Transition bremsstrahlung of relativistic particles

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High frequency ($\omega \gg \omega_{pe} = (4\pi n e^2/m_e)^{1/2}$) bremsstrahlung of electromagnetic waves in a plasma or a condensed medium are investigated in the quasi-classical limit. Scattering of virtual photons by the self-charge of the test particle (ordinary or Compton bremsstrahlung) as well as by the dynamic polarization (transition bremsstrahlung) is taken into acount. The frequency ranges in which interference occurs between the radiation corresponding to the ordinary Compton bremsstrahlung and the radiation corresponding to transition bremsstrahlung are studied. The limits of applicability of the Bethe-Heitler formulas [Proc. Roy. Soc. **146**, 83 (1934)] for a plasma are determined and it is shown that in the low-frequency range the bremsstrahlung in a plasma is determined completely by a new transition mechanism, and the Bethe-Heitler formula is not valid.

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1. INTRODUCTION

Bremsstrahlung in a plasma is of interest in connection with its possible use to generate and amplify waves in a wide frequency range $\omega_{pe} \leq \omega \leq \varepsilon/\hbar$, where ω_{pe} is the electron plasma frequency and ε is the particle energy. It is also known that in installations used for magnetic plasma containment, an appreciable fraction of the internal energy of the plasma can be lost in the form of bremsstrahlung. In addition, the bremsstrahlung instabilities of the epithermal particles, together with the various types of instabilities existing in a plasma, can hinder considerably the stability and containment of the plasma.

According to contemporary concepts, bremsstrahlung as well as other types of radiation can play a definite role in the generation of radio emission from astrophysical objects, such as pulsars.[2–4] Powerful directional beams of electrons can produce in this case coherent radiation amplified as a result of the motion energy of the beam.[5,6] A new type of scattering—transition scattering—was considered in[7].

It was shown earlier[5,8] that bremsstrahlung is produced in a plasma as a result of conversion of virtual quanta into real ones upon scattering either by the self-charge of the particle or by the dynamic polarization of the plasma that surrounds the charges in the plasma. The latter bremsstrahlung mechanism is possible only in the presence of a medium and cannot exist in vacuum. The scattering associated with it is frequently called nonlinear,[9] since the charge of the dynamic polarization of the plasma depends nonlinearly on the electric fields. A more accurate and general designation is "transition scattering," and has been investigated in detail earlier.[7] The purpose of the present study was to investigate a new mechanism of relativistic-particle bremsstrahlung, which can be called transition bremsstrahlung. It is the result of transition scattering of virtual quanta produced in particle collisions.

Transition scattering is possible from heavy particles, and in particular also in the approximation in which their mass is assumed to be infinite.[7,9] Transition bremsstrahlung therefore differs qualitatively from the usual bremsstrahlung, since it occurs also when both colliding particles have infinite mass. It turns out to be significant also in electron-ion collisions which produce, as shown below, radiation of the same order as and sometimes also much stronger than ordinary bremsstrahlung. The results obtained in the present paper show that the unjustified neglect of transition radiation (as has been the practice until now) can lead to incorrect estimates of the role of multiple scattering in bremsstrahlung, a scattering first taken into account in[10]. Particular attention is paid in the present paper to bremsstrahlung in a plasma, for in this case all the calculations can be carried through to conclusion without the use of any model representations for the dielectric constant. The results can be qualitatively extended to include other condensed media, provided that the plasma approximation can be used for the dielectric constant. In a solid-state plasma, the results can be qualitatively used by replacing the average electron thermal velocity v_{Te} by the velocity v_F on the Fermi surface.

A general analysis of bremsstrahlung in a plasma is presented in[5,8]. The actual analysis of the radiation intensity in the general case entails great difficulties. We confine ourselves here to the case of bremsstrahlung of electromagnetic waves in the frequency spectrum region $\omega \approx kc \gg \omega_{pe}$ from relativistic fast particles. Although in this frequency region the plasma has little effect on the propagation of transverse waves, it can greatly influence the matrix elements of the Compton and transition scattering of virtual quanta into real ones, and also the spectral distribution of the radiation intensity. We shall consider separately the influence of plasma on both the ordinary and transition bremsstrahlung, and on their interference.

The influence of the polarization of the medium on bremsstrahlung was investigated earlier by Melrose,[3] but his results were qualitatively incorrect because of the neglect of the transition bremsstrahlung.

