Phys. Rev. 131, 1530 (1963).

[5]E. A. Turov, M. I. Kurkin, and V. V. Nikolaev, Zh. Eksp. Teor. Fiz. 64, 283 (1973) [Sov. Phys. JETP 37, 147 (1973)].

[6]Yu. M. Bun'kov and B. S. Dumesh, Zh. Eksp. Teor. Fiz. 68, 1161 (1975) [Sov. Phys. JETP 41, 576 (1975)].

[7]L. L. Buishvili and M. D. Zviadadze, Fiz. Met. Metallov. 30, 681 (1970) [Phys. Met. Mettallogr. 30, No. 4, p. 8(1970)].

[8]L. L. Buishvili, M. D. Zviadadze, and G. R. Khutsishvili, Zh. Eksp. Teor. Fiz. 63, 1765 (1972) [Sov. Phys. JETP 36, 933 (1973)].

[9]A. Abragam, Principles of Nuclear Magnetism, Oxford Univ. Press, 1961.

[10]B. S. Dumesh, Pis'ma Zh. Eksp. Teor. Fiz. 14, 511 (1971). [JETP Lett. 14, 350 (1971)]; M. P. Petrov. G. A. Smolenskiĭ, and G. Ch. Syrnikov, Pis'ma Zh. Eksp. Teor. Fiz. 14, 514 (1971) [JETP Lett. 14, 353 (1971)]; A. A. Petrov, M. P. Petrov, and V. P. Chekmarev, Fiz. Tverd. Tela (Leningrad) 17, 2640 (1975) [Sov. Phys. Solid State 17, 1755 (1976)].

Translated by D. ter Haar

Optical observation of antiferromagnetic resonance in CoCO₃

A. S. Borovik-Romanov, V. G. Zhotikov, N. M. Kreĭnes, and A. A. Pankov

Institute of Physical Problems, USSR Academy of Sciences
(Submitted December 3, 1975)
Zh. Eksp. Teor. Fiz. 70, 1924–1935 (May 1976)

The interaction of light with the spin system during antiferromagnetic resonance (AFMR) was studied in weakly-ferromagnetic CoCO₃. The variation of the intensity of the light passing through the optical system, which consisted of a polarizer, the crystal, a compensator, and an analyzer, was measured during excitation of AFMR in the crystal. In order to amplify the weak optical signals, modulation of the microwave power was employed. It was found that under conditions of complete compensation of constant light, the alternating optical signal at resonance was absent. It appears only with discompensation of the transmitted light by means of the compensator. The shape and position of the optical signal coincide with the shape and position of the AFMR signal. The observed optical signal is produced by a change of magnetic birefringence in the crystal; this is due to a decrease, during resonance, of the mean value of the projection of the antiferromagnetic vector on its equilibrium direction.

PACS numbers: 76.50.+g

INTRODUCTION

Observation of uniform magnetic resonance by an optical method in magnetically ordered media was first carried out by Dillon on the ferromagnetic crystal CrBr₃.[1] This method was based on use of the large Faraday effect in the crystals under study and was as follows: on passage of the magnetic field through resonance, the optical system, consisting of the specimen placed between crossed polaroids, cleared because of the appearance of an alternating component of the magnetic moment along the light beam. By means of light it was also possible to observe the spectrum of magnetostatic modes in the specimen.[1] Dillon and his coworkers showed theoretically and experimentally, by means of a Fabry–Perot interferometer,[2] that the observed optical signal in crossed polaroids occurs at the displaced frequency $\omega + \Omega$, where ω is the frequency of the incident light and Ω is the frequency of the ferromagnetic resonance. Thus in this case there emerges from the system, light modulated by the ferromagnetic-resonance frequency. Later, optical detection of ferromagnetic resonance was carried out by many authors on ferrites (see, for example,[3]). But a direct proof of the fact that the observed optical signal in crossed polaroids occurs at the displaced frequency $\omega + \Omega$ is found only in the spectral experiments of Dillon.[2]

We thought it would be interesting to see an optical signal during excitation of magnetic resonance in antiferromagnetic crystals. We chose as our materials the well-studied weak ferromagnets CoCO₃ and MnCO₃. For optical detection of antiferromagnetic resonance (AFMR) we made use of the presence of magnetic birefringence in these crystals.[4]

The crystals CoCO₃ and MnCO₃ are rhombohedral; their crystalline symmetry is described by space group $\bar{D}_{3d}^6$. The elementary cell of CoCO₃ and MnCO₃ contains two magnetic ions.[3] At low temperatures these compounds change to an ordered antiferromagnetic state of the easy-plane type. The magnetic moments of the sublattices are canted and form a spontaneous ferromagnetic moment m, which lies in the basal plane (111) and is perpendicular to the antiferromagnetic vector l.[6] The value of m is in MnCO₃ 0.2% and in CoCO₃ about 5% of the nominal value of the sublattice magnetization. According to the data of[7], in CoCO₃ in the absence of an external magnetic field the vector l lies in a vertical plane of symmetry at a certain angle (different from 90°) to the third-order axis. In the basal plane the crystallographic anisotropy of these compounds is small,[6,8] and therefore in a magnetic field H (> 0.5 kOe), regardless of its direction, the vector m always sets itself along the field.

