served.[4,5] The rapid motion of the generation region in the streamer protects the crystal against optical breakdown and makes it possible to attain radiation intensities higher by several orders of order than in ordinary semiconductor lasers. In addition, high speed motion of a generation region of small size makes it easy to obtain, using different masks on the resonator mirrors, a controlled sequence of ultrashort light pulses.

We note in conclusion that under certain operating conditions of avalanche diodes with p–n junctions, ionization impact fronts are observed in which the intensity of the electric field falls off rapidly behind the front.[13] In this case, under suitable conditions, one can also expect light generation of the type considered above.

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Inelastic scattering of gamma quanta by hydrogen-like atoms

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The differential and total cross sections for inelastic scattering of γ quanta by hydrogen-like atoms that involves electron transitions from the $1s$ state to the $2s$ or $2p$ state are analytically calculated with allowance for terms of the order of α^2Z^2 inclusive. The differential cross section formulas obtained with such an accuracy are valid for small angle scattering. The results are applicable in the photon-energy region $\omega \gg m\alpha Z$ (m is the electron mass) and overlap with the results obtained by Gorshkov, Mikhaĭlov, and Sherman {Zh. Eksp. Teor. Fiz. 66, 2020 (1974) [Sov. Phys. JETP 39, 995 (1974)]} for energies $m\alpha Z \ll \omega \ll m$.

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1. INTRODUCTION

The derivation of the nonrelativistic Coulomb Green function stimulated the theoretical study of two-photon processes involving bound electrons. Computations on elastic and inelastic scattering of photons by hydrogen atoms were carried out in the dipole approximation[1–6] and without the use of this approximation,[7,8] which allowed the consideration of the entire nonrelativistic photon-energy region $\omega \ll m$ (m is the electron mass) and the processes forbidden in the dipole approximation.[8] The cross sections for elastic and Compton scattering in the relativistic photon-energy region have been obtained in a number of papers.[9–12] In these papers the cross sections for scattering processes with the transfer to the nucleus of any momentum q, including $q \sim m$, are computed. In[9,10] numerical computations of elastic scattering from the K shell of mercury[9] and of Compton scattering from the K shell of lead[10] are carried out. Analytic calculations in the first approximation in the Coulomb field have been carried out for elastic[11] and Compton[12] scattering.

In the present paper we consider the inelastic scattering of γ quanta by hydrogen-like atoms, accompanied by atomic-electron transitions from the K- to the L-shell (Raman scattering). In the photon-energy region $\omega \sim I$ (I is the ionization energy of the atom), where the dipole approximation is valid, transitions are allowed not to the entire L shell, but only to the $2s$ state. The scattering process involving transitions to the $2p$ states is forbidden. In the relativistic energy region $\omega \sim m$, both processes occur at the same rate. Simple formulas are obtained for the differential and total cross sections for the indicated processes up to terms of the order of α^2Z^2 inclusive in the entire photon-energy region $\omega \gg \eta$, where $\eta = m\alpha Z$ is the mean K-electron momentum. The formulas for the differential cross

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sections are derived only for small angles $\theta \lesssim \eta/\omega$. The differential cross sections are maximal in this scattering-angle region, and it is precisely this region of angles that makes the dominant contribution to the total cross sections for the processes. In relativistic computations, one should carry out separate calculations for the transitions to the $2p_{1/2}$ and $2p_{3/2}$ levels, in view of the difference between the wave functions of these states.

2. THE WAVE FUNCTIONS

Let us derive the wave functions of electrons in the $1s$, $2s$, and $2p_j$ states in momentum space with allowance for terms of order $\alpha^2 Z^2$ inclusive. Since the dominant contribution to the matrix elements to be computed is made in the integration by the region of small $f \sim \eta$, let us discard the terms of the type $(\alpha Z)^2 f/m$ and write these wave functions in the following form:

$$\langle \mathbf{f}|1s\rangle = N_1 \left\{1+\frac{\tilde{f}}{2m}+\sigma\left(\ln\frac{\varepsilon}{2\eta}+\int_\varepsilon^\infty \frac{d\lambda}{\lambda}\right)\right\}\left(-\frac{\partial}{\partial\eta_1}\right)\langle \mathbf{f}|V_{i(\eta_1+\lambda)}|0\rangle u_0, \qquad (1)$$

$$N_1=\eta^{3/2}\pi^{-1/2}(1+{}^5/_8\alpha^2Z^2),\ \sigma=\alpha^2Z^2/2,\ \eta=m\alpha Z,$$

while η_1 is a parameter on which the derivative acts, and which should, after the differentiation, be set equal to η. The quantity $\tilde{f} = \alpha \mathbf{f}$, where α is the Dirac matrix; u_0 is the normalized—by the condition $\bar{u}_0 u_0 = 1$—bispinor for the stationary electron. The formula (1) was derived earlier by Gorshkov, Mikhaĭlov, and Polikanov.[13]

