Hadron scattering at attainable and asymptotic energies in pomeron theory with $\alpha(0)>1$

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Experimental data at maximum accelerator energies require a value $\alpha(0)>1$ for the pomeron. In this case the experimentally observed "geometric scaling" (which, however, should vanish with increasing energy) can be explained in a natural manner. A number of other consequences of the theory for experimentally observable quantities are considered. The problem of s-channel unitarity is also discussed. It is shown that the asymptotic behavior is not of the Froissart type if $\Delta=\alpha(0)-1\leq g_{11}$, where g_{11} is the pomeron–pomeron transition vertex.

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1. INTRODUCTION

With recent publication of many new data on hadron interactions at high energies, interest in the properties of the Pomeranchuk singularity has increased. The most important characteristic of this singularity is the parameter $\alpha(0)$, which describes its position in the j-plane at $t=0$. In the case when the bare (unrenormalized) singularity is a pole, two contradictory variants of the theory are known, with asymptotically constant[1] and with increasing[2] total interaction cross sections. It is difficult then, however, to reconcile the data on the growth of the total cross section with increasing energy, on the slope of the elastic-scattering differential cross section, and on the cross section of the diffraction production of particles. It is not quite clear at present whether this is connected with incorrect allowance for the contribution of the branch points or to the value of $\alpha(0)$. Nonetheless, the reconciliation of the calculations with the experimental data becomes much simpler in a theory within which the pomeron has $\alpha(0)>1$.[3]

2. CONTEMPORARY ENERGIES

We consider the consequences that ensue for the measured quantities in the theory with $\alpha(0)>1$. All the results of this section were obtained with only non-enhanced diagrams taken into account. The contribution of the enhanced diagrams at presently attainable energies is apparently small because of the smallness of the pomeron interaction constants. At asymptotic energies, allowance for these diagrams becomes important, and they will be therefore be considered here in connection with the unitarity problem.

1. We note first that the so-called "geometric scaling" (GS) observed in pp scattering[4] finds a natural explanation. In this phenomenon the partial scattering amplitude $f(b, s)$ (b is the impact parameter) depends at high energies s only on a single variable $b^2/B(s)$, where $B(s)$ is the slope of the differential elastic-scattering cross section.

The contribution of the pomeron to the scattering amplitude, in the impact-parameter representation, is given by

$$\rho_0(b,y)=\frac{g_{\alpha1}g_{\beta1}e^{\Delta y}}{R_0^2+\alpha' y}\exp\left[-\frac{b^2}{4(R_0^2+\alpha' y)}\right]. \quad (1)$$

Here $g_{\alpha1}(k_\perp^2)g_{\beta1}(k_\perp^2)=g_{\alpha1}g_{\beta1}\exp(-R_0^2k_\perp^2)$ is the residue of the pomeron in elastic $\alpha\beta$ scattering; α' is the slope of the trajectory of the Pomeranchuk pole; $y=\ln(s/s_0)$; and $\Delta=\alpha(0)-1$.

In the pole approximation we have $B(y)\approx 2(R_0^2+\alpha' y)$. We see therefore from (1) that in a wide energy range, where the quantity

$$z=g_{\alpha1}g_{\beta1}e^{\Delta y}/(R_0^2+\alpha' y) \quad (2)$$

varies little with energy, i.e.,

$$\Delta\approx 2\alpha'/B(y), \quad (3)$$

GS will be observed. With increasing energy, an appreciable deviation from GS takes place. On the other hand, in πp and Kp scattering where $B(y)$ is smaller than in pp scattering, a much higher energy is required to satisfy relation (3), i.e., for GS to be present. The same holds also for the reaction $pp\to pX$, where R_0^2 is approximately half as large as in the elastic case.