2. PROBABILITY OF EMISSION OF ELECTROMAGNETIC WAVES

Let us obtain the emission probability and the spectral distribution of the intensity of bremsstrahlung in a plasma. We assume for simplicity that the plasma consists of nonrelativistic thermal electrons and ions. We consider bremsstrahlung generated in a plasma by an ultrarelativistic electron in pair collisions with plasma ions. In addition, bremsstrahlung is produced also in pair collisions between two ultrarelativistic beam electrons and the plasma thermal electrons. However, according to Bethe and Heitler,[1] at $Z \gg 1$, where Z is the ion charge multiplicity, the principal radiation is produced in the plasma in collisions with ions. The transition bremsstrahlung produced by collision between two ions, one at rest and the other relativistic, can also be described by the results obtained below, since these results do not depend on the incident-particle mass.

In the quasiclassical approximation, we have previously obtained[8] a general expression for the total matrix element of the bremsstrahlung of waves with arbitrary polarization, produced in pair collisions of two relativistic particles of the beam. In the particular case when one of the particles is an immobile ion and the other a relativistic electron, we have from[8] for the total matrix element of the bremsstrahlung of transverse waves (with polarization across the propagation direction) ($\hbar = 1$)

$$\mathbf{M}=\mathbf{M}^{(C)}+\mathbf{M}^{(t)}, \tag{1}$$

where

$$\mathbf{M}^{(C)} = \left(\pi^2 \frac{\partial}{\partial\omega}(\omega^2\varepsilon_{\mathbf{k}}^{t})_{\omega=\omega_{\mathbf{k}}^{t}}\right)^{-1/2} \frac{e_e^2 e_i \sqrt{2}}{m_e\gamma|\mathbf{k}|\varkappa^2\varepsilon_{\varkappa}^{l}(\omega-\mathbf{kv})} \times \left([\mathbf{k}\times\boldsymbol{\varkappa}] + [\mathbf{k}\times\mathbf{v}]\frac{(\mathbf{k}\boldsymbol{\varkappa} - \omega(\boldsymbol{\varkappa}\mathbf{v})/c^2)}{\omega-\mathbf{kv}}\right) \tag{2}$$

is the Compton-radiation matrix element, and

$$\mathbf{M}^{(t)} = \left(\pi^2 \frac{\partial}{\partial\omega}(\omega^2\varepsilon_{k}^{t})_{\omega=\omega_{\mathbf{k}}^{t}}\right)^{-1/2} \frac{e_e^2 e_i \sqrt{2}}{m_e|\mathbf{k}|\omega\varkappa^2\varepsilon_{\varkappa}^{l}} \Bigg\{[\mathbf{k}\times\boldsymbol{\varkappa}]\frac{1-\varepsilon_{-\varkappa-k}^{l(e)}}{\varepsilon^{l}_{-\varkappa-k}} + \varkappa^2(1-\varepsilon_{\varkappa}^{l(e)})\Bigg\{\frac{[\mathbf{k}\times\boldsymbol{\varkappa}]}{(\boldsymbol{\varkappa}+\mathbf{k})^2\varepsilon^{l}_{-\varkappa-k}} + \frac{(\boldsymbol{\varkappa}+\mathbf{k},\mathbf{v})}{(\boldsymbol{\varkappa}+\mathbf{k},\mathbf{v})^2\varepsilon^{t}_{-\varkappa-k}-(\boldsymbol{\varkappa}+\mathbf{k})^2c^2}\left([\mathbf{k}\times\mathbf{v}]-\frac{[\mathbf{k}\times\boldsymbol{\varkappa}](\mathbf{k}+\boldsymbol{\varkappa},\mathbf{v})}{(\boldsymbol{\varkappa}+\mathbf{k})^2}\right)\Bigg\}\Bigg\} \tag{3}$$

is the matrix element of the nonlinear transition radiation.

In formulas (2) and (3), e_e and e_i are the charges of the electron and ion, m_e, $\mathbf{v}$, and ε are the rest mass, velocity, and energy of the relativistic electron, $\gamma = \varepsilon/m_e c^2$, ε_k^l and ε_k^t are the longitudinal and transverse dielectric constants of the plasma, and $\varepsilon_k^{l(e)}$ is the electronic part of the longitudinal dielectric constant. In (2) and (3) we have introduced also the four-dimensional vectors

$$k_i=\left\{\mathbf{k},\frac{\omega}{c}\sqrt{\varepsilon_k^t}\right\};\quad \varkappa_i=\{\boldsymbol{\varkappa},0\};$$
$$(\varkappa+k)_i=\{\boldsymbol{\varkappa}+\mathbf{k},\ (\boldsymbol{\varkappa}+\mathbf{k},\ \mathbf{v})\};$$
$$i=1,\ 2,\ 3,\ 4,$$

where $\omega=\omega_k^t$ and $\mathbf{k}$ are the frequency and wave vector of the emitted photon, and $\boldsymbol{\varkappa}$ is the recoil momentum transferred from the electron to the ion.

We note also that Eq. (3) derived from the general formula of[8] includes the condition $\omega \gg kv_{Te}$, where v_{Te} is the average thermal velocity of the plasma electrons.