In the $2s$ state

$$\langle \mathbf{f}|2s\rangle = N_2 \left\{\left(1+\eta_2\frac{\partial}{\partial\eta_2}\right)\left[1+\frac{\tilde{f}}{2m}+\sigma\left(\ln\frac{\varepsilon}{\eta}+\int_\varepsilon^\infty\frac{d\lambda}{\lambda}\right)\right]\right.$$
$$\left.+\frac{3}{8}\alpha^2Z^2\left(\eta_2\frac{\partial}{\partial\eta_2}\right)\right\}\left(-\frac{\partial}{\partial\eta_2}\right)\langle \mathbf{f}|V_{i(\eta_2+\lambda)}|0\rangle u_0, \qquad (2)$$
$$N_2=\frac{\eta^{3/2}}{\sqrt{8\pi}}\left(1+\frac{23}{32}\alpha^2Z^2\right),\qquad \eta_2=\frac{\eta}{2}\left(1+\frac{\alpha^2Z^2}{8}\right),$$

while in the $2p_j$ state

$$\langle \mathbf{f}+\mathbf{q}|2p_j\rangle = N_j\left(-\eta_2\frac{\partial}{\partial\eta_2}+A_j\right)\left\{1+\frac{\tilde{f}+\tilde{q}}{2m}+\sigma_j\left(\ln\frac{\varepsilon}{\eta}+\int_\varepsilon^\infty\frac{d\lambda}{\lambda}\right)\right\}$$
$$\times(\mathbf{u}_{j1M}\nabla_q)\langle \mathbf{f}+\mathbf{q}|V_{i(\eta_2+\lambda)}|0\rangle,$$
$$N_{1/2}=\frac{\eta^{3/2}}{\sqrt{6}}\left(1+\frac{97}{96}\alpha^2Z^2\right),\qquad N_{3/2}=\frac{\eta^{3/2}}{\sqrt{6}}\left(1+\frac{47}{96}\alpha^2Z^2\right), \qquad (3)$$
$$A_j=\begin{cases} {}^3/_8\alpha^2Z^2 & \text{for } j={}^1/_2 \\ 0 & \text{for } j={}^3/_2\end{cases},\qquad \sigma_j=\begin{cases}\sigma & \text{for } j={}^1/_2 \\ \sigma/2 & \text{for } j={}^3/_2\end{cases},$$
$$\eta_2=\begin{cases} {}^1/_2\eta(1+\alpha^2Z^2/8) & \text{for } j={}^1/_2 \\ {}^1/_2\eta & \text{for } j={}^3/_2\end{cases}$$

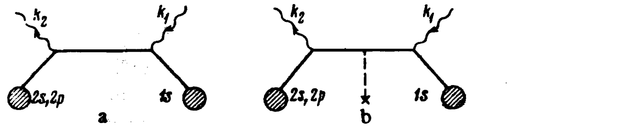
$\langle \mathbf{f}|V_{i\eta}|0\rangle$ is the Fourier transform of the Yukawa potential:

$$\langle \mathbf{f}|V_{i\eta}|0\rangle = 4\pi/(\mathbf{f}^2+\eta^2).$$

Here we use the notation[8]

$$a\mathbf{u}_{j1M} = u_{j1M}(\mathbf{a}) = \begin{pmatrix}\Omega_{j1M}(\mathbf{a})\\ 0\end{pmatrix} = a\begin{pmatrix} C^{jM}_{1,\,M-1/2,\,1/2,\,1/2}\,Y_{1,\,M-1/2}(\mathbf{v}) \\ C^{jM}_{1,\,M+1/2,\,1/2,\,-1/2}\,Y_{1,\,M+1/2}(\mathbf{v}) \\ 0 \\ 0\end{pmatrix},$$

$\mathbf{v} = \mathbf{a}/a$. When operating on the matrix elements



$\langle \mathbf{f}|V_{i(\eta+\lambda)}|0\rangle$ with any of the operators entering into the wave functions and not containing the parameter λ, we must set $\lambda = 0$ in these matrix elements. After integrating over λ we should set $\varepsilon = 0$ in the final expression.