2. It is seen from (1) that as y increases the partial amplitude $\rho_0(b, y)$ exceeds unity and violates unitarity. With increasing cross section, however, the contribution of the rescattering increases. The mutual screening of the elastic and inelastic channels leads to a decrease of the amplitudes and unitarity can be restored. This is seen with the non-enhanced diagrams of Fig. 1 as an example. This sum, calculated in the eikonal approximation, takes in the b representation the form

$$\rho(b, y)=1-\exp[-\rho_0(b, y)]. \quad (4)$$

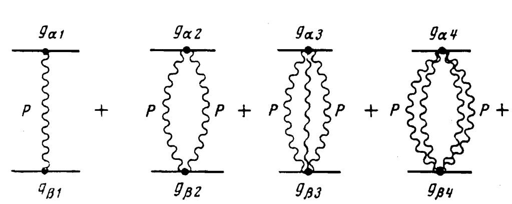

FIG. 1.

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The asymptotic form of $\rho(b, s)$ is that of a black disk of radius $2(\alpha'\Delta)^{1/2}y$ [5]:

$$\rho(b, y)\approx\theta(4\alpha'\Delta y^2-b^2) \tag{5}$$

or in the $(\omega, k_\perp)$ representation

$$\rho(\omega, k_\perp)=4\alpha'\Delta/(\omega^2+4\alpha'\Delta k_\perp^2)^{1/2}.$$

3. The energy dependence of the total cross section and the total inelastic cross section corresponding to (4) is given by expressions

$$\sigma_{tot}(y)=8\pi(R_0^2+\alpha'y)\varphi(z), \tag{6}$$

$$\sigma_{in}(y)=4\pi(R_0^2+\alpha'y)\varphi(2z), \tag{7}$$

$$\varphi(z)=C+\ln z-\mathrm{Ei}(-z). \tag{8}$$

Here z is defined in (2), $C\approx 0.577$ and $\mathrm{Ei}(-z)$ is the integral exponential function. The energy dependence of the slope $B(y)$ is seen from the expression

$$B(y)=2(R_0^2+\alpha'y)\varphi^{-1}(z)\int_0^z\varphi(x)\frac{dx}{x}. \tag{9}$$

It is understood from (6)–(9) that σ_{tot}/B, σ_{el}/σ_{tot}, etc. are practically constant at the FNAL and ISR accelerator energies ($y\approx 5$–8), where (3) is satisfied. These ratios begin to change with increasing energy.

It follows from (8) and (9) that as $y\to\infty$ the slope $B(y)\to\alpha'\Delta y^2$. It is therefore seen from (5) and (9) that the GS phenomenon will appear again at asymptotic energies. This agrees with the general result obtained by Auberson, Kinoshita, and Martin.[6]

The ratio of the real part of the forward scattering amplitude to the imaginary part is

$$\varepsilon=\frac{\mathrm{Re}\,f(y,0)}{\mathrm{Im}\,f(y,0)}=\frac{\pi}{2}\left\{\frac{\alpha'}{R_0^2+\alpha'y}+\frac{1}{\varphi(z)}\left(\Delta-\frac{\alpha'}{R_0^2+\alpha'y}\right)(1-e^{-z})\right\}. \tag{10}$$

At energies where GS holds, ε is almost constant and is close in magnitude to $\Delta\pi/2$. Asymptotically ε is equal to π/y.

4. Attention was called in[7] to the fact that the rapid growth of σ_{tot}, which follows from (1), contradicts the available data obtained in experiments with cosmic rays at energies up to $s\approx 10^9$ GeV^2. These data correspond to absorption of protons by air nuclei with $\bar{A}=14.4$. The inelastic proton-nucleus interaction cross sections were calculated in[7] in the Glauber approximation. It is seen from (5), however, that at energies as high as $s=10^9$ GeV^2 the interaction radius grows enough to become comparable with the dimensions of the nucleus. The parton clouds of most nucleons in the nucleus then turn out to overlap in the transverse plane, and the model of multiple scattering by the nucleons of the nuclei no longer holds.[8]

5. In the case when the pomeron makes a contribution to the amplitude with spin flip, the polarization of the elastic scattering has a small part that is due to the vacuum exchange and does not depend on the energy in a wide range. Analogously, the spin-spin correlations in the total cross sections depend little on the energy.

