Dislocation dragging by electrons in a magnetic field

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The electronic part of the drag force acting on dislocations in metals at a low temperature is investigated for magnetic fields of arbitrary orientation, i.e. for arbitrary angles Φ between the magnetic field and dislocation axis. It is shown that for all but the smallest angles Φ the drag force is the same as in the absence of a magnetic field. A strong angular dependence (including nonmonotonous ones) of the drag force can be observed in the small-angle region ($\Phi < a/R$, where a is the interatomic distance and R is the Larmor radius). Quantum oscillations of the deceleration force as a function of the magnetic field strength are observed. They are due to oscillations of the electron state density and their amplitude may reach several percent of the smooth part.

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1. INTRODUCTION

1. A dislocation moving with a specified velocity V through a crystal can be regarded as a source of sound waves, the connection between the frequency ω of which and the wave vector **q** of which is given by the relations $\omega = \mathbf{q}\cdot\mathbf{V}$. Each of these waves is absorbed by the crystal via some mechanism whereby they interact with the thermal phonons, electrons, impurities, and other scatterers.

In metals at low temperatures, an important role is played by absorption of sound by conduction electrons. If we calculate the power dissipated by the dislocation as a result of its interaction with the electron system, and divide this power by the velocity V, then we obtain the electronic component of the dislocation drag force. Inasmuch as the absorption of the energy of a phonon of given frequency and given wave vector has been thoroughly investigated, the problem of calculating the dislocation deceleration force reduces to integration of the sound absorption coefficient with respect to the vector **q** with a known weight.[1] The weight is determined by the number of the dislocation-generated phonons with wave vectors lying between **q** and **q** + d**q**, which in turn is expressed in terms of the Fourier transform of the known[2] dislocation deformations.

We confine ourselves to the motion of an individual straight-line dislocation. It is obvious that its velocity must be regarded as perpendicular to the dislocation axis (the y axis). It is also understandable that the vector **q** of all the dislocation-generated sound waves is perpendicular to the y axis. In the absence of a magnetic field, an electron moving in straight lines between collisions draws energy from the "dislocation phonon" wave as a result of the deformation interaction. If the sound wavelength is much shorter than the electron mean free path l, then the absorption has a collisionless character, since the electron interacts many times with the sound prior to being scattered. Inasmuch as the elasticity-theory problem of dislocation deformations contains only one parameter with the dimension of length, the Burgers vector **b**, it can be stated that the characteristic wave vector of the dislocation phonons is $q_m \sim 1/b$. Since $l \gg b$ always, in the absence of a magnetic field the absorption has a collisionless character and accordingly the drag force does not contain parameter l.[1,1]

2. When the magnetic field is turned on, so long as the Larmor radius R of the electron is much larger than the mean free path, the picture remains the same. In a strong magnetic field, however, when the electron begins to move along circles of smaller radius $R \ll l$ (in a plane perpendicular to the vector H), the character of the absorption can change substantially. In fact, the electron interacts with the wave $\Omega\tau$ times more frequently (Ω is the cyclotron frequency and τ is the time interval between collisions) than without a magnetic field before it becomes scattered. Therefore the absorption increases by a factor $\Omega\tau = l/R$ times. This increase will continue so long as the "phonon frequency" $q_m V$ can be regarded as small in comparison with the collision frequency ν (and all the more with Ω) i.e., so long as the sound-wave front can be regarded as having become rigid. With increasing dislocation velocity, the characteristic period $1/q_m V$ of the sound oscillations becomes smaller than τ, and consequently the effective energy pick-off by the electrons will take place in times on the order of $1/q_m V$. Accordingly, the absorption will turn out to be only $\Omega/q_m V$ times larger and not $\Omega\tau$ times larger than in the absence of the field. Finally, when $q_m V \gg \Omega$, owing to the rapid changes of the deformations during the time of the revolution of the electron on the orbit, the magnetic field ceases to influence the absorption.

3. The qualitative arguments presented above pertain only to the case when there is no drift motion of the electron in a direction perpendicular to the plane of the sound-wave front. This can occur only when the magnetic field H is oriented along the dislocation axis. On the other hand, if the vector H is inclined to the y axis even through a small angle, then the absorption becomes immediately collisionless, since the principal role is assumed by the indicated drift motion.

Indeed, in the case of a parallel (to the y axis) orientation of the magnetic field, the electron has time to land $\Omega\tau$ times in the region of equal phase of the sound wave prior to becoming scattered, so that the energy pick-off is quite effective. If the magnetic field is inclined a small angle Φ to the y axis, then during one revolution the electron will be displaced across the wave front a distance on the order of $v_H\Phi/\Omega$, where v_H is a projection of the electron velocity of the H direction. In order of magnitude, v_H coincides with the Fermi velocity v_F. If the average displacement exceeds the characteristic sound-wave length q_m^{-1}, i.e., $\Phi \gtrsim (q_m R)^{-1}$, then the electron drift along **q** upsets the condition of their in-phase interaction with the sound oscillations, and as a result the magnetic field exerts no influence on the absorption. However, even at angles smaller than $(q_m R)^{-1}$, the shift of the center of the Larmor circle during the time between collisions (which is $\Omega\tau$ times larger than the shift during one revolution) can exceed the sound wavelength. This occurs at $\Phi \gg (q_m l)^{-1}$, as a result of which the effect of the magnetic field on the drag force, which is appreciable at $\Phi = 0$, becomes much weaker in the region $(q_m l)^{-1} < \Phi < (q_m R)^{-1}$, and disappears completely at $\Phi \gtrsim (q_m R)^{-1}$.

It follows therefore that only in the case of almost parallel orientation of the magnetic field relative to the dislocation axis should one expect an influence of the field on the smooth part of the drag force. Of course, the estimate given above for the angles is a rough one, and we shall show below that at a certain chosen neutral orientation of the dislocation axis, the dislocation velocity, and the vector H there are curious nonmonotonic variations of the drag force with changing disorientation angle Φ and velocity V. But these phenomena take place, all the same, only in the region of very small values of the angles Φ, so that they can apparently be observed only under special conditions.

