Quantum oscillations of surface impedance

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Quantum oscillations of the surface impedance under anomalous-skin-effect conditions, in the high-frequency limit—when the frequency ω of the electromagnetic field is much greater than the collision frequency τ^{-1}—and in a magnetic field parallel to the surface of the metal, were studied theoretically by Azbel' {Zh. Eksp. Teor. Fiz. **34**, 969 and 1158 (1958) [Sov. Phys. JETP **7**, 669 and 801 (1958)}. But the expression for the conductivity obtained by him is incorrect, because in it, no allowance was made for the contribution of the "instantaneous-equilibrium" density matrix to the current; this contribution does not vanish in the quantum case. As is shown in the present paper, the amplitude of the oscillations of conductivity and of surface impedance is smaller by a factor $\omega\tau$ (at not too large $\omega\tau$) than that obtained by Azbel'.

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Quantum oscillations of the surface impedance in the high-frequency limit $\omega\tau \gg 1$ have been investigated theoretically in a paper of Azbel'[1] under anomalous-skin-effect conditions, when the following inequalities are satisfied:

$$\delta/v_F\tau \ll 1, \quad \omega\delta/v_F \ll 1, \quad \delta/r_H \ll 1, \tag{1}$$

here ω is the frequency of the electromagnetic field, v_F is the Fermi velocity, τ is the relaxation time, δ is the skin depth, and r_H is the Larmor radius. There it was shown that in a magnetic field that is parallel (or almost parallel) to the surface of the specimen, the chief contribution to the oscillations is made by electrons that do not collide with the surface, and consequently the oscillations of the impedance are actually determined by the oscillations of the conductivity tensor in an infinite medium. To calculate the conductivity, use was made in[1] of an equation for the density matrix that contains a collision integral describing relaxation to an "instantaneous equilibrium" state. But in the calculation of the current, no allowance was made for the contribution of the instantaneous-equilibrium part of the density matrix; this does not vanish in the quantum case, because of the diamagnetism of the electron gas. This is incorrect. Allowance for the contribution of the instantaneous-equilibrium density matrix to the current, as will be shown below, decreases the amplitude of the quantum oscillations by a factor $\omega\tau$ as compared with the result of[1].

We shall obtain an expression for the conductivity tensor in an infinite medium. As in[1], we shall start from the equation for the density matrix

$$\partial\hat{f}/\partial t + i[\hat{H}, \hat{f}] + I(\hat{f}) = 0, \tag{2}$$

where $\hat{H} = \varepsilon(\hat{\mathbf{p}} - e\mathbf{A}/c)$ is the Hamiltonian of the electron, $\hat{\mathbf{p}} = -i\partial/\partial\mathbf{r} - e\mathbf{A}_0/c$ is the kinematic momentum in a constant magnetic field with vector potential $\mathbf{A}_0(-Hy, 0, 0)$, and A is the vector potential of the electromagnetic field, which will be supposed to vary with the coordinates and the time according to the law $e^{-i\omega t + i\mathbf{kr}}$. The collision integral $I(\hat{f})$, as in[1], we shall write in the form

$$I(\hat{f}) = -[\hat{f} - \hat{f}_0(\hat{H})]/\tau = \hat{f}'/\tau, \tag{3}$$

where $\hat{f}_0$ is the equilibrium density matrix. The current density is determined in the usual manner:

$$j^\alpha = \mathrm{Sp}\{(\hat{f}' + \hat{f}_0(\hat{H}))\hat{j}^\alpha\}. \tag{4}$$

The current-density operator $\hat{j}^\alpha(\mathbf{R})$ in the quasiclassical approximation has the form

$$\hat{j}^\alpha(\mathbf{R}) = e\hat{v}^\alpha\delta(\mathbf{R} - \hat{\mathbf{r}}) - c^{-1}e^2A_\beta(\hat{m}^{-1})_{\alpha\beta}\delta(\mathbf{R} - \hat{\mathbf{r}}), \tag{5}$$

where $v_\alpha = \partial\varepsilon/\partial\hat{p}_\alpha$ is the velocity operator, $(\hat{m}^{-1})_{\alpha\beta} = \partial^2\varepsilon/\partial\hat{p}_\alpha\partial\hat{p}_\beta$ is the inverse-effective-mass operator, e is the charge of the electron, and c is the velocity of light. In deriving (5) we have considered that in the quasiclassical limit any Hermitan operator is determined by its classical analog, and therefore the order of the operators in (5) can be arbitrary.