6. When multiple production of particles is considered, an explanation can be found for the experimentally observed phenomenon called the "Koba, Nielsen, and Olesen" (KNO) scaling.[9] The topological cross section satisfies the relation

$$\langle n\rangle\sigma_n/\sigma_{in}=\Psi(n/\langle n\rangle). \tag{11}$$

Here $\langle n\rangle$ is the average multiplicity of the produced particles; σ_n is the particle production cross section; $\Psi(n/\langle n\rangle)$ is a certain function that is independent of energy. To verify the validity of (11) it suffices to show that the moments $\langle n^k\rangle/\langle n\rangle^k$ are independent of energy.

The diagrams that contribute to σ_n are obtained by cutting the diagrams in Fig. 1 in accordance with the rules of Abramovskiĭ, Gribov, and Kancheli.[10,11] Averaging n^k over the contributions of these diagrams we obtain

$$\sigma_{in}\langle n^k\rangle=\int e^{-\rho_0(b,y)}\left(ay\rho_c\frac{d}{d\rho_c}\right)^k(e^{\rho_c(b,y)}-1)e^{-\rho_0(b,y)}d^2b. \tag{12}$$

Here $\rho_c(b, y)=2\,\mathrm{Im}\rho_0(b, y)$ is the Green's function of the cut pomeron; $(\exp\{\rho_c\}-1)$ is the sum over the number of cut pomerons; the operator $(ay\rho_c d/d\rho_c)^k$ extracts the value of n^k for each diagram; ay is the number of particles in one cut pomeron; the factor $\exp\{-\rho_0\}$ takes into account the absorption corrections.

After integrating in (12) we obtain

$$\sigma_{in}\langle n^k\rangle=4\pi(R_0^2+\alpha'y)(ay)^k\Phi_{k-1}(2z). \tag{13}$$

where z is defined in (2) and

$$\Phi_k(x)=\int_0^x e^{-t}\left(t\frac{d}{dt}\right)^k e^t\,dt.$$

From (13) and (7) we get

$$\langle n^k\rangle/\langle n\rangle^k=[\varphi(2z)]^{k-1}\Phi_{k-1}(2z)/(2z)^k. \tag{14}$$

Thus, in the energy region where z changes little, i.e., where the GS phenomenon takes place, the moments (14) are independent of energy. Allowance for the Poisson distribution in each "comb" leads to corrections that decrease with energy, the first of which, of order y^{-1}, is equal to

$$k(k-1)\Phi_{k-2}(2z)\varphi^{k-1}(2z)/2ay(2z)^k.$$

Thus, the KNO scaling can be only approximate. With increasing energy, z increases and both types of scaling are violated. It is interesting that asymptotically GS is restored, but the KNO scaling is not.

7. At high energies σ_n the distribution in the number of produced particles will have oscillations[10] with period $\Delta n=ay$. It follows also from (13) and (7) that the average multiplicity increases like

$$\langle n\rangle=c+ay2z/\varphi(2z).$$

8. Obviously, since $\alpha(0)>1$, the "Feynman scaling" will also be violated. The inclusive cross section in the central region should increase with energy like

$$d\sigma/dy \propto e^{\Delta y}. \tag{15}$$

Dubovikov and Ter-Martirosyan[12] have shown that (15) remains valid also after the enhanced diagrams are added.