4. Oscillatory effects in the drag force can be divided into two groups—classical and quantum. The former are connected with the fact that the electron in a magnetic field can be regarded as a classical oscillator with frequency Ω. If the frequency of the dislocation phonon $q_m V$ is equal to or a multiple of the frequency of the oscillator Ω, then one can expect the appearance of resonances in the drag force under the condition $q_m V = s\Omega$ $(s = 1, 2, \ldots)$. Actually, in the case of dislocations this effect is more that of a threshold than of resonance. Thus, when a definite value of the velocity V is reached, resonance sets in first with the highest-frequency phonons, followed by phonons with smaller values of q, until the next threshold is reached, etc. The described phenomenon was investigated in detail by Natsik and Potemina.[4]

It should be noted, however, that there will be no distinct threshold at $q = q_m$, owing to the absence of an abrupt edge in the spectrum of the dislocation phonons, but it will occur at $q = 2p_F/\hbar$, as a result of the Kohn anomaly of the electron-phonon interaction.[1,7] In addition, the resonant action of the sound on the electron will be effective only when the plane of the orbit is perpendicular to the wave front of all the phonons, and consequently the classical oscillatory effects vanish even at small inclination angles Φ. Indeed, if the component of the average electron velocity along the normal to the wave front differs from zero, then the resonance condition includes a Doppler frequency shift of the order of $q_m v_F \Phi$. Writing down the condition for the smallness of this shift relative to the uncertainty of the acoustic cyclotron resonance $\nu = \tau^{-1}$, we obtain the limitation $\Phi \ll (q_m l)^{-1}$ on the angles. When this inequality is satisfied it is possible, in principle, to observe classical oscillations of the drag force. In the opposite case, averaging over the electron velocities causes these singularities to vanish.

The quantum-oscillation effects in our problem are connected with the fact that the number of electrons on the Fermi surface is an oscillating function of the magnetic field (oscillations of the density of states). Inasmuch as the electron part of the drag force is proportional to the number of electrons, it will also oscillate with changing magnetic field. To be sure, these changes are small in comparison with the smooth part of the absorption, but on the other hand they are insensitive to the orientation of the field and to the uncertainty of the quantity q_m (which, incidentally, is of the order of q_m itself). The existence of quantum oscillations is determined mainly only by the relation between the temperature and the value of the magnetic quantum $\hbar\Omega$.

2. DISLOCATION DRAG FORCE IN THE ABSENCE OF A MAGNETIC FIELD

There are two methods of calculating the absorption of the power of the external perturbation of the conductivity of a metal by electrons. The first is the classical method and is connected with the use of the traditional kinetic equation for the electron distribution function. The second, which we shall call quantum, consists in calculating the energy losses in terms of the quantum transitions of the electrons under the influence of an external perturbation. In this section we use both methods to find the dislocation drag force F in the case when the magnetic field is equal to zero, and discuss the difference between the results of these calculations.

1. The initial formula for the dissipation function is of the form

$$Q = \int d\mathbf{r}\, \overline{\langle \dot{U} \chi \rangle}. \tag{1}$$

Here Q is the rate of change of the energy of the moving dislocation per unit time; $U = \Lambda_{ik} u_{ik}$ is the potential of the interaction of the electron with the elastic deformations generated by the dislocation; the dot denotes partial differentiation with respect to time and χ is the deviation of the electron distribution function from the equilibrium Fermi function f_0; the bar denotes averaging over the time and the angle brackets denote integration over the generalized momentum electrons. The function χ satisfies the well known linearized kinetic equation[5]

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla + \hat{\nu}\right)\chi = -\dot{U}\frac{\partial f_0}{\partial \varepsilon}, \tag{2}$$

the right-hand side of which is the non-adiabatic part of the deformation interaction. The induction interaction of the electrons with the moving deformations can be disregarded, since $q_m R \gg 1$[6].

The deformation-potential model, of course, cannot

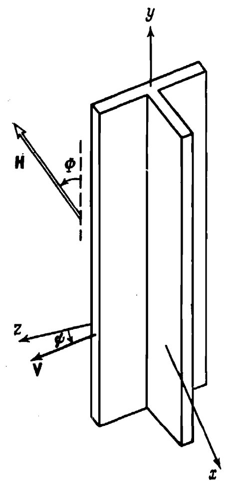

FIG. 1.

claim a rigorous quantitative description of the interactions of the electron with the dislocation core. The deformations u_{ik} near the nucleus become of the order of unity and the potential U can no longer be regarded as a perturbation. Nonetheless, we shall use the deformation potential also at large q, bearing in mind the fact that the results obtained within the framework of this model give correct dependences on all the parameters (dislocation velocity, dislocation orientation, magnetic field, etc.) with accuracy to numerical factors on the order of unity.

We emphasize that we confine ourselves to dislocation velocities V that are small in comparison with the speed of sound. Therefore the deformation field is of the form $u_{ik}(\mathbf{r}, t) = u^0_{ik}(\mathbf{r} - \mathbf{V}t)$, where $u^0_{ik}(\mathbf{r})$ is the solution of the corresponding static problem of elastic theory. If the dislocation velocity is comparable with the speed of sound, then it becomes necessary to investigate the complete dynamic system of equations,[2] the solution of which is determined both by the dislocation density and by the dislocation flux density.

Without loss of generality, but to simplify the form of some of the coefficients, we consider the dragging of a screw dislocation in a metal with a spherical Fermi surface. The components of the tensor of the elastic deformations around a screw dislocation with an axis along y (Fig. 1) are[2]

$$u_{xx}=u_{yy}=u_{zz}=u_{xz}=0, \quad u_{xy}=\frac{b}{4\pi}\frac{z}{x^2+z^2}, \quad u_{yz}=-\frac{b}{4\pi}\frac{x}{x^2+z^2}.$$

It is convenient to write down the potential of the deformation interaction in the form of a Fourier plane-wave expansion

$$U(\mathbf{r}-\mathbf{V}t)=\sum_{\mathbf{q}} U(\mathbf{q})\exp[i\mathbf{q}(\mathbf{r}-\mathbf{V}t)], \quad U(\mathbf{q})=\frac{ib}{q^2}(\Lambda_{yz}q_x-\Lambda_{xy}q_z),$$
$$\sum_{\mathbf{q}}=\int\frac{d^2\mathbf{q}}{(2\pi)^2} \qquad (3)$$

This means that the moving deformations can be represented as an assembly of "dislocation phonons." Their frequency is equal to $\omega_{\mathbf{q}} = \mathbf{q}\cdot\mathbf{V}$, and their "number" is determined by the amplitude of the Fourier transform $U(\mathbf{q})$.