We expand the instantaneous-equilibrium density matrix $\hat{f}_0(\hat{H})$ through terms linear in A:

$$\hat{f}_0(\hat{H}) = \hat{f}_0(\hat{H}_0) + \hat{f}_0', \tag{6}$$

where $\hat{H}_0 = \varepsilon(\hat{\mathbf{p}})$. We substitute the expression for $\hat{f}_0'$ into (2). Solving equation (2) in the linear approximation, we find $\hat{f}'$, and we then calculate the current density j^α by formula (4). We omit these simple calculations and present only the final expression for the conductivity tensor $\sigma_{\alpha\beta}(\omega, \mathbf{k})$:

$$j^\alpha = \sigma_{\alpha\beta}(\omega, \mathbf{k})A_\beta,$$

$$\sigma_{\alpha\beta}(\omega, \mathbf{k}) = -\frac{e^2}{c}\sum_{\lambda\lambda'}\left\{\langle\lambda|\hat{v}^\alpha(-\mathbf{k})|\lambda'\rangle\langle\lambda'|\hat{v}^\beta(\mathbf{k})|\lambda\rangle\frac{\omega}{\varepsilon_\lambda - \varepsilon_{\lambda'} - \omega - i\tau^{-1}}\right.$$

$$\left.\times\frac{f_\lambda - f_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}} + \langle\lambda|\hat{v}^\alpha(-\mathbf{k})|\lambda'\rangle\langle\lambda'|\hat{v}^\beta(\mathbf{k})|\lambda\rangle\frac{f_\lambda - f_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}} + \langle\lambda|(\hat{m}^{-1})_{\alpha\beta}|\lambda\rangle f_\lambda\right\}, \tag{7}$$

where $\lambda = n, p_x, p_z$ is a complete set of quantum numbers of the electron (we neglect spin) in a constant magnetic field, $\langle\lambda|\hat{v}^\alpha(\mathbf{k})|\lambda'\rangle$ are the matrix elements of the operator $\hat{v}^\alpha e^{i\mathbf{k}\cdot\mathbf{r}}$ with respect to the wave functions of the electron in a magnetic field, $\varepsilon_\lambda = \varepsilon_n(p_z)$ are the eigenvalues of the energy, and $f_\lambda = f(\varepsilon_\lambda)$ is the equilibrium distribution function.

In the quasiclassical approximation, which is correct for metals, since the number of filled Landau levels $n_F \gg 1$, while $k \propto \delta^{-1} \ll p_F$ (p_F is the Fermi momentum),

the matrix elements of the operators are determined by the Fourier time components of the corresponding classical quantities. We shall suppose that $\mathbf{k}=\mathbf{k}(0, k_y, 0)$; that is, $\mathbf{k}\perp\mathbf{H}$. In this case we have

$$\langle n, p_x, p_z|\hat{v}^\alpha e^{i\mathbf{k}\hat{r}}|n+l, p_x', p_z'\rangle = v_l^\alpha(\mathbf{k}, \varepsilon_n, p_z)\,\delta_{p_z p_z'}\delta_{p_x p_x'}$$

$$= \delta_{p_z p_z'}\delta_{p_x p_x'}\frac{1}{T}\int_0^T dt\, v^\alpha(t)\exp\left\{i\mathbf{k}\int_0^t \mathbf{v}(t_1)\,dt_1 + il\Omega t\right\}, \tag{8}$$

$$\langle\lambda|(\hat{m}^{-1})_{\alpha\beta}|\lambda\rangle = [(\hat{m}^{-1})_{\alpha\beta}]_0 = \frac{1}{T}\int_0^T dt\,(m^{-1})_{\alpha\beta},$$

where $T=2\pi/\Omega$ is the period of the classical motion; Ω is the cyclotron frequency.

