$\omega_H^{2/3}(\rho/c)^{1/2} \gg \omega_L \gg \omega_H^{1/2}(\rho/c)^{-1/2}$,

and this in turn is possible under the condition $\omega_H \rho/c \gg 1$. These inequalities indicate the range of values of ω_H and ω_L in which both spatial localization and good visibility of the effect are possible.

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[1)]The phase factor is introduced in (5) to simplify the system of equations (6).

[2)]The region of applicability of these equations is restricted by the condition (1), by the inequality $v \ll 1$, and by the requirement, common to all modifications of the geometrical-optics method, that the parameters of the medium vary slowly in terms of the wavelength (for more details see[9]).

[1]M. N. Cohen, Astrophys. J. **131**, 664 (1960).
[2]V. V. Zheleznyakov and E. Ya. Zlotnik, Astron. Zh. **40**, 633 (1963) [Sov. Astron. **7**, 485 (1964)].
[3]V. V. Zheleznyakov, Radioizluchenie Solntsa i planet (Radio Emission of the Sun and Planets), Nauka, 1964.
[4]D. B. Melrose, Aust. J. Phys. **27**, 43 (1974).
[5]J. E. Titheridge, J. Atm. Terr. Phys. **33**, 1533, 1541 (1971).
[6]I. J. Kantor, D. B. Rai, and F. De Mendonca, IEEE Trans. **AP-19**, 246 (1970).
[7]Yu. A. Kravtsov, Dokl. Akad. Nauk SSSR **183**, 79 (1968).
[8]O. N. Naĭda, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **13**, 1496 (1970); **14**, 1843 (1971); **17**, 896 (1974).
[9]O. N. Naĭda, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **17**, 1833 (1974).
[10]V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in Plasma), Nauka, 1967 [Pergamon, 1974].
[11]E. T. Whittaker and G. N. Watson, Modern Analysis, Part II, Cambridge, 1927.

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Relaxation of spin waves with small wave vectors

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It is shown that at small values of the wave vector **k**, the usual kinetic equation for the occupation numbers is not applicable for the calculation of the damping decrement of spin waves $\gamma_{3c}(\mathbf{k})$ due to three-magnon dipole coalescence processes. The effect of four-magnon exchange interaction on spin wave coalescence proceses is taken into account by the diagram technique in all orders of perturbation theory. The expression for $\gamma_{3c}(\mathbf{k})$ for small values of **k** considerably differs from the results derived from the usual kinetic equation. In particular, it is found that three-magnon coalescence processes give a much larger contribution to magnon damping with $\mathbf{k}=0$ than do four-magnon dipole processes.

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At $T \ll T_c$ (T_c is the Curie temperature), many equilibrium and nonequilibrium properties of magnetically ordered crystals are determined by the spin waves.[1,2] In particular, the damping of the spin waves determines the threshold of parametric excitation of the spin system by a variable magnetic field. The relaxation of the magnetization to its equilibrium value, and consequently, the width of the line of magnetic resonance in such crystals are also determined by the damping of the spin waves which, in turn, is due to their interaction with one another, with phonons, impurities, defects of the crystal structure, etc.

In what follows, we shall consider an ideal ferrodielectric and confine ourselves to the case of spin-spin interactions only, to wit, the exchange and relativistic dipole interactions. In this case, the Hamiltonian of the spin waves has the form[1,2]

$$\mathcal{H}=\mathcal{H}_0+V_{3c}^d+V_{3sp}^d+V_{4s}^{ex}+V_{4s,c}^d+W, \qquad (1)$$

where $\mathcal{H}_0$ is the Hamiltonian of the free spin waves, V_{3c}^d and V_{3sp}^d are the contributions from three-magnon processes of coalescence and splitting, due to dipole interaction, V_{4s}^{ex} is the contribution from a four-magnon exchange scattering, $V_{4s,c}^d$ is the contribution from four-magnon dipole processes, and W denotes processes of higher order in the creation and annihilation operators of the spin waves, which, at $T \ll T_c$, make a small contribution. We do not include in the Hamiltonian (1) the interactions of spin waves with the vibrations of the crystal lattice, impurities and defects, both from considerations of simplicity and because the experiment allows us to separate the contribution of the "characteristic" mechanisms of magnon damping due only to spin-spin interactions.[2,3]

