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Translated by J. G. Adashko

Parametric generation of coherent radiation in a spatially incoherent pumping field

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Parametric generation in a spatially incoherent pumping field (radiation emission by a multimode ruby laser) is investigated experimentally. It is shown that at a low ratio $l_r/l_n(l_r \text{ and } l_n \text{ are the correlation lengths for waves at resonance and not at resonance with respect to the pumping wave), the incoherence of the nonreasonant waves has little effect on the spectral and spatial characteristics of resonant wave emission. The dependence of the self-excitation threshold and of the transformation coefficient on the correlation length <math>l_n$ and on the mirror transparency is investigated for parametric generation of light. Transformation of spatially incoherent pumping into frequency-tunable emission with a divergence angle close to the diffraction value (which is smaller by approximately one order of magnitude than the pump divergence) and a transformation coefficient close to that for coherent pumping (up to 20% of the number of photons in the resonance wave and 40% of the photon flux) is obtained.

PACS numbers: 42.65.Dr

A theoretical analysis of the processes of parametric amplification and generation^[1-4] shows that under certain conditions either amplification or generation of nearly-coherent radiation can occur in an incoherent pumping field. The incoherence of the pump radiation has the least effect on the degree of coherence of this radiation (we shall call this the signal or resonant-wave radiation) if the correlation length of the pump wave with the resonant wave (l_r) is much less than either the length over which noticeable amplification takes place, or the correlation length of the pump with the idling (or nonresonant) waves (l_n) .^[4,5] The condition under which the incoherence of the pump has little effect on the energy characteristics of the amplification or self-excitation of the oscillations in the case of resonant-wave feedback can be determined from the expression for the spatial increment p_{\star} for the average field of this wave. If the synchronism conditions are satisfied and $l_r \ll l_n$, this expression can be written in the form

$$p_{+} = -(\alpha_{r} + \alpha_{n} + l_{n}^{-1})/2 + [(\alpha_{n} + l_{n}^{-1} - \alpha_{r})^{2}/4 + \gamma_{0}^{2}]^{\frac{1}{2}}.$$
 (1)

Here α_r and α_n are the absorption coefficients of the resonant and nonresonant wave; γ_0 is the self-action coefficient¹; the correlation length of the nonresonant wave and the pump wave can be estimated from the formula

$$l_n = (l_{n\tau}^{-2} + l_{nd}^{-2} + l_{na}^{-2})^{-\gamma_n}.$$
 (2a)

The quantities $l_{n\tau}$, l_{nd} , and l_{na} in (2a) are respectively the temporal, diffraction, and aperture correlation lengths and are defined by the relations

$$l_{n\tau} = \tau_{s} / |v_{n3}|, \quad l_{nd} = k_{s} \rho_{s}^{2} / |1 - k_{s} k_{n}^{-1}|, \quad l_{na} = \rho_{s} / \beta_{n},$$
 (2b)

where $\tau_3 = 2(\Delta\Omega_3)^{-1}$ is the characteristic time scale determined by the width of the pump frequency spectrum $2\Delta\Omega_3$; $\nu_{n3} = \nu_n^{-1} - \nu_3^{-1}$ is the mismatch of the longitudinal components of the group velocities v_n and v_3 of the non-resonant wave and the pump wave (group mismatch); k_n and k_3 are the wave numbers of the same waves in a nonlinear crystal; $\rho_3 = 2/k_3\Delta\theta_3$ is the characteristic scale of the spatial inhomogeneities of a pump wave with divergence $2\Delta\theta_3$ (at the e^{-1} level); β_n is the angle between the directions of the group velocities of the non-resonant wave and the pump wave. The length l_r is determined by expressions analogous to (2).

It is seen from (1) that two limiting regions can be separated in the dependence of the growth rate p_{\star} on the correlation length l_n and on the pump power density, namely, the region where $\gamma_0 l_n \ll 1$ and the region of quasi-coherent interaction, when $\gamma_0 l_n \gtrsim 1$.

In the quasi-coherent interaction region, when besides the condition

(3a)

there is also satisfied the condition that the fluctuations of the amplified wave be small

$$\gamma_0 l_r \leq (\gamma_0 d)^{-1}, \tag{3b}$$

the growth rate, the bandwidth, and the divergence of the resonant waves are close^[4] to their values in coherent pumping.²⁾ The bandwidth and the divergence of the radiation from the nonresonant wave are close to the corresponding parameters of the pump wave. These singularities of the parametric wave interaction have been demonstrated^[6,7] for the case of generation in a pump field with temporal incoherence, i.e., the case when $l_n \ll l_{na}$.