If we solve the kinetic equation (2) in the approximation of the relaxation time $\tau = 1/\nu$ and substitute this solution in (1), then, taking the definition $Q \equiv \mathbf{F}\cdot\mathbf{V}L_y$ into account (L_y is the dislocation length) we obtain for the drag force $\mathbf{F}$ per unit dislocation length the relation

$$\mathbf{F}\mathbf{V}=\frac{B\hbar}{4m}\int\frac{dq_x\,dq_z}{(2\pi)^2}(\mathbf{q}\mathbf{V})^2\frac{(\zeta_{xy}q_z-\zeta_{yz}q_x)^2}{q^4}\int do\frac{\nu}{\nu^2+(\mathbf{q}\mathbf{v}-\mathbf{q}\mathbf{V})^2}, \qquad (4)$$

where

$$B=3N_e b^2 m\varepsilon_F/2\pi\hbar,$$

$\zeta_{ik} = \Lambda_{ik}/\varepsilon_F$, the quantities Λ_{ik} are assumed to be independent of the electron momentum, m is the effective mass, N_e is the electron density, and the last integral in (4) is taken with respect to the solid angle in electron-momentum space.

Recognizing that the relation $l = v_F/\nu \gg q_m^{-1}$, is always satisfied, then the expression under the sign of the integral with respect to the solid angle can be replaced by the function $\pi\delta(\mathbf{q}\cdot\mathbf{v} - \mathbf{q}\cdot\mathbf{V})$. After explicit integrations in (4) we obtain

$$F(0)=BV\frac{\pi\hbar q_m}{32p_F}[\zeta_{xy}^2+\zeta_{yz}^2+2(\zeta_{xy}\cos\psi-\zeta_{yz}\sin\psi)^2], \quad q_m=\int_0^{q_m}dq, \qquad (5)$$

where ψ is the angle between the vector $\mathbf{V}$ and the z axis (see Fig. 1). In the integration with respect q it was assumed that the modulus of the wave vector of the dislocation phonons has an upper bound $q_m \sim b^{-1}$. More realistically, however, is a model in which there is no abrupt edge in the phonon spectrum. Let the smooth function $G(q/q_m)$ describe the profile of the distribution of the number of dislocation phonons as a function of the magnitude of their wave vector q, and let q_m determine the position of the diffuse edge. This assumption alters the result (5) as follows:

$$F_{\text{cl}}(0)=F(0)\int_0^\infty G(x)\,dx. \qquad (6)$$

2. Let us examine the results of the quantum method of calculation. In the Born approximation the general quantum-mechanical formula for the electron component of the dislocation drag force in the absence of electron scattering takes the form

$$\mathbf{F}_{\text{qu}}\mathbf{V}W=2\pi\sum_{a,b;\mathbf{q}}\mathbf{q}\mathbf{V}|U(\mathbf{q})|^2|\langle b|e^{i\mathbf{q}\mathbf{r}}|a\rangle|^2(f_a-f_b)\delta(E_b-E_a-\hbar\mathbf{q}\mathbf{V}). \quad (7)$$

Here a and b are the complete sets of the quantum numbers of the electron in the initial and final states (without the spin); E_a is the energy of an electron with wave function $|a\rangle$; $f_a \equiv f_0(E_a)$ is the Fermi distribution function; the summation is over all values of a and b and over the components of the two-dimensional vector $\mathbf{q}$; W is the volume of the crystal, and the factor 2 is due to the summation over the spins. We note that the numerical coefficient in formula (7) is due to the dependence of the interaction potential on the coordinates and on the time in the form $U(\mathbf{r}-\mathbf{V}t)$.

The summation over the final states is carried out in elementary fashion with the aid of δ symbols in the matrix elements. The difference $f_0(E_a) - f_0(E_a + \hbar\mathbf{q}\cdot\mathbf{V})$ between the Fermi functions can be expanded in terms of the small shift of the arguments, after which the result takes the form

$$\mathbf{F}_{\text{qu}}\mathbf{V}=\frac{B\pi\hbar}{4m}\int\frac{dq_x\,dq_z}{(2\pi)^2}(\mathbf{qV})^2\frac{(\zeta_{xy}q_z-\zeta_{yz}q_x)^2}{q^4}\int do\,\delta\left(\mathbf{qv}-\mathbf{qV}-\frac{\hbar q^2}{2m}\right). \quad (8)$$

This relation is the limiting value of expression (4) as $\nu\to 0$. In comparison with (4), the argument of the δ function acquired a new term $\hbar q^2/2m$ which is the "recoil frequency." Since $\hbar q\sim\hbar q_m\sim p_F$, the magnitude of this term is of the same order as the Doppler frequency qv_F, i.e., it is large. Consequently, the final result for F, generally speaking, does not agree with formula (6). Indeed, elementary integration with respect to the angles yields

$$F_{\text{qu}}(0)=F(0)\int\limits_0^\infty dx\,G(x)\Theta\left(\frac{q_0}{q_m}-x\right)=F(0)\int\limits_0^{q_0/q_m}dx\,G(x),\quad q_0=2p_F/\hbar. \quad (9)$$

Comparing (9) and (6) we conclude that the results of the classical and quantum calculations differ by an amount

$$F_{\text{cl}}-F_{\text{qu}}=F(0)\int\limits_{q_0/q_m}^\infty dx\,G(x).$$

This fact is due to Kohn threshold of the electron-phonon interaction.[7] The point is that the absorption of phonons having momenta $\hbar q > 2p_F$ is forbidden by the energy and momentum conservation laws. Thus, in the absence of a magnetic field, the use of the classical kinetic approach leads to an incorrect result for the dislocation drag force, since the main contribution to the force F is made by dislocation phonons the "tail" of the distribution $G(q/q_m)$. The abrupt termination of the spectrum of the dislocation phonons at $q=2p_F/\hbar$ is due to the Kohn singularity of the electron-phonon interaction.

3. DRAG FORCE IN A MAGNETIC FIELD NOT PARALLEL TO THE DISLOCATION AXIS

1. We proceed to calculate the drag force in a constant inhomogeneous magnetic field H oriented at an angle Φ to the dislocation axis (Fig. 1). In this geometry, the complete set of quantum numbers includes two continuous quantum numbers—the momenta p_x and p_H (the electron momentum along H), and also the magnetic quantum number $n=0, 1, 2, \ldots$ The matrix elements

$$|\langle b|e^{i\mathbf{qr}}|a\rangle|^2 \quad (10)$$

$$=\delta(p_{xb}, p_{xa}+\hbar q_x)\,\delta(p_{Hb}, p_{Ha}+\hbar q_H)\,M^2_{n_bn_a}(\hbar q_\perp^2/2m\Omega),$$

$$M_{nm}(t)=t^{(n-m)/2}L_m^{n-m}(t)\,e^{-t/2}$$

contain products of δ symbols, which eliminate the summation over the final momenta; $q_H=q_z\sin\Phi$ is the projection of the vector $\mathbf{q}$ on the direction of H; $q_\perp=\sqrt{q_x^2+q_z^2\cos^2\Phi}$ is the modulus of the projection of the vector $\mathbf{q}$ in a plane perpendicular to H; $L_m^n(t)$ is a generalized Laguerre polynomial normalized to unity.

