$$\omega\tau < n_F/kr_H \sim p_F/k \qquad (19)$$

is satisfied, the oscillations of the conductivity tensor are determined by the value of $\tilde{\sigma}_{\alpha\beta}^{(0)}$. As is evident from the expression (13) for $\tilde{\sigma}_{\alpha\beta}^{(0)}$, the oscillations of the conductivity, and consequently also the oscillations of the impedance, decrease with increase of frequency.

It should be mentioned that the collision integral (3), as is well known, does not conserve the number of particles. In order to insure fulfillment of the law of conservation of particles, it is necessary to introduce a nonequilibrium correction to the chemical potential. This will lead to the occurrence of additional ("diffusion") terms in the conductivity tensor. An expression for the conductivity tensor in a quantizing magnetic field, with allowance for diffusion terms in the case of an isotropic and quadratic dispersion law, was obtained in[3,4]. But under anomalous-skin-effect conditions these diffusion terms are negligibly small. We note also that the diffusion terms disappear if $\tau \to \infty$, and consequently formula (7) for $\tau = \infty$ gives an exact expression for the conductivity tensor of an electron gas without collisions.

In conclusion, I wish to express thanks for discussions of this article to Yu. A. Romanov, and especially to V. Ya. Demikhovskiĭ; numerous discussions with him facilitated the writing of the paper.

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Effect of the superconducting transition on the frequency and damping of transverse phonons in lead

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The phonon frequencies and damping in lead in the [111] direction are measured at 4.2, 20.4, 78 and 300°K. At 4.2°K a softening is observed for phonons with wave vectors in the $aq/2\pi = 0.35$–0.50 range, whereas strong broadening of one-phonon resonance is observed at $aq/2\pi = 0.50$–0.867. A possible mechanism of the phenomenon is discussed.

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Information on the frequencies and damping of phonons in metals at low temperatures, when the anharmonicities of the lattice are negligibly small, is of considerable interest from the point of view of the study of electron-phonon interactions (EPI). This is particularly true for superconductors at temperatures close to T_c. In this case EPI can bring about qualitative changes in the phonon characteristics of the system when the sample goes into the superconducting state and a new state of the electronic subsystem is produced.[1,2]

The purpose of the present paper was to investigate these possible changes in lead. Since the EPI in lead is very strong, we can expect the effects to be observable. The main measurements were carried out on transverse phonons in the [111] direction at temperatures 4.2 and 20.4 °K. No such investigations were performed previously for lead in the interval 5 °K < T < 80 °K,[3,4] and the data of[5] at 5 °K are not sufficiently complete.

EXPERIMENTAL PROCEDURE AND MEASUREMENT RESULTS

The measurements were performed with a three-axis neutron spectrometer by the constant momentum-transfer method (Q = const) with a fixed incident-neutron wave length $\lambda_0 = 1.611$ Å. To make the primary beam monochromatic, we used the (200) plane of a Cu single crystal with mosaic angle $\eta_m \sim 5'$. The (220) plane of single-crystal lead with $\eta_a \sim 9'$ was used as the analyzer. The intrachannel and pre-detector collimation were weakened by (α_0, $\alpha_3 \sim 1°$), and collimators with divergence $\alpha_1 \alpha_2 \sim 15'$ were placed in front of and behind the sample. The sample temperature in the cryostat was monitored with a special thermocouple and with superconducting titanium-vanadium foils with fixed superconducting transition temperatures.

A cylindrical sample (diameter ~25 mm, $l \sim 40$ mm) with axis in the [110] direction was cut from a single crystal of pure (99.9999%) lead. The mosaic angle of the sample was $\eta_s \sim 10'$, the resistivity ratio was $\rho_{300\,°K}/\rho_{7.3\,°K} \sim 2000$, the transition temperature was $T_c = 7.2\,°K$, and the width of the transition was less than 0.01 °K.

The reduction of the experimental data and the estimate of the measurement errors were carried out by the procedure proposed in[6]. In this case the average error in the determination of the frequencies was ± (1–2)%, that in the determination of the wave vector of the phonon was ± (0.2–0.5)%, and the error in the width $2\Gamma_0$ of the experimental peaks (the total width at half height) was ± (8–10)%. The spectrometer resolution function was calculated in accordance with[7,8], and the mosaic angle of the sample was taken into account in accordance with[9]. To be able to determine directly from the experimental peaks the temperature dependences of the frequencies and widths of the single-phonon resonances, all the measurements for each phonon were carried out at six values of the apparatus parameters. The measurement conditions (the resolution of the spectrometer and the choice of the direction of scanning in reciprocal space) excluded the contribution of the longitudinal phonons to the observed neutron resonances.