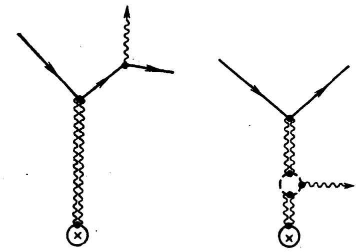

FIG. 1. Diagrams of ordinary and transition bremsstrahlung.

Not being interested in polarization effects, we get from (1)–(3) for the total bremsstrahlung probability of the transverse waves, summed over the two photon polarization directions,

$$w(\boldsymbol{\varkappa},\ \mathbf{k})=(2\pi)^6|\mathbf{M}^{(C)}+\mathbf{M}^{(t)}|^2\delta(\omega-\mathbf{kv}-\boldsymbol{\varkappa}\mathbf{v}). \tag{4}$$

The expression under the δ-function sign in (4) is the consequence of the energy conservation law in the quasiclassical limit. It follows from (4) that interference sets in between the Compton and the transition bremsstrahlung and that, strictly speaking, the two effects cannot be separated. It is typical that what are added here are not the probabilities but the matrix elements for the scattering of virtual quanta into real ones.

It follows from (2)–(4) that in the considered quasiclassical limit the probability of the bremsstrahlung is equal to the probability of the scattering of the beam electron by the potential φ of the immobile ion,

$$\varphi(\boldsymbol{\varkappa})=e_i/\varkappa^2\varepsilon^l(\boldsymbol{\varkappa},\ 0),$$

multiplied by the probability of the emission in scattering. This will be violated in the quantum limit ($\hbar\omega_k^t \sim \varepsilon$) both for vacuum[12] and for a plasma. However, the transition bremsstrahlung in the cases of practical interest is generated in the quasiclassical region, since the value of the polarization charge ($\varepsilon_k^{l(e)}-1$) decreases with frequency at frequencies exceeding certain critical values.

Figure 1 shows diagrams corresponding to the matrix elements (2) and (3). The expressions for the individual matrix elements are determined from these diagrams in accordance with the rules given in[5], the circle with the cross in the figure corresponding to an immobile heavy ion.

3. INFLUENCE OF PLASMA POLARIZATION ON BREMSSTRAHLUNG INDUCED BY COMPTON SCATTERING

We proceed to consider the influence of plasma on the angular distribution of the intensity of the radiation produced separately by each of the indicated processes. Although interference occurs between them, analysis shows that in our problem this interference is significant in a limited region of frequency, in view of the different dependences of the matrix elements $\mathbf{M}^{(C)}$ and $\mathbf{M}^{(t)}$

on the frequency. We consider first the radiation produced by Compton scattering of virtual quanta into real ones. Starting from (2) and (4) we have for the angular dependence of the spectral intensity of the radiation in Compton scattering

$$\frac{dI^{(C)}_{\omega,x}}{dx\,d\omega}=\frac{2e_i^2r_e^2\omega^2}{\gamma^2(\omega-kvx)^2}\left(1-\frac{(1-x^2)(\omega^2+\omega_{pe}^2\gamma^2)}{2\gamma^2(\omega-kvx)^2}\right)$$
$$\times\left\{\ln\frac{1+\varkappa_{max}^2d^2}{1+d^2(\omega-kvx)^2/v^2}-\frac{\varkappa_{max}^2d^2}{1+\varkappa_{max}^2d^2}\left(1-\frac{(\omega-kvx)^2}{v^2\varkappa_{max}^2}\right)\right\},\qquad(5)$$

where $x=\cos\vartheta$, ϑ is the angle between $\mathbf{k}$ and $\mathbf{v}$, $r_e=e_e^2/m_ec^2$ is the classical radius of the electron, $\varkappa_{max}$ is the maximum recoil momentum from the electron to the ion, d is the radius of the Debye sphere in the plasma: $d=[T/4\pi ne^2(1+Z)]^{1/2}$ (at $T_e=T_1=T$; Z_e is the plasma-ion charge and differs from the charge e_i the plasma ion that takes part in the considered radiation process).

Of practical interest is the case when $\varkappa_{min}\ll\varkappa_{max}$ and $\varkappa_{max}d\gg1$. Taking into account also the directivity of the angular distribution of the radiation (in a small angle along $\mathbf{v}$), and discarding small terms, we get in place of (5)

$$\frac{dI^{(C)}_{\omega,x}}{dx\,d\omega}=\frac{2e_i^2r_e^2\omega^2}{\gamma^2(\omega-kvx)^2}\left(1-\frac{(1-x^2)(\omega^2+\gamma^2\omega_{pe}^2)}{2\gamma^2(\omega-kvx)^2}\right)$$
$$\times\left(\ln\frac{\varkappa_{max}d^2}{1+d^2(\omega-kvx)^2/v^2}-1\right).\qquad(6)$$