In the three-Reggeon region the inclusive cross section also increases with energy[13]:

$$d\sigma/dtdx = G_{PPP}(t)\,e^{\Delta y}/(1-x)^{1+\Delta+2\alpha' t}. \tag{16}$$

3. s-CHANNEL UNITARITY AND ASYMPTOTIC REGIME

1. The unitarity problem is of course not confined to the eikonal expansion. A generalization to a larger class of non-enhanced diagrams was carried out by Cardy.[14] He has shown that if the vertices $g_{\alpha n}$ and $g_{\beta n}$ for the emission of n pomerons by particles α and β admit of a unique analytic continuation to the complex-plane, then the θ function in (5) acquires a factor $g_{\alpha 0}g_{\beta 0}$, i.e., the disk may be "gray."

Inclusion of enhanced diagrams greatly complicates the unitarity problem. Following Cardy, it is convenient to replace the sum of the pomeron diagrams in Fig. 1 by the diagram shown in Fig. 2, which K. A. Ter-Martirosyan named "froissaron." It is possible to combine froissarons into more complicated diagrams, for example those shown in Fig. 3. The vertex g_{00} was introduced by Cardy. This is a result of an analytic continuation of the vertices g_{mn} of the conversion of m pomerons into n.

It is important to note that the summation in Fig. 3 is carried out with $m=n=1$. The corresponding term in the Lagrangian of the pomeron field $g_{11}\Psi^+\Psi$ has the form of a mass term, and is therefore already contained in Δ. When the froissaron diagrams are constructed it is therefore necessary to redefine the quantity Δ and replace it by

$$\Delta_0 = \Delta - g_{11}. \tag{17}$$

It is easily seen that the froissaron diagrams violate unitarity. For example, the diagram of Fig. 3(a) corresponds at $t=0$ to the singularity ω^{-6} ($\omega=j-1$), i.e., its contribution to σ_{tot} increases like y^5. Cardy has noted[14] that there is an appreciable cancellation in the sum of the diagrams in Figs. 3(a) and 3(b). He proposed a diagram-summation procedure in which each enhanced diagram corresponds to a cancelling diagram.

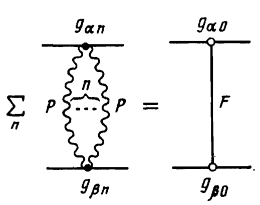

FIG. 2.

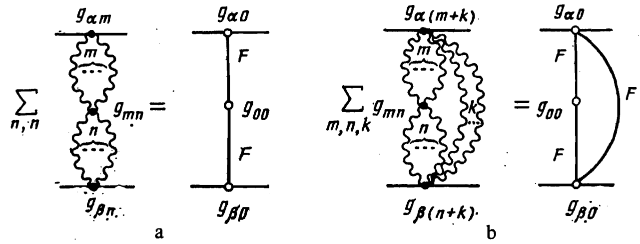

FIG. 3.

However, these cancellations are insufficient to ensure unitarity. Indeed, if the behavior of the Green's function of the froissaron at distances $b \sim 2(\alpha'\Delta_0)^{1/2}y$ is correctly taken into account, then it turns out that the sum of the diagrams in Figs. 3(a) and 3(b) increases like $\sqrt{y}$ at $b \approx 2(\alpha'\Delta_0)^{1/2}y-(\alpha'/\Delta_0)^{1/2}\ln y$.

2. It is nevertheless possible to propose another method of summing the diagrams. It is easily seen that iteration of the diagram of Fig. 3(a) in the s channel, carried out in analogy with Fig. 1, leads to a unitary answer. We denote the procedure of "eikonalization" of the $\rho(b, y)$ diagram by the symbol $\mathrm{E}[\rho(b, y)]$. The function $\mathrm{E}(\rho_0)$ takes the form (4) in the eikonal approximation. It is clear furthermore that the operation $\mathrm{E}(\rho)$ can be carried out with any diagram or sum of diagrams which are irreducible in the s channel, i.e., they cannot be cut by a vertical line without crossing pomeron lines at the same time. Taking the foregoing into account, the results of the summation of all the diagrams can be written in the form

$$T(b, y) = \mathrm{E}\Big[\sum_k \rho_k(b, y)\Big]. \tag{18}$$

Dubovikov and Ter-Martirosyan[12] have shown that the requirement that the sum $\sum\rho_k$ be positive can be satisfied if g_{00} is small enough and the solution $T(b, y)$ (18) takes the form of a froissaron:

$$T(b, y) \approx \theta(4\alpha'\Delta_0 y^2 - b^2). \tag{19}$$

We report here briefly the method proposed in[12] for classifying the ρ_k diagrams, and show that solutions that are not of the Froissart type are also possible.