The degree of coherence of the radiation can also be greatly increased in the case of generation by various types of stimulated scattering (SS).^[8-14] In particular, in stimulated Raman scattering (SRS) lasers, the analog of the condition (3a) is satisfied^[4] if the width $2\Delta\Omega_3$ of the pump spectrum is smaller than the spontaneous-scattering line width $2\Delta\Omega_4$.³⁾ As shown in a number of papers, ^(8,10-14) in this case coherent Stokes radiation can be generated with a large conversion coefficient. In a broad band pump field at $\Delta\Omega_3/\omega_3 \gtrsim 1$, the analog of the condition (3) is violated, and this leads to a significant change of the value of the conversion coefficient. ^(8,11)

At $\Delta\Omega_3/\Delta\omega_s \ll 1$, the analog of the condition (3a) is satisfied because the group velocity of the molecular oscillations (which are equivalent to the resonant wave in the case of parametric generation) is practically equal to zero. In the field of coherent Stokes radiation these oscillations are therefore correlated at each point with the field of the incoherent pump, which depends slowly enough on the time. An analog of condition $(3b)^{[4]}$ can be satisfied either when an appreciable role is played by the opposing interaction, or else when there is appreciable spatial incoherence of the pump at a small gain of the Stokes wave over a distance on the order of the characteristic longitudinal dimension of pump-field inhomogeneity. The situation in this case is analogous in many respects to the situation in lasers with small-scale active-medium inversion inhomogeneities that do not scatter or distort greatly the structure of the coherent-radiation field. In particular, just as in a laser, the degree of spatial coherence of the generated radiation is determined in SRS by the geometry of the system (for example, by sufficiently high diffraction losses of the higher transverse modes of the excited oscillations).

We present here the results of an investigation of parametric lasing in the field of sufficiently narrowband pumping with an appreciable spatial incoherence.⁴⁾ The conditions (3) can be satisfied in this case only for definite angles between the group velocities of the interacting waves ($\beta_n \approx 0$, $\beta_r \neq 0$). The mechanism that produces the spatial coherence of a resonant wave with given frequency differs qualitatively from the mechanism in SRS and is connected to a considerable degree with the specific "parametric" spreading. The investigations were carried out for an ordinary single-resonator parametric laser (SRPL) and for SRPL with nonresonant pump reflection (SRPL with mirror in the pump path).

1. DESCRIPTION OF EXPERIMENTAL SETUP AND DETERMINATION OF THE OBSERVATION THRESHOLD

The SRPL was pumped by a ruby laser $(\lambda_3 = 0.6943 \ \mu)$ that was Q-switched with a Pockels cell. The laser emission in this case was multimode with divergence $2\Delta\theta_3 \approx 6'$ and spectrum width $2\Delta\Omega_3 \approx 0.5 \ \mathrm{cm^{-1}}$ (the analogous parameters are defined here and below at the e^{-1} level). The laser pulse durations were different in different measurement runs, but within the range 50-80 nsec. The pump energy W_3 incident on the SRPL could be smoothly varied with an attenuator up to the maximum value $W_3^{\max} = 1$ J employed in our study. The maximum pump-beam radius was about 4 mm. Its dimensions on entering the SRPL could be varied with the aid of a diaphragm (round or rectangular). The divergence of the pump beam remained practically the same when its dimensions were varied.

The investigated SRPL consisted of an LiIO₃ crystal placed in a resonator with flat mirrors. The distance L between the entrance mirror (reflectivity $R_{.}$) and exit mirror ($R_{.}$) was about 60 mm. The orientation of the SRPL resonator was determined by the angle θ_R between the normal to the mirrors and the direction of the pump wave vector \mathbf{k}_3 in the narrow-synchronism-angle plane xz passing through \mathbf{k}_3 and the optical axis of the crystal. The LiIO₃ crystal, with length d = 35 mm, was so cut that its optical axis made an angle 23° to the normal with the working faces. The working faces of the crystal were made transparent in a region near 1 μ in such a way that the transmission coefficient of the crystal was $T_c = 0.95$.

In the SRPL system with the pump reflection, the pump mirror was located behind the exit mirror (R_*) of the resonator. Its reflection coefficient at the pump radiation wavelength was $R_{*3} = 0.98$.

The registration system made it possible to record simultaneously the following pump-radiation and resonant-wave parameters:

1) The energy of the pump pulses incident on the SRPL; the waveform, duration, spatial spectrum and cross section of the incident pump beam and of the pump beam passing twice through the parametric generator.

2) The energy, waveform, and duration of the pulse, and also the spatial and frequency spectra, the propagation direction, and the wavelength of the resonant SRPLon the side of the exit mirror (R_{\star}) of the resonator, and the cross section of the resonant wave beam—on the side of the entrance mirror (R_{\star}) .⁵⁾

The temporal correlation length $l_{n\tau}$ and the diffraction correlation length l_{nd} were large enough under the experimental conditions $(l_{n\tau} \approx 8 \times 10^2 \text{ cm}, l_{nd} = 40 \text{ cm} \text{ at } \lambda_{\tau} = 1 \text{ and } \lambda_n = 2 \mu$). The pump incoherence, as can be seen from (1), could therefore exert an appreciable influence on the excitation conditions only if the angle β_n between the group velocity of the pump and the nonresonant wave



FIG. 1. Emission spectrograms of SRPL with pump mirror at certain values of the angle θ_R and at approximately double the threshold for the observation of the wave of the first type. The numbers without indices designate emission spectra of type I: $1-\theta_R = 1^\circ 40'$; $3-\theta_R = 1^\circ 51'$; $4-\theta_R = 1^\circ 54'$; $5-\theta_R = 1^\circ 57'$. The corresponding type-II spectra are labeled by the index a. The spectrum 2b corresponds to $\theta_R = 1^\circ 46'$; it was obtained without a pump mirror as a result of several shots. The different densities of the remaining spectrograms are due mainly to inaccurate incidence of the radiation on the spectrograph slit. 6- reference line of gas laser ($\lambda = 1.15 \ \mu$).