After elementary integrations with respect to p_{xb}, p_{Hb}, and p_{xa} (with allowance for the degeneracy of the electron spectrum with respect to p_x), formula (7) takes the form

$$\mathbf{FV}=\frac{B\pi^2\hbar}{2mp_F}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\mathbf{qV}\frac{(\zeta_{xy}q_z-\zeta_{yz}q_x)^2}{q^4}\sum_{s=-\infty}^{\infty}\sum_{n=0}^{\infty}M^2_{n+s,n}\left(\frac{\hbar q_\perp^2}{2m\Omega}\right)$$
$$\times\int\limits_{-\infty}^{\infty}dp_H\left\{f_0\left[\left(n+\frac{1}{2}\right)\hbar\Omega+\frac{p_H^2}{2m}\right]-f_0\left[\left(n+\frac{1}{2}\right)\hbar\Omega+\frac{p_H^2}{2m}+\hbar\mathbf{qV}\right]\right\}$$
$$\times\delta\left(s+\frac{p_Hq_H}{m\Omega}+\frac{\hbar q_H^2}{2m\Omega}-\frac{\mathbf{qV}}{\Omega}\right). \quad (11)$$

In the derivation of this formula we have changed from summation over n_b to summation over the "classical" number $s=n_b-n_a$. It determines the energy distances between the Landau levels in units of $\hbar\Omega$.

We note one important feature of formula (11). In contrast to relation (8), the recoil energy in (11) is contained not only in the argument of the δ function, but also in the matrix elements. Thus, the "longitudinal" part of the recoil energy $(\hbar q_H)^2/2m$ renormalizes the frequency of the dislocation phonon in the energy conservation law, and the "transverse" part $(\hbar q_\perp)^2/2m$ determines the argument of the functions M. For the dislocation phonons $(\hbar q)^2/2m$ can be of the order of ε_F. This circumstance makes the limiting transition $H\to 0$ from expression (11) to formula (8) nontrivial. The solution of this equation reduces to derivation of a correct asymptotic expression for the squared matrix elements $M^2_{n+s,n}$, when $n>s>\hbar q^2/m\Omega\gg 1$. Following Szegö[8] we can show that the asymptotic form of M coincides with the asymptotic expression for Bessel functions:

$$M_{n+s,n}(t)\approx J_s[\sqrt{t(4n+2s+2-t)}]. \quad (12)$$

We shall show below how to use this relation to obtain the result (8) independent of the magnetic field.

2. The initial formula (11) describes both smooth variations of the drag force as a function of H, and quantum oscillations of the force F due to oscillations of the density of the electron states on the Fermi surface. The analysis of the quantum oscillations is the subject of Sec. 5 of the present paper. Here we discuss the influence of the magnetic field on the smooth part of the dislocation drag force.

To investigate the smooth function $F(H)$, formula (11) should be simplified in the following manner. The difference between the Fermi "steps" is replaced by the expression

$$\hbar\mathbf{qV}\delta[(n+1/2)\hbar\Omega+p_H^2/2m-\varepsilon_F].$$

This δ function will be used to calculate the sum over n, which will in turn be replaced by an integral. Changing from integration with respect to p_H to integration with respect to the polar angle $\theta(\cos\theta=p_H/p_F)$, we obtain

$$\mathbf{FV}=\frac{B\pi^2\hbar}{2m\Omega}\int\frac{d^2\mathbf{q}}{(2\pi)^2}(\mathbf{qV})^2\frac{(\zeta_{xy}q_z-\zeta_{yz}q_x)^2}{q^4}\sum_{s=-\infty}^{\infty}\int_0^{\pi}d\theta\sin\theta$$
$$\times J_s^2\left[q_\perp R\sqrt{\sin^2\theta+\frac{s+1}{2}\frac{\hbar\Omega}{\varepsilon_F}-\left(\frac{q_\perp}{q_0}\right)^2}\right]$$
$$\times\frac{1}{\pi}\frac{\nu/\Omega}{(\nu/\Omega)^2+(s+q_HR\cos\theta+\hbar q_H^2/2m\Omega-\mathbf{qV}/\Omega)^2}. \quad (13)$$

We have used here the asymptotic form (12) for the matrix elements, and we have "smeared out" the δ function with the energy conservation law with a relative collision frequency ν/Ω.

Even a cursory glance at formula (13) leads to the first statement of our paper, namely, that the drag force is independent of the magnetic field for practically all angles Φ. It suffices to assume that $q_HR \sim qR\sin\Phi \gg 1$, in order for the sum over s to contain contributions from many terms. This corresponds to calculation of the asymptotic form of expression (11) as $H\to 0$. Replacing the sum over s by an integral, and the Bessel function by an asymptotic form of the type

$$J_s^2(t)\approx\Theta(t^2-s^2)/\pi(t^2-s^2)^{1/2}, \quad (14)$$

we see that the integral with respect to q diverges at the upper limit. This means that the main contribution to this integral is made by large $q\sim q_0$, so that in all the estimates containing q it is necessary to substitute q_0. After employing this procedure, all the integrals in (13) are calculated in terms of elementary functions and the answer is

$$F=F_{\text{qu}}(0), \quad (15)$$

where $F_{\text{qu}}(0)$ is given by formula (9). Thus, for practically all angles between the vector H and the y axis, satisfying the inequality

$$\sin\Phi\gg 1/q_0R, \quad (16)$$

the drag force coincides with its value at $H=0$. Since the parameter $1/q_0R$ is quite small, we arrive at the conclusion that a dependence of the force F on the magnetic field can be observed only at very small angles Φ.

In concluding this section, we will recall that the result (15) is valid also at $\Phi=0$ if the velocity of the dislocation is high enough. This conclusion was derived by Natsik and Potemina,[4] and is due to the following circumstance. At $\Phi=0$ the relative Doppler frequency shift $q_HR\cos\theta$ and the longitudinal component of the recoil energy $\hbar q_H^2/2m\Omega$ vanish identically. If at the same time $V\gg\Omega/q_0$, then many terms contribute to the sum over s (13), meaning that all the arguments used in the derivation of (15) are valid.