The matrix elements $v_l^\alpha(\mathbf{k})$ for $kr_H \gg 1$ are calculated by the method of stationary phase. As is evident from (8), they differ significantly from zero if $l \lesssim kr_H \ll n_F$, since $k \ll p_F$. Consequently an energy difference of the form $\varepsilon_{n+l}(p_z)-\varepsilon_n(p_z)$ in (7) may be replaced by $l\Omega(p_z)$. When this is taken into account, the expression (7) for the conductivity tensor can be transformed to the form

$$\sigma_{\alpha\beta}(\omega,\mathbf{k}) = -\frac{e^2}{c}\left|\frac{eH}{c}\right|\frac{1}{2\pi^2}\sum_{n,l}\int dp_z\left\{v_l^\alpha(-\mathbf{k})v_{-l}^\beta(\mathbf{k})\,\frac{\omega}{l\Omega-\omega-i\tau^{-1}}\right. \tag{9}$$

$$\left.\times\frac{f(\varepsilon_n+l\Omega)-f(\varepsilon_n)}{l\Omega} + v_l^\alpha(-\mathbf{k})v_{-l}^\beta(\mathbf{k})\,\frac{f(\varepsilon_n+l\Omega)-f(\varepsilon_n)}{l\Omega} + [(\hat{m}^{-1})_{\alpha\beta}]_0 f(\varepsilon_n)\right\}.$$

In formula (9) the summation extends over all l. The terms with $l=0$ are understood in the sense of the limit as $l\to 0$.

In the expression for the conductivity obtained in[1], the last two terms in wavy brackets in formula (9) are missing; these give the contribution of the instantaneous-equilibrium density matrix $\hat{f}_0(H)$ to the current. Their sum vanishes on approach to the classical limit. It is incorrect, however, to discard these terms in the calculation of the oscillations of the conductivity. In fact, in[1], where only the first term in (9) was considered, it was shown that the principal contribution to the oscillating part is made by terms with $l=0$. Consequently, according to[1] the oscillations of $\sigma_{\alpha\beta}$ are determined by the oscillations of the quantity $\sigma'_{\alpha\beta}$:

$$\sigma_{\alpha\beta}' = \frac{e^2}{c}\frac{\omega}{\omega+i\tau^{-1}}\left|\frac{eH}{c}\right|\frac{1}{2\pi^2}\sum_n\int dp_z v_0^\alpha(-\mathbf{k},\varepsilon_n)v_0^\beta(\mathbf{k},\varepsilon_n)\frac{\partial f}{\partial\varepsilon}. \tag{10}$$

But if we take account also of the terms with $l=0$ in the second term of formula (9), then instead of (10) we get

$$\sigma_{\alpha\beta}^{(0)}(\omega,\mathbf{k}) = \frac{e^2}{c}\left(\frac{\omega}{\omega+i\tau^{-1}}-1\right)\left|\frac{eH}{c}\right|\frac{1}{2\pi^2}\sum_n\int dp_z v_0^\alpha(-\mathbf{k},\varepsilon_n)v_0^\beta(\mathbf{k},\varepsilon_n)\frac{\partial f}{\partial\varepsilon}. \tag{11}$$

When $\omega\tau\gg 1$, the quantity $\sigma_{\alpha\beta}^{(0)}$ is smaller by a factor $\omega\tau$ than $\sigma'_{\alpha\beta}$. If $\tau=\infty$, then the terms with $l=0$ in the first and second terms of formula (9) almost completely cancel each other. But at not too large values of $\omega\tau$, such as are ordinarily attained experimentally, the quantity $\sigma_{\alpha\beta}^{(0)}$ makes the principal contribution to the oscillations of the conductivity. In order to prove this, it is necessary to estimate the order of magnitude of $\tilde{\sigma}_{\alpha\beta}^{(0)}$, the oscillating part of $\sigma_{\alpha\beta}^{(0)}$, and to compare it with the contribution to the oscillations from the neglected terms. In the classical limit for $\omega\ll\Omega$ we have[2]

$$\sigma_{\alpha\beta}^{cl} \sim a_{\alpha\beta}\frac{e^2N}{mc}\frac{\omega}{\omega+i\tau^{-1}}\frac{1}{kr_H}, \tag{12}$$

where N is the electron concentration, the coefficients $a_{\alpha\beta}\sim 1$, and m is the effective mass.