The AFMR spectrum of the carbonates CoCO₃ and MnCO₃ has two branches, which differ significantly with
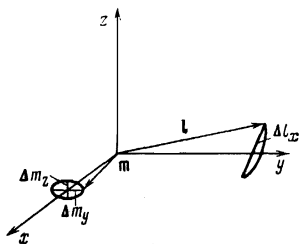
FIG. 1. Schematic motion of the vectors m and l in low-frequency resonance.

respect to frequencies and to the form of the oscillations of the vectors m and l.[9] Hereafter we shall speak only of the low-frequency branch.

Figure 1 depicts schematically the motion of the vectors m and l in the low-frequency resonance. The vector m precesses about the direction of the external magnetic field $H^{1)}$ in such a way that its end-point describes an ellipse with semiaxis ratio

$$\frac{\Delta m_y}{i\Delta m_x} = \left| \left(\frac{H+H_D}{H} \right)^{1/2} \right|$$

The vector l oscillates in the basal plane, not leaving it, and

$$\Delta l_x/\Delta m_y = |2H_E/(H+H_D)|$$

(H_E is the exchange field and H_D the Dzyaloshinskiĭ field).

In the paramagnetic state, the crystals $CoCO_3$ and $MnCO_3$ are optically uniaxial. On transition to the antiferromagnetic state, they develop an additional magnetic birefringence Δn_M. According to[4] there are two contributions to Δn_M: an isotropic, independent of the direction of the antiferromagnetic vector l with respect to the crystallographic axes, and an anisotropic. The elements of the dielectric permittivity tensor for these crystals contain both isotropic and anisotropic terms[2]:

$$\varepsilon_{xx} = \varepsilon_\perp{}^0 + \lambda_2 l^2 + \lambda_4 l_x{}^2 + \lambda_7 l_z l_y + \lambda_8 (l_x{}^2 - l_y{}^2),$$
$$\varepsilon_{yy} = \varepsilon_\perp{}^0 + \lambda_2 l^2 + \lambda_4 l_z{}^2 - \lambda_7 l_z l_y + \lambda_8 (l_x{}^2 - l_y{}^2),$$
$$\varepsilon_{zz} = \varepsilon_\parallel{}^0 + \lambda_1 l^2 + \lambda_3 l_z{}^2,$$
$$\varepsilon_{xy} = \varepsilon_{yx} = \lambda_7 l_z l_x + 2\lambda_8 l_x l_y,$$
$$\varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2}\lambda_5 l_z l_x + \lambda_6 l_x l_y.$$
$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\lambda_5 l_z l_y + \frac{\lambda_6}{2}(l_x{}^2 - l_y{}^2).$$

It has been found experimentally that in $CoCO_3$ the anisotropic contribution is quite large and is comparable in value with the isotropic.[4] In the ordered state there also develops a difference $n_x - n_y$ of the refractive indices in the basal plane (the plane perpendicular to the optic axis), amounting to $\sim 27 \cdot 10^{-5}$, and the crystal becomes optically biaxial. But because of the fact that $n_x - n_y$ is small in comparison with the natural birefringence $n_z - n_y$, which amounts to $25 \cdot 5 \cdot 10^{-2}$, the angle between the new optic axes is small and amounts to only $\sim 3°$ (see[10]). Since during excitation of resonance the orientation of the antiferromagnetic vector with respect to the crystallographic axes changes, the anisotropic part of the magnetic birefringence should also change.

We shall calculate the nature of a light signal passing,

at a moment of excitation of uniform low-frequency resonance, through a crystal that possesses anisotropic magnetic birefringence. We shall consider an optical system consisting of a source of monochromatic light with wavelength λ, a polarizer, the crystal, an analyzer, and a photoreceptor. We shall restrict ourselves to the case in which the light is propagated along the high-order axis C_3 of the crystal. Then only the magnetic birefringence in the basal plane enters into consideration. The position of the principal axes of the tensor in this plane, for a given crystal, is determined by the position of the antiferromagnetic vector l. Since in uniform resonance all the spins in the specimen oscillate with the same phase, therefore for light that is passing through the crystal the oscillation of the vector l is equivalent to oscillation of the principal axes of the tensor with respect to their equilibrium positions, with the resonance frequency.