The measurements were performed in the $(1\bar{1}0)$ plane of the reciprocal lattice and the [111] direction at temperatures 300, 78, 20.4, and 4.2 °K.[1)] Figure 1 shows the frequencies of the transverse phonons in the [111] direction, determined from the data of these measurements, and the results of Fürrer and Halg.[5] We see that the agreement between the results at 300, 78, and 4.2 °K is quite good. More complete measurements than those of[5] at $T = 4.2\,°K$, and in particular measurements at $T = 20.4\,°K$ have revealed an interesting behavior of the frequencies and of the phonon widths. The usual increase of the frequency with decreasing temperature is observed for all the investigated phonons in the temperature interval 300–20.4 °K. On going from 20.4 to 4.2 °K, the frequencies of the phonons with $aq/2\pi = 0.35$, 0.40, 0.45, and 0.50 (a is the lattice constant and **q** is the wave vector of the phonon) decrease, whereas for the remainder of the investigated phonons $\omega(4.2\,°K) > \omega(20.4\,°K)$. Figure 2 shows a plot of the ratio $\omega(4.2\,°K)/\omega(20.4\,°K)$ against $aq/2\pi$, illustrating this result (for the sake of clarity, a smooth dashed curve was drawn through the experimental points).

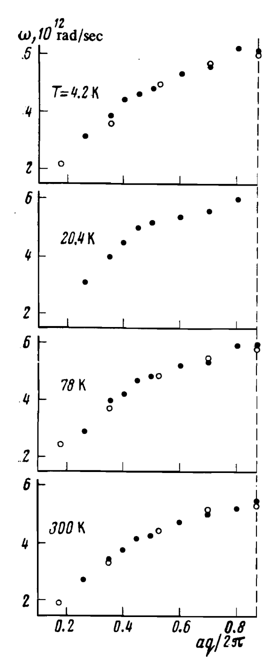

FIG. 1. Dispersion of transverse phonons in lead in the [111] direction at different temperatures: ●—our data, ○—data of [5]. The dashed line shows the band boundary.

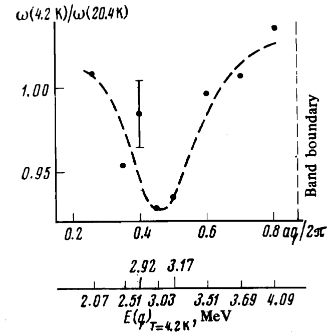

FIG. 2. Variation of $\omega(4.2\text{ K})/\omega(20.4\text{ K})$ with the wave vector of the phonons: the lower scale is that of the equivalent phonon energy $E(q)$.

The widths of the single-phonon resonances also behave differently for different phonon groups when the temperature decreases from 20.4 to 4.2 °K. For phonons with $aq/2\pi = 0.50$, 0.60, 0.70, 0.80, and 0.867, a rapid increase of the resonance width is observed with decreasing temperature, whereas the width of the neutron groups for $aq/2\pi = 0.26$, 0.35, 0.40, and 0.45 decreases with decreasing temperature in the entire investigated temperature interval. It should be noted that the quantities $2\Gamma_0(4.2\,°K)$ of the broadened phonons not only exceeded the values of $2\Gamma_0(20.4\,°K)$ but turned out to be of the order of or larger than the widths at $T = 78\,°K$ (the Debye temperature for lead is ~ 80 °K). Figure 3 shows the dependence of the ratio of the observed widths of the single-phonon resonances on the wave number q:

$$2\Gamma_0(4.2\,\text{K})/2\Gamma_0(20.4\,\text{K}) = f(aq/2\pi)$$

(just as in Fig. 2, a smooth dashed curve is drawn through the experimental points).

We note that the observed resonances are broadened as a result of the finite resolution of the apparatus. The broadening affects the line shape in a complicated manner. Therefore the reconstruction of the true line shape from the observed peak is in general a rather complicated task.[7–9] In the simplest case, when the

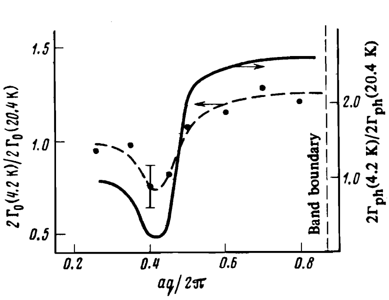

FIG. 3. Ratio of the experimental (points) and intrinsic (solid curve) phonon widths at temperatures 4.2 and 20.4 °K vs the wave vector.