4. REGION OF SMALL INCLINATION ANGLES

1. Before we proceed to a more detailed investigation of the region of small angles Φ, we return to the limiting case $\Phi=0$. It was investigated earlier in[4,9]. We recall that the magnetic field is assumed to be classically strong ($\Omega\tau\gg 1$), inasmuch as the opposite limiting case is trivial: $F=F_{\text{qu}}(0)$ for any orientation of the magnetic field. Thus, if $\Phi=0$, then at high dislocation velocities $V\gg\Omega/q_0$, as already noted above, the drag force is equal to its value at $H=0$. On the other hand, if $V\ll\nu/q_0$ then, as seen from (13), the main contribution to the sum over s is made by the term with $s=0$. This contribution is calculated in elementary fashion and yields the result $F_{\text{qu}}(0)\,\Omega\tau/\pi$, i.e., an increase of the force by a factor $\Omega\tau/\pi$. Thus, at $\Phi=0$ the range of the dislocation velocities breaks up naturally into three sections: low velocities ($V\ll\nu/q_0$), large velocities ($V\gg\Omega/q_0$) and intermediate velocities ($\nu/q_0\ll V\ll\Omega/q_0$). Collision dominated and noncollision absorption of the dislocation phonons predominate at low and high velocities, respectively.

The intermediate case was investigated by Natsik and Potemina,[4] so that there is no need to dwell on it in detail. We note only that in no velocity region does the drag force increase logarithmically with velocity in the form given by Kravchenko.[9] The apparent reason is that in formula (1) of[9] the value of $\mathring{U}$ was determined by a sum of two terms, one of which is indirectly the dislocation flux density. Yet the dislocation flux density begins to influence the deformation significantly only at large velocities V, on the order of the speed of sound. In addition, to the extent that the flux density enters the right-hand side of one of the equations of the complete system, its effect on the deformation rate manifests itself not directly, but via the solution of this system. It is therefore not clear whether the force will increase logarithmically with the velocity if we solve exactly the complete dynamic system of equations for the dislocation deformations.

Formula (13) contains also some oscillatory effects. One of them, albeit extremely weak (its relative magnitude can be at the most $1/q_0R$), is connected with the oscillations of the Bessel function. This effect leads to the appearance of a very small ripple on the investigated plots, and we shall disregard it.

Another effect is connected with the difference between the sum in (13) and the integral. If $\Phi=0$ and the condition $\mathbf{qV}/\Omega=s$ is satisfied, the corresponding term can make a resonant contribution to the sum. This is the usual classical (cyclotron) resonance, when the frequency of the external action $\omega_{\mathbf{q}}=\mathbf{q}\cdot\mathbf{V}$ is a multiple of the natural frequency Ω of the oscillations of the absorbing system. To be sure, since this condition is satisfied by relatively few wave vectors $\mathbf{q}$, for a given value of s it follows that this resonance becomes greatly smeared out in the integration with respect to $\mathbf{q}$. If the integral were abruptly cut off at a certain value of q_m, then the threshold effect described in[4] would take place. Actually, however, the integration with respect to $\mathbf{q}$ is cut off by the function $G(q/q_m)$ on the side of large q, not abruptly, but smoothly, over a width on the order of q_m itself. This circumstance leads to a very small amplitude of this effect. Nonetheless, the threshold values in the $F(V)$ dependence, as well as the formulas of Natsik and Potemina which describe them,[4] should exist when the dislocations reach velocity values $V=s\Omega/q_0$. This is due to the Kohn anomaly at $q=q_0=2p_F/\hbar$, which we have already discussed in Sec. 2. The conduction electrons

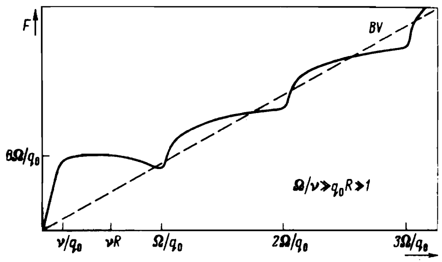

FIG. 2. Dependence of the drag force on the velocity at $\Phi = 0$.

absorb the energy of only dislocation phonons with $q \leq q_0$.[2)] The last statement, in the case $H \neq 0$, is clearly demonstrated by formula (13). If $\Phi = 0$, then the argument of the δ function does not depend on the angle θ, and s turns out to be of the order of qV/Ω. Therefore the quantity $s\hbar\Omega/\varepsilon_F$ can be neglected in the argument of the Bessel functions. If the asymptotic form (14) is used, the integral with respect to θ can be calculated explicitly and leads to the expression $(qR)^{-1}\Theta(q_0 - q)$. It limits the integration with respect to q to the value q_0. The inclination of the magnetic field to the dislocation axis, even through a very small angle, makes observation of this resonance effect impossible. Indeed, at $\Phi \neq 0$, when the Doppler frequency shift $q_z v_F \sin\Phi$ exceeds the frequency ν, i e., $\Phi \gtrsim 1/q_0 l$, the averaging over the angle θ smears out the singularity completely. The dependence of the drag force F on the velocity V at $\Phi = 0$[4] is shown in Fig. 2.

2. We proceed now to investigate the smooth part of the drag force in the region of small angles Φ. As seen from (13), in this case the contribution of the term with $s = 0$ is separated in natural fashion. The summary contribution of all other terms yields as before a quantity $F_{qu}(0)$ that does not depend on H. Inasmuch as the angle Φ is small, we put $\cos\Phi = 1$ and $\sin\Phi = \Phi$. Then the term with $s = 0$ takes the form

$$F_0=\frac{BV\varepsilon_F}{\pi\hbar\nu}\int_0^1 t\,dt\int_{-1}^1\frac{d\mu\,\Theta(1-t^2-\mu^2)}{1+\pi q_0Rt\sqrt{1-t^2-\mu^2}}\oint\frac{d\varphi\cos^2(\varphi-\psi)\,(\zeta_{xy}\cos\varphi-\zeta_{yz}\sin\varphi)^2}{1+t^2(q_0V/\nu)^2D^2},$$
$$D(t,\mu,\varphi)=\cos(\varphi-\psi)-\mu\frac{v_F\Phi}{V}\cos\varphi-t\frac{v_F\Phi^2}{V}\cos^2\varphi. \qquad (17)$$

We have used in (17) the interpolation formula $J_0^2(x) \approx (1+\pi x)^{-1}$; φ is the azimuthal angle in the space of the wave vectors q; $t = q/q_0$ and $\mu = \cos\theta$; the function $G(q/q_m)$ was replaced by unity for the sake of simplicity.