Let x and y be the equilibrium positions of the principal axes in the basal plane, x' and y' the instantaneous positions of these axes during oscillations in resonance (see Fig. 2), which are determined by a variable angle ψ. According to the model under consideration, the value of the angle ψ varies with the AFMR frequency Ω according to the law $\psi = \psi_0 \cos\Omega_t$. We shall suppose that the light wave incident on the crystal is linearly polarized along y; that is,

$$E_{x\,\text{inc}} = 0, \quad E_{y,\,\text{inc}} = E_0 \cos \omega t.$$

A simple calculation shows that the expression for the amplitude of the wave passing through the crystal and through the analyzer, which is turned at an angle φ with respect to the polarizer, will have the form

$$
\begin{aligned}
E_\varphi = & E_0 \cos \varphi \cos(\omega t + \gamma) - \tfrac{1}{2} E_0 \psi_0{}^2 \cos \varphi [\cos(\omega t + \gamma) - \cos \omega t] \\
& + \tfrac{1}{2} E_0 \psi_0 \sin \varphi [\cos(\omega t + \Omega t + \gamma) + \cos(\omega t - \Omega t + \gamma) - \cos(\omega t + \Omega t) \\
& - \cos(\omega t - \Omega t)] - \tfrac{1}{2} E_0 \psi_0{}^2 \cos \varphi [\cos(\omega t + 2\Omega t + \gamma) + \cos(\omega t - 2\Omega t + \gamma) \\
& - \cos(\omega t + 2\Omega t) - \cos(\omega t - 2\Omega t)].
\end{aligned}
\tag{1}
$$

Here $\gamma = 2\pi d\lambda^{-1}(n_x - n_y)$ is the phase difference that accumulates in the specimen of thickness d because of the presence of the magnetic birefringence $n_x - n_y$.

Formula (1) shows that there emerges from the crystal in resonance, along with light at the original frequency ω, light at the shifted frequencies $\omega \pm \Omega$ and $\omega \pm 2\Omega$; or equivalently, the emerging light is modulated by the AMFR frequency and a multiple of it. We shall now calculate the intensity I_φ of the light that passes through the whole system. We remark that in the ex-

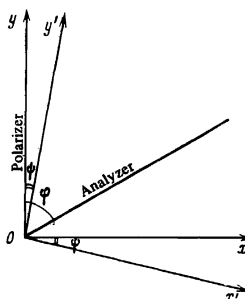FIG. 2. Position of principal axes of the crystal in uniform resonance.

pression obtained for the intensity, we have omitted terms of the form

$$\tfrac{1}{4}E_0^2\psi_0 \sin 2\varphi \cos^2(\omega t+\gamma)\cos \Omega t$$

which are linear in the angle ψ_0 of deflection of the spins. They oscillate with frequency Ω (in our case 36 GHz). The photoreceptor that we have cannot accept such a high modulation frequency. As a result, the signal corresponding to these terms is averaged with respect to the frequency Ω in the photoreceptor and makes no contribution to the received intensity. But for those Ω's that the photoreceptor can pass, the signal from them can be observed. To terms of the second order in ψ_0, we obtain the following expression:

$$I_\varphi=E_0^2 \cos^2 \varphi+2E_0^2\psi_0^2 \sin^2 \frac{\gamma}{2}\,(\sin^2 \varphi-\cos^2 \varphi\,). \qquad (2)$$

The second term of formula (2) for the light intensity occurring at resonance (that is, that proportional to ψ_0^2) consists of two terms, which behave differently in their dependence on the mutual positions of the polaroids. From a comparison with the expression (1) it is evident that the first term ($\sim \sin^2\varphi$) corresponds to light at frequency $\omega \pm \Omega$, the second ($\sim \cos^2\varphi$) at frequency ω. It is evident that in completely crossed polaroids, only light at the shifted frequency $\omega \pm \Omega$ passes through the system; and in parallel polaroids, only light at the basic frequency ω. From analysis of expression (2) it follows that for arbitrary φ the intensity of light at frequency ω decreases (a minus sign in front of $\cos^2\varphi$), but in return light appears at frequency $\omega \pm$ (a plus sign in front of $\sin^2\varphi$). The maximum decrease of the intensity of light at frequency ω is equal to the maximum increase of intensity of light at the shifted frequencies (namely, $2E_0^2\psi_0^2 \sin^2(\gamma/2)$).

For comparison with experiment, it is convenient to put formula (2) into the form

$$I_\varphi=E_0^2 \cos^2 \varphi-2E_0^2\psi_0^2 \sin^2(\gamma/2) \cos 2\varphi.$$

The picture described above is similar to that considered by Dillon for FMR on the basis of the Faraday effect. But in the case that we have considered, the reason for the modulation of the light is the magnetic birefringence.

A calculation was made for the case in which the light incident on the crystal is polarized at angle 45° to the y axis. The intensity of the light that passes through the crystal in this configuration is given by the following expression:

$$I_\varphi=\tfrac{1}{2}E_0^2(1+\cos 2\varphi \cos \gamma)+2E_0^2\psi_0^2 \sin^2(\gamma/2)\cos 2\varphi. \qquad (3)$$

Here also φ is the angle between the polarizer and the analyzer. Analysis of this expression shows that in completely crossed polaroids, the light due to the AFMR passes at the basic frequency; in parallel, at the shifted. We note that the calculation made above is correct for a crystal of any thickness; that is, γ is arbitrary.

We shall now turn to experiment.

