# Tearing instability in plasma configurations

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The development of tearing instability in plasma containing a neutral diffusive layer and a magnetic field with a small but finite component perpendicular to the sheet is discussed. The effect of this component on electron orbits in the neighborhood of the neutral sheet is to stabilize the electron tearing mode even for very small amplitudes of the normal field. The development of the ion tearing mode of given wavelength is found to be possible only in the "gap" corresponding to a certain restricted range of values of the normal magnetic-field component for which its effect on ion orbits in the neutral sheet can still be neglected whilst the stabilizing contribution of magnetized electrons to the plasma permittivity is already small. It is shown that gaps of this kind can appear only when the current in the sheet is large enough. When the value of the normal magnetic-field component lies below the instability region, the plasma states are metastable with respect to the excitation of the ion tearing mode.

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## INTRODUCTION

Processes of reconnection and annihilation of a magnetic field play a key role in a broad range of physical phenomena, for example, in solar flares, [1,2] reconnection processes in the earth's magnetosphere, [3] processes in neutral and current-carrying sheets of laboratory plasma, [4] and instabilities of thermonuclear plasmas in tokamaks (tearing instability[5]). Reconnection processes in the tail of the magnetosphere are of particular interest at present because the extensive experimental studies of plasma phenomena in the magnetosphere, which have been carried out with the aid of satellites, enable us to look forward to an acceptable theory that would be of general interest in physics.

The description of spontaneous reconnection processes in terms of the simplest one-dimensional neutral-sheet model, first put forward by Coppi, Laval, and Pellat, [6] encounters serious difficulties because the tearing instability mode, [6,7] which is characteristic for such configurations, can be stabilized for very small perturbation amplitudes, [8–10] and a global rearrangement of the neutral-sheet configuration during the instability development will not occur.

Moreover, experimental data show that the topology of the magnetic field in the region of field inversion cannot usually be described within the framework of the simple one-dimensional model. In particular, the rearrangement of the magnetic-field configuration and the formation of the neutral line in the tail of the earth's magnetosphere occur in those regions where a finite magnetic field component, perpendicular to the neutral sheet, was present prior to this. [11–14] This rearrangement occurs spontaneously in a very short time, and is probably connected with the development of the tearing instability. Studies of the stability of a neutral sheet in the presence of a finite magnetic-field component perpendicular to the sheet are therefore essential.[1)] Cases for which the normal component to the sheet changes sign, or when it is constant or quasiconstant along the sheet, are best considered separately because the dynamics of particles in the neighborhood of neutral lines has a number of distinctive features. This last (and simplest) case is discussed in the present paper.

Preliminary analysis[15] has shown that the most important effect here is the perturbation of particle trajectories in a narrow region near the neutral sheet by the normal component $B_n$ of the magnetic field which, firstly, violates the conditions for the development of instability associated with the resonance interaction of these particles[2)] with oscillations and, secondly, because of the capture of the particles by the normal field into Larmor orbits, there is a greater stabilizing contribution to the real part of the permittivity (but this contribution may decrease with increasing $B_n$). It turns out that the electron instability mode is stabilized for small values of $B_n$ whilst the development of the ion instability of given wavelength is possible only in a certain "gap," i.e., a finite interval of values of $B_n$ in which the motion of the ions is still only slightly perturbed, whereas the stabilizing effect of the electrons is already small. The existence of such "gaps" is possible only for a sufficiently thin sheet (sheet with a sufficiently large current).

Plasma states lying under the set of gaps corresponding to wavelengths that can be excited in a given configuration are metastable with respect to the development of the ion tearing instability (a small increase in the normal component resulting in a finite perturbation takes the plasma into the unstable region). The ion instability mode is more rapid by a factor of $(M/m)^{1/4}$ as compared with the electron mode. Its development (if the gap exists) should therefore lead to a rapid spontaneous rearrangement of the magnetic-field topology in the region of the neutral sheet, which may have a radical effect on the dynamics of the plasma configuration. The role of the ion tearing instability as a mechanism of spontaneous reconnection during the transition to the explosive phase of a magnetic substorm is discussed briefly in the Conclusions.

## 1. CASE OF EQUILIBRIUM STATE AND BASIC EQUATIONS FOR PERTURBATIONS

The simplest two-dimensional model of a magnetic field near a neutral sheet is the homogeneous current sheet model (with the current flowing along the $y$ axis) and a constant magnetic-field component at right-angles to the sheet. In particular, it was shown previously in[16,17] that this model could be used to achieve a correct description of certain phenomena in the earth's magnetosphere:

$$\mathbf{B}=B_0(x)\mathbf{e}_z+B_n\mathbf{e}_x, \tag{1}$$

where $\mathbf{e}_x$ and $\mathbf{e}_z$ are unit vectors along the axes $x$ and $z$.

However, attempts to obtain a rigorous description of the equilibrium state of even this simple two-dimensional plasma configuration encounter considerable mathematical difficulties in the solution of the self-consistent set of stationary equations for the field and particle.[18-22]

When the stability problem is tackled by the variational method,[23] it is possible to obtain the necessary and sufficient conditions for equilibrium that are very general but, at the same time, applicable to the case of arbitrary two-dimensional equilibrium. These conditions involve the equilibrium scalar and vector potentials and the distribution function corresponding to the particular equilibrium state in the form of a function of the constants of motion:

$$f_{0j}=F_j(H_{0j};\ P_{yj}), \tag{2}$$

where the energy integral $H_{0j}$ and the conserved component of the generalized momentum $P_{yj}$ are given by

$$H_{0j}={}^1/_2 m_j\mathbf{v}^2+e_j\varphi_0(x,\ z),\quad P_{yj}=m_jv_y+e_jc^{-1}A_0(x,\ z).$$

The situation is substantially simplified if we are interested not in the two-dimensional equilibrium itself but in the influence of the normal component of the magnetic field on the stability of the sheet. We shall show that this influence is largely due to the change in the character of the particle trajectory trapped near the neutral sheet and providing the main contribution to the interaction between the plasma particles and perturbations. The precise form of the distribution of these particles is found to be unimportant for the interaction, so that it can be assumed to have the Maxwellian shape, and distortion due to the presence of a weak current can be neglected in the same way as it is in the well-known problem of drift instability of particles trapped in a magnetic field with weak magnetic mirrors.[24] Thus, when the normal component of the magnetic field is small both inside and outside the neutral sheet, we can assume in the analysis of stability that the particle distribution and the profile of the magnetic-field component parallel to the sheet take the form obtained by Harris[18] for one-dimensional equilibrium:

$$\mathbf{B}=B_0\,\mathrm{th}\,(x/L)\,\mathbf{e}_z+B_n\mathbf{e}_x,$$
$$F=\frac{n_0}{\mathrm{ch}^2(x/L)}\left(\frac{\alpha_j}{\pi}\right)^{3/2}\exp\{-\alpha_j(v_x^2+(v_y-u_j)^2+v_z^2)\}, \tag{3}$$

where

$$\alpha_j=v_{Tj}^{-2}=\frac{m_j}{2T_j},\quad u_j=cT_j\frac{d}{dx}\ln n(x)/e_jB_z(x),$$

$$\frac{u_i}{T_i}+\frac{u_e}{T_e}=0,\quad L=\left(\frac{\omega_{pj}^2}{c^2}\frac{u_j^2}{v_{Tj}^2}\eta_j\right)^{-1/2},\quad \omega_{pj}^2=\frac{4\pi e_j^2n_0}{m_j},$$
$$\eta_j=\frac{T_i+T_e}{T_j}.$$