It is seen from (6) that for ultrarelativistic electron velocities the bremsstrahlung in the frequency region $\omega\approx kc\gg\omega_{pe}$ in the case of the Compton radiation mechanism has sharp directivity. The photon is emitted predominantly forward along the direction of motion of the electron. The radiation-directivity cone apex angle $\Delta\vartheta$, in which the radiation intensity has a sharp maximum, is equal to

$$\Delta\vartheta\approx(1/\gamma^2+\omega_{pe}^2/\omega^2)^{1/2}.\qquad(7)$$

To the contrary, as follows from (5), waves with frequency on the order of the plasma frequency, i.e., $\omega\approx\omega_{pe}\gg kc$, are emitted, with high accuracy, isotropically relative to the direction of motion of the electron.

In the frequency region $\omega_{pe}\ll\omega\ll\gamma\omega_{pe}$, the apex angle of the directivity cone becomes broader in comparison with the radiation in vacuum where, as is well known, [12] $\Delta\vartheta\approx\gamma^{-1}$. This effect was taken into account by Melrose.[3] On the other hand, the decrease of the total intensity of the bremsstrahlung at $\omega\ll\omega_{pe}\gamma$ was investigated by Ter-Mikaelyan[11] and was called the density effect in bremsstrahlung. The same results were obtained later by Melrose[3] and called the Razin-Tsytovich effect in bremsstrahlung. The role of the plasma, however, does not reduce to this effect alone.

Besides the transition bremsstrahlung, which will be investigated below, a change takes place in the screening effect and leads to a change in the expressions under the logarithm sign, and furthermore at frequencies much higher than $\omega_{pe}\gamma$. Formula (6) takes correct account of the screening in the plasma. In this connection, it is necessary to discuss also in greater detail the question of the values of the largest ($\varkappa_{max}$) and the smallest ($\varkappa_{min}$) recoil momenta in the plasma. In a reference frame connected with the fast electron, the lowest frequency ω'_{min} of a quantum emitted in the direction $\mathbf{v}$ is

$$\omega'_{min}=\frac{\omega}{2\gamma^2}\left(1+\gamma^2\frac{\omega_{pe}^2}{\omega^2}\right).\qquad(8)$$

In vacuum we have correspondingly

$$\omega'_{min}=\omega/2\gamma^2,\qquad(9)$$

i.e., $\omega=\gamma\omega_{pe}$ is the threshold frequency at which the character of the Doppler shift of the frequency in the plasma changes, as does also the character of the spectral distribution of the intensity (6).

The quantity $\varkappa_{min}$ can be obtained from (4)

$$\varkappa_{min}(\vartheta)=(\omega-kv\cos\vartheta)/v.\qquad(10)$$

It follows from (8) and (10) that at $\omega\gg\omega_{pe}$ the smallest momentum

$$\varkappa_{min}(0)=\frac{\omega'_{min}}{v}\approx\frac{\omega}{2\gamma^2c}\left(1+\gamma^2\frac{\omega_{pe}^2}{\omega^2}\right)\qquad(11)$$

is given up by the electron to the ion in the case when the photon is emitted in the direction of $\mathbf{v}$. To determine $\varkappa_{max}$ we recognize that in the general case the recoil momentum is equal to

$$\varkappa=\mathbf{p}-\mathbf{p}_1-\mathbf{k},\qquad(12)$$

where $\mathbf{p}$ and $\mathbf{p}_1$ are the momenta in the initial and final states of the electron. We shall take it into account here, first, that the emitted photon is concentrated mainly inside a narrow cone, and second, that in the quasiclassical limit we have $|\mathbf{k}|\ll|\mathbf{p}|$, while inside the directivity cone we have $|\mathbf{p}-\mathbf{p}_1-\mathbf{k}|\ll|\mathbf{p}|$. These two conditions, together with (7), denote in fact that the final electron also moves at a small angle to $\mathbf{v}$. By starting from these considerations, we obtain after some calculations from (12)

$$\varkappa_{max}\approx m_ec(1+\gamma^2\omega_{pe}^2/\omega^2)^{1/2}.\qquad(13)$$

The following conditions are easily satisfied for all values of γ and ω

$$\frac{\omega}{2\gamma^2c}(1+\gamma^2\omega_{pe}^2/\omega^2)\ll m_ec(1+\gamma^2\omega_{pe}^2/\omega^2)^{1/2}\ll m_ec\gamma.\qquad(14)$$

The left-hand side of (14) corresponds to the condition $\varkappa_{min}\ll\varkappa_{max}$, which was used to derive the intensity (6) from (5). It follows also from (13) that we practically always have $\varkappa_{max}d\gg1$.