The functions $\langle nl|\mathbf{f}\rangle$ are normalized by the condition

$$\int \langle nl|\mathbf{f}\rangle\langle \mathbf{f}|nl\rangle \frac{d^3f}{(2\pi)^3} = 1.$$

3. THE AMPLITUDES OF THE PROCESSES

Let us denote by $k_i = (\omega_i, \mathbf{k}_i)$ the four-dimensional momenta of the initial ($i = 1$) and the final ($i = 2$) photons and by $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$ the momentum imparted to the nucleus. The energy conservation law for the processes of photon scattering by a bound electron that involve the transition of the electron from the $1s$ state to the $2s$ or $2p_j$ state can be written in the form $\omega_1 + \varepsilon_1 = \omega_2 + \varepsilon_2$, where ε_1 and ε_2 are the energies of the bound states of the initial and final electrons, these energies being given in the approximation under consideration by the formulas: $\varepsilon_1 = m - \eta^2/2m$, $\varepsilon_2 = m - \eta^2/8m$.

The amplitudes of the scattering processes under study are determined by the sum of the two Feynman diagrams shown in the figure and two similar diagrams with the phonons lines interchanged. The hatched blocks in this figure indicate that the electron wave functions in the Coulomb field are evaluated with allowance for terms of order $(\alpha Z)^2$ inclusive, while the dashed line in Fig. (b) denotes a one-time interaction with the Coulomb field in the intermediate state. It can be shown that diagrams with two or more Coulomb lines in the intermediate electron state make to the amplitude a contribution that is of higher order than is required by the approximation under consideration.[14]

The contribution of the diagram a to the amplitude can be written in the form

$$I_a = 4\pi\alpha\int\langle 2l|\mathbf{k}_1+\mathbf{f}_1\rangle\hat{e}_2\frac{\hat{k}_1+\hat{f}_1+m}{(k_1+f_1)^2-m^2}\hat{e}_1\langle \mathbf{f}_1|1s\rangle\frac{d^3f_1}{(2\pi)^3}, \qquad (4)$$

where the $\mathbf{e}_i$ are the photon polarizations and $\hat{A} = \gamma_0 A_0 - \boldsymbol{\gamma}\cdot\mathbf{A}$.

The contribution to the amplitude of the diagram b can be written in the form

$$I_b = -(4\pi)^2\alpha(\alpha Z)\int\langle 2l|\mathbf{f}_2\rangle\hat{e}_2\frac{\hat{k}_2+\hat{f}_2+m}{(k_2+f_2)^2-m^2}\frac{\gamma_0}{(\mathbf{q}+\mathbf{f}_1-\mathbf{f}_2)^2}\frac{\hat{k}_1+\hat{f}_1+m}{(k_1+f_1)^2-m^2}$$
$$\times\hat{e}_1\langle \mathbf{f}_1|1s\rangle\frac{d^3f_1}{(2\pi)^3}\frac{d^3f_2}{(2\pi)^3}. \qquad (5)$$

Using the expressions (1)–(3) for the wave functions, going over from the Dirac matrices and bispinors to the Pauli matrices σ and spinors χ for the s states and to the spherical spinsors Ω_{j1M} for the p states, and integrating over $\mathbf{f}_1$ and $\mathbf{f}_2$, we obtain for the amplitudes of the scattering involving small momentum transfers, $q \sim \eta$, to the nucleus the formulas given below:

$$A = I_a + I_b + (k_2 \rightleftarrows -k_1,\ \mathbf{e}_2 \rightleftarrows \mathbf{e}_1) = 4\pi r_0 F.$$

For scattering involving the $1s \to 2s$ transition

$$F(1s \to 2s) = \frac{2^{1/2}\eta^4}{a^4}(\mathbf{e}_2\mathbf{e}_1)\chi_2^+\left\{\frac{q^2}{a^2}\left[L_1 + i\frac{\sigma[\mathbf{n}\times\mathbf{q}]}{4m}\right] + \frac{\alpha^2 Z^2}{4}L_2\right\}\chi_1, \tag{6}$$

$$L_1 = 1 - \frac{q^2}{4m^2} + \frac{\alpha^2 Z^2}{2}\left(\frac{15}{16} + \ln\frac{a^2}{2\eta^2} + \frac{q^2-\mu^2}{q\mu}\operatorname{arctg}\frac{q}{\mu}\right),$$

$$L_2 = -\frac{5}{4} + \frac{\mu^4}{a^4} - \frac{a^2}{2q\mu}\operatorname{arctg}\frac{q}{\mu},$$