The dimensionless parameter

$$b=B_n/B_0\ll 1,$$

which characterizes the magnetic-field component perpendicular to the sheet is one of the small parameters of the problem. Another small parameter is the ratio of the Larmor radius of the particles to the characteristic thickness of the neutral sheet:

$$\varepsilon_j=\rho_j/L,\quad \rho_j=m_jv_{Tj}c/|e_j|B_0.$$

When the finite size of the parameter $b$ is taken into account (the problem is only slightly two-dimensional), the characteristic size of the sheet inhomogeneity

$$L_z=|\partial\ln n(x,\ z)/\partial z|^{-1}$$

is, at worst, not greater than $Lb^{-1}$, which, for small $b$, enables us to seek the perturbations of the vector and scalar potentials in the quasiclassical form in $z$[3]:

$$\left.\begin{array}{l}A_{1y}=A_1(x)\\ \varphi_1=\varphi_1(x)\end{array}\right\}\exp\left(-i\omega t+i\int^z k(z')\,dz'\right). \tag{4}$$

The condition for the validity of (4) is

$$2\pi k^{-1}<L_z<Lb^{-1}$$

and this provides an upper bound for the wavelength of the developing perturbations

$$m=kL\gg b. \tag{5}$$

When $b\neq$ const, or the plasma parameters vary along the sheet, the inequality given by (5) is replaced by the more stringent inequality

$$m>m^*=2\pi L/L_z^*, \tag{6}$$

where $L_z^*$ is the corresponding characteristic inhomogeneity size.

We shall now linearize the Vlasov equations for the perturbations of the scalar and vector potentials $A_1(x,z)$ and $\varphi_1(x,z)$, subject to the following assumptions: a) the phase velocities of the perturbations are small in comparison with the velocity of light, i.e., $|\omega/kc|\ll 1$; b) the characteristic instability-development time is small in comparison with the characteristic time of the particle motion, i.e., $|\omega/kv_T|\ll 1$; and c) the wavelength of the perturbations under consideration is much greater than the Debye wavelength. This enables us to neglect the displacement currents, the components $A_{1x}$ and $A_{1y}$ of the vector potential,[23] and the perturbations of the space charge (so that the quasineutrality condition can be employed), respectively.

The correction to the distribution function in the linear approximation is

$$j_{1j}=\frac{e_j}{c}\left(F_{Pj}{}'A_1+F_{Hj}{}'(\varphi_1 c)+F_{Hj}{}'\left[-i\omega\int_{-\infty}^{0}d\tau(v_y(\tau)A_1(\tau)-\varphi_1(\tau)c)\right]\right), \tag{7}$$

where

$$F_{Pj}{}'=\frac{\partial F_j(H_{0j};P_{yj})}{\partial P_{yj}},\quad F_{Hj}{}'=\frac{\partial F_j(H_{0j};P_{yj})}{\partial H_{0j}}.$$

Integration with respect to the time $\tau=t'-t$ in the last term in (7) is carried out along the unperturbed particle trajectory, which is calculated with allowance for the influence of the normal magnetic-field component upon it.

Using the quasineutrality equation, which relates the perturbations of the scalar and vector potentials, we obtain the following equation for the $y$ component of the perturbation of the vector potential, $A_1=A_{1y}$, which has the form of the Schrödinger equation:

$$\left[\frac{d^2}{dx^2}-(k^2+V_0(x)+V^{<}(x,\omega,k))\right]A_1(x)=0. \tag{8}$$

The coefficients in this equation are determined by the following equations in which $\varepsilon_j(\tau)=\exp\{-i\omega\tau+ik[z'(\tau)-z]\}$, $q=4\pi e_j^2/c^2$, $V^{<}=\tilde{V}^{<}+\delta\tilde{V}^{<}$:

$$V_0(x)=-q\sum_j\int F_{Pj}{}'v_y\,d^3v, \tag{9a}$$

$$\tilde{V}^{<}=\sum_j\tilde{V}_j^{<}=-q\sum_j\int F_{Hj}{}'v_y\,d^3v\left[-i\omega\int_{-\infty}^{0}v_y(\tau)\varepsilon_j(\tau)\,d\tau\right], \tag{9b}$$

$$\delta V^{<}=-q\left(\sum_j\int F_{Hj}{}'v_y\,d^3v\left[-i\omega\int_{-\infty}^{0}\varepsilon_j(\tau)\,d\tau\right]\right)$$
$$\times\left(\sum_j\int F_{Hj}{}'d^3v\left[-i\omega\int_{-\infty}^{0}v_y(\tau)\varepsilon_j(\tau)\,d\tau\right]\right)\left(\sum_j\int F_{Hj}{}'d^3v\left[1+i\omega\int_{-\infty}^{0}\varepsilon_j(\tau)\,d\tau\right]\right)^{-1} \tag{9c}$$

The expression for the perturbed current $j_{1y}$ obtained with the aid of (7) can be written in the form given by (8) only under an important simplifying assumption, i.e., the local approximation in $x$, which involves the substitution $A_1[x'(\tau)]\to A_1(x)$ in (7). This approximation is possible because the characteristic size $l_x$ of the projection of the particle orbit onto the $x$ axis is small in comparison with the characteristic size $L_x\sim L$ of the inhomogeneity at right-angles to the neutral sheet. The quantity $l_x\sim|x'-x|_{\max}$ amounts to $\varepsilon_j L$ in the external region, and $\varepsilon_j^{1/2}L$ or $bL$ (see below) in the internal region near the plane of the neutral sheet, respectively.

The energy eigenvalue in (8) is $E=-k^2$, and the potential energy takes the form of a shallow potential well $V_0(x)$ with a narrow hump $V^{<}$ at the center (in the neighborhood of the neutral sheet). As already noted, the shape of this well is not essentially modified when a small ($b\ll 1$) vertical magnetic field is superimposed, and can be determined with the aid of the simple distribution given by (3). Under these conditions, $F'_{Hj}=-F_j/T_j$, $F'_{Pj}=u_jF_j/T_j$, and we have

$$V_0(x)=-2L^{-2}\,\mathrm{ch}^{-2}\,(x/L). \tag{10}$$

The potential well defined by (10) is referred to as the Teller potential.

It follows from the results reported in our previous paper[10] that perturbations with $k_y\neq 0$ correspond to a still shallower potential well

$$V_0(x)=-2L^{-2}\,\mathrm{ch}^{-2}\,(x/L)\cos^2\theta',\quad \mathrm{tg}\,\theta'=k_y/k_z;$$

and it is readily shown that the form of the potential hump $V^{<}$ remains unaltered. When $\theta'\neq 0$, the localized perturbation is more readily squeezed out of the potential well by the hump, and the instability is reduced more rapidly than in the case $k_y=0$. As noted above, we can therefore confine our attention to the analysis of the most unstable perturbations with $k_y=0$.

The potential $V^{<}$ consists of two terms: $V^{<}=\tilde{V}^{<}+\delta V^{<}$. In a previous paper,[15] we took into account only the first of these two terms, which arose when perturbations of the scalar $\varphi_1$ were neglected. The correction $\delta V^{<}$ is connected with the inclusion of $\varphi_1$, and leads to a small change in some of the numerical coefficients.[4)]

All the expressions containing the index < are connected with integrals over the trajectory. It was shown earlier[7,10,25] that particles magnetized by the main field outside the narrow neighborhood of the neutral sheet usually provide a negligible contribution to the potential $\tilde{V}^{<}$. The same conclusion may be extended to other expressions of the same structure in $\delta V^{<}$. The detailed shape of the potential barrier $V^{<}(x)$ is described by a function that is nonzero only in the relatively narrow neighborhood of the neutral sheet. The height and width of the potential barrier depend on the magnetic-field component at right-angles to the sheet because the particle trajectories along which the integration in (8) is carried out are very sensitive to this component. The parameters of the barrier $V^{<}$ can be found for different limiting cases.