Using (11) and (13), we obtain

$$\frac{\varkappa_{max}}{\varkappa_{min}}\approx\frac{2m_ec^2\gamma^2}{\omega(1+\gamma^2\omega_{pe}^2/\omega^2)^{1/2}}.\qquad(15)$$

In the limit $\omega\gg\gamma\omega_{pe}$ expression (15) coincides fully with the analogous relation for $\varkappa_{max}/\varkappa_{min}$, which is contained under the logarithm sign in the Bethe-Heitler formula.[1]

Integrating in (6) over all the photon emission angles, and also neglecting small terms, we have for the spectral distribution of the radiation intensity

$$\frac{dI_\omega^{(c)}}{d\omega}=\frac{8e_i^2r_e^2\omega^2}{3(\omega^2+\gamma^2\omega_{pe}^2)}\left\{\ln\frac{\varkappa_{max}^2d^2}{1+\varkappa_{min}^2d^2}\right.$$

$$-\varkappa_{min}^2d^2\left(2-\frac{3}{2}\ln\frac{4\omega^2(1+\varkappa_{min}^2d^2)}{c^2\varkappa_{min}^2(1+4\omega^2d^2/c^2)}\right.$$

$$\left.\left.+\frac{3-2\varkappa_{min}^2d^2}{\varkappa_{min}d}\operatorname{arctg}\frac{2\omega d/c}{1+2\omega\varkappa_{min}d^2/c}\right)\right\}, \tag{16}$$

where $\varkappa_{min}$ and $\varkappa_{max}$ are given by (11) and (13). Formula (16) is general, in the sense that it takes into account the influence of the plasma both as a result of the Debye screening and in the Doppler frequency displacement.

We proceed to an analysis of the spectral intensity of the radiation (16) as a function of the degree of screening by a plasma in different regions the frequency spectrum.

1. Weak screening: $\varkappa_{min}d\gg 1$. This condition can be satisfied according to (11) at the frequencies

$$\omega\gg 2\gamma^2\frac{c}{v_{Te}}\omega_{pe}(1+Z)^{1/2}, \tag{17}$$

where v_{Te} is the average thermal velocity of the plasma electrons ($v_{Te}\ll c$). Using (11) and (13) and neglecting in (16) the small terms in accordance with the inequality (17), we obtain

$$\frac{dI_\omega^{(c)}}{d\omega}=\frac{16e_i^2r_e^2}{3}\left(\ln\frac{2m_ec^2\gamma^2}{\omega}-\frac{19}{12}\right). \tag{18}$$

The result (18) coincides practically fully with the Bethe-Heitler formula, [1] since the difference between the numerical value of the term that follows the logarithmic term (19/12 instead of 1/2) in[1] is due entirely to the fact that in the quasiclassical limit the value $\varkappa_{max}$ (13) can be estimated only accurate to a numerical factor on the order of unity. Thus, the condition for the applicability of the Bethe-Heitler formula[1] for bremsstrahlung in a plasma is the rather stringent condition (17), and not the condition $\omega\gg\omega_{pe}\gamma$, as might seem at first glance. The reason is that only if the condition (17) is satisfied do we have $d\gg r_{max}$, where r_{max} is the largest impact parameter and the Debye screening of the Coulomb field of the ion has little effect on the transition of the virtual quanta into real quanta when the former are scattered by the self-charge of the incident fast electron.

2. Strong screening: $\varkappa_{min}d\ll 1$. If it is recognized that we practically always have $\varkappa_{max}d\gg 1$, then in the case of strong screening of the plasma the largest $r_{max}\approx 1/\varkappa_{min}$ and the smallest $r_{min}\approx 1/\varkappa_{max}$ impact parameters of the collision satisfy the condition $r_{min}\ll d\ll r_{max}$. Under these conditions the formula for the bremsstrahlung should contain d in place of r_{min}. Indeed, at the frequencies

$$\omega_{pe}\ll\omega\ll 2\gamma^2\frac{c}{v_{Te}}\omega_{pe}\sqrt{1+Z} \tag{19}$$

we have from (13) and (16) for the spectral intensity

$$\frac{dI_\omega^{(c)}}{d\omega}=\frac{16e_i^2r_e^2}{3}\frac{\omega^2}{\omega^2+\gamma^2\omega_{pe}^2}\left\{\ln\frac{m_ecv_{Te}}{\omega_{pe}(1+Z)^{1/2}}-\frac{1}{2}\right\}. \tag{20}$$

This result differs from that obtained by Ter-Mikaelyan and Melrose in the character of the dependence of the expression under the logarithm sign on the frequency ω and on the energy γm_ec^2.