$a^2 = q^2 + \mu^2$; $\mu = 3\eta/2$; $r_0 = \alpha/m$; χ_1 and χ_2 are the Pauli spinors for the initial and final electrons; $\mathbf{n} = \mathbf{n}_1 + \mathbf{n}_2$; $\mathbf{n}_i = \mathbf{k}_i/\omega_i$.

For scattering involving the $1s \to 2p_j$ transition

$$F(1s \to 2p_j) = \frac{4(6\pi)^{1/2}\eta^5}{a^6}\left\{\overset{+}{\Omega}_{j1M}(\mathbf{q})\left[(\mathbf{e}_2\mathbf{e}_1)\left(L_j + \frac{i\sigma\mathbf{s}_1}{4m\omega_1}\right) + i\frac{\sigma\mathbf{s}_2}{4m\omega_1}\right]\right.$$

$$\left. + \frac{a^2}{6m}\left[i\frac{(\mathbf{e}_2\mathbf{e}_1)}{4m}\left(\left(\overset{+}{\Omega}_{j1M}(\mathbf{n}_1)\mathbf{s}_1 + \overset{+}{\Omega}_{j1M}(\mathbf{s}_1)\mathbf{n}_1\right)\sigma\right) + \frac{\overset{+}{\Omega}_{j1M}(\mathbf{s}_3)}{\omega_1}\right]\right\}\chi, \tag{7}$$

$$L_{1/2} = 1 + \frac{\alpha^2 Z^2}{2}\left[\frac{31}{24} - \frac{\mu^2}{2a^2} + \frac{a^2}{4q^2} - \frac{2q^2+\mu^2}{12\eta^2}\right.$$

$$\left. + \ln\frac{a^2}{2\eta^2} + \left(\frac{q}{\mu} - \frac{\mu}{q} - \frac{a^4}{4\mu q^3}\right)\operatorname{arctg}\frac{q}{\mu}\right],$$

$$L_{3/2} = 1 + \frac{3\alpha^2 Z^2}{8}\left[\frac{41}{36} - \frac{5}{4}\frac{a^2}{\mu^2} + \frac{a^2}{4q^2} + \ln\frac{a^2}{2\eta^2}\right.$$

$$\left. - \frac{1}{3}\ln 2 + \left(\frac{q}{\mu} - \frac{\mu}{q} - \frac{a^4}{4\mu q^3}\right)\operatorname{arctg}\frac{q}{\mu}\right],$$

$\mathbf{s}_1 = [\mathbf{n}_1\times\mathbf{q}]$, $\mathbf{s}_2 = (\mathbf{e}_2\mathbf{q})[\mathbf{q}\times\mathbf{e}_1] - (\mathbf{e}_1\mathbf{q})[\mathbf{q}\times\mathbf{e}_2]$, $\mathbf{s}_3 = (\mathbf{e}_2\mathbf{q})\mathbf{e}_1 - (\mathbf{e}_1\mathbf{q})\mathbf{e}_2$.

4. DIFFERENTIAL AND TOTAL CROSS SECTIONS

The differential cross sections for the processes, averaged over the initial, and summed over the final, electron polarizations ν, are determined by the following expression:

$$d\sigma = \frac{1}{4\omega_1\omega_2}\frac{1}{2}\sum_\nu |A|^2\frac{d^3k_2}{(2\pi)^3}2\pi\delta(\omega_1+\varepsilon_1-\omega_2-\varepsilon_2) = \frac{r_0^2}{2}\sum_\nu |F|^2\frac{\omega_2}{\omega_1}d\Omega_2. \tag{8}$$