EXPERIMENTAL APPARATUS AND SPECIMENS

In our research we investigated the change of the magnetic birefringence of light in a crystal during passage of the magnetic field through the AFMR line. For amplification of the weak optical signals, we used modulation of microwave power. The detailed scheme of the experimental apparatus is shown in Fig. 3. Microwave radiation ($\Omega/2\pi=35$–40 GHz) from a GZ-30 klystron generator, modulated in power by rectangular pulses with duty factor 2, was fed through a valve and a calibrated attenuator to a shorted waveguide. The repetition frequency of the pulses could be varied from 1 to 20 kHz. The specimen being investigated was attached to a plunger and placed inside the shorted waveguide, at an antinode of the magnetic component of the microwave field. The reflected microwave signal was detected with a crystal detector and entered either the oscillograph or the Y_2 coordinate of the double-pen XY recorder. The signal that entered the X coordinate of this recorder was proportional to the magnetic field.

The optical scheme for measurement of the birefringence Δn of light in the crystal was similar to that described in[4]. A beam of light ($\lambda=6328$ Å)[3] from an LG-56 helium-neon laser passed through the system, which consisted of a polarizer, the crystal under study, a Berek calcite compensator, and an analyzer, and entered a PM-79 photomultiplier. The signal registered by the photomultiplier (the voltage drop across the load of the PM) could contain both a constant component, resulting from discompensation of the light, and a variable, changing with the modulation frequency of the klystron. The constant component of the signal was measured by means of an F-30 digital voltammeter. The variable part of the light signal was amplified by a U2-6 narrow-band amplifier, tuned to the modulation frequency of the klystron, and entered either the oscillograph or, through a synchronous detector, the Y_1 coordinate of the recorder. Thus the optical scheme of the apparatus permitted measurement of the change of
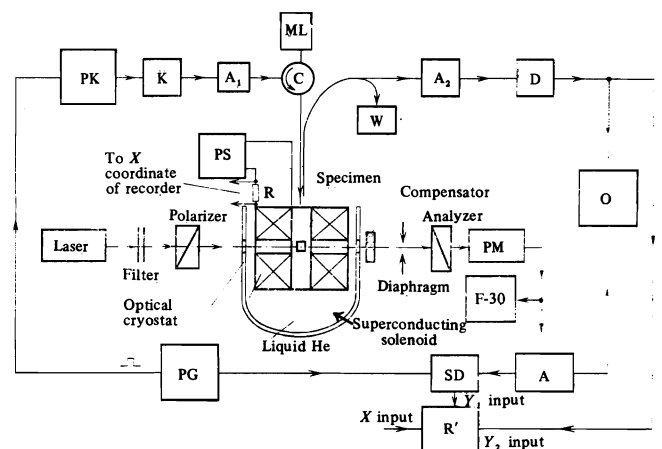

FIG. 3. Schematic drawing of the experimental apparatus. PK, power supply for klystron; K, klystron; A_1 and A_2, attenuators; C, circulator; W, wave meter; D, detector; ML, matched load; O, double-beam oscillograph; A, narrow-band amplifier; SD, synchronous detector; R', double-pen XY recorder; PG, rectangular-pulse generator; PS, power supply for solenoid; F-30, digital voltammeter.

birefringence Δn both in the static mode, by the method of direct compensation of the path difference (by means of the Berek compensator) and also in the dynamic mode (at the moment of passage through resonance), by the value of the alternating signal. The sensitivity to change of Δn for direct current was $\sim 10^{-5}$ to 10^{-6}; for alternating current, significantly higher: $\sim 10^{-8}$ to 10^{-9} (depending on the specimen thickness).

On the basis of the minimum value of the parasitic constant light signal, when it passed through an optimally compensated optical system, it was possible to judge the quality of the specimens under study and the accuracy of their adjustment. The final adjustment of the specimen in the apparatus was made by change of the position of the cryostat with respect to the laser beam. In the best cases, when the light was propagated along the optic axis of the crystal, the degree of compensation of the system was $\sim 0.03\%$ of the total intensity of the incident light; in the worst cases, $\sim 1\%$.

All the experiments were conducted in an optical helium cryostat at temperatures ~ 1.5 to 2.0 K, in superfluid helium (since it does not scatter light). The light to the specimen was fed through windows in the cryostat and special openings in the waveguide.

The magnetic field was produced by a superconducting solenoid and could be directed either along the light beam or in the plane perpendicular to it. Resonance was observed for $CoCO_3$ in a field ~ 1 to 2 kOe, and for $MnCO_3$ in a field ~ 10 to 12 kOe (depending on the frequency of the microwave power supplied).

The $MnCO_3$ and $CoCO_3$ crystals that we used were grown in the Institute of Crystallography of the USSR Academy of Sciences.[4] Chosen specimens were oriented either on the basis of the external faceting, or by means of x rays with accuracy $1-2°$. The specimens were made in the form of disks or of rectangular parallelopipeds. The surfaces were optically polished. Altogether, four specimens of $CoCO_3$ and one of $MnCO_3$ were investigated in the research. The main experimental results for $CoCO_3$ were obtained on a specimen cut in the form of a thin disk of diameter ~ 1.2 mm and thickness $d = 0.3$ mm. The plane of the disk coincided with the basal plane of the crystal. The $MnCO_3$ specimen had the form of a rectangular parallelopiped with axes $z \| C_3$, $x \| C_2$ and linear dimensions ~ 2 mm. For $CoCO_3$, the specimen cut in the form of a disk had the narrowest line ($\Delta H \sim 30$ Oe). The other specimens had
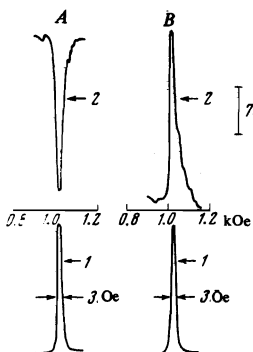