Formula (20) is valid regardless of whether $\gamma>c/v_{Te}$ or $\gamma<c/v_{Te}$. It follows from (20) that in the case of strong screening, i.e., when the condition (19) is satisfied, the bremsstrahlung for the Compton mechanism is maximal at the high-frequency interval $\gamma\omega_{pe}\ll\omega\ll 2\gamma^2\omega_{pe}c/v_{Te}$, and decreases rapidly at $\omega<\gamma\omega_{pe}$. Even at $\omega\gg\omega_{pe}\gamma$ the result differs significantly from that obtained by Ter-Mikaelyan[11] since the expression under the logarithm sign does not depend on ω or γ. The decrease of the radiation intensity $\omega\ll\omega_{pe}\gamma$ is a universal property of the emission of an ultrarelativistic particle in a plasma and was first observed in[19] for a synchroton radiation of an ultrarelativistic particle.

It follows from (17)–(20) that the frequency $\omega=2\gamma^2\omega_{pe}c/v_{Te}$ is the lower limit of the applicability of the formula of Bethe and Heitler[1] in the case of ordinary bremsstrahlung in a plasma. In the frequency region $\omega_{pe}\gamma\ll\omega\ll 2\gamma^2\omega_{pe}c/v_{Te}$, the expression under the logarithm sign changes, and does not cease to depend on ω and γ, while at $\omega\ll\omega_{pe}\gamma$ the coefficient in front of the logarithm also decreases. The change in the expression under the logarithm sign in the interval $\omega\ll 2\gamma^2\omega_{pe}c/v_{Te}$ is due to the fact that the minimum value of $\varkappa$ actually begins to be determined by the Debye screening, since $\varkappa_{min}\ll 1/d$.

4. BREMSSTRAHLUNG CONNECTED WITH CONVERSION BY DYNAMIC POLARIZATION OF THE PLASMA (TRANSITION BREMSSTRAHLUNG)

We consider now transition bremsstrahlung produced when virtual quanta are converted by scattering from the dynamic polarization of the plasma into electromagnetic waves. We simplify the matrix element (3) that describes the considered radiation, by using the condition $\omega\gg\omega_{pe}$. Analysis shows that at high particle energies the main contribution to (3) is made in this case by the term containing $\varepsilon_{\varkappa+}^t$. This means that the bremsstrahlung in this process proceeds mainly as a result of transverse virtual quanta of the fast particle. This is true only for waves with high frequencies $\omega\gg\omega_{pe}$. After certain simplifications we obtain from (3) for this case

$$\mathbf{M}^{(t)}\approx-\frac{e_e^2e_i(\omega[\mathbf{k}\times\mathbf{v}]-c^2[\mathbf{k}\times\boldsymbol{\varkappa}])}{\pi\omega^{1/2}m_e|\mathbf{k}|(1+\varkappa^2d^2)(\omega^2-\omega_{pe}^2-(\boldsymbol{\varkappa}+\mathbf{k})^2c^2)}. \tag{21}$$

It follows from (4) and (21) that the angular distribution of the transition bremsstrahlung of the fast particle, in contrast to ordinary bremsstrahlung, does not have sharp forward directivity in the general case. In our case, the vector directed forward at the small angle (7) is not the wave vector $\mathbf{k}$ of the emitted quantum, but the vector $\boldsymbol{\varkappa}+\mathbf{k}$. The general result for the angular distribution of the intensity is too cumbersome to be written out here. We present only the total spectral radiation intensity integrated over the angles:

$$\frac{dI_\omega^{(t)}}{d\omega}=2e_i^2r_e^2\frac{\omega\omega_{pe}^2}{cv_{Te}^2}(1+Z)\int\limits_{\omega/v}^{\varkappa_{\max}}\frac{d\varkappa}{\varkappa^2(\varkappa^2-k^2)}$$
$$\times\left\{\frac{1}{1+(\varkappa-k)^2d^2}-\frac{1}{1+(\varkappa+k)^2d^2}+\frac{1}{4\varkappa^2k^2d^4}\left(\frac{3}{2}-\frac{\varkappa^2}{k^2}\right)\right.$$
$$\left.\times\left[4\varkappa kd^2-(1+(\varkappa^2+k^2)d^2)\ln\frac{1+(\varkappa+k)^2d^2}{1+(\varkappa-k)^2d^2}\right]\right\}, \tag{22}$$

where $\varkappa_{\max}$ is determined from (13).

Without presenting the value of the integral in (22) we proceed to investigate the intensity of the transition bremsstrahlung in different intervals of the frequency spectrum. To this end we use the conditions $\varkappa_{\max} \gg \varkappa_{\min}$ and $\varkappa_{\max}d \gg 1$.

