Cyclotron resonance with non-extremal orbits in indium

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Experiments on cyclotron resonance with nonextremal orbits (CRNO) are performed with indium. CRNO due to electrons on orbits separated because the electron trajectories are cut off by the sample thickness can be used to study non-extremal electron groups. The effective masses of the electrons with non-extremal orbits and the Doppler splitting of the CRNO lines are measured in a magnetic field inclined to the sample surface. The measured dependence of the effective mass on non-extremal orbit diameter is explained within the framework of a model wherein the electron Fermi surface of indium consists of interconnected arms that narrow down towards their junctions.

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INTRODUCTION

Cyclotron resonance (CR) with electrons belonging to non-extremal sections of the Fermi surface and separated by the fact that the electron trajectory is cut off by the thickness of the sample (of a thin metallic plate) was first observed in bismuth.[1] At frequencies $\Omega = eHn/m^*c$ (n is the order of the resonance) corresponding to the frequencies of the revolution of the electrons on an orbit with diameter $D = 2p_x c/eH$ equal to the plate thickness, this resonance makes possible a direct experimental investigation of the non-extremal carriers in metals. Lur'e and Peschanskiĭ[2] have developed the theory of cyclotron resonance with non-extremal orbits (CRNO), in which it was established that the singularity of the impedance of a thin metallic plate has a logarithmic character in the case of CRNO. Measurements of the effective masses of the electrons of non-extremal sections of the Fermi surface (FS) of bismuth, carried out in[3] by the CRNO method, have yielded a lucid and exact description of the deviation of the energy spectrum of the electrons from quadratic.

The CRNO method can be used in principle also to study other metals in which the effect of cutting off the cyclotron resonance is observed. A convenient object for study by the CRNO method is indium, the electronic Fermi surface of which is made up of arms that taper down towards the ends.

We describe in this paper experiments on the observation and investigation of CRNO in indium, the purpose of which was to obtain quantitative characteristics of the electrons of the non-extremal sections of the Fermi surface.

EXPERIMENT

The indium samples were grown from the melt in a dismountable optically-polished quartz mold by the method described in[4]. In the course of preparation of thin single crystals of indium, which has high adhesion to the smooth surface of quartz, we used a coating of finely-dispersed SiO_2.[1)] The coating was produced by depositing on the surface of the mold a "fog" of solid particles of pyrolytic SiO_2 produced as a result of burning a mixture of silico-ethyl ether with small amount (~5%) of ethyl alcohol added, at a limited air supply. The deposited SiO_2 film, after treatment with superheated steam, became strong enough and acquired low wettability by liquid indium. The best results in the preparation of the single crystals were obtained by using a composite coating, with a thin layer of lamp black from burning gasoline, deposited over the SiO_2 film.

The samples were disks measuring 18 mm in diameter and $D = 119 \pm 2$, 114 ± 2, and 114 ± 2 μ thick. The average sample thickness was determined after the end of its investigation by weighing and measuring the average diameter. For the 119-μ sample, the normal N to the flat surface agreed within ~2° with the $[1\bar{1}0]$ direction, with N ∥ [100] for the other two samples.

The quality of the investigated single crystals is characterized by the parameter $\omega\tau$, which amounted to 85–90 for the 142-μ sample and 70–80 for the thinner samples for the electrons of the central section of the Fermi-surface tube of indium in the third zone.

The value of the parameter $\omega\tau$ was determined in accordance with[5] from the width of the first-order CR line at the temperature $T = 0.67$ °K.

The single crystal lying on the quartz substrate on which it was grown was placed in a strip resonator included in the circuit of an autodyne traveling-wave-tube spectrometer.[6] The magnetic field H applied to the sample was modulated at frequencies 12.5 and 75 Hz synchronized with the line voltage. A signal proportional to $\partial R/\partial H$, where R is the active component of the surface impedance of the sample, was recorded with a two-coordinate potentiometer as a function of the magnetic field.