We divide $\sum_k \rho_k$ into three groups:

$$\sum_k \rho_k = \rho_0 + D(T) + C(T). \tag{20}$$

Here $\rho_0(b, y)$ is defined in (1), where Δ must be replaced by Δ_0 from (17). The group $D(T)$ contains diagrams that are irreducible in either the s or the t channels. This is a sequence of skeleton diagrams constructed from the exact Green's functions $T(b, y)$. The group $C(T)$ contains the diagrams shown in Fig. 4. In the (ω, k^2) representation, $C(\omega, k_\perp^2)$ takes the form[12]

$$C(\omega, k_\perp^2) = T(\omega, k_\perp^2) - [T^{-1}(\omega, k_\perp^2) + g_{00}]^{-1}. \tag{21}$$

We note that the addition of enhanced diagrams to the series shown in Fig. 1 has led to the renormalization

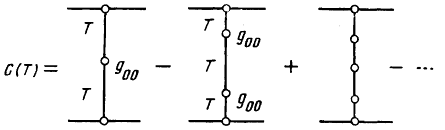

FIG. 4.

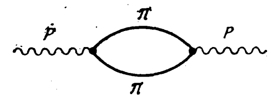

FIG. 5.

$\Delta - \Delta_0$. Thus, at very high energies, where the enhanced diagrams reach the asymptotic form, a change of the regime takes place. One cannot exclude here the interesting possibility that $g_{11} \geqslant \Delta$, i.e., $\Delta_0 \leqslant 0$. The basis for this may be an estimate of g_{11} in the one-pion-exchange model. Simple calculation of the diagram of Fig. 5 yields for g_{11}

$$g_{11} \approx \frac{(\sigma_{tot}^{\pi N})^2}{16\pi^3 \sigma_{tot}^{NN}} [\mu^2 - 2\mu^2 \ln(\mu^2 R^2) + R^{-2}]. \tag{22}$$

Here μ is the pion mass; $R^2/2$ characterizes the dependence of $\sigma_{tot}^{\pi N}$ on the square of the mass of the virtual pion. If we choose $R^2 = 1\ (\text{GeV}/c)^{-2}$, then we obtain $g_{11} \approx 0.08$, i.e., a quantity of the same order as Δ.

We turn now to Eqs. (18) and (20), and consider the case $\Delta_0 \leqslant 0$. In this case the Froissart regime is impossible in the asymptotic region. Let the total cross section increase like

$$\sigma_{tot}(y) \propto y^{\eta}. \tag{23}$$

Then $T(\omega, k_\perp^2 = 0) \propto \omega^{-\eta-1}$. If the contribution of $D(b, y)$ does not increase exponentially with y, then $C(\omega, k_\perp^2)$ should have no singularities in the right-hand ω half-plane. This calls at least for

$$\eta \leqslant 1. \tag{24}$$

In this case the inclusive cross section in the central region and the average multiplicity are given by

$$d\sigma/dy_1 \propto y_1^{\eta}(y - y_1)^{\eta}, \tag{25}$$

$$\langle n \rangle \sim y^{\eta+1}. \tag{26}$$

A possible realization, in the (b, y) representation, of a solution with $\eta = 1$ is a ring of constant width with radius increasing in proportion to y.