## 2. POTENTIAL BARRIER PARAMETERS

### A. Plane one-dimensional neutral sheet

In the absence of the normal magnetic-field component ($b=0$), we can neglect the effect of the main magnetic field $B_{0z}(x)$ on the motion of the particles[6,7] when we evaluate the integrals in the expressions for $V^{<}$ in the narrow neighborhood of the neutral layer. This is valid for particles of type $j$ in a sheet of size $|x|<d_j=\varepsilon_j^{1/2}L$ because the magnetic field is then always small enough to ensure that the Larmor radius of the particles either exceeds or is comparable with the width of the sheet. It can be shown that, with the exception of a small group of particles with velocities practically parallel to the $y$ axis, all the particles are captured in the sheet and execute rapid oscillations (with characteristic frequency $\Omega_{Bj}\sim v_{Tj}d_j^{-1}$) between almost impermeable magnetic walls at $x=\pm d_j$. The motion of the particles along the $y$ axis, averaged over these oscillations, is found to be weakly perturbed by these fast motions along the $x$ axis for $|x|<d_j$, and can therefore be looked upon as free motion taking place in the absence of the main field $B_{0z}(x)$.[25]

The main contribution to the integrals in (9b) and (9c) is provided by the semiresidues in the velocity integrals describing the interaction of the perturbations with freely moving resonance particles:

$$V_j^< = \frac{\omega_{pj}^2}{c^2}\left(-\frac{i\pi^{1/2}\omega}{kv_{Tj}}\right). \qquad (11)$$

The correction $\delta V^<$ is totally unimportant in this case, and the height of the potential barrier $V^< = \Sigma_j \tilde{V}_j^<$ is determined by the resonance absorption of the energy of the oscillations by plasma particles from the region $|x| < d_j$. Since the tearing mode which we are considering here is, in fact, a negative-energy mode, [10] Landau damping resulting from this resonance interaction between tearing perturbations and electrons or ions leads to the growth of these perturbations (electron or ion instability branch).

## B. Suppression of resonance interaction due to Larmor rotation in a vertical field

The magnetic-field component perpendicular to the sheet (we shall refer to it as the vertical component since the sheet itself is located horizontally) disturbs the resonance interaction between the particles and the perturbation if the particle executes one or more revolutions in the vertical magnetic field during the perturbation growth time:

$$\Omega_j \tau_j > 1, \qquad (12)$$

where $\tau_j = |\operatorname{Im} \omega_j|^{-1}$ is the reciprocal of the instability growth rate and $\Omega_j = e_j B_n / m_j c$ is the cyclotron frequency of particles of type $j$ in the vertical magnetic field $B_n = bB_0$.

Hence, it follows that the electron instability branch with growth rate $\operatorname{Im}\omega_e \sim kv_{Te}\varepsilon_e^{3/2}$ is stabilized by a very weak vertical field $b \sim \varepsilon_e^{5/2}$. [8] The effect of the vertical magnetic field on the tearing instability mode is not, however, confined to the removal of the Landau resonance. It is shown in[15] that particles captured into cyclotron orbits in the vertical field provide a major contribution to the potential energy.

When $\Omega_{Bj} > \Omega_j$ (or $b < \varepsilon_j^{1/2}$), the Larmor rotation of the particles in their cyclotron orbits in the field $B_n$ can be regarded as slow in comparison with the frequency of oscillations between the magnetic walls and, precisely as in the preceding section, the motion of the particles can be averaged over these fast oscillations. The average trajectories over which the integrals in (9) are evaluated can then be taken to be the slow cyclotron motions of the particles over the Larmor orbits in the vertical field. [15]

In this approximation, the expressions given by (9) for a plasma with distribution function given by (3) can be evaluated as follows:

$$\tilde{V}_j^< = \frac{\omega_{pj}^2}{c^2} 4\alpha_j^2 \int_0^\infty v_\perp^3 \exp(-\alpha_j v_\perp^2) \sum_{n=-\infty}^{+\infty} \frac{\omega}{\omega - n\Omega_j} J_{n,j}'^2 dv_\perp, \qquad (13a)$$

$$\delta V^< = \left(\sum_j \frac{\omega_{pj}^2}{c^2} 4\alpha_j^2 \int_0^\infty v_\perp^2 \exp(-\alpha_j v_\perp^2) \sum_{n=-\infty}^{+\infty} \frac{\omega}{\omega - n\Omega_j} J_{n,j} J_{n,j}' dv_\perp\right)^2$$

$$\times\left(\sum_j \frac{\omega_{pj}^2}{c^2} 2\alpha_j \left(1 - 2\alpha_j \int_0^\infty v_\perp \exp(-\alpha_j v_\perp^2) \sum_{n=-\infty}^{+\infty} \frac{\omega}{\omega - n\Omega_j} J_{n,j}^2 dv_\perp\right)\right)^{-1}, \qquad (13b)$$

where $J_{n,j} = J_n(kv_\perp/\Omega_j)$ is the Bessel function of order $n$ and $J'_{n,j} = \partial J_{n,j}(q)/\partial q$ is the derivative of the Bessel function with respect to its argument. The particle distribution function in (13) is taken to be Maxwellian because corrections due to the slight anisotropy in $F_j$ (and, as before, the corrections to $F_j$ due to the fact that $b \neq 0$) are unimportant when the integrals in (13) are evaluated.

In the limit of a weak vertical field, when condition (12) is still not satisfied, summation of the contributions due to all the harmonics [neglecting the effects of the vertical field on particle orbits in (13)] leads to (11). In the opposite case, i.e., when (12) is satisfied, the particles are already captured by the vertical field into Larmor orbits, and the main contribution to the integrals is provided by the terms with $n = 0$

$$\tilde{V}_j^< = 2\frac{\omega_{pj}^2}{c^2}\frac{d}{d\lambda_j}(\lambda_j \exp(-\lambda_j) I_1(\lambda_j)), \qquad (14a)$$

$$\delta V^< = \left[\sum_j \frac{\omega_{pj}^2}{c^2}(2\alpha_j\lambda_j)^{1/2}\frac{d}{d\lambda_j}(\exp(-\lambda_j) I_0(\lambda_j))\right]^2$$

$$\times\left[\sum_j \frac{\omega_{pj}^2}{c^2} 2\alpha_j (1 - \exp(-\lambda_j) I_0(\lambda_j))\right]^{-1}, \qquad (14b)$$

where $\lambda_j = \frac{1}{2}k^2\tilde{\rho}_j^2$, $\tilde{\rho}_j = v_{Tj}\Omega_j^{-1}$ is the radius of the Larmor rotation in the normal field, and $I_n$ is a modified Bessel function of order $n$. We recall that the formulas given by (14) are valid only for $|x| < d_j$.

According to (13), the potential $V^<$ is continuously transformed as the vertical field increases from the resonance expression given by (11) to the actual potential (14) (corresponding to a high potential barrier), and does not vanish.