observed resonance is close to Gaussian, its width Γ_0 is connected with the natural width of the phonon Γ_{ph} by the relation $\Gamma_{ph}^2 = \Gamma_0^2 - \Gamma_a^2$, where Γ_a is the apparatus broadening calculated with the aid of the resolution function. In all other cases, the procedure of extracting the natural widths is not reliable enough.[2)] We therefore deemed it advantageous to show in Fig. 3 that ratio of the experimental widths $2\Gamma_0(4.2\ °K)/2\Gamma_0(20.4\ °K)$ measured at equal apparatus parameters. The dependence of Γ_a on the temperature is connected essentially with the change of $d\omega(\mathbf{q})/d\mathbf{q}$ with temperature, which in our case is negligible. We can therefore assume, with sufficient degree of reliability, that the function $f(aq/2\pi)$ is similar to the function

$$f_1(aq/2\pi) = 2\Gamma_{ph}(4.2\ K)/2\Gamma_{ph}(20.4\ K).$$

The crude procedure of extracting the natural widths (as in the Gaussian case) yields the estimated $f_1(aq/2\pi)$ curve shown solid in Fig. 3. Naturally, the effects are more strongly pronounced on the natural widths.

To determine the mechanisms that lead to the broadening of the peaks, measurements were made at 4.2 °K, with the sample placed in a magnetic field $H > H_c$ ($H \sim 1300$ Oe). Figure 4 shows the neutron resonance for the phonon $aq/2\pi = 0.60$, obtained in measurements with and without a magnetic field. We see that application of a field $H > H_c$ leads to a noticeable narrowing of the resonance, accompanied by an increase of the peak intensity. In the case of a phonon with $aq/2\pi = 0.26$ we observed a slight broadening of the resonance and a decrease of its peak intensity in measurements with a magnetic field.

DISCUSSION OF RESULTS

The observed effects seem to be connected with the transition of the sample to the superconducting state. To treat the results we can use the analogy between the experiment on absorption of ultrasound and excitation of natural lattice vibrations in the presence of inelastic scattering of neutrons in superconductors. In the latter case the role of α_S/α_N ($\alpha_{S,N}$ are the ultrasound absorption coefficients in the superconducting and normal states) is played by the ratio of the natural phonon widths $\Gamma_{phS}/\Gamma_{phN}$.

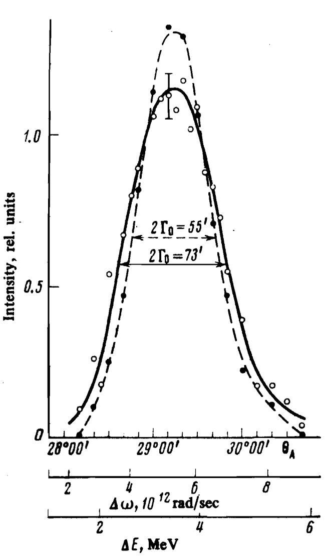

FIG. 4. Single-phonon resonance at $aq/2\pi = 0.60$ measured in a magnetic field $H > H_c$ (●) and without a field (○), and shown as a function of the analyzer angle θ_A, the energies transfered to the electron ΔE, and the transfer frequency $\Delta\omega$.

A theory of the absorption of longitudinal ultrasonic oscillations of high frequency was developed in the BCS approximation for the isotropy model by Privorotskiĭ[11] and by Bobetic.[12] Bobetic[12] calculated the behavior of α_S/α_N in the temperature interval from 0 °K to T_c as a function of the ultrasound frequency. At $\hbar\omega < 2\Delta(T)$ and $T = 0$ there is no absorption, and with increasing temperature, $0 < T < T_c$, finite absorption appears on account of scattering by thermally excited quasiparticles. When $\hbar\omega = 2\Delta(T)$, the absorption increases jumpwise, this being due to the possibility of production of excitation pairs at $\hbar\omega \geq 2\Delta(t)$. It was noted in[12] that this jump can be smeared out by the anisotropy of the Fermi surface.

For transverse modes, analogous effects due to Umklapp processes should also be observed. Indeed, as shown above (Fig. 3), in the case of phonons with $\hbar\omega > 2\Delta(T)$ the width is larger than in the normal state ($T = 20.4$ °K), and for phonons with $\hbar\omega < 2\Delta(T)$ it becomes somewhat smaller. The character of the change of $f_1(aq/2\pi)$ is similar to the dependence of α_S/α_N on ω given in[12]. It can thus be assumed that the observed effect is connected with the transition of the sample to the superconducting state and to the appearance of the gap, as a result of which the intensity of states of the quasiparticles near the Fermi surface increases. This is confirmed also by measurements performed at 4.2 °K when the sample is placed in a magnetic field $H > H_c$ (Fig. 4).