was large enough. The value of the angle β_n and the corresponding value of β_r for the resonant wave depends on the angle θ_r between the pump-wave and resonant-wave vectors:

$$\beta_n = \frac{\lambda_n}{\lambda_r} \theta_r - \beta_0(\theta_3), \quad \beta_r = \theta_r + \beta_0(\theta_3).$$
(4)

Here $\lambda_r(\theta_3, \theta_r)$ and $\lambda_n(\theta_3, \theta_r)$ are the lengths of the resonant and nonresonant waves, which are in synchronism with the pump at the given angles θ_3 and θ_r ; θ_3 is the angle between the pump wave vector and the optical axis of the nonlinear crystal; $\beta_0(\theta_3)$ is the birefringence angle of the pump wave and corresponds to θ_3 . It follows from (1)-(4) that an SRPL system with collinear interaction ($\theta_r = 0$, $|\beta_r| = |\beta_n| = \beta_0$) should have a high self-excitation threshold. In this case $l_{na} = l_{ra} = 0.4$ cm (in LiIO₃ for $\lambda_3 = 0.69 \mu$ and $\theta_3 = 23^\circ$ we have $\beta_0 = 3.5^\circ$) and the spatial growth rate becomes small: $p_* \approx \gamma_0^2 l_{na} \ll \gamma_0$.

An experimental investigation of an SRPL system in which the interaction was close to collinear ($\theta_r = 0$) has shown that the theoretical estimates are correct. The peak value of the average pump power density S_{0n} at the lasing observation threshold turned out to be quite high $(S_{0n} \simeq 60 \text{ MW/cm}^2)$. Such values of the average pump power density were close to the breakdown threshold of lithium iodate (traces of the breakdown in the working volume of the crystal were observed approximately after 10-15 flashes. So low a breakdown threshold is apparently due to the fact that the power density in the pumpbeam inhomogeneities is much higher than the value averaged over the beam).

The main investigations were carried out at angles θ_R for which β_n was close to zero ($\beta_n \leq 5$ ', $l_n \geq 10$ cm; in this case $\beta_r \approx 5^\circ$, $l_r \approx 0.2$ cm). At $\beta_n = 0$, the observation threshold S_{0n} for the usual SRPL system turned out to be 6 MW/cm². For the SRPL system with the pump mirror, it was lower by an approximate factor 1.5, in agreement with the theoretical estimates.⁶⁾

All the investigated parametric lasers were characterized by a sharp dependence of the observation threshold on the mismatch of the resonator mirrors: a deviation of 3''-5'' from parallelism increased the threshold by 1.5-2 times. We note also a very weak (at $\beta_n = 0$) dependence of the observation threshold in the system with the pump mirror on the rotation of this mirror (within ±10') relative to the position perpendicular to the pump beam. These two singularities in the behavior of the threshold are in good agreement with theoretical estimates carried out when solving the problem of self-excitation of SRPL without allowance for the multimode character of the pump beam. ^[15-18]

2. SPECTRAL AND SPATIAL CHARACTERISTIC OF RESONANT-WAVE EMISSION

A typical picture of the spectral and angular (in the narrow synchronism-angle plane) distributions of the resonant-wave radiation is shown in Fig. 1 for the ordinary SRPL scheme (spectrogram 2b).⁷⁾ We note the following features of the characteristics of this radiation:

1) The propagation direction of the average frequency ω_{r0} of the spectrum made an angle $\psi_{r0} \approx 10'$ with the direction normal to the resonator mirrors.

2) The width of the directivity pattern (in the narrow synchronism-angle plane xz) at a fixed frequency is close to the diffraction value and equals approximately $2\Delta \overline{\theta}_{rx}^{e} \approx 0.5$ ', whereas the total width of the directivity pattern of the resonant radiation⁸⁾ in this plane was equal to $2\Delta \overline{\theta}_{rx}^{e} \approx 12$ '. In the broad synchronism angle plane yz, the divergence is $2\Delta \theta_{ry}^{e} \approx 10$ ' at a pump-beam radius $a_{y} = 0.4$ cm in this plane.

3) The frequency spectrum of the resonant wave emission is very broad, $2\Delta \tilde{\Omega}_r^{\varphi} \approx 300 \text{ cm}^{-1}$.

The characteristic of the lasing in the SRPL system with pump mirror depends essentially on the reflection coefficients of the resonator mirrors and on the degree of the excess over the observation threshold.

At large resonator-mirror reflection coefficients ($R_{\star} \approx 0.96$, $R_{-} \approx 0.98$) in the region where the pump power density exceeds slightly its value S_{0n} at the observation threshold ($S_{0m}/S_{0n} < 1.5$), the propagation direction of the generated waves on the resonator-mirror surfaces coincided with the normal to these surfaces-radiation of type I. At a level approximately 1.5 times the observation threshold, in addition to the radiation of type I, there appears radiation of type II, the spectral and angular characteristics of which are similar to the characteristics of the usual SRPL system (see Fig. 1). With increasing pump power, the intensity of type-II radiation increases whereas the coefficient of pump conversion into type-I radiation does not increase (Fig. 2, curve 1).