The expressions in (14) can be written in a simple form if we use the asymptotic representation of $I_n$ for $\lambda_j > 1$, and an expansion into a series for $\lambda_j < 1$. Depending on $b$, the potential $V^<$ has a double hump shape with maxima at $\lambda_e \sim 1$ and $\lambda_i \sim 1$, respectively. Inclusion of the corrections $\delta V^<$ is found to be important only for the descending part of the electron hump of the potential with $\lambda_e < 1$. Using (14), we can readily obtain the expressions for the total height $V^<$ of the potential barrier:

$$V^< \approx \omega_{pj}^2 k^2 \tilde{\rho}_j^2 \varkappa_j / c^2, \quad k\tilde{\rho}_j < 1, \qquad (15)$$

for the descending part of the electron and ion humps, respectively. In this expression, $\varkappa_i = \eta_i$ and the numerical coefficient $\varkappa_e = (1 + \eta_e)/2$ arises when the corrections $\delta V^<$ are included. On the descending parts of the potential humps

$$V^< \approx V_j^< \approx \omega_{pj}^2 / k\tilde{\rho}_j c^2, \quad k\tilde{\rho}_j > 1. \qquad (16)$$

Since henceforth we shall be largely concerned with states on the boundary of stability with $\omega = 0$, we can omit the terms corresponding to the ion semiresidues in the expressions (15) and (16) for electrons ($j = e$).

### C. Magnetized neutral sheet

The approximation in which the magnetic field between the impermeable walls bounding the neutral sheet can be neglected, and only its vertical component taken into account, is violated when the Larmor radius in the vertical field becomes less than the size of the layer (and the Larmor frequency is correspondingly greater than the frequency of the oscillations between the magnetic fields), i. e.,

$$\tilde{\rho}_j<d_j, \quad \Omega_j>\Omega_{Bj}. \tag{17}$$

The inequality given by (17) is equivalent to $b>\varepsilon_j^{1/2}$. Averaging over the fast oscillations across the plane of the neutral sheet in the form in which we have used it above is then no longer valid. On the other hand, the plasma can then be regarded as magnetized throughout (magnetized neutral sheet), and we can use the drift trajectories of the particles when we calculate the effective potential $V^<$. The particle-trajectory parameters in the field near the neutral sheet have been calculated by Tverskoĭ[16] for this model. In particular, the first and second adiabatic invariants are given by:

$$I_1=\frac{p^2}{B_M}, \quad I_2=\int_0^{x_M} p\,ds=\frac{pL}{B_0B_nB_M^{1/2}}\int_{B_n}^{B_M} dB\,B^2\left(\frac{B_M-B}{B^2-B_n^2}\right)^{1/2},$$

where $p$ is the total momentum and $B_M$ is the modulus of the magnetic field at the point of reflection of the particle from the magnetic mirror.

To evaluate the integrals with respect to time in the expressions given by (9), we shall average over the fast (in comparison with the instability development time) motions, i. e., over the cyclotron rotation of the particles and their oscillations between the magnetic mirrors. For simplicity, we shall neglect the corrections $\delta V^<$ in this section.

Although the drift velocity $v_{Dy}$ of the particles in the neighborhood of the neutral sheet may be high (see[16]), we shall, as before, include only the cyclotron rotation and ignore the drift velocities when we evaluate the integrals over the trajectories in the expressions for the current $j_{1y}$ [given by (9)]. This apparent contradiction is connected with the fact that, when the drift velocities are included in the evaluation of the total current, it is necessary to include also the so-called magnetization current.[26] As a result, for the almost isotropic plasma distribution which we have chosen, the final expression will contain a small average-mass velocity of the plasma $\langle v\rangle\sim u_j$, the inclusion of which is unimportant. From integration with respect to the velocities, we pass on to the integration with respect to the total momentum and the field magnitude at the point of reflection. The time average of the oscillations between the magnetic mirrors is given by

$$\langle\psi\rangle=\tau^{-1}\int_0^{\tau}\psi[B(x(t),z(t))]\,dt$$

$$=\left(\int_{B_n}^{B_M}\frac{B\,dB}{[(B^2-B_n^2)(1-B/B_M)]^{1/2}}\right)^{-1}\int_{B_n}^{B_M}\frac{B\psi(B)\,dB}{[(B^2-B_n^2)(1-B/B_M)]^{1/2}}. \tag{18}$$

Moreover, the argument of the Bessel function in (18) is small in the region in which the drift approximation is valid, and we can therefore confine our attention to the first term in the expansion of it in terms of the small argument. The result is

$$V_j^<(x)=\frac{15}{8}\frac{\omega_{pj}^2}{c^2}\lambda_j\int_{h_{Mx}^{1/2}}^{\infty}\frac{dh_M}{h_M^{2}}\left(\frac{1}{h_{Mx}}-\frac{1}{h_M}\right)^{-1/2}\left\langle\frac{1}{h^2}\right\rangle; \tag{19}$$

$$b=\frac{B_n}{B_0}, \quad h=\frac{|\mathbf{B}|}{B_n}, \quad h_M=\frac{B_M}{B_n}, \quad h_{Mx}=1+\frac{x^2}{b^2L^2}.$$

It is readily seen that, according to (19), the height of the potential barrier in the case of a magnetized neutral sheet is no different from that given by the simple estimate (15) used previously in[15]. This estimate is therefore valid in both cases. However, in the case of a magnetized neutral layer, the main mass of particles can no longer penetrate the magnetic walls at $x=\pm d_j$.

It is clear from (19) that an expansion of the particle-localization region corresponds to an increase in the width of the potential $V_j^<(x)$ (which is now determined by the parameter $bL$) as compared with the unmagnetized case. We thus finally have

$$\Delta_j=\begin{cases}\varepsilon_j^{1/2}L, & b<\varepsilon_j^{1/2} \text{ (unmagnetized neutral sheet)}\\ bL, & b>\varepsilon_j^{1/2} \text{ (magnetized neutral sheet).}\end{cases} \tag{20}$$

It is important to note that calculations which we have carried out in a simple magnetohydrodynamic approximation, valid for the description of the particle motion in the neighborhood of the neutral sheet in the case of a magnetized layer, yield the same results as those obtained above on the basis of the kinetic equation.[27]

## 3. DISPERSION RELATION

In Sec. 1, we obtained equation (8) for the perturbations of the vector potential $A_1(x)$, which has the form of a Schrödinger equation with energy $E=-k^2$ and an effective potential in the form of the Teller well $V_0(x)$ at the center of which there are two superimposed narrow and tall humps $V_e^<(x)$ and $V_i^<(x)$ due to the electron and ion contributions of the singular region near the neutral sheet, respectively (Fig. 1).

Without introducing additional assumptions about the

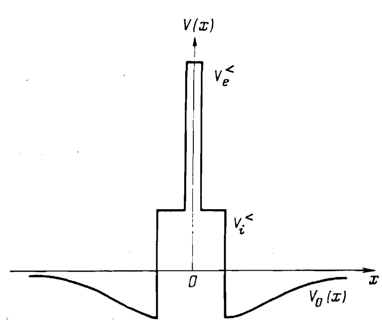


FIG. 1. Shape of the effective potential $V(x)$. The half-width of the humps $V_e^<$ and $V_i^<$ at the center of the Teller well $V_0(x)$ is $\Delta_e$ and $\Delta_i$, respectively.