The damping of spin waves, i.e., the approach to an equilibrium occupation number $n_k=\langle a_k^+ a_k\rangle$ ($\langle\,\rangle$ denotes averaging) is determined in second-order perturbation theory by the usual kinetic equation

$$\frac{dn_k}{dt}=2\pi\left\{\sum_{lm}|V_{lm}|^2\delta(\varepsilon_l-\varepsilon_m)-\sum_{l'm'}|V_{l'm'}|^2\delta(\varepsilon_{l'}-\varepsilon_{m'})\right\}. \qquad (2)$$

Here $\delta(x)$ is the delta function, V_{lm} are the matrix elements of the interaction operator in the representation in which the Hamiltonian $\mathcal{H}_0$ is diagonal, l and m denote

the initial and final states for those processes in which the spin waves with wave vector k are created and l' and m' are the initial and final states for processes in which they are annihilated.

In the case of a weak departure from equilibrium, the expression on the right side of (2) can be linearized according to $\Delta n_k = n_k - \bar{n}_k$ ($\bar{n}_k$ is the equilibrium value of n_k) and we can find the damping decrement $\gamma(\mathbf{k})$ as a function of $\mathbf{k}$, T and the external magnetic field H. Thus, for example, the damping of the magnons due to three-wave coalescence processes, determined by this method, is of the form

$$\gamma_{3c}(\mathbf{k}) = \frac{1}{2\pi^2}\int d\mathbf{p}\,|V_d(\mathbf{k}+\mathbf{p};\,\mathbf{k},\,\mathbf{p})|^2[n(\varepsilon_p)-n(\varepsilon_p+\varepsilon_k)]\,\delta(\varepsilon_k+\varepsilon_p-\varepsilon_{k+p}), \tag{3}$$

where $V_d(\mathbf{k}+\mathbf{p};\,\mathbf{k},\,\mathbf{p})$ is the amplitude of the specified process, ε_k is the energy of the spin wave with wave vector $\mathbf{k}$.[1,2] (Here and elsewhere, we use the system of units in which Planck's constant, Boltzmann's constant and the Bohr magneton are taken to be equal to unity.)

It is known that the damping of magnons as $\mathbf{k}\to 0$ is determined in the case considered only by the dipole interaction, since the exchange interaction does not make a contribution to the damping of the spin waves with sufficiently small k (because of its short-range nature). Here it follows from (2) that the contribution to the wave damping with $\mathbf{k}=0$ from V^d_{3c} and V^d_{3sp} is equal to zero, since the processes of coalescence and splitting corresponding to these interactions are forbidden by the laws of conservation of energy and of the wave vector that is contained in the collision integral of the kinetic equation (2) (see, for example, (3)). Thus, in second-order perturbation theory, the contribution to the damping of waves with $\mathbf{k}=0$ from spin-spin interactions is determined only by the four-magnon dipole scattering, and the calculation of this contribution from Eq. (2) gives[1]

$$\gamma^d_{4s}(0) \sim \frac{M_0^2}{T_c}\left(\frac{T}{T_c}\right)^2, \tag{4}$$

where M_0 is the saturation magnetization. For example, for yttrium iron garnet (YIG) at room temperature ($M_0 = 140$ Ga, $T_c = 550$ °K) we obtain, in units of the magnetic field, $\gamma^d_{4s}(0) \sim 10^{-3}$ Oe, which is much less than the observed values.

Of course, the prohibition, imposed by the conservation laws on the participation of three-wave processes in the relaxation of magnons with $\mathbf{k}=0$ is valid only with accuracy to the next order of perturbation theory. Therefore, the real contribution of such processes to $\gamma(\mathbf{k}\to 0)$ is determined by corrections of higher order. However, it will be shown below that for three-magnon coalescence processes, the order of the perturbation theory at small $\mathbf{k}$ diverges in the temperature and magnetic field regions achievable experimentally. Account of this circumstance leads, as we shall see, to an effective increase in the contribution to damping of magnons with small $\mathbf{k}$ from three-magnon processes. In particular, the contribution from three-magnon coalescence processes to the damping of magnons with small $\mathbf{k}$ turns out to be much greater than (4).