The integration with respect to t is carried out explicitly if account is taken of the following circumstance. For the angles $\Phi \ll 1$ of interest to us, the last term (proportional to Φ^2) in the function D is much smaller than the second (Doppler) term and can be neglected.[3)] This means that we can disregard the dependence of the function D on t. After making the change of variable $y = t(1 - t^2 - \mu^2)^{-1/2}$, the integral with respect to y can be evaluated in terms of elementary functions:

$$F_0=\frac{B\hbar\Omega}{16\pi^2p_F}\oint d\varphi\cos^2(\varphi-\psi)\,(\zeta_{xy}\cos\varphi-\zeta_{yz}\sin\varphi)^2\int_{-1}^1\frac{d\mu}{|D(\mu,\varphi)|}$$
$$\times\frac{\pi\sqrt{1-\mu^2+(\nu/q_0VD)^2}+(2V|D|/\pi\nu R)\ln(V|D|/\pi\nu R\sqrt{1-\mu^2})}{1-\mu^2+(VD/\pi\nu R)^2+(\nu/q_0VD)^2} \qquad (18)$$

We did not write out small increments to F_0, the relative values of which do not exceed $1/q_0R$.

In the analysis of (18) we must distinguish between three regions of variation of the dislocation velocity: the section of low velocities ($V \ll \nu/q_0$), intermediate velocities ($\nu/q_0 \ll V \ll \nu R$) and high velocities ($V \gg \nu R$).

a) $V \ll \nu/q_0$. For inclination angles $\Phi \ll 1/q_0 l$ we can neglect in the interval with respect to μ the terms proportional to $V|D|/\pi\nu R \ll 1$, and also the quantity $1 - \mu^2$ in comparison with the parameter $\nu/q_0V|D| \gg 1$. The integral with respect to μ is then equal to $2\pi q_0 V\tau$, and the final result becomes

$$F_0=BV\frac{\Omega\tau}{16}\,[\zeta_{xy}^2+\zeta_{yz}^2+2(\zeta_{xy}\cos\psi-\zeta_{yz}\sin\psi)^2]=\frac{F_{qu}(0)\,\Omega\tau}{\pi}. \qquad (19)$$

Under these conditions, the drag force is $\Omega\tau/\pi$ times larger than the value at $H = 0$. At larger inclination angles $\Phi \gg 1/q_0 l$ we can eliminate from the function D the first term $\cos(\varphi - \psi)$. Further asymptotic calculation of the integrals of (18) makes it possible to represent the angular dependence $F_0(\Phi)$ by the following interpolation formula:

$$F_0=\frac{BV\hbar\Omega}{8\pi\varepsilon_F\Phi}\left[\zeta_{xy}^2-\zeta_{yz}^2+\cos2\psi\frac{\zeta_{xy}^2+5\zeta_{yz}^2}{3}-\frac{4}{3}\sin2\psi\,\zeta_{xy}\zeta_{yz}+\sin^2\psi\,\zeta_{yz}^2L(\Phi)\right]$$
$$\times\ln\frac{q_0l\Phi}{1+(\Omega\tau\Phi/\pi)},\qquad L(\Phi)=\ln(q_0l\Phi)\left[1+\frac{\Omega\tau\Phi/\pi}{1+(\Omega\tau\Phi/\pi)}\right]. \qquad (20)$$

For all its complexity, this expression is valid only in the logarithmic approximation. We see that the term F_0 in the drag force decreases rapidly when H deviates from the dislocation axis. When Φ reaches the value $\Phi \approx 1/q_0R$, the value of F_0 becomes comparable with $F(0)$, i.e., it becomes of the order of the total contribution of the remaining terms. At inclination angles larger than $1/q_0R$, the total force is equal to $F_{qu}(0)$, just as in the absence of a magnetic field. A schematic plot of F_0 against Φ is shown in Fig. 3a.

b) Region of intermediate velocities $\nu R \gg V \gg \nu/q_0$. If $\Phi = 0$, then $D = \cos(\varphi - \psi)$, and in the integrals of (18) we can neglect the parameters $\nu/q_0V|D|$, $V|D|/\pi\nu R \ll 1$ and obtain

$$F_0=\frac{B\hbar\,\Omega}{12p_F}\,[\zeta_{xy}^2+\zeta_{yz}^2+(\zeta_{xy}\cos\psi-\zeta_{yz}\sin\psi)^2]. \qquad (21)$$

This case of relatively slow motion of the dislocations

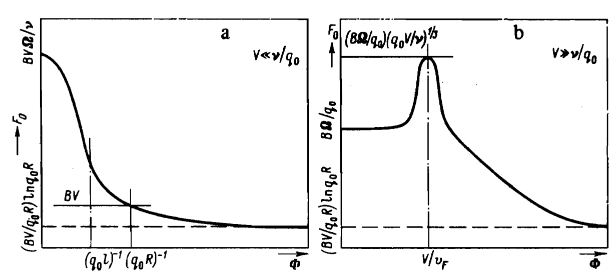

FIG. 3. Dependence of the principal term of the drag force on the angle Φ: a) $q_0V \ll \nu$, b) $q_0V \gg \nu$.

corresponds to the plateau region of Fig. 2, when the drag force does not depend on the velocity V. We note that relation (21) is valid also for small angles $\Phi \ll V/v_F$, when the second term in the D function in (17) does not exceed unity.

We shall discuss the behavior of the $F_0(\Phi)$ curves at angles $\Phi \gtrsim V/v_F$ first for the special geometry $\psi=0$ and $\psi=\pi$, when the dislocation moves in the plane containing the dislocation axis and the magnetic field. In this situation, it is reasonable to single out in the integrals with respect to μ and φ in (18) the contribution of the region adjacent to the curve, which is determined from the condition that the function $D(\mu, \varphi)$ vanish. The values of the integrand at the points on the "singular" curve are proportional to $q_0 V\tau$, i.e., they are large. The characteristic width of the band within which this function assumes values of the order of the value at the maximum is determined from a comparison of the quantities $1-\mu^2$ and $(\nu/q_0 VD)^2$, and turns out to be of the order of $1/q_0 l\Phi$. At the same time, the remaining part of the rectangle $0 \leqslant \varphi \leqslant 2\pi$, $-1 \leqslant \mu \leqslant 1$ makes a contribution approximately equal to $(V/v_F\Phi)\ln(q_0 l\Phi)$ to the integrals of (18). Thus, the deceleration force is equal to $F_0 \sim (BV\hbar\Omega/\varepsilon_F\Phi)\ln(q_0 l\Phi)$, and the contribution of the singular point is not exclusively separated in the integral. Exceptions are those values of Φ for which $v_F\Phi \approx V$. In this case the singular line is localized near the value $\mu=1$, and its width becomes much larger, of the order of $(q_0 V\tau)^{-2/3}$, so that the relative contribution of the singular region to the integrals increases, and the force increases to a value $F_0 \sim (B\Omega/q_0)(q_0 V\tau)^{1/3}$.