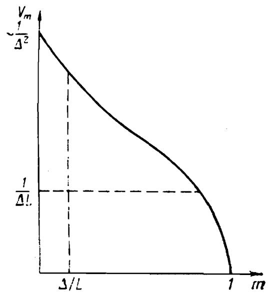


FIG. 2. The dispersion curve [corresponding to the solution of (24)] relating the position of the energy level $E=-m^2L^{-2}$ in the potential well of height $V_m$ and the width $\Delta$ of the potential hump at the center of the well.

shape of the potential hump, the dispersion relation between the perturbation frequency and wavelength can be obtained only in the limit where the presence of the hump leads only to a slight level shift. Using the wave functions for the unperturbed state given in[10], let us calculate the matrix elements corresponding to the perturbing potential $V^<(x)$, and the correction to the unperturbed energy level $E_0=-L^{-2}$. The result is

$$1-k^2L^2=\frac{L}{2}\int_{-\infty}^{+\infty}dx V^<(x)\,\mathrm{ch}^{-2}\frac{x}{L}. \tag{21}$$

For a narrow hump on a symmetric potential, the dispersion relation can also be obtained for the case where the level shift is no longer small, but the level is still located inside the well. In other words, in this case, both humps can be looked upon as transparent for the perturbations:

$$\Delta_e^2V_e^<+\Delta_i^2V_i^<\ll 1, \tag{22}$$

where $\Delta_{e,i}$ are the widths of the two humps (see Sec. 2). In this case, the contributions of $k^2$ and $V_0(x)$ to (8) can be neglected in the internal region $x\ll\Delta_{e,i}$, and the logarithmic derivative of the potential perturbation is

$$[A_1^<(x)]_x'/A_1^<(x)=\int_0^x (V_e^<(x)+V_i^<(x))\,dx.$$

The solution in the external region can be expressed in terms of the Legendre function

$$A_1^>(x)=P_1^{-kL}\left(\mathrm{th}\frac{x}{L}\right).$$

If we match the logarithmic derivatives $A_1^{\gtrless}(x)$ at a point well outside the internal potential humps (i.e., for $\Delta_i\ll x\ll L$), we obtain the dispersion relation in the form

$$\frac{L}{2}\int_{-\infty}^{+\infty}(V_e^<(x)+V_i^<(x))\,dx=\frac{1-m^2}{m}. \tag{23}$$

In this expression, $m=kL$ and we have used the well-known result

$$\left[\frac{dP_1^{-m}(\mu)}{d\mu}\Big/P_1^{-m}(\mu)\right]_{\mu=0}=\frac{1-m^2}{m}.$$

We thus see that, according to (23), the Teller well contains the single energy level $E_0=-L^{-2}$ in the absence of the potential hump. The appearance of the hump leads to a gradual shift of the energy level toward the surface of the well, and when the barrier becomes completely opaque [i.e., when (22) is violated] it leaves the well altogether.

The dispersion relation that provides the correct description of the exclusion of all the levels from the well can be obtained by further simplifying the shape of the potentials $V^<$ and by approximating the actual shape of the humps by step functions with nonzero constants for $|x|<\Delta_j$.[7] In actuality, the exclusion of a level can be due to either potential hump. It is therefore sufficient to obtain the dispersion relation for the case of a single hump $V^<$ of width $\Delta$. In the interior, where $|x|<\Delta$, the solution of (8) is $A^<(x)\sim\cosh(V^<)^{1/2}x$ (for $V^<\gg L^{-2}$) whereas, as already noted, the solution for the external region is given by

$$A^>(x)\sim P_1^{-m}\left(\mathrm{th}\frac{x}{L}\right).$$

If we match the logarithmic derivatives at the point $x=\Delta\ll L$, we obtain

$$\chi\,\mathrm{th}\,\chi=\frac{\Delta}{L}\frac{1-m^2}{m}\Big/\left(1+\frac{\Delta}{L}\frac{1-m^2}{m}\right), \tag{24}$$

where $\chi=(V^<)^{1/2}\Delta$.

A graphical solution of this equation is shown in Fig. 2. The approximate analytic solution is

$$V_m^<=\begin{cases}(1-m^2)/m\Delta L; & m>\Delta/L\\ \sim 1/\Delta^2, & m\ll\Delta/L\end{cases}. \tag{25}$$

## 4. ANALYSIS OF THE DISPERSION RELATION

It follows from the preceding discussion that there are two effects due to the presence of the magnetic-field component perpendicular to the sheet which have a stabilizing influence on instability development: firstly, there is the suppression of resonance between perturbation and particle and, secondly, there is the exclusion of the localized perturbation from the region of the neutral sheet [i.e., the exclusion of the energy level from the effective potential well $V_0(x)$].

Let us first estimate the critical values of the normal component of the magnetic field for which these two effects begin to be important for a given (ion or electron) perturbation mode with given energy level $E=-k^2=-m^2/L^2$ (or wavelength $2\pi L/m$). Since each of these effects is mostly due to the contribution of one of the plasma components, we shall analyze them in terms of the simplified dispersion relation (24) obtained for this case. The resonance interaction is disturbed when (12) is satisfied. This condition includes the instability growth rate $\gamma_j$, which we shall obtain as a function of wavelength from the dispersion relation given by (24). To do this, let us substitute the estimated height of the potential hump taken from (11) into (24):

$$\gamma_j(m)=mV_{m,j}L^2\varepsilon_j^2\eta_jv_{Tj}/\pi^{1/2}L;$$

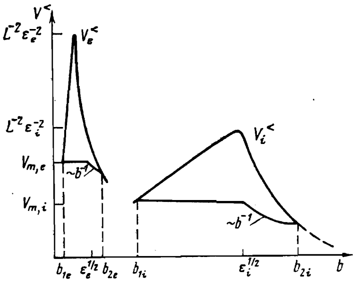

FIG. 3. Effective potentials formed in the neighborhood of the neutral sheet due to the Larmor rotation of particles in the normal field. For $b_{1j}(m) < b < b_{2j}(m)$, the energy level $E = -m^2L^{-2}$ is absent from the potential well ($V_j^< > V_{m,j}$) and the instability is suppressed. In the gap (if present), the ion tearing mode may develop between $b_{2e}(m)$ and $b_{1i}(m)$. When $b > \varepsilon_i^{1/2}$, the expansion of the potential hump $\Delta_j \approx bL$ must be taken into account, and this leads to a narrowing of the gap.

where $V_{m,j}$ is the formal solution of (24) for $V^<$.

We now estimate the magnitude of the parameter $|\omega/kv_T|$. The fact that this parameter is small was used in the derivation of the original equations in Sec. 1, and this is indicated by the fact that $|\omega/kv_T| < \varepsilon$ even for small $m$. This suggests that, contrary to[7], long-wave perturbations with $m < \varepsilon^{1/2}$ can be discussed within the framework of the theory which we have developed.

Having found the growth rate $\gamma_j(m)$, we can readily obtain the critical vertical magnetic-field component for which the resonance between the perturbations and particles of type $j$ is suppressed:

$$b_{1j}(m) = mV_{m,j}\varepsilon_j^3\eta_j L^2 \approx \begin{cases} m\varepsilon_j^2\eta_j, & m<\varepsilon_j \\ (1-m^2)\varepsilon_j^{2}\eta_j. & m>\varepsilon_j \end{cases} \quad (26)$$

Hence, it follows that, for example, even for a very weak vertical field $b = b_{1e}(m)$, the electrons cannot interact with perturbations with given $m$. The remaining weak (in comparison with the previous electron contribution) interaction of these perturbations with ions cannot lead to instability because the height of the potential hump $V_e^<$ turns out to be equal to $V_{m,e}$ for this value of $b = b_{1e}(m)$ and, according to (24), this is sufficient to exclude the energy level from the potential well.

When the normal component is increased further, the height of the potential hump at first increases in proportion to $b$, in accordance with (16), and then, when $b > m\varepsilon_e$, it falls in inverse proportion to $b^2$, in accordance with (15) (Fig. 3). It follows that, for a certain value $b = b_{2e}(m)$, the energy level $E = -m^2L^{-2}$ reappears in the potential well $V_0(x)$.