The summation over the spin states for the $1s \to 2s$ transition can be carried out in the usual manner. As a result, we obtain

$$d\sigma(1s \to 2s) = 32r_0^2(\mathbf{e}_2\mathbf{e}_1)^2\frac{\eta^8 q^4}{a^{12}}\left\{1 + \alpha^2 Z^2\left[\frac{3}{2} - \frac{a^2}{4\eta^2} + \ln\frac{a^2}{2\eta^2}\right.\right.$$
$$\left.\left. + \left(\frac{q}{\mu} - \frac{\mu}{q}\right)\operatorname{arctg}\frac{q}{\mu} + \frac{a^2}{2q^2}\left(\frac{\mu^4}{a^4} - \frac{5}{4} - \frac{a^2}{2\mu q}\operatorname{arctg}\frac{q}{\mu}\right)\right]\right\}\frac{\omega_2}{\omega_1}d\Omega_2. \tag{9}$$

The summation over the components M of the total angular momentum j for the transitions to the $2p_{1/2}$ and $2p_{3/2}$ states is simplified by the use of the formula[8]

$$\sum_\nu (\mathbf{a}\Omega_{j1M})(\overset{+}{\Omega}_{j1M}\mathbf{b}) = \frac{1}{4\pi}\left\{\left(j + \frac{1}{2}\right)\mathbf{ab} + (-1)^{j+1/2}i\mathbf{a}[\sigma\mathbf{b}]\right\}. \tag{10}$$

For these transitions we obtain

$$d\sigma(1s \to 2p_{1/2}) = 24r_0^2(\mathbf{e}_2\mathbf{e}_1)^2\frac{\eta^{10}q^2}{a^{12}}\left\{1 + \alpha^2 Z^2\left[\frac{11}{12} + \frac{3}{16}\frac{a^2}{\mu^2}\right.\right.$$
$$\left.\left. - \frac{\mu^2}{2a^2} + \frac{a^2}{4q^2} + \ln\frac{a^2}{2\eta^2} + \left(\frac{q}{\mu} - \frac{\mu}{q} - \frac{a^4}{4\mu q^3}\right)\operatorname{arctg}\frac{q}{\mu}\right]\right\}\frac{\omega_2}{\omega_1}d\Omega_2, \tag{11}$$

$$d\sigma(1s \to 2p_{3/2}) = 48r_0^2(\mathbf{e}_2\mathbf{e}_1)^2\frac{\eta^{10}q^2}{a^{12}}\left\{1 + \frac{3}{4}\alpha^2 Z^2\left[\frac{7}{18} - \frac{a^2}{2\mu^2}\right.\right.$$
$$\left.\left. - \frac{1}{3}\ln 2 + \frac{a^2}{4q^2} + \ln\frac{a^2}{2\eta^2} + \left(\frac{q}{\mu} - \frac{\mu}{q} - \frac{a^4}{4\mu q^3}\right)\operatorname{arctg}\frac{q}{\mu}\right]\right\}\frac{\omega_2}{\omega_1}d\Omega_2. \tag{12}$$

The cross sections summed over the photon polarizations differ from the formulas (9), (11), and (12) only by the replacement of the factor $(\mathbf{e}_2\cdot\mathbf{e}_1)^2$ by $(1+\cos^2\theta)/2$, where $\cos\theta = \mathbf{n}_1\cdot\mathbf{n}_2$ and $\mathbf{n}_i = \mathbf{k}_i/\omega_i$.

In the region $q \sim \eta$, where the differential cross sections are maximal, all the above-given expressions are of the same order of magnitude. However, there exists such a region of momentum-transfer values $q \sim q_{\min} = \omega_1 - \omega_2 = 3\eta^2/8m$ where the differential cross sections for the processes involving the $1s \to 2p_j$ transitions are considerably larger than the differential cross section for the process involving the transition $1s \to 2s$. In this region, the dominant term in the amplitude of the latter of the indicated processes becomes of the order of the correction term. Therefore, we should take into account in the cross section the square of the correction term in the amplitude of the indicated process at such values of q; the formulas (11) and (12) for the differential cross sections for the processes involving the $1s \to 2p_j$ transitions remain valid in this limit.

For $q \sim q_{\min}$ the formulas for the differential cross sections assume the following form:

$$d\sigma(1s \to 2s) = 32r_0^2(2/3)^8\left(\frac{q^2}{\mu^2} - \frac{3}{16}\alpha^2 Z^2\right)^2 d\Omega_2, \tag{13}$$

$$d\sigma(1s \to 2p_{1/2}) = 24r_0^2(2/3)^{10}\frac{q^2}{\mu^2}(1 - 0.53\alpha^2 Z^2)\,d\Omega_2, \tag{14}$$

$$d\sigma(1s \to 2p_{3/2}) = 48r_0^2(2/3)^{10}\frac{q^2}{\mu^2}(1 - 1.11\alpha^2 Z^2)\,d\Omega_2. \tag{15}$$

It should be noted that the differential cross section for the first process vanishes in our approximation at $q = 3\sqrt{3}\,\eta^2/8m$.