FIG. 4. Example of a record of the microwave absorption line (curve 1) and of the alternating light signal (curve 2) at resonance in $CoCO_3$, for two values of the path differences Γ: A, $\Gamma = \pi/6$; B, $\Gamma = 7\pi/6$.

significantly broader lines, ~ 200 to 400 Oe. But besides the quality of the specimen, broadening of the AFMR line may be due to large size of the specimens, and also to incorrect form of their surface. For $MnCO_3$ the value of ΔH was ~ 50 Oe.

EXPERIMENTAL RESULTS

In this research we investigated the intensity of the alternating (at the frequency of modulation of the klystron) component of the light signal during ferromagnetic resonance in $CoCO_3$ and $MnCO_3$ crystals, and its dependence on various parameters.

1. $CoCO_3$. The main experimental results were obtained in an orientation in which the wave vector of the light was directed along the C_3 axis, and the magnetic field lay in the basal plane and was directed vertically (see below, insert in Fig. 5; the x and y axes in the plane were not determined). Two cases were studied: when the polarization of the incident light coincided with the direction of the magnetic field, and when it was directed at an angle of 45° to it. In both cases the polarizer and analyzer were crossed. In the first case, no alternating optical signal was detected, to within accuracy $\sim 5 \cdot 10^{-6}$ of the incident light. In the second case, with maximum compensation of the constant optical signal by means of the compensator, the alternating optical signal is also absent at the moment of resonance. But it appears after a certain discompensation of the system is introduced by rotation of the compensator. Figure 4 shows examples of the record of the alternating light signal $I_\sim$ and of the microwave power at the moment of passage of the field through antiferromagnetic resonance, for different positions of the compensator, for a $CoCO_3$ specimen.

As is easily observed, the form of the record of the optical signal completely reproduces the curve of microwave absorption in the specimen. The amplitude and phase of the alternating optical signal $I_\sim$ vary with the value of the path difference Γ, measured in radians, produced in the specimen and the compensator. The phase of the alternating light signal is considered in relation to the phase of the modulation of the microwave power by the rectangular phase. Figure 5 shows the variations of the intensity of the constant light signal $I_=$ (dark points) and of the alternating $I_\sim$ (light points) with path difference Γ, measured by means of the compensator. What is actually plotted in the figure is the change of path difference $\Gamma - 2\pi n$ from the value of Γ corresponding to the minimum of the constant light.

The dependence obtained experimentally can be described analytically by the formulas

$$I_= = I_{0=} \sin^2 (\Gamma/2) - \text{curve } 2,$$
$$I_\sim = I_{0\sim} \sin \Gamma - \text{curve } 1. \tag{4}$$

(The first formula agrees with the theoretical dependence of the intensity of the transmitted light on the compensator position in crossed polaroids; see, for example, [11].) From Fig. 5 it is seen that the maxima of the amplitude of $I_\sim$ correspond to $\Gamma = 2\pi n + \pi/2$ or $\Gamma = 2\pi n + 3\pi/2$ and amount to $\sim 2 \cdot 10^{-4}$ of the amplitude $I_{0=}$
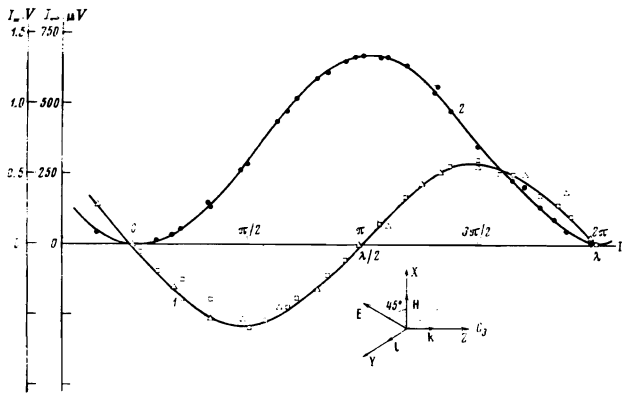
FIG. 5. Variation of intensity of constant (I_m) and alternating $(I\sim)$ light signal with path difference Γ produced in compensator and specimen. $\square$ and $\triangle$ are data from different series of experiments.

of the constant signal. For complete compensation or recompensation of the light, the alternating light signal is absent. For a fixed position of the compensator, the value of $I\sim$ varies linearly with the microwave power fed to the waveguide and with the intensity of the light incident on the crystal. We obtained all the results described above with modulation of the microwave power at frequency 1 kHz. Raising of the modulation frequency to 20 kHz had no effect on the results obtained, either qualitatively or quantitatively.