Subsequent behavior of the potential is determined by ions whose motion leads to the formation of a second potential hump which is lower by a factor of $m_e/M_i$ as compared with the first. Thus, the resonance between the ions and the perturbations, and the exclusion of the level $E = -m^2L^{-2}$ by the ion potential hump, occur for $b = b_{1i}(m)$ and, when $b > b_{2i}(m)$, the level may reappear in the shallow well because of the reduction in height of the ion potential barrier. The critical value $b_{2j}(m)$ can be found from (24), with the barrier height given by (15):

$$V_j^< = m^2\varkappa_j/b^2L^2\eta_j,$$

and its width is given by (20). Using the approximate analytical expression for $V_{m,j}$ given by (25), we have

$$b_{2j}(m) = \begin{cases} m\varepsilon_j^{1/2}\left(\dfrac{\varkappa_j}{\eta_j}\right)^{1/2}, & m<\varepsilon_j^{1/2} \\ \dfrac{m^{3/2}}{(1-m^2)^{1/2}}\varepsilon_j^{1/4}\left(\dfrac{\varkappa_j}{\eta_j}\right)^{1/2}, & \varepsilon_j^{1/2}<m<\varepsilon_j^{1/6}\left(\dfrac{\eta_j}{\varkappa_j}\right)^{1/3} \\ \dfrac{m^3}{1-m^2}\dfrac{\varkappa_j}{\eta_j}, & \varepsilon_j^{1/6}\left(\dfrac{\eta_j}{\varkappa_j}\right)^{1/3}<m<\left(\dfrac{\eta_j}{\varkappa_j+\eta_j}\right)^{1/2} \end{cases} \quad (27)$$

Thus, for $b_{1e}(m) < b < b_{2e}(m)$ and $b_{1i}(m) < b < b_{2i}(m)$ (Fig. 4), there is no localized solution of (8) with $k^2 = m^2L^{-2}$, which corresponds to plasma stability against perturbations of the above wavelength. This means that the electron branch of the instability exists, in agreement with[8,9], only for very weak vertical fields. Contrary to the statements in[9], the ion branch can be excited only for $b_{2e}(m) < b_{1i}(m)$ and provided

$$b_{2e}(m) < b < b_{1i}(m). \quad (28)$$

This discrepancy is due to the fact that the effect of electrons trapped near the neutral sheet on the localized perturbation was ignored in[8,9].

For vertical fields $b > b_{2i}(m)$, we again have the possibility of a localized perturbation, the resonance of which with the untrapped (transit) magnetized particles in the external region (see[10]) can result in the development of instability. However, for $b > b_{2i}$, the condition $b \ll 1$, ensuring the validity of our theory, is usually violated and we shall not, therefore, consider this region here.

## 5. DEVELOPMENT OF ION TEARING INSTABILITY

As shown in the previous section, the ion tearing mode can develop only when there is a gap between $b_{2e}(m)$ and $b_{1i}(m)$, and the region in which the ion tear-

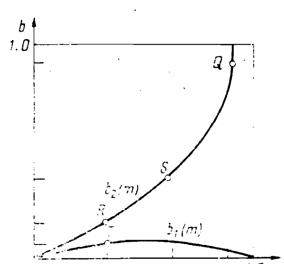

FIG. 4. Critical amplitudes of the normal field, $b_1(m)$ and $b_2(m)$, as functions of the wavelength of the perturbations $m = kL$. When $b_{1j}(m) < b < b_{2j}(m)$, the energy level $E = -m^2L^{-2}$ is excluded from the potential well and the development of the $j$-th instability mode with $k = mL^{-1}$ is, in principle, impossible. The expressions for the electron and ion modes differ only by the indices. The points separating regions on which the various approximations have to be used have the following coordinates: $T(\varepsilon^{1/2}, \eta\varepsilon^{5/2})$, $R(\varepsilon^{1/2}, \varkappa^{1/2}\eta^{-1/2}\varepsilon)$, $S(\varepsilon^{1/6}\eta^{1/3}\varkappa^{-1/3}, \varepsilon^{1/2})$, $Q\{[\eta/(\eta+\varkappa)]^{1/2}, [\eta/(\eta+\varkappa)]^{1/2}\}$.

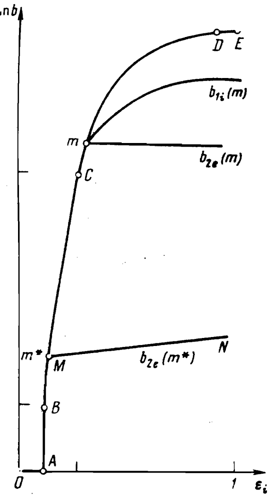


FIG. 5. In terms of the coordinates $(b,\ \varepsilon_i)$ to each $m=\text{const}$, there corresponds a gap of definite shape. We show the gap shape for

$$\mu^{1/10}\varkappa_e^{-3/10}\eta_e^{1/5}<m<[\eta_e/(\varkappa_e+\eta_e)]^{1/2},$$

where *OABCDE* is the envelope of the family of gaps for all possible wavelengths. Regions corresponding to the utilization of the different approximations described in the text are separated on the envelope by the following points:

$$A(\mu^{1/6}\varkappa_e^{1/3}\eta_e^{-1/2}\eta_i^{-1/2},0),$$
$$B(\mu^{1/6}\varkappa_e^{1/3}\eta_e^{-1/2}\eta_i^{-1/2},\ \ln\mu^{2/3}\eta_e^{-3/2}\varkappa_e^{5/6}),$$
$$C(\mu^{1/10}\varkappa_e^{1/5}\eta_e^{-3/10}\eta_i^{-1/2},\ \ln\mu^{3/10}\eta_e^{-2/5}\varkappa_e^{1/10}).$$

When the wavelength of the tearing perturbations is restricted by the size of the system $m>m^*$, the ion tearing instability corresponds to the region covered by the broken line *NMCDE*.

ing instability can grow is determined by the set of inequalities given by (28). The appearance of the gap is possible only for an anisotropy exceeding a certain critical value:

$$\varepsilon_i>\mu^{1/6}\varkappa_e^{1/3}(\eta_e\eta_i)^{-1/2},\qquad \mu=m_e/M_i,\quad \eta_j=(T_i+T_e)/T_j,\quad \varkappa_e=(1+\eta_e)/2. \tag{29}$$

When (29) is satisfied, the gap appears for long-wave perturbations

$$m<\varepsilon_e^{*1/2}=\mu^{1/3}\eta_e^{-1/2}\varkappa_e^{1/6}.$$

Further increase in the anisotropy $\varepsilon_i$ results in an expansion of the region occupied by the gap toward greater $m$ and $b$. It is clear from (26) and (27) that the gap parameters are very dependent on the wavelength $m$ of the perturbations. When analytic expressions for the critical fields $b_{2e}(m)$ and $b_{1i}(m)$ are available [see (26) and (27)], the determination of the shape of the gap in the space of the variables $(b,\ \varepsilon_i,\ m)$ presents no fundamental difficulties although it is quite laborious.

Figure 5 shows sections through the gap by the $m=\text{const}$ planes. A particular gap shape corresponds to each $m$ and begins with certain critical values $b^*(m)$, $\varepsilon_i^*(m)$. Eliminating $m$ from the expressions for $b^*(m)$ and $\varepsilon_i^*(m)$, we obtain the curve $b^*(\varepsilon_i^*)$, which is the envelope of the family of gaps for different $m$.