In computing the total cross sections, we can replace with the aid of the formula

$$d\Omega_2 = 2\pi\frac{q\,dq}{\omega_1\omega_2}$$

the integration over the solid angle by an integration over the momentum q imparted to the nucleus in the limits from $q = q_{\min}$ to $q = q_{\max} = \omega_1 + \omega_2$. Performing this integration, we obtain

$$\sigma(1s \to 2s) = \sigma_0\cdot{}^2/_5(2/3)^6\frac{\eta^2}{\omega_1^2}\left[1 - \alpha^2 Z^2\left(0.82 + 1{,}69\frac{m^2}{\omega_1^2}\right)\right], \tag{16}$$

$$\sigma(1s \to 2p_{1/2}) = \sigma_0\cdot{}^1/_5(2/3)^6\frac{\eta^2}{\omega_1^2}\left[1 + \alpha^2 Z^2\left(0.87 - 0.75\frac{m^2}{\omega_1^2}\right)\right], \tag{17}$$

$$\sigma(1s \to 2p_{3/2}) = \sigma_0\cdot{}^2/_5(2/3)^6\frac{\eta^2}{\omega_1^2}\left[1 - \alpha^2 Z^2\left(0.53 + 0.75\frac{m^2}{\omega_1^2}\right)\right], \tag{18}$$

where $\sigma_0 = 8\pi r_0^2/3$ is the cross section for Thomson scattering.

The dominant terms in the differential and total cross sections for the considered processes coincide with the corresponding formulas obtained by Gorshkov, Mikhaĭlov, and Sherman[8] for the photon-energy region $\eta \ll \omega \ll m$.

The above-given formulas allow us to calculate the cross sections for the indicated scattering processes in the region of energies $\omega \gg \eta$ for small and medium Z (up to $Z \sim 50$); for large Z the higher-order terms discarded in the cross sections should be taken into account.

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Excitation of large-amplitude solitary waves in a plasma

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A method is developed for generating solitary space charge waves in a plasma by employing an electron beam that is strongly modulated in velocity. At a current of 7 mA, mean beam energy 200 eV, and modulation depth ~0.5, solitons with a mean velocity of $3.3 \cdot 10^9$ cm/sec and amplitude up to 2 kV can be generated by the method. The amplitude attained is about 0.65 of the amplitude limit at which the plasma electrons are trapped. The details of the soliton generation process are elucidated. It is shown, in particular, that the fast beam electrons whose energy exceeds the mean value transfer to the solitons an energy proportional to the modulation depth. The effect of varying the beam and plasma parameters as well as the geometrical dimensions of the plasma on soliton formation is investigated.

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INTRODUCTION

Excitation of large-amplitude space-charge waves is of great interest for applications such as particle acceleration, plasma heating, and others. At a given propagation velocity w it is possible to excite in the plasma a wave with amplitude φ_m not exceeding a value

$$\varphi_m = mw^2/2e \tag{1}$$

(e and m are the charge and mass of the electron), such that the wave begins to capture the plasma electrons. (We neglect here the thermal velocity of the electrons, which we assume to be small in comparison with w.) However, the wave can become essentially nonlinear at amplitudes much lower than φ_m. As follows from the theory of stationary waves in a plasma layer or cylinder, [1] the space-charge waveform is sinusoidal only at low amplitudes, and in the limit of large amplitudes the wave goes over into solitary waves (solitons), the amplitudes of which can reach values φ_m.

Excitation of silitons in a bounded plasma was investigated experimentally in[2–5]. In[2–4] the excitation was produced by applying voltage pulses to an electrode immersed in the plasma. Another method of soliton excitation is described in [5], where the effect was obtained in a beam-plasma system by modulating the initial beam velocity.

The present paper is devoted to a further investigation of the formation and propagation of solitons in a beam-plasma system. With rather modest means (initial beam energy 200 eV, current 7 mA), the soliton amplitude, is shown by measurements, reaches 2 kV at a propagation velocity 3.3×10^9 cm/sec. This gives grounds for hoping that for moderate beam parameters it is possible to excite in this manner solitons that are of practical interest.

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