We made an attempt to investigate the form of individual light pulse, corresponding to a rectangular pulse of microwave power. By means of an S7-7 stroboscopic oscillograph, in the accumulation mode, we succeeded in recording on the recorder a light pulse and a pulse of microwave power of duration 50 μsec. This experiment enables us to assert qualitatively that the time of lag of the light pulse in comparison with the high-frequency does not exceed ~ 2 μsec. More definite deductions cannot be made at the present time, because the frequency characteristics of the scheme that we used do not permit acceptance of pulses with front duration less than 1 μsec.

We investigated the alternating light signal in CoCO$_3$ also in another experimental configuration, namely when $k_{light} \perp C_3 \perp H$. In this case also it exists only for incomplete compensation of the light, and its value and phase depend on the degree of discompensation. This configuration has not yet been studied in detail.

2. MnCO$_3$. With the MnCO$_3$ crystal, the experiment was done for two configurations:

a) $k_{light} \parallel C_3 \perp H$, b) $k_{light} \perp C_3 \perp H$. In the first case, within the limits of accuracy of our apparatus, no optical resonance signal was detected. In the second case, a very weak signal, with amplitude ~ 5 μV, was observed. Qualitatively, its behavior coincides with that observed for CoCO$_3$. In all experiments with CoCO$_3$ and MnCO$_3$, a change of the magnetic field to the opposite direction was made. No change was detected either in the amplitude or in the phase of the alternating light signal.

DISCUSSION OF EXPERIMENTAL RESULTS

Our experimental results show that in the case of complete compensation of the constant light, both for vertical polarization of the incident light (i.e. $E \parallel H$) and for polarization at angle 45° to the direction of H, there is no alternating optical signal. This means that we have not succeeded in observing the effect sought, which was described in the Introduction and is due to uniform oscillations of the magnetization at resonance. But with discompensation of the light by means of the compensator, there appears a comparatively large alternating optical signal. The nature of this signal is other than of that sought. Its large value impeded our search for the calculated signal, whose value is apparently significantly smaller than the value of that detected.

The alternating optical signal that we observed can be explained if we take into account that at the moment of resonance, there occurs a decrease of the magnetic birefringence of the whole specimen. This decrease occurs because of a decrease of the component of the vector l directed perpendicular to the magnetic field during oscillations of l with respect to its equilibrium position. Hereafter, for definiteness, we shall suppose that the magnetic field H is directed along the x axis. Then in equilibrium the component l_y of the vector l is nonzero; that is, in this case $l_{0y} = |l|$.

We again consider an optical system consisting of a polarizer, the crystal, a compensator, and an analyzer. Light is incident in the direction perpendicular to the basal plane of the specimen, in which there is magnetic birefringence caused by the phase difference γ. According to reference[4], it is proportional to the square of the component l_y of the antiferromagnetic vector:

$$\gamma = \alpha l_y^2. \qquad (5)$$

The intensity of the light that passes through the whole system is given by the expression (see[11])

$$I = E_0^2 [\cos\varphi - \sin 2\chi \sin 2(\chi-\varphi) \sin^2(\Gamma/2)]. \qquad (6)$$

Here χ is the angle between the polarizer and the y axis, φ is the angle between the polarizer and the analyzer, and Γ is the phase difference accumulated in the specimen and the compensator: $\Gamma = \gamma + \Delta\Gamma$.

In the excitation of resonance in the crystal, energy from the spin oscillations that are uniform over the specimen is transferred to nonuniform oscillations—spin waves with wave vector $k \neq 0$ are excited. The total number of spin waves excited is proportional to the square of their amplitude; this is conveniently expressed in terms of the mean square angle of deflection of the spins from the equilibrium position, $\psi^2/2$. In the case of uniform oscillations, the angle ψ thus introduced coincides with the angle introduced earlier, which describes the amplitude of oscillations of the principal axes of the tensor. It is obvious that with excitation of oscillations, the mean value of the projection of the antiferromagnetic vector on the equilibrium direction

A. S. Borovik-Romanov et al.

decreases:

$$l_y = l_{0y}(1 - \psi^2/2). \tag{7}$$

Change of the mean value of l_y leads to a change of the phase difference $\Delta\gamma = \gamma_0\psi^2$ (to terms of the second order in ψ), where $\gamma_0 = \alpha l_{0y}^2$. By use of formula (6), we obtain the following expression for the change produced by AFMR in the intensity of the light that passes through the system:

$$\Delta I = 2E_0^2\gamma_0\psi^2 \sin 2\chi \sin 2(\chi - \varphi) \sin \Gamma. \tag{8}$$

The law of change of ψ^2 in time is prescribed by the law of change of the microwave power fed to the specimen. In our experiment, as was indicated above, the microwave power for excitation of resonance is supplied in the meander mode with frequency Π (~ 1000 Hz). On representing the meander in the form $1 - \cos\Pi t$, we get $\psi^2 = \frac{1}{2}\psi'^2(1 + \cos\Pi t)$. In formula (8) we replace ψ^2 by $\frac{1}{2}\psi'^2(1 + \cos\Pi t)$ and separate out from it the part proportional to $\cos\Pi t$:

$$I_\sim = E_0^2\gamma_0\psi'^2 \sin 2\chi \sin 2(\chi - \varphi) \sin \Gamma \cos \Pi t. \tag{9}$$