When the reflection coefficient of the output mirror is



FIG. 2. Experimental plots of the efficiency of a parametric laser, determined from the number of photons of the resonant wave ($\eta_{\rm pr}$, solid lines), and of the instant of the start of the nonlinear regime (T_n , dashed lines), against the excess of the pump energy density w_{0m} over its value w_{0n} corresponding to the threshold of the observation of the resonant wave in the resonator with R = 0.84. The plots were obtained at $\beta_n = 0$ for certain values of the reflection coefficient R_+ of the exit mirror: $1-R_+=0.96$, R=0.84; $2-R_+=0.88$, R=0.74; $3-R_+=0.84$, R=0.70; $4-R_+=0.64$, R=0.5. SRPL system with pump mirror ($\beta_r = 5^{\circ}18^{\circ}$, $\lambda_r = 1.045 \ \mu$, $W_{0n} = 0.35 \ J/cm^2$).

decreased to $R_* \lesssim 0.92$, no type-II radiation is observed in the entire tuning range (see Sec. 3), at least up to double the observation threshold. Some characteristics of the type I radiation are given in Table I and in Fig. 3. We note the following characteristic features:

1. The width of the frequency spectrum of the resonant-wave emission is smaller by approximately two orders of magnitude than in the system without the pump mirror, and depends little on the excess of the pump power over threshold (Fig. 4). (The intensity modulation observed in the spectrum is apparently due to the selecting properties of the resonator-mirror substrates.)

2. The lasing direction coincides with the normal to the resonator mirrors $(\theta_r = \theta_R)$.

3. The width of the directivity pattern in the xz plane is smaller by almost one order of magnitude than the width of the pump-radiation pattern in the same plane, and is close to the diffraction value. The energy con-



FIG. 3. Typical plots of the resonant-wave emission spectrum at two values of the excess over the observation threshold: $a - S_{0m}/S_{0n} \approx 1$, $b - S_{0m}/S_{0n} \approx 1.9$. SRPL with pump mirror ($R_{+} = 0.84$, R = 0.7, $\beta_{n} = 0$, $\beta_{r} = 5^{\circ}18^{\circ}$).

tained in the principal maximum of the directivity pattern did not exceed 10% of the total resonant-wave energy. The width $2\Delta\theta_{ry}^{e}$ of the directivity pattern in the yz plane depends on the radius a_{y} of the pump band in this plane $(2\Delta\theta_{ry}^{e} \approx 3.5' \text{ at } a_{y} = 0.15 \text{ cm and } 2\Delta\theta_{ry}^{e} \approx 10' \text{ at}$ $a_{y} \approx 0.4 \text{ cm}).$

4. When the pump power density exceeds the observation-threshold value by more than 1.3 times, a nearly-periodic modulation of the intensity appears in the plane of the narrow synchronism angle. The spatial period of the modulation decreases with increasing excess over threshold.

The peculiarities described above and the values of the resonant-wave radiation characteristics and the ordinary SRPL and the SRPL with pump mirror can be explained mainly in the approximation of the interaction of single-mode beams, and have little connection with the multimode character of the pump.¹⁰

It follows from a theoretical analysis carried out in the diffusion approximation^[15-18] that in a laser without a pump mirror, at $\beta_{rn} = \beta_r - \beta_n \neq 0$, the maximum of the beam of the excited waves propagates along a direction close to the direction of the average group velocity of the resonant and nonresonant waves. The largest growth rate is possessed by a wave with frequency ω_{r0} , the beam of which moves in the crystal along a certain "optimal" trajectory. The θ_{r0} corresponding to this wave

TABLE I. Characteristics of type I emission of SRPL with pump mirror at certain values of the excess of the average pumppower density over its value at the registration threshold.

S _{0m} /S _{0n}	η _{nr}	$2\Delta\Omega_{r3}^{e}$, cm ⁻¹	$2\Delta\Omega_{r3},$ cm ⁻¹	$2r_{\rm x}^{e}$, cm	2∆θ ^e rx	$2\Delta \theta^e_{rxt}$	27 ^e y	2Δθ ^e _{γy}	$2\Delta \theta_{ryd}^s$	2∆θ ^e rsel	2∆ ^{out}	N _{sn}
1.3	0.02	3.5	2.4	(0.3) 0.3	0.7	(0.7') 0.7'	1.2	3.1'	(0.24')	1.4'	1.7'	108
1.6	0.08	4.0	3.1	(0.4) 0.2	1.4	(0.6') 1.2'	1.2	3.6'	(0.24')	1.5'	2.2'	80
2.2	0.17	4.3	4.1	(0.5) 0.12	1.5	(0.5') 1.9'	1.3	3.8'	(0,2')	1.6'	3.1'	55

Note. The values $\Delta \theta_{ryd}^e$ of the diffraction divergences of the radiation of the resonant wave were obtained on the basis of measurements of the radii of its beam r_x and r_y^e (the corresponding values of r_x^e , r_y^e , $\Delta \theta_{ryd}^e$ and $\Delta \theta_{ryd}^e$ are given in parentheses) and by measuring the minimum values of the dimensions of the inhomogeneities of the intensity of this radiation over the beam cross section (the corresponding values are given without parentheses). $\Delta \theta_{ry}^e$ and $\Delta \theta_{ry}^e$ are the corresponding experimentally measured divergences of the resonant wave, $\Delta \theta_{ryd}$ and $\Delta \theta_{ry}^{out}$ are the theoretical values of the angle divergence in the yz plane, due to the divergence of the parametric laser mode of radius $r_{0y}^{[15,18]}$ and to excitation of fields that travel out of the interaction region. $\Delta \Omega_{r_3}$ and $\Delta \Omega_{r_3}^e$ are the theoretical and experimental widths of the emission spectra of the resonant wave.