The use of a strip resonator that could be tuned during the course of the helium experiment[7] has made it possible to carry out the measurements at any frequency in the range 8.8–9.5 GHz. By rotating the strip in the horizontal plane we obtained the required polarization of the microwave currents on the sample surface.

It should be noted that the observation of the CRNO would be quite difficult against a background of the intense CR excited at the limiting point of the Fermi surface. Our experiments were therefore performed under conditions that did not favor the excitation of the

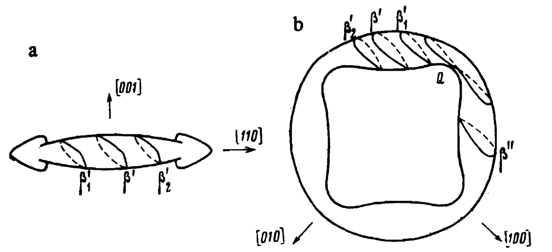

FIG. 1. Arrangement of the orbits on the electron Fermi surface of indium in the third zone. The following are shown: a) the non-extremal equivalent orbits β_1' and β_2' corresponding to one order of CRNO in the $(1\bar{1}0)$ plane; b) the nonequivalent orbits β_1' and β_2' corresponding to one order of CRNO in the (001) plane; Q—the stopping point—is the center of the orbit with self-intersection, β' and β'' are the extremal orbits.

CR at the limiting point, i.e., with the high-frequency currents I on the sample surface having a linear polarization perpendicular to the direction of the magnetic field, I ⊥ H.

The magnetic field was produced by an electromagnet, the field intensity was measured with a Hall pickup calibrated with an NMR magnetometer for each plot of $\partial R/\partial H$ against the magnetic field.

RESULTS OF EXPERIMENTS

We investigated two crystallographic planes, $(1\bar{1}0)$ and (001). The plane most convenient for the investigation of CRNO is $(1\bar{1}0)$, in which only one effective mass is observed.[8] The electronic Fermi surface, and also the electron orbits—extremal β' and nonextremal β_1' and β_2'—are shown schematically in Fig. 1a. The presence of superconductivity at $H \leqslant 200$ Oe did make it impossible to observe in our samples the cutoff of the CR for electrons with small effective masses, i.e., at H ∥ [110] or near this direction. The CR cutoff field exceeds 200 Oe only at magnetic field

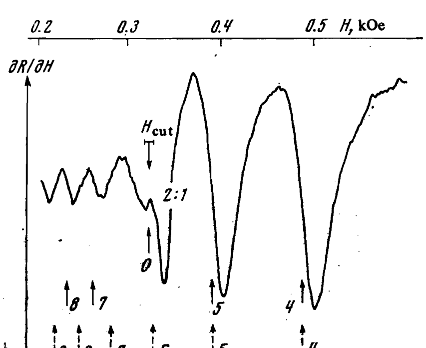

FIG. 2. Plot of CR in single-crystal indium 119 μ thick; $\measuredangle$(H, [001]) = 11°, f = 9.05 GHz, the symbol 2 : 1 marks the location where the gain of the circuit is doubled. Here and in the other figures, the arrows under the curves indicate the positions of the resonances, and the numbers indicate the orders of the resonance. The lower dashed arrows mark the calculated positions of the CR lines on the central section of the Fermi-surface tube without cutoff. O—resonant peak corresponding to cut off of the nonresonant orbits, H_{cut}—cutoff field. The sample temperature is T = 1.71 °K.

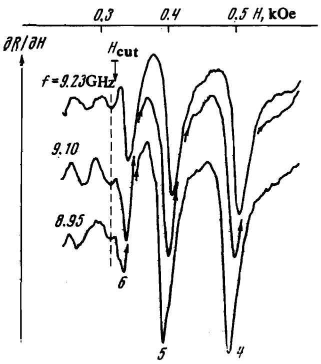

FIG. 3. Plot of CR at different frequencies f of the measuring microwave field, for a sample 119 μ thick; T = 1.2 °K.

directions making small angles (up to ~ 10°) with the [001] axis. The electron effective mass is strongly anisotropic if H is near the [001] axis. In addition, the CR amplitude increases rapidly with increasing effective mass. This restricted the angle interval at which the CRNO was investigated to a value ~ 2°.