We note that in a known case[2] in which the bare value of Δ is equal to the critical value, $\Delta = \Delta_c$, the sum (20) has the extreme right-hand singularity at $\omega = 0$. This is therefore one of the cases considered above. This gives rise to the requirement

$$\Delta_c \leqslant g_{11}, \tag{27}$$

which agrees with the estimates of Δ_c from[2] and of g_{11} from (22).

We note that various asymptotic-regime types may not appear at attainable energies. The contribution of the non-enhanced diagrams leads to a Froissart growth of the cross sections already at energies 10^6–10^{10} GeV. The enhanced diagrams whose contribution leads to a change in the energy dependence reach their asymptotic regime at higher energies.

4. CONCLUSION

The pomeron theory with $\alpha(0) = 1$ has many attractive properties and explains the general regularities of the scattering of hadrons at attainable energies. A number of subtle effects observed in experiment in recent years, however, has made it necessary to consider a variant of the theory with $\alpha(0) > 1$. At the existing energies this causes small deviations of order Δy, which make it possible to explain the experimental data. The asymptotic picture of the scattering is radically altered.

We list the main consequences for the experimentally measured quantities:

1) Owing to a "fortuitous" conjunction of the parameters, GS scaling obtains at FNAL and ISR accelerator energies. With further increase of energy, the GS will vanish and will appear again in the asymptotic regime.

2) The total cross sections for hadron scattering grow at an increasing rate and reach the Froissart regime at energies 10^6–10^{10} GeV. So far, this rapid growth does not contradict any existing data.

3) The ratio of the real part of the forward scattering amplitude to the imaginary part should reach a value $\sim \Delta\pi/2$ with increasing energy and should remain constant in a wide energy range.

4) If the pomeron contributes to the amplitude with spin flip, then the elastic polarization should have a small energy-independent part in the energy region where GS holds.

5) An explanation is found for the approximate KNO scaling of the topological cross sections in the energy region where z is approximately constant. The average multiplicity of the produced particles acquires an additional factor $2z/\varphi(2z)$, and deviates only slightly from the $\ln(s/s_0)$ dependence at accelerator energies.

6) Exact Feynmann scaling cannot take place. The inclusive cross section increases like $e^{\Delta y}$ in the central and three-Reggeon regions.

At asymptotic energies, the unitarity problem arises. A procedure is proposed for summing the Cardy-Gribov froissaron diagrams and makes it possible to obtain an answer that is unitary in explict form. It is shown that solutions that are not of the Froissart type are possible only if $\Delta \geqslant g_{11}$. One such solution is the strong-coupling variant,[2] where $\Delta = \Delta_c$ is the critical value.

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[1]V. N. Gribov, Yad. Fiz. 17, 603 (1973) [Sov. J. Nucl. Phys. 17, 313 (1973)].
[2]A. A. Migdal, A. M. Polyakov, and K. A. Ter-Martirosyan, Zh. Eksp. Teor. Fiz. 67, 84 (1974) [Sov. Phys. JETP 40, 43 (1975)]; H. D. I. Abarbanel and J. B. Bronzan, Phys. Rev. D9, 2397 (1974).
[3]P. D. B. Collins, F. D. Gault, and A. Martin, Phys. Lett. 47B, 171 (1973); Nucl. Phys. B80, 135 (1974); Nucl. Phys. B83, 241 (1974).
[4]V. Barger, Proc. 17th Intern. Conf. on High Energy Physics, London, 1974.
[5]H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1456 (1970).
[6]G. Auberson, T. Kinoshits, and A. Martin, Phys. Rev. D3, 3185 (1971).
[7]V. Barger, F. Halzen, T. K. Gaisser, C. J. Noble, and G. V. Yodh, Phys. Rev. Lett. 33, 1051 (1974).
[8]O. V. Kancheli, Pis'ma Zh. Eksp. Teor. Fiz. 18, 469 (1973) [JETP Lett. 18, 277 (1973)].
[9]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).
[10]V. A. Abramovski, V. N. Gribov, and O. V. Kancheli, Proc. 16th Intern. Conf. on High Energy Physics, Chicago-Batavia, 1972.
[11]K. A. Ter-Martirosyan, Phys. Lett. 44B, 377 (1973).
[12]M. S. Dubovikov and K. A. Ter-Martirosyan, Intern. Conf. on Fundamental Interactions of Elementary Particles, Coral Gables, 1976.
[13]Yu. M. Kazarinov, B. Z. Kopeliovich, L. I. Lapidus, and I. K. Potashnikova, JINR E2-9218, Dubna, 1975.
[14]J. L. Cardy, Nucl. Phys. B75, 413 (1974).