For theoretical estimates of $\Delta\Omega_{r_3}$ and $\Delta\Omega_{r_3}$, the number of passes N_{sn} was determined from measurements of the time T_n and from estimates of the instant t_{thr} of the appearance of the self-excitation threshold, obtained in the plane-wave approximation.



FIG. 4. Width of spectrum of **SR**PL with pump mirror $(2\Delta\Omega_{r3})$ (see the caption of Fig. 3 for the parameters) vs the excess of the pump power over the observation threshold. When plotting the theoretical (solid) curve, the number of runs N_{sr} was determined in the manner given in the note to the table.

differs from the angle θ_R by an amount ψ_{r0} ($\psi_{r0} = \theta_{r0}$ $-\theta_R$). The waves of other frequencies $\omega_r = \omega_{r0} + \Omega$, depending on the $\theta_r(\omega_r)$ tuning curve, will have different values of $\psi_r(\Omega)$, and will therefore propagate along other, "non-optimal" trajectories with a smaller gain. Since the dependence of the latter on the frequency is relatively weak in a rather wide range of pump beams, a large number of waves is excited, with frequencies ω_r and with propagation directions θ_r determined by the corresponding tuning curve. In the usual SRPL scheme without a pump mirror, in the case of broad beams (in the narrow-synchronism-angle plane), one should therefore observe a broad frequency spectrum $2\Delta \tilde{\Omega}_{r}$ and a broad angle spectrum $2\Delta \overline{\theta}_{rx}$. The radiation divergence in the xz plane at a fixed frequency $2\Delta \theta_{rr}$ should be approximately equal to the diffraction value.

Estimates based on the diffusion-approximation formulas yield for the conditions of our experiment

 $2\Delta \tilde{\Omega}_r \approx 250 \text{ cm}^{-1}$, $2\Delta \bar{\theta}_{rx} \approx 0.2'$, $2\Delta \bar{\theta}_{rx} \approx 9'$, $\psi_{r0} = 50'$.

It is easily seen that the theoretical estimates of the width of the spectrum and of the angle divergence agree well with the experimental results. The discrepancy in the theoretical and experimental values of the angle ψ_{r0} are apparently connected with the difference between the pump profile and the quadratic profile used in the theory, and also with the fact that the parameters of the pump beam lie at the applicability limit of the diffusion approximation for the determination of the value of ψ_{r0} . [15, 18] In the SRPL with pump mirror, the largest growth rate is possessed by the waves that are in synchronism both in the forward and in the return passes through the nonlinear crystal. It is easily seen that these waves should have a propagation direction that coincides on the resonator-mirror surfaces with the normal to these surfaces $(\theta_{r0} = \theta_R)$. Waves for which the normal to the mirrors and the propagation direction make an angle comparable with the width of the synchronism angle $\delta \theta_{rs} = 2\sqrt{2}\gamma_0/k_r\beta_{rn}$ are amplified during only one pass through the nonlinear element. Therefore the angle divergence $2\Delta\theta_{rx}$ in the narrow synchronism angle plane should be close to the diffraction value. The width of the lasing frequency spectrum $2\Delta\Omega_{r3}$ is in this case equal to $2\Delta \hat{\Omega}_r$ at the given propagation direction of the generated waves¹¹⁾ and is given for broad pump

beam^[15,18] by the relation¹²⁾

$$\Delta\Omega_{r3} = \sqrt{2\delta\Omega_{rs}} / \sqrt{\Gamma_0 N_{sn}}$$
⁽⁵⁾

The spectrum width calculated on the basis of (5), as seen from the table, agrees well with the experimentally measured values for radiation of type I.

It is seen from Table I that the divergence of the resonant wave increases somewhat with increasing pump power in the plane of the narrow synchronism angle. Even in this case, however, it is close to the diffraction value when account is taken of the dimensions of the intensity modulations that appear in the beam. The reason for their appearance is not clear. It is possible that they are connected with the large-scale pump-beamintensity inhomogeneities. However, the quasi-periodic character of the intensity modulations indicates that their appearance can be due to singularities of the nonlinear generation regime.

The table lists the experimental values of the angular divergence of the radiation in the broad synchronism angle plane yz, as well as the values calculated without allowance for the multimode character of the pumping. A comparison of these values, as well as the experimentally obtained dependence of the divergence (in the yz plane) of the resonant-wave radiation on the pump beam dimensions in this plane, show that this divergence is connected mainly with excitation of the fields that travel away from the interaction region.¹³⁾ The divergence of the resonant-wave radiation in the yz plane is of the same type in the SRPL without pump mirror. (The theoretically calculated values of the divergence $2\Delta \theta_{rv}^{out} \approx 8'$, corresponding to the beam radius $a_v = 0.4$ cm used in the investigation of this scheme, agree well with the experimental value.)