On calculating $\tilde{\sigma}_{\alpha\beta}^{(0)}$ by means of Poisson's formula, we get

$$\tilde{\sigma}_{\alpha\beta}^{(0)} \sim a_{\alpha\beta}\frac{e^2N}{mc}\frac{\omega}{\omega+i\tau^{-1}}\frac{i}{\omega\tau}\frac{1}{kr_H}\frac{1}{n_F^{1/3}}. \tag{13}$$

We shall now estimate the contribution to the oscillations from the terms with $l\neq 0$ in the second term of formula (9); this we shall denote by $\tilde{\sigma}_{\alpha\beta}^{(1)}$. We shall again use Poisson's formula and shall go over to integration over ε and p_z. By performing the integration over p_z by the method of stationary phase, one can put $\tilde{\sigma}_{\alpha\beta}^{(1)}$ into the form

$$\tilde{\sigma}_{\alpha\beta}^{(1)} = \frac{e^2}{c}\frac{1}{\pi^2}\sum_{s=1}^{\infty}\sum_m\sum_{l\neq 0}\int d\varepsilon$$

$$\times f(\varepsilon)\left\{\mathrm{Re}\exp\left[2\pi i n(\varepsilon)s \pm i\frac{\pi}{4}\right]\frac{F_{\alpha\beta}^l(\varepsilon,s)-F_{\alpha\beta}^l(\varepsilon-l\Omega,s)}{l\Omega}\right\}_{p_z=p_z^m}, \tag{14}$$

where

$$F_{\alpha\beta}^l(\varepsilon,s) = \left(s\left|\frac{d^2n}{dp_z^2}\right|\right)^{-1/2} m^*(\varepsilon,p_z)v_l^\alpha(-\mathbf{k},\varepsilon,p_z)v_{-l}^\beta(\mathbf{k},\varepsilon,p_z),$$

$m^*(\varepsilon, p_z)$ is the cyclotron mass, the index m in (14) enumerates the extremal cross sections of the Fermi surface, and p_z^m is the momentum corresponding to the mth extremal cross section.

The matrix elements $v_l^\alpha(\mathbf{k},\varepsilon)$ are oscillatory functions of the parameter kr_H, with period $\sim 2\pi$. The other factors in $F_{\alpha\beta}^l$ vary more smoothly with energy. The important contribution to (14) comes from the terms with $l\lesssim kr_H$; therefore if the condition

$$(k/p_F)kr_H \sim (k/p_F)^2 n_F \ll 1 \tag{15}$$

is satisfied, the finite-difference relation in (14) can be replaced by a derivative, and we get

$$\tilde{\sigma}_{\alpha\beta}^{(1)} = \frac{e^2}{c}\frac{1}{\pi^2}\sum_{s=1}^{\infty}\sum_m\sum_{l\neq 0}\int d\varepsilon\, f(\varepsilon)\,\mathrm{Re}\exp\left[2\pi i n(\varepsilon)s \pm i\frac{\pi}{4}\right]\frac{\partial F_{\alpha\beta}^l}{\partial\varepsilon} \tag{16}$$

(we note that the expression (8) for the matrix element is also correct only when the inequality (15) is satisfied). The sum over l in (16) is easy to calculate, since

$$\sum_{l\neq 0} v_l^\alpha(-\mathbf{k})v_{-l}^\beta(\mathbf{k}) = (v^\alpha v^\beta)_0 - v_0^\alpha(-\mathbf{k})v_0^\beta(\mathbf{k}). \tag{17}$$

By use of (17) it is easy to obtain from (16)

$$\tilde{\sigma}_{\alpha\beta}^{(1)} \sim a_{\alpha\beta}\frac{e^2N}{mc}\frac{1}{n_F n_F^{1/3}}. \tag{18}$$

