the left-hand side of the equation on the electron concentration in the passive region.

Equation (35) must be solved simultaneously with (14) and with the particle-number balance equation

$$\int\limits_0^\infty d\varepsilon\, g(\varepsilon)G(\varepsilon)+\int\limits_0^\infty d\varepsilon\, g(\varepsilon)R(\varepsilon)=0. \tag{36}$$

At helium temperatures, the equilibrium carrier density can be neglected and we can write

$$R(\varepsilon)=-f(\varepsilon)/\tau_R(\varepsilon). \tag{37}$$

Unfortunately, there is no information whatever concerning the dependence of the lifetime τ_R on ε. Judging from the data of Ulbrich[2] for GaAs at low excitation levels, when the intensity is $I\approx 10^{-3}$ W/cm^2 and the density is $n\approx 10^{13}$ cm^{-3}, the effective value is $\tau_R\approx 10^{-7}$ sec. It is assumed in[1] that $n\approx 5\times 10^{16}$ cm^{-3} for the same material at an excitation intensity $I\approx 1.3\times 10^4$ W/cm^2. The corresponding value is

$$G_0=\frac{I}{\hbar\nu}\frac{1}{d}=1.1\cdot 10^{27}\ \text{cm}^{-3}\text{sec}^{-1}. \tag{38}$$

Here $\hbar\nu=2.4$ eV is the energy of optical quantum and $d=0.3\ \mu$ is the thickness in which the electron production takes place (the ambipolar-diffusion length). It is easily seen that from these data we get $\tau_R\approx 10^{-10}$ sec.

In any case, τ_R exceeds the time that the electron stays in the active region $\tau_{PO}^{*}\approx 10^{-11}$ sec. Therefore $n^{*}\ll\hat{n}$, i.e., almost all the electrons are in the passive region. We can assume that $\hat{n}=n$ in (35).

We compare now the balance equation (35) with the balance equations used in[1,2,4]. The left-hand side of (35) contains two terms corresponding to different mechanisms of energy relaxation in the passive region. The term Q_A, which describes the acoustic relaxation, coincides with that calculated in[2] (the acoustic relaxation was not taken into account in[1,4] at all). However, the term $\hat{Q}_{ee}$ differs in principle from the corresponding term in[1,2,4], which takes in our notation the form

$$\tilde{Q}_{PO}(T_e)=n\hbar\Omega_0\tau_{PO}^{-1}\exp\{-\hbar\Omega_0/T_e\}. \tag{39}$$

This expression, as well as $\hat{Q}_{ee}$, depends on T_e mainly exponentially, with the same argument of the exponential. But $\tilde{Q}_{PO}$ and $\hat{Q}_{ee}$ are determined essentially by different relaxation times, $\tilde{Q}_{PO}$ by the time of spontaneous emission of the optical phonon and $\hat{Q}_{ee}$ by the time of electron-electron scattering. The physical reason is the following.[6] The relaxation of the electron energy in the passive region proceeds in two stages: at first, in a time on the order of τ_{ee}, energy is transferred from the passive to the active region, and this energy is then given up to the lattice in a time on the order of τ_{PO}. The total relaxation time is the sum of these times, and is therefore determined by the longest time (the "bottleneck" of the process). In Stratton's theory the *ee* scattering predominates over the *PO* scattering at all energies, i.e., $\tau_{PO}\ll\tau_{ee}$, so that the bottleneck is the relaxation of the energy to the lattice. Under our assumptions, *PO* scattering predominates in the active region, i.e., $\tau_{PO}\ll\tau_{ee}$, so that the bottleneck is the energy transfer from the passive to the active region.

It should be noted, to be sure, that despite the difference in principle between $\hat{Q}_{ee}$ and $\tilde{Q}_{PO}$, their numerical values for GaAs are quite close. If it is assumed that $\chi_{\min}$ is determined by the ratio of the electron-electron collision parameter at $\varepsilon=\hbar\Omega_0$ and by the Debye length, then we get

$$\chi_{\min}^2=\frac{n}{n_D},\quad n_D=\frac{9}{4}\frac{1}{8\pi}\left(\frac{\hbar\Omega_0}{e^2}\right)^3. \tag{40}$$

For GaAs we obtain $n_D=3\times 10^{18}$ cm^{-3}, i.e., in the considered concentration region we actually have $\chi_{\min}\ll 1$. Expression (30) can now be represented in the form

$$\hat{Q}_{ee}(T_e)=n\frac{\hbar\Omega_0}{\bar{\tau}_{ee}}\left(\frac{\hbar\Omega_0}{T_e}\right)^{1/2}\exp\left\{-\frac{\hbar\Omega_0}{T_e}\right\}. \tag{41}$$

where

$$\frac{1}{\bar{\tau}_{ee}}=\frac{1}{2\sqrt{\pi}}\frac{1}{\tau_{ee}}\bigg|_{n=n_D}.$$

For GaAs we obtain $\bar{\tau}_{ee}^{-1}=0.85\times 10^{13}$ sec^{-1}, which is very close to τ_{PO}^{-1}, so that (39) and (41) differ by an inessential factor $(\hbar\Omega_0/T_e)^{1/2}$, which ranges from two to six.