This is the qualitative picture of the function $F_0(\Phi)$. To illustrate it by means of formulas, we note that at angles $\Phi \gg V/v_F$ the formula (20) and the reasoning presented in subsection a will remain valid. Formula (20) describes the right-hand side of the $F_0(\Phi)$ line. Let us investigate the behavior of $F_0(\Phi)$ in the immediate vicinity of the maximum. If $\Phi \ll 1/\Omega\tau$ (and all the more, if $\Phi \approx V/v_F$), the parameter $V|D|\pi\nu R$ in the integral with respect to μ in (18) can be neglected throughout. This simplifies noticeably the form of the integrand, and the integral with respect to μ becomes elliptic:

$$I(\Phi)=\int_{-1}^{1} d\mu \ldots = \pi \int_{-1}^{1} \frac{d\mu}{\sqrt{(\nu/q_0 V)^2 + D^2(1-\mu^2)}}. \qquad (22)$$

It is easily seen that at $\Phi = V/v_F$, the main contribution to $I(\Phi)$ is made by the vicinity of the point $\mu=1$ with characteristic width $1-\mu \lesssim (\nu/q_0 V\cos\varphi)^{2/3}$ and

$$I(V/v_F)=\frac{\pi(q_0V\tau)^{1/3}}{2^{1/3}\cos^{2/3}\varphi}\int_0^\infty \frac{dt}{\sqrt{1+t^3}} = \pi(q_0V\tau)^{1/3}\frac{2F(\pi/2, \sin 75^\circ) - F(\varphi_0, \sin 75^\circ)}{2^{1/3}3^{1/4}\cos^{2/3}\varphi}$$

where $F(\varphi, k)$ is an incomplete elliptic integral of the first kind

$$\varphi_0 = \arccos\frac{\sqrt{3}-1}{\sqrt{3}+1}.$$

Calculating the integral with respect to the angle φ, we obtain ultimately

$$F_{0\,max} = \alpha\frac{B\hbar\Omega}{p_F}(q_0V\tau)^{1/3}(3\zeta_{yz}^2 + 7\zeta_{xy}^2); \qquad (23)$$

$$\alpha = \frac{3^{3/4}}{160\cdot 2^{1/3}\pi^{1/2}}\frac{\Gamma(7/6)}{\Gamma(2/3)}[2F(\pi/2, \sin 75^\circ) - F(\varphi_0, \sin 75^\circ)] \approx 0.016.$$

Thus, at the point $\Phi = V/v_F$, the $F_0(\Phi)$ curve has a maximum which is approximately $(q_0 V\tau)^{1/3}$ times larger than at $\Phi=0$. A simple analysis shows that this maximum is quite narrow, and its width is determined by the inequality $|1 - v_F\Phi/V| \lesssim (q_0V\tau)^{-2/3}$. If this inequality is violated, then F_0 decreases rapidly, to the value of F_0 from (21) at small angles Φ, and in accordance with relation (20) at $\Phi > V/v_F$.

The appearance of this maximum can be explained in the following manner. As indicated earlier, the shift of the electron in the direction perpendicular to the wave fronts (along q_z) during one revolution is $v_F\Phi/\Omega$. Inasmuch as the dislocation moves with velocity V, the planes of the front are displaced during this time by a distance V/Ω. If both shifts cancel each other, i.e., $V = v_F\Phi$, then the electron lands many times in the region of equal phase of the sound field and effectively absorbs the dislocation phonons. If the angle Φ is increased or decreased, the condition for synchronous interaction is violated and the drag force decreases. The nonmonotonic dependence of F_0 on the angle Φ is shown in Fig. 3b. This is a manifestation of the known "inclination effect"[10] in the absorption of frequency sound.

In the derivation of the formulas in subsection (b), a number of assumptions were made. Thus, we have assumed that the function $D(t, \mu, \varphi)$ in (17) does not depend on the variable t. Actually, so long as all the values of μ contribute to the integral with respect to μ, the third (proportional to t) term in the D function is much smaller than the second term and can be neglected. At $\Phi = V/v_F$, the main contribution to the integral is made by $\mu=1$, and the difference between the first two terms of D is of the order of $\cos\varphi(1-\mu) \sim \cos\varphi(q_0V\tau)^{-2/3}$ and can be small. In the integration with respect to t in (17), an essential role is played by the values

$$t \lesssim (q_0V\tau|D|)^{-1} \sim [q_0V\tau\cos\varphi(1-\mu)]^{-1}.$$

Therefore a comparison of the third term in the D function with the difference of the first two leads to the parameter $(1-\mu)^2 q_0 l \sim (v_F/V)(q_0V\tau)^{-1/3}$. If it is larger than unity, i.e.,

$$(q_0V\tau)^{1/3} \ll v_F/V, \qquad (24)$$

then the third term is negligibly small in comparison with the difference between the first two terms and it can be disregarded. In other words, the criterion (24) limits the applicability of formula (23). From this it is seen, for example, that at the maximum at $\Phi = V/v_F$ the drag force does not exceed $F_0 \sim B\hbar\Omega/mV$.

If the inequality (24) is reversed, then the resonance condition $D=0$ depends already also on the integration variable t. The physical meaning of this circumstance is that when short-wave quanta are absorbed the energy conservation law covers also the recoil energy. The integration with respect to t (the modulus of $\mathbf{q}$) will smear out the resonant maximum almost completely.