THE NONAPPLICABILITY OF ORDINARY KINETIC EQUATION FOR PROCESSES OF COALESCENCE OF SPIN WAVES WITH SMALL k

The kinetic equation is applicable, as is well known, to time scales that are much greater than the characteristic interval of time τ_0 for which all the distribution functions of higher orders begin to be determined by specification of the single-particle functions, i.e., the occupation numbers.[4] Since the characteristic time scale of change of n_k is the corresponding time of damping $\tau_k = \gamma_k^{-1}$, then the criterion of applicability of the kinetic equation is written down in the form of the inequality $\tau_k \gg \tau_0$. Thus, in the use of the usual kinetic equation, it is assumed that the results obtained with its help are true with accuracy to the ratio τ_0/τ_k.

We shall show that as $\mathbf{k}\to 0$ the ratio τ_0/τ_k is not a small parameter for three-magnon coalescence processes, and therefore the question of the applicability of the kinetic equation and its accuracy demands additional study. Actually, in its physical meaning, τ_0 is the time within which the correlation between the occupation numbers of the spin waves, which arise in the process of their interaction,[4] vanishes. Consequently, τ_0 can be determined from the damping of correlation functions of the type

$$\langle V_k(t)\,V_k(0)\rangle = \sum_{nm}|V_{nm}|^2\rho_n \exp\{i(E_n-E_m)t\}, \tag{5}$$

where

$$V_k(t) = \exp(i\mathcal{H}_0 t)\,V_k \exp(-i\mathcal{H}_0 t).$$

Here V_k is the considered interaction, responsible for the processes of coalescence of magnons with wave vector k[1]:

$$V_k = \sum_{12} V_d(1,2,k)\,a_1^+ a_2 a_k \Delta(k_1-k_2-k) + \text{H.c} \tag{6}$$

and ρ_n is the probability of finding the system in a state with energies E_n.

Writing down the unperturbed Hamiltonian in the form

$$\mathcal{H}_0 = \sum_k \varepsilon_k a_k^+ a_k,$$

we can easily obtain the result from (6) that as $\mathbf{k}\to 0$, for three-magnon coalescence processes, $E_n - E_m = \varepsilon_0$, i.e., the correlator (5) is not damped but oscillates with constant frequency ε_0. This means that in first approximation, the correlation time increases without limit as $\mathbf{k}\to 0$ and consequently the ratio τ_0/τ_k ceases to be a small parameter. The latter fact also indicates the nonapplicability of the usual kinetic equation.

For investigation of the corrections to the kinetic equation (2) and for calculation of the damping in higher orders of perturbation theory, it is convenient to use the diagram technique. It is well known that at small

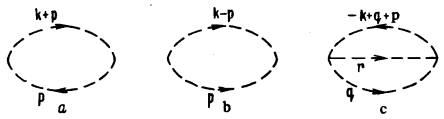

FIG. 1.

departures from equilibrium $\gamma(\mathbf{k}) = \mathrm{Im}\Sigma(\varepsilon_k, \mathbf{k})$, where $\Sigma(\varepsilon_k, \mathbf{k})$ is the mass operator for the equal-time retarded Green's function of the magnon.[1] Since the diagram technique has been worked out for temperature Green's functions, then the problem actually reduces to the estimation of the mass operator for the temperature Green's function, i.e., to the determination of $\Sigma(\varepsilon_n, \mathbf{k})$, where $\varepsilon_n = 2\pi i n T$, $n = 0, 1, 2, \ldots$. Then $\Sigma(\varepsilon, \mathbf{k})$ is found by analytic continuation of $\Sigma(\varepsilon_n, \mathbf{k})$ with a discrete set of points on the upper half-plane of the complex variable ε.[1,5] In particular, the calculation of the damping by the kinetic equation (2) is equivalent to calculation of the mass operator in second-order perturbation theory. Thus, for example, the calculation of the damping decrement from the kinetic equation for processes of coalescence, splitting or scattering of spin waves is equivalent to finding the contribution to the mass operator from the second order diagram, shown in Fig. 1 (the dashed lines here correspond to the zero Green's functions).