The right-hand side of (35) contains two terms corresponding to two methods whereby energy can proceed to the passive region. Only the second term was taken into account in[1,4], and only the first in[2]. The first term corresponds to the energy brought by the electron that lands in the passive region at the end of the process of relaxation by optical phonons, while the second term corresponds to the energy transferred by the electron in the active region to the electrons in the passive region via electron-electron collisions during the time of this relaxation. The second term predominates at large concentrations

$$n\gg n_c\hbar\Omega_0/\varepsilon_0. \tag{42}$$

In the experiments with GaAs,[1,2] depending on the quantum energy $\hbar\nu$, we obtain $\varepsilon_0/\hbar\Omega_0=10$ to 20, and (42) changes into the condition $n\gtrsim 5\times 10^{16}$ cm^{-3}. In[2] the concentration was much lower, so that inclusion of only the first term is perfectly justified. The second term can be taken into account in[11], if at all, only at the highest excitation levels.

To analyze the solution of the balance equation (35) and (36), we assume that $\tau_R(\varepsilon)\sim\varepsilon^r$. Substituting in (37) the distribution (10), we obtain from (36)

$$n=G_0\bar{\tau}_R u^r, \tag{43}$$

where $\bar{\tau}_R$ is a certain constant with the dimension of time, and $u\equiv T_e/\hbar\Omega_0$. Using (41), we can express the balance equation (35) at $T_e\gg T$ in the form

$$\frac{1}{\bar{\tau}_A}(pu^{-1/2}+du^{-3/2})+\frac{1}{\bar{\tau}_{ee}}u^{1/2}e^{-u}=\frac{G_0}{n}\left[\Delta y+y_0\frac{n}{n_c}\right], \tag{44}$$

where

$$\bar{\tau}_A{}^{-1}=\tau_{PA}{}^{-1}+\tau_{DA}{}^{-1}.\quad p=\tau_{PA}{}^{-1}/\bar{\tau}_A{}^{-1},\quad d=\tau_{DA}{}^{-1}/\bar{\tau}_A{}^{-1},$$
$$\Delta y=(\varepsilon_{k0}-\hbar\Omega_0)/\hbar\Omega_0<1,\qquad y_0=\varepsilon_0/\hbar\Omega_0.$$

For GaAs we have $\bar{\tau}_A^{-1}=1.94\times10^8$ sec^{-1}, $p=0.29$, and $d=0.71$. From (43) and (44) we see that at low concentration, when the term y_0n/n_c can be neglected, the quantity G_0 drops out of (44), i.e., T_e should not depend on the excitation level. This is not observed in experiment. On the other hand if the term y_0n/n_c predominates, then the concentration n drops out of (44) and we can obtain the dependence of T_e on G_0, i.e., on I. The energy relaxation mechanisms become comparable at $u=14.6$, i.e., $T_e=29$ °K. At $T_e>29$ °K the electron-electron relaxation mechanism predominates, and (44) leads to an exponential dependence of T_e^{-1} on G_0. At $T_e<29$ °K, T_e^{-1} is a power-law function of G_0, i.e., $T_e^{-1}\sim G_0^{-2}$ (the term with the coefficient d makes practically no contribution). The temperature $T=29$ °K at which the change of mechanisms takes place corresponds according to (44) to $G_0\approx10^{24}$ cm^{-3} sec^{-1}, i.e., $I\approx10$ W/cm^2. A similar picture was indeed observed in experiment,[1] where noticeable deviations from an exponential dependence set in near 30 °K at intensities close to 100 W/cm^2. It is not clear, however, why such a fair agreement takes place, for at the concentrations that obtain in the experiment an important role is played not only by the term y_0n/n_c, but also by the term Δy.

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Anti-Stokes resonant Raman scattering of light by excitons in CdS crystals

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A new mechanism for Raman scattering of light, due to the interaction between bound and free excitons in the intermediate state is suggested. We have observed and investigated the continuous stimulated light emission due to recombination of bound excitons following excitation by light from a mercury lamp, as well as the Raman scattering stimulated by this emission at the free-exciton frequency in the CdS crystal.

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1. It is known that the direct radiative recombination of free excitons in semiconductors usually leads to a relatively weak radiation. This is due primarily to three causes: 1) owing to the large absorption coefficient K at the free-excition frequency ($K=2\times10^5$ cm^{-1} [1]) the observed radiation emerges mainly from a layer of thickness $\approx K^{-1}$; 2) the intensity of the radiation depends to a great degree on the state of the semiconductor surface, which determines the lifetime of the exciton in the near-surface region[2,3]; 3) for most free excitons direct radiative recombination is forbidden: the law of momentum conservation can be satisfied when they recombine either as a result of interaction with the lattice, with impurities, or with the surface of the crys-