Thus, an experimental investigation of the angular and frequency characteristics of the SRPL radiation wave shows that, at least at $l_r \approx 0.2$ cm and $l_n \gtrsim 10$ cm, the multimode character of the pump, as expected, has little effect on these characteristics. Almost all the features of the angular and frequency spectra agree well with the results of a theoretical diffraction-approximation analysis of the linear operating regime of the laser without allowance for the multimode character of the pumping. Within the framework of this analysis, besides the already noted modulation of the intensity in the resonant-wave beam, it is impossible to explain only the excitation of the type-II wave in the system with pump mirror. However, as already noted, radiation of this type is excited also in the case of singlemode pumping, i.e., it is not connected with the multimode character of the pump.

3. ENERGY CHARACTERISTICS OF PARAMETRIC LASERS

It was shown above that at $l_r \ll l_n$ the multimode character of the pump has little effect on the spectral and angular characteristics of the SRPL resonant-wave emission. Further experiments performed with the SRPL with pump mirror have shown that the influence



FIG. 5. Dependence of the instantaneous value of the quantum efficiency $\eta_0(t)$ of an SRPL with pump mirror on the time at two values of the excess of the pump power density S_{0m} over its value at the registration threshold: a) $S_{0m}/S_{0n}=1.5$, b) $S_{0m}/S_{0n}=2.1$ ($R_{+}=0.84$, R=0.7, $\beta_{\tau}=5^{\circ}18^{\circ}$, $\lambda_{\tau}=1.045$ μ).

of the multimode character of the pump on the energy characteristics of the laser is also weak if the angle β_n between the group velocities of the nonresonant wave and the pump wave is close to zero, so that $l_n \gamma_0 > 1$. The experimentally measured quantity was the instantaneous value of the quantum efficiency¹⁴⁾ $\eta_0(t)$ and the efficiency η_{nr} ¹⁵⁾ with respect to the number of photons of the resonant wave. As seen from Fig. 5, the peak value of $\eta_0(t)$, obtained at $\beta_n \approx 0$ and at double the threshold, is 58%. The maximum value of η_{nr} was obtained also at β_n close to zero. It turned out to be 19% at approximately double the observation threshold (Fig. 2). The resonant-wave energy W_r was 0.032 J in this case.¹⁶⁾

Let us examine in greater detail the influence of certain factors on the energy characteristics of a parametric laser at $\beta_n = 0$. Figure 2 shows the dependence of η_{nr} and of the time T_n of the start of the nonlinear regime¹⁷⁾ on the excess of the pump energy density w_{0m} over its observation-threshold value w_{0n} for four values of the reflection coefficient R_{+} . As expected, with increasing pump power the resonant-wave quantum-number efficiency increases monotonically for all values of R_{\star} . This growth is rapid in the initial section and then slows down. This character of the dependence is connected with the fact that in the initial stage the efficiency increases both as a result of a narrow increase of the lasing-pulse duration (rapid decrease of the development time T_n) and because of the increase of the instantaneous value of the quantum efficiency. When the lasing-pulse duration becomes comparable with the pumppulse duration, the increase of η_{nr} is due mainly to the increase of the instantaneous value of the quantum efficiency.

In a high-Q resonator (at a feedback coefficient $R \gtrsim 0.8$), saturation is observed in the plot of η_{nr} against the excess of the pump energy density over its threshold value (curve 1, Fig. 2). The decrease in the growth rate of η_{nr} begins approximately when the threshold is exceeded by a factor 1.5. It turns out that at these pump levels radiation of type II is produced in a high-Q resonator. The energy of the type-II radiation was not measured. From an analysis of photographs of the directivity patterns of both types it can be concluded, however, that further increase of the pump leads to a rapid increase of the type-II radiation, which reaches (at $w_{0m}/w_{0n} \approx 2$) the energy of the type-I radiation, and subsequently exceeds it. The type-I radiation energy

also increases in this case, but much more slowly, and this leads to a weak variation of η_{nr} in the region $w_{0m}/w_{0n} > 2$. As already noted, type-II radiation was not produced when the transparency of the SRPL resonator mirrors was increased. As seen from Fig. 2, with increasing transparency of the exit mirror the maximum value of η_{nr} increases. This increase is due to the redistribution of the useful and harmful losses with changing R_* , as follows from the corresponding theoretical relation¹⁸⁾

$$\eta_{n_r} \approx \frac{1-R_+}{1-R} \eta_n. \tag{6}$$

It is easily seen that with decreasing R_{\star} , in view of the increase of the observation threshold, the growth of the efficiency is limited by the finite value of the crystal breakdown threshold. The maximum value η_{nr} = 0. 19 was obtained for the most transparent of the experimentally employed exit mirrors (R_{\star} = 0. 64) and, as seen from Fig. 2, is still not the optimum. The experimentally obtained values of η_0 and η_{nr} are close to the maximum values obtained in experiments with singlemode beams. ^{(15, 18, 22) (19)} It can thus be concluded that at β_n = 0 the small-scale transverse structure of the pump beam has little effect not only on the angular and frequency characteristics of the resonant wave, but also on the energy characteristics of the lasing.