In such an approach, the inapplicability of the kinetic equation for coalescence processes at small $\mathbf{k}$ means that the contribution from the diagram in Fig. 1(a) ceases to be a good zero approximation in the diagram series for the mass operator corresponding to this process, $\Sigma_{3c}(\varepsilon, \mathbf{k})$, shown in Fig. 2. Here, the complete Green's functions $G(\varepsilon_n, p) = (\varepsilon_p - \varepsilon_n - \Sigma(\varepsilon_n, p))^{-1}$ correspond to the dashed lines, and Γ means the renormalized vertex. It is not difficult to establish the fact that as $\mathbf{k} \to 0$, the diagram series for Σ_{3c} actually diverges; This can be done by direct calculation of the diagrams of higher orders. The contribution from diagrams of fourth order dominates the contribution from diagrams of second order shown in Fig. 1(a) (in some region of T and H).

CALCULATION OF THE MASS OPERATOR

Thus, the problem of the determination of the damping decrement of the spin wave due to three-magnon coalescence processes reduces to the calculation of the mass operator Σ_{3c} shown in Fig. 2. We first consider the renormalization of the Green's functions $G = (\varepsilon_p - \varepsilon_n - \Sigma)^{-1}$ entering into Σ_{3c}. For this, we first note that here, in the consideration of the total mass operator Σ, we can confine our attention to the contribution from diagrams of second order, shown in Fig. 1, since the perturbation theory works well in this region of intermediate wave vectors $\mathbf{p}$, which make the principal contribution to Σ_{3c}. Second, from the diagrams of second order shown above, we can in what follows take into account the contribution to Σ from the diagrams in Fig. 1(c) only, which corresponds to a four-magnon exchange scattering. The fact is that the contribution to the damping of the spin wave from exchange scattering increases, with increase in the wave vector much more rapidly than the contributions from dipole processes as is well known,[1] and it is clear from the following that the contribution from the exchange is decisive in the fundamental region of integration in Σ_{3c} over the intermediate wave vectors $\mathbf{p}$. Finally, we can write the renormalized Green's function in the form

$$G(\varepsilon_n, \mathbf{p}) = (\varepsilon_p - \varepsilon_n - i\,\mathrm{Im}\,\Sigma_{ex}(\varepsilon_n, p))^{-1}, \tag{7}$$

where Σ_{ex} is the exchange part of the total mass operator in second-order perturbation theory, graphically shown in Fig. 1(c) (the real part of Σ_{ex} in the considered problem is unimportant, since it leads only to renormalization of the exchange constant in the spectrum of spin waves.[1]).

We now proceed to the renormalization of the vertex Γ and show that, under the experimental conditions, the noted renormalization can be neglected, i.e., in the calculation of the mass operator Σ_{3c}, we can limit ourselves to renormalization of the Green's functions (7) and in place of Γ there remains the bare vertex $V_d(k+p; k, p)$. For this purpose, we consider first Σ_{3c} with account only of the bare vertex $V_d(\mathbf{k}+\mathbf{p}; \mathbf{k}, \mathbf{p})$, the dependence of which on $\mathbf{k}$ and $\mathbf{p}$ is known.[1] Using (7), it is not difficult in such a case to obtain, after corresponding analytic continuation,[1)] the following expression for the imaginary part of the desired mass operator Σ_{3c} (we denote it in this approximation by $\hat{\Sigma}_{3c}$):

$$\mathrm{Im}\,\hat{\Sigma}_{3c}(\varepsilon_k, \mathbf{k}) = \frac{1}{4\pi^3}\int \frac{d\mathbf{p}\,|V_d(\mathbf{k}+\mathbf{p}; \mathbf{k}, \mathbf{p})|^2 [\gamma_{ex}(\mathbf{p}) + \gamma_{ex}(\mathbf{k}+\mathbf{p})]}{(\varepsilon_k + \varepsilon_p - \varepsilon_{k+p})^2 + [\gamma_{ex}(\mathbf{p}) + \gamma_{ex}(\mathbf{k}+\mathbf{p})]^2}$$
$$\times [n(\varepsilon_p + \varepsilon_k) - n(\varepsilon_{p+k} - \varepsilon_k) - n(\varepsilon_p) + n(\varepsilon_{p+k})], \tag{8}$$
$$n(\varepsilon_p) = [\exp(\varepsilon_p/T) - 1]^{-1},$$

and for ε_p we limit ourselves in further estimates to the simple dispersion law $\varepsilon_p = H + Dp^2$ (D is the exchange constant). Moreover, $\gamma_{ex}(p) = \mathrm{Im}\,\Sigma_{ex}(\varepsilon_p, p)$, i.e., $\gamma_{ex}(p)$ is the damping decrement due to four-magnon exchange scattering, and for those $\mathbf{p}$ which make a fundamental contribution in the integration in (8),

$$\gamma_{ex}(p) \sim T_c (T/T_c)^{3/2} (ap)^3,$$

where a is the lattice constant.[1,6]