In our case $\chi = 45°$, $\varphi = 90°$, and

$$I_\sim = -E_0^2\gamma\psi'^2 \sin \Gamma \cos \Pi t. \tag{10}$$

Meanwhile the intensity of the constant light varies as

$$I_= = E_0^2 \sin^2(\Gamma/2). \tag{11}$$

As is seen from Fig. 5, the experimental data are well described by the formulas (10) and (11) obtained above; this attests to the correctness of the picture presented. It must be emphasized that the value of the alternating signal is proportional to the square of the deflection of the vector l, just as in the case considered in the Introduction.

The experimental data enable us to estimate the value of the angle ψ' at the moment of resonance, since the ratio of the maximum amplitude of the alternating signal to the constant is $\Delta\gamma = \gamma_0\psi'^2$. The estimate gives the value

$$\psi' \approx 2\cdot 10^{-2} \text{ rad}, \quad \psi'^2 \approx 4\cdot 10^{-4}.$$

As is clear from the preceding, the value of ψ'^2 describes the change of the component of the vector l perpendicular to the field. Consequently, measurement of the value of the alternating light signal is one of the methods of measuring the values of Δl_y by means of birefringence. In like manner, one measures the change of the component of magnetization directed along the field in paramagnetic resonance, on the basis of the Faraday effect.[12] The value of ψ'^2 is proportional to the total number of all the spin waves excited in the system at resonance (including even the spin waves with $k \neq 0$).

The angle of deflection of the spins during uniform precession at resonance can also be calculated from the value of the amplitude h of the microwave field in which the specimen is located:

$$\psi' = \Delta m/m_0 = \varkappa h/m_0;$$

here Δm is the change of moment at resonance, $\varkappa$ is the high-frequency susceptibility, and h is the amplitude of the microwave field. On using the expression for the low-frequency susceptibility from reference[13] and the formula for the low-frequency AFMR branch, we finally get for ψ': $\psi' = 2h/\Delta H$, where ΔH is the width of the resonance line. In our case, $h \sim 0.01$ Oe and $\Delta H \simeq 30$ Oe for $CoCO_3$. Hence $\psi' \approx 7\cdot 10^{-4}$ rad.

Comparison of the estimates made shows a difference by a factor 1000 in the value of ψ'^2. If we suppose, as was indicated above, that the value of ψ'^2 describes the number of spin waves excited in the system, then the estimates obtained imply that under AFMR conditions, the number of spin waves with $k \neq 0$ excited in the spin system exceeds by a factor 1000 the number of spin waves with $k = 0$, corresponding to uniform precession. In other words, we can say that an accumulation of energy occurs in the spin system at the moment of resonance. Thereafter, the energy is transferred to the lattice and to the helium bath. From the form of the light pulse, one can make an estimate of the relaxation time of the whole combined process: it is less than 2 μsec.

We shall make some remarks regarding the phenomenon that we have not yet succeeded in detecting, namely modulation of the light by the microwave frequency. From the estimates presented above it follows that the effect caused by this phenomenon is at least 1000 times weaker than the one we observed. Hence it is clear that detection of it in polarization experiments is experimentally complicated because of the necessity to separate it from a background of stronger signal. The only convincing experiment will be observation of the line at a shifted frequency by means of a spectral instrument—a Fabry–Perot interferometer.

It should also be mentioned that the effect of light modulation due to birefringence may be in principle unobservable in substances in which the determining reason for the birefringence is magnetostriction, for the following reason. Oscillation of the vector l with the microwave frequency should be accompanied by deformation of the lattice. But as was shown in[14,15], the presence of a magnetostrictive gap in the AFMR spectrum of hematite (α-Fe_2O_3) indicates that the deformation of the lattice cannot occur in such short times and change with the microwave frequency. Because in $CoCO_3$ the magnetostriction may be large, for the reason indicated above the modulation of light by means of birefringence may not occur in it. But we note that a magnetostrictive gap in the AFMR spectrum of $CoCO_3$ has so far not been detected.

All the considerations presented above are unrelated to modulation of light by means of the Faraday effect. But in $CoCO_3$ at resonance, the vector m precesses along a strongly elliptic orbit, with semiaxis ratio $(\Delta m_y/\Delta m_z)^2 \sim 30$, so that the alternating component of

1007   Sov. Phys. JETP, Vol. 43, No. 5, May 1976

A. S. Borovik-Romanov et al.   1007

the vector m along the direction of the light in our experiment is very small, and apparently the resulting Faraday modulation of the light should also be small.

In closing, the authors thank P. L. Kapitsa for his interest in the research, and A. A. Solomko, Yu. A. Gaĭdaĭ, V. N. Venitskii, and V. V. Eremenko for discussion of the results.

[1)]The external magnetic field H is directed along a binary axis (the x axis); the z axis coincides with C_3.

[2)]In [4] an error was made in the coefficients of the coefficients of the nondiagonal components of the tensor ε. This error has been corrected in the present paper.