The multimode character of the pump begins to influence the energy characteristics of the SRPL when β_n differs from zero. This manifests itself in the fact that when β_n is increased the threshold pump energy increases, and both the efficiency with respect to the number of quanta of the resonant wave (Fig. 6) and the peak efficiency (Fig. 5) decrease. The indicated relations can be easily explained by using some results of the theory of wave amplification in an incoherent-pumping field, which were described in the Introduction. As follows from (1) and (2), the length l_n of the correlation between the nonresonant wave and the pump wave de-





creases with increasing β_n . Consequently, the spatial growth rate p_{\star} of the average resonant field also decreases. Thus, an increase of the angle β_n leads to a decrease of the efficiency of the interaction of parametrically amplified waves in an incoherent-pumping field.

It should be noted that, as follows from the experimental results, the influence of the multimode character and the decrease of the efficiency is stronger the smaller the feedback coefficient R. Thus, for example, for equal excesses of the pump power density over its observation-threshold value, at the optimum value (at $\beta_n = 0$) in an SRPL with R = 0.7, the efficiency with respect to the number of quanta of the resonant wave decreases by a factor of 1.5 in comparison with the optimal value when the angle β_n is increased to 35' ($l_n = 2.4$ cm), whereas in an SRPL with R = 0.5 the decrease is almost threefold.

These results are in qualitative agreement with the theoretical estimates, which can be easily obtained in the approximation in which the field structure of the resonant wave is given and its radiation is assumed to be coherent. Thus, for the ordinary SRPL the decrease $\Delta \eta_0$ of the quantum efficiency in the stationary state, due to the incoherence of the pump, can be expressed approximately (at $\Delta \eta_0 \ll \eta_0$) in the form

$$\Delta \eta_0 = (1-R)/8\gamma_0^2 l_n^2. \tag{7}$$

For the SRPL with pump reflection, the expression for $\Delta \eta_0$ differs from (7) by a factor on the order of unity. Thus, the decrease of the influence of the incoherence of the pump on the value of η_{nr} , which is observed with increasing *R*, agrees with the theoretical relation.

To conclude this section, we consider certain results of an investigation of the tuning characteristics of the SRPL, which were made in passing while investigating the influence of the multimode character of the pump on the energy characteristics.

The change that must be made in the angle β_n in these experiments can be effected, as follows from (4a), either by changing the angle θ_R between the normal to the resonator mirrors and the direction of the wave vector of the pump, or by changing λ_r when the angle θ_3 is varied.²⁰⁾

We obtained in the experiments the functions $\lambda_r(\theta_3)$ at constant θ_R for two values $\theta_R (\theta_R^{(1)} = 1^{\circ}11', \text{ and } \theta_R^{(2)} = 1^{\circ}46')$ and of $\lambda(\theta_R)$ at constant θ_3 for $\theta_3^{(1)} = 23^{\circ}05'$ and $\theta_3^{(2)} = 23^{\circ}30'$. The experimental points were in the main in good agreement with those calculated from the values of the refractive index given in^[21] in the entire tuning range (for the resonant wave, from 0.88 to 1.2 μ ; for the nonresonant wave this corresponds to the region from 3.2 to 1.68 μ).^[15] Some discrepancy between the experimental and theoretical relations, observed in the λ_r interval from 0.88 to 0.92 μ , is apparently due to the fact that the values of the refractive index of lithium iodate were not determined accurately enough for the nonresonant wave, which falls in the crystal absorption band.

The fact that lasing was obtained for λ_n in the 3- μ re-

gion, where the absorption coefficient of the nonresonant wave is quite high ($\alpha \approx 0.5 \text{ cm}^{-1[22]}$), gives grounds for assuming that in the investigated SRPL with pump mirror it is possible to obtain effective lasing in the entire transparency band of the lithium iodate crystal.^[23,24] For example, at $\lambda_3 = 0.6943 \mu$, effective generation of the resonant wave can be obtained at least in the regions $0.79 \leq \lambda_r \leq 2.7 \mu$ and $5.73 \geq \lambda_r \geq 0.93 \mu$).

The main conclusions of our investigations can be formulated as follows.

The spectral and spatial characteristics of the resonant-wave radiation in an incoherent pump field are close to the corresponding characteristics of the radiation generated in a coherent pump field, at least in the parameter regions $l_r/l_n < 1$ and $l_r \gamma_0 < 1$. In SRPL with nonresonant pump reflection it is possible to obtain in this case radiation with a divergence in the narrow synchronism angle plane close to the diffraction value, with a pump-beam radius a_x in this plane at least up to 0.4 cm. To obtain a resonant-wave radiation coherent in the plane of the broad synchronism angle at large values of a_y ($a_y \gtrsim 0.2$ cm), it is necessary to use additional selectors.

The efficiency of conversion of the incoherent pump into coherent radiation is close to the efficiency of conversion of coherent pumping at sufficiently large values of the parameter $\gamma_0 l_n$. When this parameter is decreased to values on the order of unity, the efficiency of conversion decreases, and the degree of this decrease depends on the feedback coefficient.