With the help of (8), we estimate the interval of intermediate wave vectors $\mathbf{p}$ which gives the principal contribution to the mass operator Σ_{3c}. Making the substitution $x = Dp^2/T$ and neglecting the dependence $V_d(\mathbf{k}+\mathbf{p}; \mathbf{k}, \mathbf{p})$ on the angular variables, it is not difficult to rewrite (8) at $\mathbf{k} = 0$ in the form

$$\mathrm{Im}\,\hat{\Sigma}_{3c} \sim \frac{V_d^2}{T_c}\left(\frac{T_c}{T}\right)(e^{H/T} - 1)\int \frac{dx\, x^2 e^{x+H/T}}{(\delta^2 + x^3)(e^{x+H/T} - 1)^2}, \tag{9}$$

where

$$\delta \sim \left(\frac{H}{T}\right)^{3/2}\left(\frac{T_c}{T}\right)^2 \sim \left(\frac{H}{\gamma_{ex}T}\right)^{3/2}. \tag{10}$$

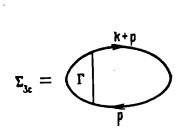

FIG. 2.

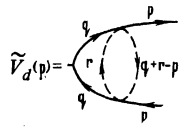

FIG. 3.

Here $\gamma_{ex}^T \sim T_c(T/T_c)^4$ is the mean damping due to four-magnon exchange scattering.[1]

It follows from (9) that at $H \gg \gamma_{ex}^T$, i.e., when $\delta \gg 1$, the principal contribution to the integral is made by the thermal intermediate states ($x \sim 1$). In this case, the series of perturbation theory for Σ_{3c} is seen to converge in the parameter γ_{ex}^T/H and such a situation was considered in Ref. 7. In the present work, we consider the opposite case, $\delta \ll 1$, which is frequently realized under experimental conditions (YIG, room temperature).

We now show that at $\delta \ll 1$ renormalization of the vertex makes a small contribution in the calculation of Σ_{3c}. At $\delta \ll 1$, the principal contribution to (9) is made by $x \sim \delta$. Therefore, as $\mathbf{k} \to 0$, the problem reduces to the estimate of $\Gamma(\mathbf{p})$ in the corresponding region of phase space, i.e., for $p \sim p_0$, where p_0 is determined by the condition $\gamma_{ex}(p_0) \sim H$. In this range of wave vectors, the principal role is played by the processes of exchange scattering,[1] which has already been taken into account in the renormalization of the Green's functions (see (7)). Consequently, even in the renormalized vertex, the principal contribution is made by the exchange corrections, and it is not difficult to establish the fact that for small input wave number $\mathbf{k}$, the greatest contribution to $\Gamma(p)$ is made by the diagram shown in Fig. 3. We estimate this diagram by noting that in the region $\gamma_{ex}(p_0) \sim H$, the contribution from it can be rewritten, obviously, in the form

$$\tilde{V}_d(p_0) \approx V_d \int \frac{dq\,dr}{H+\gamma_{ex}(q)} \gamma_{q,r}(p_0), \tag{11}$$

where $\gamma_{q,r}(p_0)$ is an expression, not integrated over the interior lines, corresponding to the diagram of Fig. 1(c) at $k=p_0$. After substitution of the well known expression for $\gamma_{q,r}$,[1] the integral (11) is easily determined and the result takes the form $\tilde{V}_d(p_0) \sim V_d \delta^{1/2}$. The latter also means that in the considered case $\delta \ll 1$, renormalization of the vertex can be neglected, i.e., we can limit ourselves to renormalization of the Green's function. It then follows that the damping decrement of the spin waves due to three-magnon coalescence processes is determined by Eq. (8).