1. A typical plot of the CR cutoff for indium samples with N ∥ $[1\bar{1}0]$ is shown in Fig. 2. The obtained dependence of $\partial R/\partial H$ on H is analogous in many respects to that previously observed in bismuth.[1,3] The amplitude of the CR on the extremal section of the Fermi-surface tube near the cut off field H_{cut} decreases abruptly, but the resonant peaks are observed in weaker fields down to the critical indium field H_c = 200 Oe at T = 1.71 °K. The periodicity of the peaks is a function of the reciprocal field, which is rigorously satisfied for CR at $H > H_{cut}$, is not observed in this case.

The value of H_{cut} was determined from experiment by varying the resonator frequency. Figure 3 shows a plot of such an experiment with an indium sample 119 μ thick. The jump of the derivative $\partial R/\partial H$, marked by the letter O on Fig. 2, corresponds to the cutoff of the extremal nonresonant orbits[19] and determines the cutoff field H_{cut}. On all the curves with different high-frequency field frequencies in the resonator, the position of this discontinuity of the magnetic field remains unchanged. The value $H_{cut} = 320 \pm 3$ Oe, determined from the derivative discontinuity O, coincides within the limits of errors with the value calculated from the central-section electron momentum p_{xc} measured in[10].

2. In the (001) plane, at a direction of H close to the [110] axis, two series of CR were observed, corresponding to the orbits β' and β''—Fig. 1b. The use of angular modulation of the field[11] has made it possible to suppress by more than 20 times the CR signals of the electrons of the orbit β'' with practically isotropic effective mass near the [110] axis, and to investigate the cutoff of the cyclotron orbits β' of the electrons with larger effective mass. Figure 4 shows the plot of the experiment performed on a sample 142 μ thick with N ∥ [001]. In the magnetic-field interval from the cutoff field $H_{cut} = 371 \pm 4$ Oe to the calculated position of the sixth-order CR on the extremal section, which is nearest to H_{cut}, two resonance peaks are observed. In Figure 4 they are marked by the symbols β_1' and β_2''. In the $(1\bar{1}0)$ plane, less than one peak of CRNO falls in the analogous field interval $H_7 < H < H_{cut}$ (see Fig. 2). This indicates

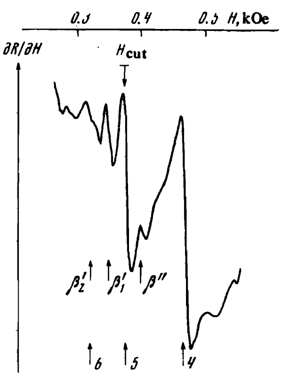

FIG. 4. Plot of CR for single-crystal indium 142 μ thick in (001) plane; $\measuredangle(H[110]) = 4°$. The resonances β_1' and β_2' correspond to the orbits β_2' and β_2' on Fig. 1b. The resonance β'' is the CR, suppressed by the angular modulation of the electrons with central orbit β''.

that the function $m^*(p_x)$ is substantially different in the planes $(1\bar{1}0)$ and (001). The possible cause of this difference will be discussed later on.

Experiments with frequency tuning have demonstrated that the derivative $\partial R/\partial H$ does not experience in the (001) plane the discontinuity corresponding to the cutoff of the nonresonant orbitals. The angle intervals in which a discontinuity of the derivative is observed are apparently close to the corresponding intervals for the radio-frequency size effect. Nor is the latter observed in the angle interval ~30° near the [110] axis.[10] The cutoff field was therefore determined from the momentum of the electrons of the central cross section p_{xc}, obtained by extrapolating the angular dependence of p_{xc}.[10] More accurate values of $H_{cut} = 371 \pm 4$ Oe were obtained from experiments with frequency tuning: with decreasing frequency of the high-frequency field in the resonator, as the fifth-order CR approaches the vicinity of the cutoff field H_{cut}, its amplitude decreases and the line shape becomes distorted.