- ¹⁾We assume that the wave amplitudes are so normalized that the parameter γ_0 is proportional to the square root of the peak value of the average pump power density S_{0m} : $\gamma_0 = \chi S_{0m}^{1/2}$, where χ is a coefficient that depends on the nonlinear properties of the crystal (see, e.g., $1^{(3,41)}$).
- ²⁾The conditions (3) pertain to the case of large gain over the crystal length d ($\gamma_0 d \gtrsim 1$), a case to which we confine ourselves here. At $\gamma_0 d < 1$, the conditions (3) should be modified somewhat. ^[4]
- ³⁾In stimulated scattering by oscillations with small but finite group velocity (for example, in SMBS). an analog of condition (3a) imposes a limitation on the pump divergence. ^[4,8]
- ⁴⁾These results are partially reported in^[5,15].
- ⁵⁾The emission energy of a laser or of a parametric laser was measured with calibrated IÉK-1 meters; the waveform and duration of the pulses were measured with FÉK-15 coaxial photocells and with 12-7 time-interval meters.
- ⁶⁾These observation-threshold values were determined for the same SRPL resonator parameters: in the wavelength region λ_{τ} from 1 to 1.2 μ , $R_{-}=0.98$, $R_{\star}=0.96$, and R=0.84 (*R* is the resonator feedback coefficient: $R=R_{\star}R_{\star}T_{C}^{2}$). The reflection coefficients of the resonator mirrors at the pump wavelength were approximately the same at 0.15.
- ⁷⁾The spectra were plotted in the focal plane of the DFS-8 spectrograph, on the entrance slit of which was applied the directivity pattern of the SRPL radiation in the *xz* plane, obtained with the aid of a lens with F = 800 mm and scanned along the slit.
- ⁸⁾The total width of the directivity pattern in all the experiments was measured in the focal plane of a lens with F = 1600 mm.

- ⁹⁾In the investigation of a similar SRPL system, the authors of^[19] failed to note the analogous broadening of the emission spectra of the resonant waves, probably because they used narrow pump beams.
- ¹⁰⁾This is confirmed also by the fact that in our experiments the results obtained with similar SRPL systems, using the second harmonic of a neodymium laser, were for the most part analogous. ^[15,18]
- ¹¹⁾Thus, in this case, the pump mirror plays the role of a selecting element. This important, in our opinion, property of the SRPL system with pump mirror was not noted in ^[20].
- ¹²⁾Here $\delta\Omega_{rs} = 2\sqrt{2}\gamma_0/|\nu_{nr}|$ is the width of the synchronism band, ν_{nr} is the group detuning, $\Gamma_0 = \gamma_0 d$, and N_{sn} is the number of passes during the time of establishment of the nonlinear regime.
- ¹³⁾From simplest geometric considerations we can derive, for the radiation divergence produced in the yz plane by the enhancement of the fields that move out of the interaction region, the estimating formula $\Delta \theta_{ry}^{out} = a_y / L N_{sn}$, which holds equally for SRPL with and without pump mirror. ^[15,18]
- ¹⁴⁾The instantaneous value of the quantum efficiency was determined from oscillograms of the pump pulses incident on the SRPL and passing through it, using the relation $\eta_0(t)$ = $[A_1(t) - A_2(t)]/A_1(t)$, where $A_1(t)$ and $A_2(t)$ are the instantaneous values of the amplitudes of these impulses.
- ¹⁵⁾The efficiency with respect to the number of quanta of the resonant wave is determined from the formula $\eta_{mr} = W_r \lambda_r / W_{0m} \lambda_3$, where W_r and W_{0m} are the resonant-wave and the pump-wave energies measured with calorimeters.
- ¹⁶ These are not the limiting values of η_{mr} and W_{r} . This follows from the fact that the curve for Fig. 2 was obtained for a pump pulse duration approximately 1.5 times smaller than the remaining curves of this figure.
- ¹⁷⁾The time T_n of the start of the nonlinear regime was reckoned from the point of the leading front of the pump pulse corresponding to the 1/2 level, up to the instant of the start of the nonlinear regime.
- ¹⁸Here η_n is the coefficient of nonlinear conversion of the pump energy. When the registration system is suitably calibrated, it can be determined experimentally from oscillograms of the pump pulses incident on the SRPL and passing through it, as the ratio of the difference of the areas of these pulses to the area of the incident pulse. At a known value of the linear losses, the quantity calculated from formula (5) should coincide with the efficiency η_{nr} with respect to the number of photons of the resonant wave. Experiment has shown good agreement of these values, i.e., that the relation (6) is correct.
- ¹⁹The high-efficiency SRPL lasing obtained in the experiment had, as noted, a much larger degree of spatial coherence than the pump. An investigation of the directivity pattern of the pump incident on the SRPL and passing through it has shown that all the spatial components of the pump spectrum take part in the lasing process to an equal degree.
- ²⁰⁾In the experiments, the angle β_n was varied mainly by rotating the crystal, i.e., by changing the angle θ_3 and by suitably changing λ_{μ} . To keep the resonator Q constant we used mirrors with a broad flat section (approximately 2000 Å) of the plot of the reflection coefficient against λ_{\star} .
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Translated by J. G. Adashko