An estimate of the terms with $l\neq 0$ in the first term of formula (9) shows that they make a contribution smaller than (18). We shall now consider the third term in (9). In the case of a quadratic dispersion law, it obviously is independent of the magnetic field. But if the dispersion law differs significantly from a quadratic, this term undergoes oscillations whose amplitude, as is easy to demonstrate, coincide in order of magnitude with $\tilde{\sigma}_{\alpha\beta}^{(1)}$. On comparing the values of $\tilde{\sigma}_{\alpha\beta}^{(0)}$ and $\tilde{\sigma}_{\alpha\beta}^{(1)}$, we find that when the condition

$$\omega\tau < n_F/kr_H \sim p_F/k \qquad (19)$$

is satisfied, the oscillations of the conductivity tensor are determined by the value of $\tilde{\sigma}_{\alpha\beta}^{(0)}$. As is evident from the expression (13) for $\tilde{\sigma}_{\alpha\beta}^{(0)}$, the oscillations of the conductivity, and consequently also the oscillations of the impedance, decrease with increase of frequency.

It should be mentioned that the collision integral (3), as is well known, does not conserve the number of particles. In order to insure fulfillment of the law of conservation of particles, it is necessary to introduce a nonequilibrium correction to the chemical potential. This will lead to the occurrence of additional ("diffusion") terms in the conductivity tensor. An expression for the conductivity tensor in a quantizing magnetic field, with allowance for diffusion terms in the case of an isotropic and quadratic dispersion law, was obtained in[3,4]. But under anomalous-skin-effect conditions these diffusion terms are negligibly small. We note also that the diffusion terms disappear if $\tau \to \infty$, and consequently formula (7) for $\tau = \infty$ gives an exact expression for the conductivity tensor of an electron gas without collisions.

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Effect of the superconducting transition on the frequency and damping of transverse phonons in lead

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The phonon frequencies and damping in lead in the [111] direction are measured at 4.2, 20.4, 78 and 300°K. At 4.2°K a softening is observed for phonons with wave vectors in the $aq/2\pi = 0.35$–0.50 range, whereas strong broadening of one-phonon resonance is observed at $aq/2\pi = 0.50$–0.867. A possible mechanism of the phenomenon is discussed.

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Information on the frequencies and damping of phonons in metals at low temperatures, when the anharmonicities of the lattice are negligibly small, is of considerable interest from the point of view of the study of electron-phonon interactions (EPI). This is particularly true for superconductors at temperatures close to T_c. In this case EPI can bring about qualitative changes in the phonon characteristics of the system when the sample goes into the superconducting state and a new state of the electronic subsystem is produced.[1,2]

The purpose of the present paper was to investigate these possible changes in lead. Since the EPI in lead is very strong, we can expect the effects to be observable. The main measurements were carried out on transverse phonons in the [111] direction at temperatures 4.2 and 20.4 °K. No such investigations were performed previously for lead in the interval 5 °K < T < 80 °K,[3,4] and the data of[5] at 5 °K are not sufficiently complete.

EXPERIMENTAL PROCEDURE AND MEASUREMENT RESULTS

The measurements were performed with a three-axis neutron spectrometer by the constant momentum-transfer method (Q = const) with a fixed incident-neutron wave length $\lambda_0 = 1.611$ Å. To make the primary beam monochromatic, we used the (200) plane of a Cu single crystal with mosaic angle $\eta_m \sim 5'$. The (220) plane of single-crystal lead with $\eta_a \sim 9'$ was used as the analyzer. The intrachannel and pre-detector collimation were weakened by (α_0, $\alpha_3 \sim 1°$), and collimators with divergence $\alpha_1\,\alpha_2 \sim 15'$ were placed in front of and behind the sample. The sample temperature in the cryostat was monitored with a special thermocouple and with superconducting titanium-vanadium foils with fixed superconducting transition temperatures.

A cylindrical sample (diameter ~25 mm, l ~40 mm) with axis in the [110] direction was cut from a single crystal of pure (99.9999%) lead. The mosaic angle of