3. A criterion that determines whether the observed CR belongs to a noncentral orbit is provided by an experimental investigation of the dependence of the CR on the inclination of the magnetic field to the sample surface. The resonant field H_n for electrons of noncentral sections, having a Fermi-velocity component v_H along the magnetic-field direction H, is shifted by the Doppler effect. As a result, a splitting or a displacement of the resonant peaks is observed.[2]

Figure 5 shows plots of CRNO in the (001) plane at different inclinations ϑ of the magnetic field to the sample surface. It is seen from Fig. 5 that at small inclinations ϑ the cyclotron resonance β_1' in the (001) plane (see Fig. 4) splits into two peaks. From the shift of the resonances, described by the formula

$$\frac{\Delta H_n}{H_n} = \frac{k v_H \sin\vartheta}{\omega} = \frac{v_H \sin\vartheta}{\omega \delta_l}$$

(where $\delta_l = 1/k$ is the inductive depth of the skin layer[2]), by assuming in accordance with[12] $\delta_l = 2.1 \times 10^{-5}$ cm, we can estimate the component v_H of the resonating electrons. The value obtained in this manner for electrons with an orbit of diameter $p_x = 0.93\, p_{xc}$ is $v_H \approx 1.0 \times 10^6$ cm/sec. A calculation performed, for comparison, with the β-arm approximated by an ellipsoid yields for v_H a value one order of magnitude larger.

It should be noted that the inclination of the magnetic field to the sample surface exerts a much larger influence on the CRNO β_2': its splitting occurs already at inclination angle $\vartheta \sim 1'$, and the amplitudes of the resonance peaks decrease to such an extent that they are practically blanked out by the noise.

The inclination of the magnetic field to the sample surface affects strongly the CRNO in the $(1\bar{1}0)$ plane. However, no clearly pronounced splitting of the peaks, as in the (001) plane, takes place, and all that is seen is a broadening of the resonances and a decrease of their amplitude.

DISCUSSION

It can be noted first of all that the results of experiments with frequency tuning, the anisotropy-investigation results presented below, and particularly the results of measurements in a magnetic field inclined to the sample surface, confirm that the CR observed at $H < H_{cut}$ is due to electrons with non-extremal orbits.

1. From the measured values of the resonant field H_n we can determine the dependence of the electron effective mass on the momemtum complex p_x. The effective masses were calculated from the formula $m^* = eH_n n/\omega c$, while the corresponding momentum components p_x, which are the diameters of the orbits in p space and correspond to orbits in coordinate space with a diameter equal to the sample thickness D were calculated from the formula $p_x = eH_n D/2c$. At $H_n < H_{cut}$, the fields H_n were assumed to be those corresponding to the inflection points in the $\partial R/\partial H(H)$ plots, marked by arrows in Fig. 2. The basis for this definition of H_n may be the following. It is noted in[3] that CRNO has the same logarithmic characteristic as the CR at the limiting point. For the latter, the indicated choice of H_n in the field region where the oscillations are practically sinusoidal yields values of m^* that coincide with the values calculated from CR of lower orders n with a clearly pronounced resonant line shape.

The result of the determination of m^* as a function of the momentum component p_x is shown in Fig. 6. The curves were plotted point by point for definite direction of the magnetic field relative to the [001] axis. The lowest curve, corresponding to the largest angle

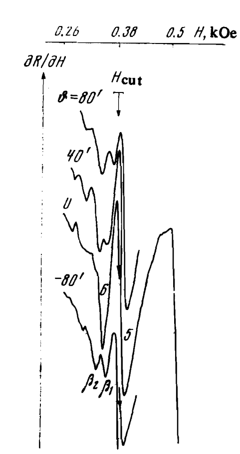

FIG. 5. Plot of CRNO at different inclinations ϑ of the magnetic field H to the surface of the 114-μ sample; $\measuredangle(H, [110]) = 10°$.