1. Radiation in the frequency interval (17). In this limit we have from (22)

$$\frac{dI_\omega^{(t)}}{d\omega}=4e_i^2r_e^2\left(\frac{2\gamma^2c\omega_{pe}}{\omega v_{Te}}\right)^2\left(\frac{\omega_{pe}c}{2\omega v_{Te}}\right)^2(1+Z). \tag{23}$$

It follows therefore that the radiation intensity is much lower in this case than in the case of ordinary bremsstrahlung (see formula (18)). Thus, at frequencies $\omega \gg 2\gamma^2\omega_{pe}c/v_{Te}$, the plasma, as follows from (18) and (23) exerts no significant influence on the bremsstrahlung in either the ordinary mechanism or the transition mechanism.

2. Radiation in the frequency spectrum

$$\sqrt{1+Z}\frac{c\omega_{pe}}{2v_{Te}}\ll\omega\ll2\gamma^2\frac{c}{v_{Te}}\omega_{pe}\sqrt{1+Z}. \tag{24}$$

In this limit we have from (22)

$$\frac{dI_\omega^{(t)}}{d\omega}=2e_i^2r_e^2\left(\frac{c\omega_{pe}}{v_{Te}\omega}\right)^2(1+Z)\left(\ln\frac{2\gamma\omega}{(\omega^2+\gamma^2\omega_{pe}^2)^{1/2}}-1\right). \tag{25}$$

By virtue of condition (24), the radiation intensity (25), just as in the preceding case, is small in comparison with the result of Bethe and Heitler.[1] The frequency $c(1+Z)^{1/2}\omega_{pe}/v_{Te}$ is in a certain sense the threshold for the transition bremsstrahlung radiation (just as the frequency $\omega_{pe}\gamma$ is the threshold for ordinary bremsstrahlung). However, whereas for ordinary bremsstrahlung the intensity decreases at $\omega < \omega_{pe}\gamma$, the intensity of the transition bremsstrahlung, as follows from (25), decreases at $\omega > c\omega_{pe}(1+Z)^{1/2}/v_{Te}$ in accordance with the law $(c\omega_{pe}/v_{Te}\omega)^2(1+Z)$.

3. Radiation in the frequency spectrum

$$\omega_{pe}\ll\omega\ll\frac{1}{2}\frac{c}{v_{Te}}\omega_{pe}\sqrt{1+Z}. \tag{26}$$

By discarding the small terms corresponding to the condition (26), we obtain from (22)

$$\frac{dI_\omega^{(t)}}{d\omega}=\frac{16}{3}e_i^2r_e^2\ln\frac{c}{v_{Te}}\frac{\gamma\omega_{pe}}{(\omega^2+\gamma^2\omega_{pe}^2)^{1/2}}. \tag{27}$$

Thus, it follows from (23), (25), and (27) that the transition bremsstrahlung of an ultrarelativistic particle, which is due to scattering by the dynamic-polarization charge in the plasma, is excited mainly within the frequency spectrum (26). The length of the electromagnetic wave radiated in this case greatly exceeds the Debye radius, i.e., $\lambda \gg d$. It is this which causes the entire polarization charge to radiate coherently and makes the intensity maximal. In the remaining frequency region, the transition bremsstrahlung is much smaller. Formula (27) differs from the formula of Bethe and Heitler in the expression under the logarithm sign, which does not contain the Planck constant, since the transition radiation is described entirely quasiclassically.

5. DISCUSSION OF RESULTS

Melrose[3] considered in a different manner the emission of electromagnetic waves by an ultrarelativistic particle in a plasma, with allowance for only the Compton radiation mechanism. Investigating qualitatively the behavior of the spectral intensity of the radiation, he has observed an abrupt decrease of the intensity at frequencies $\omega \ll \omega_{pe}\gamma$. The same effect for condensed media was discussed earlier by Ter-Mikaelyan.[11] On this basis Melrose[3] states that in a plasma, including cosmic plasma, the bremsstrahlung in the frequency interval $\omega \ll \omega_{pe}\gamma$ is negligibly small and that the radiation takes place only at $\omega \geqslant \omega_{pe}\gamma$. However, when account is taken of transition bremsstrahlung considered in the present paper and due to the dynamic polarization of the plasma, the general picture of the spectral intensity of the radiation turns out to be entirely different. In particular, as follows from a comparison of (20) and (27), which are valid under the conditions (19) and (26), respectively, at fast-electron energies $\gamma \ll c/2v_{Te}$ the bremsstrahlung in the plasma is generated in the entire region of the frequency spectrum, i.e., at $\omega < \gamma_{pe}$. In this case the radiation due to the Compton mechanism decreases rapidly at $\omega < \gamma\omega_{pe}$, but the radiation due to the dynamic polarization does not vanish.