REGION OF APPLICABILITY OF THE KINETIC EQUATION. DISCUSSION OF THE RESULTS

In the case $H \ll \gamma_{ex}^T$, i.e., at $(H/T_c)(T_c/T)^4 \ll 1$, the damping decrement due to three-wave coalescence processes of spin waves has the form (see (8))

$$\gamma_{3c}(\mathbf{k}) = \frac{1}{2\pi^3} \int \frac{d\mathbf{p}\,|V_d(\mathbf{k}+\mathbf{p};\mathbf{k},\mathbf{p})|^2 \gamma_{ex}(\mathbf{p})}{(\varepsilon_k+\varepsilon_p-\varepsilon_{k+p})^2+4\gamma_{ex}^2(\mathbf{p})} \times [n(\varepsilon_p+\varepsilon_k)-n(\varepsilon_{p+k})-n(\varepsilon_p)+n(\varepsilon_{p+k})]. \tag{12}$$

As $\gamma_{ex}(\mathbf{p}) \to 0$, the obtained expression transforms into (3) as it should, i.e., into the result which follows from the ordinary kinetic equation (2).

Comparison of (12) with (3) shows that the expression (12) for $\gamma_{3c}(\mathbf{k})$ found in the work corresponds formally to the approximation of the "broadened delta function," i.e., to the replacement of the δ function entering into (3) by the Lorentz formula. Such a replacement of the δ function is sometimes simply postulated, as has been done, for example, by Simons in the calculation of the damping of phonons[9] and by White and Sparks in the calculation of the instability threshold of spin waves.[10] However, it follows from the results of the present work that the approximation of the "broadened delta function" is correct only in those special cases in which in the calculation of the mass operator we can restrict ourselves to the renormalization of the Green's functions and neglect the renormalization of the vertex, i.e., acutally, in a restricted region of change of the characteristic physical parameters of the problem (for example, for $(H/T_c)(T_c/T)^4 \ll 1$ in our case).

We now determine the interval of values of the wave vector $\mathbf{k}$ for which the kinetic equation (2) is inapplicable, i.e., when it is necessary to use the relation (12). Since (12) enters into (3) at $\gamma_{ex}(\mathbf{p}) \to 0$, then, consequently, the expression (3) will be applicable only for such large $\mathbf{k}$ when we can neglect the exchange damping $\gamma_{ex}(\mathbf{p})$ in the denominator of the integrand expression in (12) in comparison with $\varepsilon_k + \varepsilon_p - \varepsilon_{k+p} = H - 2D\mathbf{k}\cdot\mathbf{p}$. Since the basic contribution to the integration in (12) as $\mathbf{k} \to 0$ is made by the values of $\mathbf{p}$ which satisfy the condition $\gamma_{ex}(\mathbf{p}) \sim H$, the region of applicability over $\mathbf{k}$ of the kinetic equation (2) is determined by the relation $Dkp \gg \gamma_{ex}(\mathbf{p}) \sim H$. Then we obtain the following value of the limiting wave vector k_{lim}:

$$ak_{lim} \sim (T/T_c)^{1/3}(H/T)^{2/3}. \tag{13}$$

Thus, the kinetic equation (i.e., Eq. (3)) is applicable for the calculation of the damping in three-magnon processes of confluence in the range $k \gg k_{lim}$, and for $k \leq k_{lim}$ the damping decrement is determined by the expression (12).

For damping of magnons with $k=0$, it is not difficult to obtain

$$\gamma_{3c}(0) \sim \frac{M_0^2}{T_c}\left(\frac{T}{T_c}\right)^{1/2}\left(\frac{T}{H}\right)^{1/2}. \tag{14}$$

from (12) with $k=0$. Comparing (14), with (4), we find

$$\frac{\gamma_{3c}(0)}{\gamma_{4s}^d(0)} \sim \left(\frac{T_c}{T}\right)^{1/2}\left(\frac{T}{H}\right)^{1/2} \gg 1. \tag{15}$$

It then follows that the corrected kinetic equation materially changes the point of view with regard to the role of three-magnon processes in the damping of spin waves with small wave vectors: in the framework of the approach based on the usual kinetic equation, it was assumed, in contrast with (15), that the contribution $\gamma_{3c}(0)$ can be neglected in comparison with $\gamma_{4s}^d(0)$.[1,2]

We note that as $H \to 0$ the damping should naturally remain finite. This is violated in (14) for the reason that we, for simplification of the calculations, made use of a simple dispersion law, not taking into account the dipole interaction. This means that (14) is applicable

for $H \gg M_0$, i.e., when the external magnetic field is much greater than the local fields of the crystal.