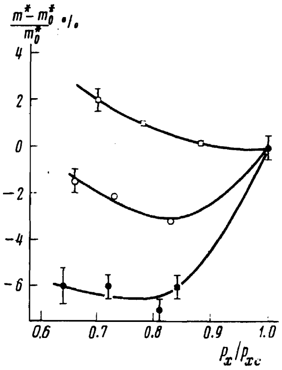

FIG. 6. Dependence of the electron effective mass m^* on p_x/p_{xc}: ● – $\varphi = \measuredangle(\mathrm{H}, [001]) = 11°$, $f = 9.05$ GHz, central-section electron mass $m_0^* = 0.605\ m_e$; ■ – $\varphi = 11°$, $f = 9.225$ GHz, $m_0^* = 0.605\ m_e$; ○ – $\varphi = 10°40'$, $f = 9.05$ GHz, $m_0^* = 0.611\ m_e$; □ – $\varphi = 10°$, $f = 9.05$ GHz, $m_0^* = 0.630\ m_e$. The quantity $p_x/p_{xc} = H/H_{\mathrm{cut}}$ is the ratio of the diameters of the electron orbits in momentum space. The data were obtained in experiments with a sample 119 μ thick.

between the direction of H and the [001] axis, indicates that the effective mass of the electrons of the non-extremal sections of the Fermi surface decrease as the section approaches the neck of the arm. This type of $m^*(p_x)$ dependence agrees with the results of the measurement of the effective masses of extremal sections at the center and on the neck of the arm. As shown in[8], the lowest effective mass of the electrons in indium is connected with an orbit situated on the neck of the arm β' near the point where the neck joins the other arm β''.

The lower curves of Fig. 6 demonstrate the variation of the dependence of the effective mass on the diameter of the orbit as the angle between the magnetic field H and the [001] axis is decreased. The increase of the effective mass and the increase of its anisotropy, which follow from the diagram, can also be explained by using for indium a Fermi-surface model consisting of arms that taper down towards their ends. Indeed, as a result of the narrowing down of the Fermi surface arms, a non-extremal orbit with self-intersection is possible at the point of their junction. When the magnetic field is rotated towards the [001] axis, the trajectory of the non-extremal section electron approaches the self-intersection point of this orbit (the stopping point). Since an orbit with self-intersection corresponds to an infinite effective mass, the approach of the electron trajectory to the stopping point causes its effective mass to increase.[13] The result represented by the graph of Fig. 6, namely the dependence of the effective electron mass $m^*(p_x, \varphi)$ on the orbit diameter in momentum space and on the angle between H and the [001] axis, can be interpreted as a consequence of the described phenomenon.

2. We discuss now the possible causes of the difference between the observed CRNO pictures in the planes $(1\bar{1}0)$ and (001).

In the $(1\bar{1}0)$ plane, the non-extremal orbits β_1' and β_2' located on both sides of the central orbit β' on the tube (Fig. 1a) are equivalent, and both groups of electrons give rise to a single CRNO. In the (001) plane, the orbits β_1' and β_2' cease to be equivalent: β_1' passes near the necks of the arms inside the "torus," while β_2' passes on the outside (Fig. 1(b)). The electrons with orbit β_1' passing near the stopping point Q—the center of the orbit with the self-intersection—have a larger effective mass than the electrons with orbit β_2' having the same diameter p_x as β_1'. This gives rise to a distinctive splitting of the resonance, wherein the same order of the resonance corresponds to two resonance peaks. The first, corresponding to the orbit β_1', has a larger effective mass m^* and a larger diameter p_x than the second, corresponding to the orbit β_2'. The resonance peak closest to H_{cut} (Fig. 4) can be assumed, in light of the foregoing, to be due to the electrons with orbit of type β_1' (Fig. 1b) and diameter $p_{x1} = (0.94 \pm 0.2)p_{xc}$. The effective mass determined from the position of this resonant peak in a magnetic field is the effective $m_1^* = (0.63 \pm 0.01)m_e$, or 11% higher than the value $m^* = 0.57\ m_e$ for the electrons of the extermal section. The second resonant peak is due to electrons having an orbit β_2' (Fig. 1b) with diameter $p_{x2} = (0.89 \pm 0.02)p_{xc}$. The value of the effective mass for these electrons is the same within the limits of errors as for the electrons of the extremal section: $m_2^* = (0.58 \pm 0.01)m_e$.