Over $AB$ $(0<m<\mu^{1/3}\varkappa_e^{1/6}\eta_e^{-1/2})$, the shape of the envelope and the region occupied by the gap are given by

$$\varepsilon_i^*=\mu^{1/6}\varkappa_e^{1/3}(\eta_e\eta_i)^{-1/2},\qquad \mu^{1/4}m\varepsilon_i^{1/2}\varkappa_e^{1/2}\eta_e^{-3/4}\eta_i^{1/4}<b<m\varepsilon_i^2\eta_i. \tag{30}$$

Over $BC$ $(\mu^{1/3}\varkappa_e^{1/6}\eta_e^{-1/2}<m<\mu^{1/10}\varkappa_e^{-3/10}\eta_e^{1/5})$, we have, correspondingly,

$$b^*=(\varepsilon_i^*)^{11/2}\mu^{-1/4}\varkappa_e^{-1}\eta_e^{5/4}\eta_i^{11/4},\qquad \mu^{1/8}\varepsilon_i^{1/4}m^{3/2}\varkappa_e^{1/2}\eta_e^{-5/8}\eta_i^{1/8}<b<m\varepsilon_i^2\eta_i. \tag{31}$$

For $CD$ $\{\mu^{1/10}\varkappa_e^{-3/10}\eta_e^{1/5}<m<[\eta_e/(\varkappa_e+\eta_e)]^{1/2}\}$,

$$b^*=(\varepsilon_i^*)^3\eta_e^{1/2}\eta_i^{3/2}(\varkappa_e+\eta_e\eta_i\varepsilon_i^{*2})^{-1/2},\qquad m^3(1-m^2)^{-1}\varkappa_e\eta_e^{-1}<b<m\varepsilon_i^2\eta_i. \tag{32}$$

Finally, over the segment $DE$, the envelope $DE$ coincides with the upper boundary of the gap $b=m\varepsilon_i^2\eta_i$ for $m=m_{\max}=[\eta_e/(\varkappa_e+\eta_e)]^{1/2}$.

Over $AB$ and $BC$, the intersections of the curves $b_{2e}(m)$ and $b_{1i}(m)$ occur always for $m<\varepsilon_i^{1/2}$ and, consequently, only for the rising branch of the curve $b_{1i}(m)=m\varepsilon_i^2\eta_i$. Over $CD$, the pattern of the intersections may become more complicated for certain values of the parameters but, since this segment does not play an important role, we shall, as before, confine our attention to the intersection of the curves for $m<\varepsilon_i^{1/2}$. Thus, over the segment $CD$

$$m_{CD}^*=\varepsilon_i^*(\eta_e\eta_i)^{1/2}(\varkappa_e+\eta_e\eta_i\varepsilon_i^{*2})^{-1/2},$$

this assumption will be valid if the plasma parameters $\varkappa$ and $\eta$ satisfy the inequality

$$\varkappa_e(\eta_e\eta_i)^{-1}>1/4,$$

which is equivalent to the condition $T_i/T_e>\sqrt{2}-1$.

The development of the ion tearing mode with given $m$ is possible in the regions occupied by the gaps defined by (30)–(32). Under real conditions, the perturbation wavelength is always restricted by the size of the system inhomogeneity $m>m^*$. In terms of the coordinates $(b,\ \varepsilon_i)$, the region in which the ion tearing instability can develop can be found as the combination of regions occupied by gaps corresponding to $m^*<m<m_{\max}<1$. Thus, in terms of these coordinates, the complete region corresponding to instability is bounded by the curves $b^*(\varepsilon_i^*)$ (envelope of gap family) and $b_{2e}(\varepsilon_i, m^*)$ (Fig. 5). As noted above, the region in which the electron mode develops

$$b<b_{1e}\approx\mu^{5/4}\varepsilon_i^{5/2}\eta_e^{-1/4}\eta_i^{5/4},$$

corresponds to very small $b$ and, for simplicity, it is not shown in Fig. 5.

It is important to remember that our development of

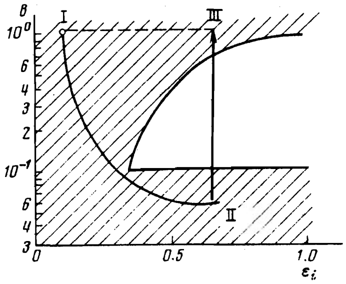


FIG. 6. Form of unstable region on the $(b,\ \varepsilon_i)$ plane for $m > m^* \sim 0.6$. The numerical values of the parameters correspond to the conditions in the tail of the earth's magnetosphere. During the first phase of a substorm, the tail is reversibly taken from the stable state I to the metastable state II. The I → II transition corresponds to the accumulation of energy in the tail. The rapid development of the ion tearing instability is associated with the sharp transition to the explosive phase of the substorm (energy dissipation phase). The tail returns to the initial state I through the re-establishment phase III → I.

the above theory was based on the assumption that the parameters $b$ and $\varepsilon_i$ were small. This means that the results for values of $b$ and $\varepsilon_i$ approaching unity (top right-hand part of Fig. 5), i.e., results obtained by extrapolation from small values of these parameters, must be regarded as only qualitative.

## CONCLUSIONS

One of the most important consequences of the theory developed above is the prediction of the possible existence of metastable states of a diffusive neutral sheet.[15] States with $b < b_{2e}(\varepsilon_i, m^*)$ for $\varepsilon_i > \varepsilon_i^*(m^*)$ (i.e., states lying under the family of gaps) are metastable. Such states are stable against small perturbations but cannot be nonlinearly unstable against perturbations of finite amplitude $\Delta b \sim b_{2e}(m^*) - b$, which take the sheet into the unstable region. The perturbation $\Delta b$ producing instability can be either an external perturbation or an intrinsic thermal fluctuation in the plasma (in the case of states near the limit of stability). Plasma may remain in this metastable state for a relatively long time but, when a nonlinear instability is initiated in a short time $\tau_0 \sim \gamma_i^{-1}$ (where $\gamma_i \sim v_{Ti}\varepsilon_i^{3/2}/L$ is the growth rate of the ion tearing mode), the plasma will take up a new stable state which, in general, depends on the form of the perturbation $\Delta b$ and, in this sense, need not necessarily coincide with the upper boundary of the unstable region.

In many cases, the state of the neutral sheet may be located in an unstable region as a result of a slow reversible evolution toward the instability boundary because, as the sheet is compressed, the increase in the anisotropy $\varepsilon_i$ and the reduction in $b$ are accompanied by a reduction in the parameter $m^* = 2\pi L/L_z^*$ and the attendant downward shift of the lower instability boundary $b_{2e}(\varepsilon_i, m^*)$.

The conditions for the appearance of the gap are very critically dependent on the equilibrium current (sheet thickness). Even condition (29), necessary for the development of only long-wave perturbations, is very stringent [for the magnetospheric tail, (29) yields $\varepsilon_i \sim 0.1$].

The development of the ion tearing mode in the neutral sheet with constant or quasiconstant $B_n$ may result in the formation of neutral lines. The dynamics of particles in the presence of such lines has a number of new distinctive features, and the subsequent process cannot, therefore, be analyzed within the framework of the above theory. Undoubtedly, however, fast spontaneous rearrangement of the magnetic-field topology near the neutral sheet is very important in the presence of an electric field. The development of the ion tearing instability in the case of a metastable sheet (spontaneous reconnection) may be the "trigger mechanism" which modifies the plasma flow and initiates intensive "stimulated reconnection" in the electric field (see[28]), which was forbidden in the magnetic-field topology of the initial state.

Although the plasma-flow distribution during this phase is determined by the applied electric field, the mechanism that is directly responsible for the tearing and reconnection of the lines of force in the neighborhood of the neutral sheet is, as before, the ion tearing mode. Estimates show[10] that, when the sheet current is large enough, the ion tearing mode is unstable in the nonlinear state as well. The high growth rate of this instability means that the lines of force at the neutral point can be rapidly ruptured and the plasma flow assumes the hydrodynamic distribution with considerable reconnection rates even in the simple model of stimulated reconnection analogous to the Parker–Sweet model.