At $\mathbf{k} \neq 0$, the calculation of the integral in (12) becomes complicated, with the exception of the case $k \ll k_{11m}$, when it is not difficult to obtain the following expression for $\gamma_{3c}(k)$:

$$\gamma_{3c}(k) = \gamma_{3c}(0) + \gamma_1 a^2 k^2, \quad \gamma_1 \sim \frac{M_0^2}{T_c}\left(\frac{T_c}{T}\right)^{7/6}\left(\frac{T}{H}\right)^{5/3}. \tag{16}$$

Estimate of k_{11m} from the relation (14) for YIG at room temperature and $H \sim 10^3$ Oe gives $k_{11m} \sim 10^4$ cm^{-1}. This value lies on the lower boundary of this region of k, which has been studied experimentally up to the present time,[2,3] which makes difficult the comparison of the obtained results with experiment. We note that the estimate $\gamma_{3c}(0)$ from the relation (14) gives $\gamma_{3c}(0) \sim 10^{-2}$ Oe for YIG at room temperature, which can be a significant fraction of the total line width of the better samples.[2] It is also possible that the experimentally observed[3] deviation from the expected linear law for the dependence of $\gamma(0)$ on T is connected with the contribution of the three-magnon coalescence processes considered in the work (according to (14), the contribution from such processes to $\gamma(0)$ changes with temperature as $T^{11/6} \sim T^2$).

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[1)]Analytic continuation is carried out by the standard method; see, for example, Ref. 8.

[1]A. I. Akhiezer, V. G. Bar'yakhtar and S. V. Peletminskiĭ, Spinovye volny (Spin waves) Nauka, 1967 [Pergamon, 1971].
[2]A. G. Gurevich, Magnitnyĭ rezonans v ferritakh i antiferromagnetikakh (Magentic resonance in ferrites and antiferromagnetics) Nauka, 1973.
[3]A. G. Gurevich and A. N. Anisim ov, Zh. Eksp. Teor. Fiz. 68, 677 (1975) [Soviet Phys. JETP 41, 336 (1975)].
[4]R. Peierls, Quantum Theory of Solids (Russian translation, IIL, 1956).
[5]A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics) Fizmatgiz, 1968 [Pergamon].
[6]J. Shy-Yih Wang, Phys. Rev. B6, 1908 (1972).
[7]V. L. Ginzburg and M. I. Kaganov, Fiz. Tverd. Tela. 10, 3293 (1968) [Soviet Phys. Solid State 10, 2603 (1969)].
[8]T. Holstein, Annals of Phys. 29, 410 (1964).
[9]S. Simons, Proc. Phys. Soc. (London) 82, 401 (1963).
[10]R. White and M. Sparks, Phys. Rev. 130, 632 (1963).

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The resistive state and pinning in deformed single crystals of niobium

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The current-voltage characteristics, pinning, and dislocation structure in deformed single crystals of niobium are studied. An effect of plastic deformation on the current-voltage characteristics near the upper critical field H_{c2} is observed. The observed features of the resistive state, the effect of deformation on the current-voltage characteristics, and the peak effect are discussed in terms of models that account for the possibility of varying the number of moving vortices and for the increase in the dynamic pinning force near H_{c2} in materials with an inhomogeneous distribution of pinning centers.

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I. INTRODUCTION

The study of the resistive state, consisting of the current-voltage characteristics of type-II superconductors in the mixed state, yields the most complete information on pinning and on the motion of the vortex lattice. In most studies of type-II superconductors the current-voltage characteristics, observed while varying the external magnetic field H, are a series of curves with an initial nonlinear portion, becoming linear with further increase of the current. The slope of the linear portion is proportional to the magnetic field.[1,2]

Since the reports by Kim and coworkers were published[1,2] it has been assumed that changes in the structural state of the superconductor (its imperfection or inhomogeneity) do not affect the shape of current-voltage characteristics, causing only their parallel shift along the current axis.[1,2] An unusual behavior, however, of the current-voltage characteristics (near H_{c2}, at least) was observed in a number of studies, for example, in deformed Nb-Ti and Nb-Ta alloys,[3-5] and in recrystallized foils of an Nb-Zr alloy.[6] We have recently reported[7] features of the current-voltage characteristics in stress-deformed single crystals of niobium near H_{c2}.

The present work is devoted to the features of the