It should be noted that phonons with $aq/2\pi = 0.40$ and 0.45, which have an energy $\hbar\omega$ higher than the maximum gap energy cited in the literature $2\Delta_{max}(4.2\ K) = 4.5k_BT_c$, remain narrow at $T = 4.2$ °K, and the first broadened phonon has an energy $\hbar\omega = 1.14 \cdot 2\Delta_{max}(4.2\ K)$.[10] This is due either to errors in the determination of the exact value of the gap by usual methods (ultrasound, heat capacity, tunneling, etc.) or else we are dealing with a contraction of the broadening limits, the possibility of which was indicated by Schuster[13] and which is connected with the openness of the Fermi surface.

The abrupt change of the phonon width on going to the superconducting state should be accompanied by a corresponding change of the frequencies which are connected with the width by the Kramers–Kronig relations for the Green's functions. Indeed, besides the usual increase of the frequencies when the temperature is lowered from 20.4 to 4.2 °K, we have observed a softening of the frequencies of the group of phonons with wave vectors $aq/2\pi = 0.35$, 0.40, 0.45, and 0.50. Attention is called to the magnitude of the softening, which averages approximately 8–10%. It is usually assumed that in the superconducting transition we have $\langle\Delta\omega\rangle/\langle\omega\rangle \leq 10^{-3}$.[14] This estimate for the frequencies averaged over the spectrum, however, may turn out to be incorrect for individual phonons.[1] (We note that lead is a metal with a strong EPI, having therefore, for example, a large Kohn anomalies initially assumed not to be observable in experiment.)

A softening in this region is apparent also from the data of Fürrer and Hälg[5] (single point: $aq/2\pi = 0.347$, $\omega(4.2\text{ K}) = 3.61 \cdot 10^{12}$ rad/sec, $\omega(78\text{ K}) = 3.71 \cdot 10^{12}$ rad/sec). The absence of measurements at temperatures directly preceding the superconducting transition, however, made it impossible for the authors of[5] to observe the softening effect described here. Nor can this effect be revealed by such integral characteristics of the frequency spectrum as the specific heat,[15] inasmuch as besides the softening phonons the spectrum also contains phonons whose frequency increases below T_c, and in addition, the specific heat is not very sensitive to details of the phonon spectrum.

It is of interest to investigate the anisotropy of the described effects. For a detailed analysis of the results of the present paper we need model calculations that take into account all the contributions made to the damping of the transverse phonons.

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1)Some of the results obtained by us are reported in [10].

2)In particular, these are apparently the causes of the very large (by almost one order of magnitude) discrepancies between the phonon natural width given in[4,5], even at $T = 300\,^\circ$K, when the contribution $2\Gamma_{ph}$ to the total width of the neutron resonance is quite large. (In addition, different procedures were used in these papers to calculate the resolution.) Our estimates of $2\Gamma_{ph}$ at $T = 300\,^\circ$K give values closer to the results of [4].

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Distinctive features of the phonon spectrum in solid helium

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The effect of the defecton-phonon interaction on the phonon spectrum in crystalline helium is theoretically considered. It is shown that a substantial renormalization of the phonon group velocities occurs in the neighborhood of the threshold frequencies corresponding to the decay of a phonon into a defecton pair. The phonon spectrum breaks off at the threshold points and disappears, when the interaction is sufficiently strong, in the region of frequencies corresponding to the continuum of the free defecton pairs. The coupling between the phonons and the bound defecton states leads to an additional modification of the spectrum. It is also found that the defecton-phonon interaction leads to a nonmonotonic dependence of the intensity of the one-phonon mode in the dynamic form factor on the transferred momentum.

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1. INTRODUCTION

The weakness of the interaction, coupled with the smallness of the atomic mass, is the source of quite a number of properties that qualitatively distinguish solid He^3 and He^4 from normal classical crystals. To these properties pertains, in particular, the fact that the energy of formation of a point defect in crystalline helium turns out to be appreciably lower than the cutoff energy (ω_D) of the phonon excitations and, as was first noted by Andreev and Lifshitz,[1] can, in principle, become zero.

The object of the present paper is to consider the possible distinctive features of the phonon spectrum of solid helium that are caused by the interaction with the defecton excitations under conditions when the minimum energy of creation of the latter is finite. We shall show