[3)]The crystals $CoCO_3$ and $MnCO_3$ are transparent for wavelength λ = 6328 Å.

[4)]The authors are very grateful to N. Yu. Ikornikova, V. M. Egorov, and V. R. Gakel' for the specimens they provided.

[1]J. F. Dillon, Jr., H. Kanimura, and J. P. Remsika, J. Appl. Phys. 34, 1240 (1963); J. F. Dillon, Jr., L. R. Walker, and J. P. Remeika, Proc. Intl. Conf. on Magnetism, Nottingham, 1964, p. 369.

[2]J. T. Hanlon and J. F. Dillon, Jr., J. Appl. Phys. 36, 1269 (1965).

[3]L. K. Anderson, J. Appl. Phys. 34, 1230 (1963).

[4]A. S. Borovik-Romanov, N. M. Kreines, A. A. Pankov, and M. A. Talalaev, Zh. Eksp. Teor. Fiz. 66, 782 (1974) [Sov.

Phys. JETP 39, 378 (1974)].

[5]R. A. Alikhanov, Zh. Eksp. Teor. Fiz. 37, 1145 (1959) [Sov. Phys. JETP 10, 814 (1960)].

[6]A. S. Borovik-Romanov and V. I. Ozhogin, Zh. Eksp. Teor. Fiz. 39, 27 (1960) [Sov. Phys. JETP 12, 18 (1961)].

[7]R. A. Alikhanov, Zh. Eksp. Teor. Fiz. 39, 1481 (1960) [Sov. Phys. JETP 12, 1029 (1961)].

[8]J. Kaczér and N. M. Kreines, Zh. Eksp. Teor. Fiz. 41, 1691 (1961) [Sov. Phys. JETP 14, 1202 (1962)].

[9]A. S. Borovik-Romanov, Antiferromagnetiki s anizotropieĭ tipa legkaya ploskost' (Antiferromagnets with Anisotropy of the Easy-Plane Type). Problemy magnetizma (Problems of Magnetism), Nauka, 1972.

[10]N. F. Kharchenko, V. V. Eremenko, and O. P. Tutakina, Zh. Eksp. Teor. Fiz. 64, 1326 (1973) [Sov. Phys. JETP 37, 672 (1973)].

[11]M. Born and E. Wolf, Principles of Optics, Oxford, Pergamon, 1970 (Russ. transl., Nauka, 1972).

[12]G. F. Imbusch and S. Geschwind, Phys. Lett. 18, 109 (1965).

[13]E. A. Turov, Fizicheskie svoĭstva magnetouporyadochennykh kristallov (Physical Properties of Magnetically Ordered Crystals), Izd. Akad. Nauk SSSR, 1963 (translation, Academic Press, 1965).

[14]E. G. Rudashevskiĭ and T. A. Shal'nikova, Zh. Eksp. Teor. Fiz. 47, 886 (1964) [Sov. Phys. JETP 20, 593 (1965)].

[15]A. S. Borovik-Romanov and E. G. Rudashevskiĭ, Zh. Eksp. Teor. Fiz. 47, 2095 (1964) [Sov. Phys. JETP 20, 1407 (1965)].

Translated by W. F. Brown, Jr.

Localization of electrons in disordered systems. The mobility edge and theory of critical phenomena

M. V. Sadovskiĭ

Metal Physics Institute, Urals Scientific Center, Academy of Sciences of the USSR
(Submitted December 3, 1975)
Zh. Eksp. Teor. Fiz. 70, 1936–1940 (May 1976)

It is shown that the most probable spatial behavior of the one-electron Green function in the region of localized states near the mobility edge in the Anderson model coincides with the spatial behavior of the correlation function in the critical region of a second-order phase transition with a zero-component order parameter.

PACS numbers: 71.50.+t

Ideas about the localization of electrons in a random field lie at the basis of the modern theory of disordered systems.[1] The most highly developed scheme for treating the problem of localization is the well-known Anderson model[2–4] describing an electron propagating in a regular lattice with random energy levels at the different sites. Most of the papers on the Anderson model are devoted to proving the localization of electron states when the ratio of the parameter W describing the spread of levels to the amplitude V of an electron transition from site to site is sufficiently large, to determining the critical ratio W_c/V, and also to determining the mobility edges E_c, i.e., the critical electron energies separating the regions of localized and delocalized states in the band.[1,2,4] It is of great interest to study the character of the electron states near the mobility edge, since the corresponding characteris-

tics essentially determine the kinetics and other electronic properties of disordered systems.[5] Attempts in this direction have been undertaken in papers by Anderson, Edwards, and Freed.[3,6,7]

There exist a number of obvious analogies between the problem of the localization of an electron near the mobility edge and the problem of describing the critical phenomena near a second-order phase-transition point. For example, as the electron energy approaches the mobility edge in the region of localized states the localization length of the electron wavefunction diverges, just as the correlation length of fluctuations at a phase-transition point diverges. This prompts the thought that the spatial behavior of electron states near the mobility edge can be described by the (scaling) dependences that are characteristic for the phase-transition problem,