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Investigation of the $\pi^+ \to e^+ + \nu_e + e^+ + e^-$ decay

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An experiment has been carried out on the $\pi^+ \to e^+ + \nu_e + e^+ + e^-$ decay. Five events were found and could essentially be interpreted as $\pi^+ \to e^+ + \nu_e + \gamma$ decays although they should also contain $\pi^+ \to e^+ + \nu_e + e^+ + e^-$ events with a probability of over 1/2. The ratio γ of the axial and vector formfactors in the $\pi^+ \to e^+ + \nu_e + \gamma$ decay is reported together with the estimated upper limits for the formfactors ξ in the $\pi^+ \to e^+ + \nu_e + e^+ + e^-$ decay and the electromagnetic radius of the pion. An upper limit has been established for the probabiity of the latter decay mode: $W_{\pi \to e\nu ee}/W_{\pi \to \mu\nu} < 4.8 \times 10^{-9}$ with a confidence level of 90% on the assumption that the decay matrix element is a constant.

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1. INTRODUCTION

No one has so far observed the

$$\pi^+ \to e^+ + \nu_e + e^+ + e^- \tag{1}$$

decay. In a previous paper,[1] we reported that the upper limit for the relative decay probability was

$$R = W(\pi^+ \to e^+ + \nu_e + e^+ + e^-)/W(\pi^+ \to \mu^+ + \nu_\mu) < 3{,}4 \cdot 10^{-8} \tag{2}$$

with a 90% confidence level. In the present paper, we report the results of a new experiment performed at the Laboratory for Nuclear Problems of the Joint Institute for Nuclear Research in which searches for the decay mode given by (1) were carried out. Preliminary results of this experiment were reported in[2].

The $\pi \to e\nu ee$ decay mode was first considered by Okun', Pontecorvo, and Rubbia[3] in connection with the possible existence of an anomalous four-lepton interaction. However, estimates based on experiments using colliding electron-positron beams[4] lead to an exceedingly low decay probability for this diagram ($R < 2 \times 10^{-15}$).

The $\pi \to e\nu ee$ decay may proceed as a result of a six-fermion interaction, the possible existence of which was first suggested by Glashow.[5] The corresponding decay diagram is shown in Fig. 1. The probability of decay due to the six-fermion interaction has been calculated by Vanzha, Isaev, and Lapidus.[6] The corresponding interaction constant is conveniently written in the form

$$G_6 = \frac{G}{\sqrt{2}} \frac{1}{\lambda^3}, \tag{3}$$

where G is the usual four-fermion interaction constant. The parameter λ has the dimensions of mass and is usually employed to characterize the constants G_6. It was found in[6] that

$$R = 1.25 \cdot 10^{-4} (m_\mu/\lambda)^6, \tag{4}$$

where m_μ is the muon mass.

The upper limit for the probability of the decay mode

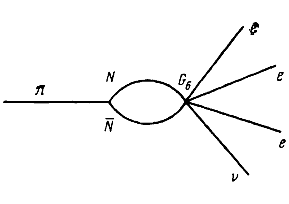

FIG. 1. Diagram representing the $\pi \to e\nu ee$ decay due to the six-fermion interaction.

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