Whether the observed CRNO belong to the orbit β_1' or β_2' can be verified by varying the frequency ω of the measuring microwave field. When the frequency is decreased, as the orbits β_1' and β_2' move away from the central orbit β', one should expect the difference between the effective masses to increase, a situation reflected in the mutual positions of the CRNO peaks in the experimental plots. Unfortunately, we can draw no definite conclusions from the results of our experiments with frequency variation, since the tuning range of the resonant frequency was not large enough.

3. The CRNO could be observed only for electrons of the Fermi-surface tubes near those crystallographic directions for which the orbit diameter is large enough (and consequently the effective mass is large). On the other hand, to investigate CRNO in rotational crystallographic directions, which are a rule are characterized by small effective masses, would call for samples one-third as thick, but of equally high quality as used in the present paper. It is not realistic at present to expect to prepare such samples.

CONCLUSION

The main result of this study is the observation, in indium, of cyclotron resonance on non-extremal orbits, a new phenomenon observed previously only in bismuth. The investigation of the CRNO has yielded a number of qualitative characteristics of the electrons of the non-extremal sections of the tubes β of the Fermi surface of indium. Measurements of the dependence of the electron effective mass on the diameter of the non-extremal orbit and studies of its anisotropy have yielded the influence exerted on the effective mass of the electrons by the proximity of their trajectories to topologically singled-out orbits with self-intersection. The investigation described here can serve as an example of the use of CRNO as a new method for studying the vicinities of singular points of non-singly-connected Fermi surface of polyvalent metals. The results of similar investigations can be used to verify Fermi-surface calculations.

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Permittivity anomaly in metal-dielectric transitions. Theory and simulation

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We formulate some dynamic percolation theory problems whose solution may be of interest in studies of the dynamic properties of disordered systems such as strongly developed doped semiconductors, ferroelectric semiconductors with a diffuse phase transition, island films, and other objects with a nonuniform conductivity and (or) inhomogeneous permittivity. Some of the problems are simulated by means of networks consisting of capacitors and resistors with randomly broken bonds and nodes. The results obtained by such simulation and also the theory of effective media developed in the paper, as well as some considerations based on percolation theory, indicate that the static permittivity of the sample should become infinite for the metal-dielectric transition. This result is used to interpret qualitatively the "polarization catastrophe" observed in the metal-dielectric transition in n-silicon.

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1. INTRODUCTION

The methods of percolation theory are widely used to describe the static conductivity of disordered systems, static hopping conductivity, etc. (see, e.g., [1,2]). In this paper we formulate percolation-theory problems whose solutions can be of interest to the study of the dynamic properties of disordered systems, strongly-doped semiconductors, ferroelectric semiconductors, and other physics objects with inhomogeneous conductivity and (or) inhomogeneous dielectric constant. Some of these problems will be investigated experimentally by simulation with a network made up of capacitors and resistors. For an analytic description of the results we use the effective-medium theory, which is a generalization of the theory developed in[3–5].

We use the results, in particular, to interpret experimental data that point to a "polarization catastrophe" (wherein the static dielectric constant becomes infinite) in the metal-dielectric transition in n-type silicon.[6] The results obtained for the flat (two-dimensional) case can be used to interpret the results of experiments with so called island films (see, e.g., [7]).

2. FORMULATION OF PROBLEM

We recall first the formulation of one of the standard problems of percolation theory. Consider an infinite network made up of identical resistors. We open in random fashion a fraction x of the nodes (sites) of this network (the site problem, see, e.g. [1]). Then at a certain critical value $x = x_c$ the network becomes open ("percolation" of the current over the network ceases). Percolation-theory methods make it possible to estab-