Thus, taking the transition mechanism of radiation into account, at $\gamma \ll c/2v_{Te}$ the bremsstrahlung in the plasma is, in the entire frequency spectrum. of the same order as given by the formula of Bethe and Heitler, in which, however, the expression under the logarithm sign is changed twice—at $\omega \approx 2\gamma^2c\omega_{pe}/v_{Te}$ and at $\omega \approx \omega_{pe}\gamma$. The value of the logarithm at $\omega \approx 2\gamma^2c\omega_{pe}/v_{Te}$ ceases to increase with decreasing frequency, and it retains at $\omega < 2\gamma^2c\omega_{pe}/v_{Te}$ the same value as at $\omega \approx 2\gamma^2c\omega_{pe}/v_{Te}$, while in the region $\omega \approx \omega_{pe}\gamma$ the expression under the logarithm sign is decreased appreciably to the value determined by (27).

Inasmuch as at frequencies ω on the order $\omega_{pe}c/v_{Te}$ an interference takes place between the two bremsstrahlung mechanisms, it follows that a certain drop in the radiation intensity is also possible in the entire frequency region. However, since the matrix element of the transition bremsstrahlung depends little on the frequency at $\omega < \omega_{pe}c/v_{Te}$, whereas the matrix element of the ordinary bremsstrahlung decreases rapidly with frequency, it follows that in the region $\omega_{pe}\gamma < \omega < \omega_{pe}c/v_{Te}$ the interference leads to a relatively small change in the logarithmic term.

In the opposite case, i.e., at very high electron energy, when $\gamma \gg c/2v_{Te}$, the intensity of the Compton scattering decreases in the frequency interval $c\omega_{pe}/2v_{Te} \ll \omega \ll \gamma\omega_{pe}$ in accordance with the law $(\omega/\gamma\omega_{pe})^2$, from $\omega \approx \omega_{pe}\gamma$ towards lower frequencies, while the transition bremsstrahlung increases like $\frac{1}{2}(c\omega_{pe}/v_{Te}\omega)^2$ and reaches saturation only at $\omega \approx \omega_{pe}c/v_{Te}$. Thus, in the case $\gamma \gg c/2v_{Te}$ in the frequency region $\omega_{pe} < \omega < c\omega_{pe}/2v_{Te}$ the radiation exists as a result of the transition mechanism. On the other hand, in the frequency interval $c\omega_{pe}/2v_{Te} \ll \omega \ll \gamma\omega_{pe}$ the bremsstrahlung in the plasma is strongly suppressed.

The described regularity of the bremsstrahlung can be used also for a solid-state plasma but only at high frequencies exceeding the atomic frequencies. If, for example, we assume that $c/v_F \approx 10^3$, where v_F is the velocity on the Fermi surface and $\gamma \approx 10^1-10^2$, then, if the transition bremsstrahlung is included, bremsstrahlung is produced in metals in the entire frequency spectrum.

It is now necessary to estimate from new viewpoints the role of multiple scattering in bremsstrahlung—an effect first investigated by Landau and Pomeranchuk without allowance for the bremsstrahlung. The transition bremsstrahlung, which has no sharp directivity, will not be suppressed by multiple scattering. Therefore, even if ordinary bremsstrahlung is suppressed, at $\omega < \omega_{pe}c/2v_{Te}$ we are left with the transition bremsstrahlung. For condensed media this condition must be replaced by $\omega < c/d$, where d is the screening radius. For a plasma it is relatively easy to satisfy the criterion under which multiple scattering has no influence at all on the bremsstrahlung. This corresponds to the case when the multiple scattering does not affect the ordinary bremsstrahlung. By determining more accurately the mean squared scattering angle in the plasma (by changing the screening in comparison, say, with gases or condensed media), we obtain an estimate that indicates more accurately than in[11] when it is possible to neglect multiple scattering:

$$\omega\left(1+\gamma^2\frac{\omega_{pe}^2}{\omega^2}\right)^2 \gg \frac{\omega_{pe}Z\gamma^2}{N_c}, \quad N_c=n_e\frac{c^3}{\omega_{pe}^3}. \tag{28}$$

We have taken into account here the fact that $n_e = Zn_i$. The criterion (28) will be satisfied for all frequencies if it is satisfied at $\omega \approx \omega_{pe}\gamma$, which yields

$$\gamma \ll \frac{N_c}{Z} = N_d\frac{c^3(1+Z)^{3/2}}{v_{Te}^3 Z}, \quad N_d=n_e d^3. \tag{29}$$

Since N_d in a plasma are usually a large number, the condition (29) is violated only at extremely high energies.

Let us discuss finally the bremsstrahlung generated by ultrarelativistic ions in a cosmic plasma. Since the radiation in the Compton mechanism is smaller by a factor $(m_e/m_i)^2$, the bremsstrahlung of an ultrarelativistic ion is due only to dynamic polarization of the plasma, which does not depend on the ion mass. The radiation occurs principally at the frequencies $\omega < c\omega_{pe}/2v_{Te}$. It appears that the bremsstrahlung of fast ions should therefore be regarded as a possible source of radio emission under astrophysical conditions.

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