The theory which we have developed may find extensive applications to processes occurring in the tail of the earth's magnetosphere. One of the most important and interesting processes in the magnetosphere is the magnetospheric substorm.[29] A substorm is a cyclic process, the initial phase of which corresponds to the accumulation of a large store of energy ($10^{21}$–$10^{22}$ erg) in the tail of the magnetosphere. The second phase is called the explosive stage and corresponds to the rapid release of the stored energy. The system returns to the initial state during the third stage.

The first phase, i.e., the accumulation of energy in the tail of the magnetosphere, has been investigated experimentally and theoretically,[11–14,29–31] but the "trigger" mechanism ensuring the rapid transition to the explosive phase remained an open question for a long time. The results reported here and in our previous papers[9,10,15] enable us to conclude that this phenomenon is probably connected with the development of the ion tearing instability in the neutral sheet of the magnetospheric tail which is in the metastable state. Experimental data[12–14] show that the evolution of the tail during the first phase of energy accumulation involves a reversible transition to a metastable state. The reduction in the normal component and in the thickness of the plasma sheet (increase in current) observed during the first phase takes the tail from the stable (I) to the metastable (II) state *under* the unstable region (Fig. 6). Since the tail remains stable during the I → II transition, this state can be taken well into the metastable region so that a considerable store of energy can accumulate in the tail.

If we estimate the characteristic length of the section of the tail in which the instability develops as being $L_z^* \sim 10L$, we can determine the values of parameters corresponding to the metastable state II. The result is $m^* \sim 0.6$, $\varepsilon_i^* \sim 0.3$, $b \sim 0.05$, which is not inconsistent with experimental data.[14] Precise measurements of a small normal component $B_n$ are difficult so that a more complete comparison with the theory is as yet impossible.

The estimated value of the time $\tau_0$ for the rearrangement of the field topology (actually, the time of operation of the trigger mechanism) is $\tau_0 \sim \gamma_i^{-1} \sim 10$ sec, and this enables us to explain the exceedingly rapid, explosive character of the transition to the energy dissipation phase.[14,20] The transition to this explosive phase is connected with the formation of a neutral line in the part of the tail of the magnetosphere that is nearest to the earth, and this corresponds to a rapid ($\tau \sim \tau_0$) "stimulated reconnection" which is accompanied by the dissipation of the stored energy and a reduction in the excess magnetic flux stored in the tail.

It is important to note that explosive processes connected with neutral lines and sheets have been the subject of many simulation experiments involving the use of laboratory plasma. In most of these experiments,[4,32,33] the neutral line was preset in advance by suitably choosing the geometry of conductors producing the magnetic field. In some experiments,[4,32] the electric field along the neutral line was produced by external sources whereas, in other experiments,[33] it arose in a self-consistent fashion as a result of the reconnection of the magnetic field in the neighborhood of the neutral line. Thus, the sharp changes in the rate of reconnection of magnetic lines of force in these experiments were due to a change in the plasma resistance as a function of current and not due to the formation of an additional neutral line.

A plane neutral sheet was also observed[4,32] during the flow of plasma in the neighborhood of a neutral line in the presence of an external electric field. The decay of the plane neutral sheet into current filaments can probably be explained in terms of collisional instability with respect to the tearing mode.[35]

An attempt to investigate collisionless instability of a neutral sheet in an inverted theta pinch is described in[34]. However, because of the low initial temperature, the plasma in the neutral sheet was compressed under the action of the magnetic pressure to such a small thickness that ion acoustic waves were efficiently excited, and this substantially increased the effective particle collision frequency. As a result, the collisionless parameter deteriorated substantially. Nevertheless, the experimental detection of tearing mode development in this installation can be satisfactorily explained by the development of collisionless instability in the limiting case of thin (in comparison with the ion Larmor radius) short neutral sheets.

Studies of the dependence of the instability threshold on the current and the normal magnetic-field component require preliminary heating of the plasma, so that more diffusive neutral sheets with low current can be obtained and a transition can be made to long systems in which small magnetic-field components perpendicular to the sheet can be investigated.

In conclusion, the authors wish to express their gratitude to R. Z. Sagdeev and C. F. Kennel for their interest in this research and for valuable advice.

1) When the normal component is present, we shall interpret the phrase "neutral sheet" as referring to the magnetic-field inversion region, the position of which can be found by neglecting the normal component of the magnetic field.

2) Henceforth, the instability mode of a diffusive neutral sheet originating in the inverse resonance Landau interaction with electrons (ions) will be referred to as the electron (ion) tearing instability.

3) For the sake of simplicity, we confine ourselves to the most unstable perturbations with $k_y = 0$, so that $k = k_z$.

4) We are indebted to R. Pellat for bringing this to our attention.

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# Quasilinear relaxation of a beam of fast ions in the tokamak

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Quasilinear relaxation of a beam of fast ions produced in a tokamak upon injection of a beam of fast neutral atoms is investigated theoretically. Relaxation of this type is assumed to be due to interaction between the ions and Alfvén waves. A beam moving along the magnetic field is considered. The shear of the magnetic force lines is neglected. It is shown that under these assumptions the quasilinear relaxation is much more rapid than the Coulomb relaxation due to pair collisions. It is concluded that the concept of a "two-component tokamak," which is based on the assumption of Coulomb relaxation of the fast ions, calls in general for revision.

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## 1. INTRODUCTION

According to present-day concepts, one of the main methods of obtaining a plasma with thermonuclear parameters in a tokamak is to inject a beam of fast neutral atoms (see, e.g., the reviews of Artsimovich[1] and Furth[2]). It is therefore important to study the dynamics of fast ions produced by ionizing these atoms, and in particular to study the relaxation of their thermodynamic non-equilibrium (non-Maxwellian) velocity distribution.

The velocity relaxation of fast ions in a tokamak has been the subject of many theoretical papers (reference to which can be found in[1,2]). Common to most hitherto performed theoretical investigations of the velocity relaxation of fast ions in a tokamak is the assumption that the only cause of this relaxation are the Coulomb collisions. One of the essential consequences of these investigations is the concept of the possibility of producing a so-called "two-component" thermonuclear tokamak reactor, i.e., a tokamak reactor whose plasma contains ions of two groups, "slow" (i.e., ions of the fundamental plasma component, obtained for example by Joule heating) and fast (injected) ions. The basis for this concept is the fact that the time of the Coulomb relaxation of the fast ions turns out to be just as long as the characteristic operating time of the two-component tokamak reactor.

It is clear from this that the concept of the two-component tokamak reactor may turn out to be untenable if it turns out that the distribution function of the injected ions is unstable and that the velocity relaxation of the ions, due to the reaction of the growing noise, turns out to be faster than the Coulomb relaxation. It is therefore important to carry out a thorough analysis of the instability of the ion beams and the associated processes of non-Coulomb relaxation. The latter call for further development of turbulence theory, particularly the quasi-linear theory.

Until recently, the linear approximation of plasma-oscillation theory has revealed no instabilities presenting any danger whatever to the problem of the two-component tokamak reaction (see, e.g., the article of Cordey and Houghton[3] and the references cited therein). The problem of the instability of ion beams in a tokamak was considered recently more fully, with allowance for the toroidal character of the magnetic field and the nonpotential character of the perturbations.[4] It was shown there that fast ions injected in a tokamak can lead to a buildup Alfvén waves. The purpose of the present paper is to investigate the quasi-linear relaxa-