The same considerations lead to the conclusion that the resonant peak of the function $F_0(\Phi)$ vanishes if the dislocation velocity V does not lie in the plane containing the magnetic field and the y axis (i.e., $\psi \neq 0$ and π). Indeed, earlier in the integrals of (17) we had on the singular curve $D=0$ (for a fixed $\Phi \geq V/v_F$) at a definite value of μ and at arbitrary angles φ. Now it can be equal to zero only in a strictly defined vicinity of the variables μ and φ. In other words, the curve on Fig. 3b should be also averaged over the position of the maximum that depends on φ. This additional integration eliminates the singularity of the deceleration force at $\Phi = V/v_F$. A more detailed investigation shows that the maximum is preserved under the condition of small deviations $\delta\psi \ll (q_0 V\tau)^{-2/3}$ of the vector V from the yz plane. For arbitrary ψ, the monotonic dependence of $F_0(\Phi)$ is described by formulas (20) and (21).

c) The region of large velocities $V \gg \nu R$. At $\Phi = 0$ in the integrals (18), the principal term is the logarithmic term and the result for it (in the logarithmic approximation)

$$F_0 = \frac{B\hbar\nu}{4mV}\ln\left(\frac{V}{\nu R}\right)(\zeta_{xy}^2+\zeta_{yz}^2) \tag{25}$$

describes the decrease of F_0 with increasing velocity V. For large angles $\Phi \gg V/v_F$, formula (20) is valid, in which the parameter $\Omega\tau\Phi/\pi$ must be regarded as larger than unity. At the resonance point, when $\Phi = V/v_F$, the values of the integrals (18) is determined mainly by the vicinity of the singular line $D=0$. In the general case the width of this region is

$$\delta\mu \leqslant \frac{(q_0 V\tau)^{-1/3}(\nu R/V)^{1/2}}{(q_0 V\tau)^{-1/6}+(\nu R/V)^{1/2}}, \tag{26}$$

and the value of the integrand on the singular curve is $\pi q_0 V\tau$. Therefore the value of the force F_0 at the maximum can be estimated as

$$F_{0\,max} \sim \frac{B\hbar\Omega}{p_F}\frac{(q_0 V\tau)^{1/6}(\nu R/V)^{1/2}}{(q_0 V\tau)^{-1/6}+(\nu R/V)^{1/2}}. \tag{27}$$

At $V \ll \nu R$ it goes over into formula (23). We note that the relation (23) determines exactly the value of the maximum also in the case $(q_0 V\tau)^{1/3} \gg V/\nu R \gg 1$. Finally, if the condition $V/\nu R \gg (qV\tau)^{1/3}$ is satisfied, it follows from the order-of-magnitude formula (27) that

$$F_{0\,max} \sim (B\hbar\Omega/p_F)\sqrt{q_0 R} \sim Bv_F(q_0 R)^{-1/2}.$$

This means that the height of the maximum decreases in comparison with (23) by a factor $(q_0 V\tau)^{-1/3} V/\nu R$.

Thus, to observe a sharp peak on the angular dependences $F(\Phi)$ it is necessary that $F_{0\,max}$ from (27) exceed the monotonic part of the drag force $F_{qu}(0)$. In addition, just as in subsection b), we must be assured that the integration with respect to t and φ in (18) do not smear out the resonant singularity. This is ensured by satisfaction of the inequalities

$$\delta\mu \gg (q_0 l)^{-1/2}, \quad \delta\psi \ll \delta\mu, \tag{28}$$

in which it is necessary to substitute $\delta\mu$ from (26).

5. QUANTUM OSCILLATIONS OF THE DRAG FORCE

1. We shall investigate the quantum effects in the drag force at low temperatures, when the thermal diffuseness of the Fermi level can be disregarded (the appropriate criteria will be given later on).

Greatest interest attaches to dislocation motion in the plane $x = \text{const}$ containing the vector H and y (V $\parallel z$ on Fig. 1). In this case it is necessary to separate from the sums over n and s in (11) the large resonant term, which must correspond to the term with $s = 0$. Indeed, if $s \neq 0$, then integration of the asymptotic forms of the matrix element (12) with respect to q_x yields a smooth function of the variable q_z, which does not become infinite anywhere and is small everywhere. The difference between the Fermi steps in (11) is a function equal to unity inside a certain interval and to zero outside this interval. The integral with respect to q_z is therefore, an integral of a small quantity between finite limits, and cannot result in any singularity.

To the contrary, the term with $s = 0$, corresponding in (11) to the square of the zero-order Bessel function, yields upon integration with respect to q_x

$$\int_{-\infty}^{\infty}\frac{dq_x}{2\pi}J_0^2\left\{\left[\frac{\hbar q_\perp^2}{2m\Omega}\left(4n+2-\frac{\hbar q_\perp^2}{2m\Omega}\right)\right]^{1/2}\right\}\frac{(\zeta_{xy}q_z-\zeta_{yz}q_x)^2}{q^4}$$
$$\approx \frac{1}{4|q_z|}(\zeta_{xy}^2+\zeta_{yz}^2), \quad \frac{\hbar q_z^2}{m\Omega}(2n+1) \ll 1. \tag{29}$$

It is obvious that the integral (29) contains a singularity of the type $1/q_z$ as $q_z \to 0$. Since this singularity is not integrable, it follows that if the interval of the integration with respect to q_z includes the point $q_z = 0$, then the corresponding term with $s = 0$ can become large. We note that in formula (29) the integration was carried out between infinite limits, i.e., the fact that $|q_x| \leq q_0$ was not taken into account. This is permissible, since the main contribution to the integral (29) is made by values $|q_x| \sim |q_z|$. We shall henceforth be interested in terms with $n \approx \mathcal{N} \approx \varepsilon_F/\hbar\Omega$, and for these terms the inequality (29) takes the form $|q_x| \sim |q_z| \ll 1/R \ll q_0$.

Let us examine in greater detail the singular term F_r. We introduce the symbols $\mathcal{N}$ and Δ for the integer and fractional parts of the quantity $(\varepsilon_F/\hbar\Omega) - 1/2$, so that $\varepsilon_F/\hbar\Omega \equiv \mathcal{N} + \Delta + 1/2$; the expression

$$v_n \equiv v_F\left(\frac{\mathcal{N}+\Delta-n}{\varepsilon_F/\hbar\Omega}\right)^{1/2} \tag{30}$$

stands for the quantized values of the modulus of the projection (on the direction of the magnetic field H) of the velocity of electrons having the Fermi energy.

An analysis of formula (11) shows that at $V \parallel z$ the δ function and the difference between the Fermi functions in the curly brackets do not depend on q_x. This enables us to carry out independent integration with respect to q_x in (29) and with respect to p_H with the aid of a δ function. We can separate in the integral with respect to

p_H the "resonant" values of the longitudinal (along H) electron momentum:

$$p_H = \frac{mV}{\sin\Phi}\left(1 - \frac{\hbar q_z \sin^2\Phi}{2mV}\right). \qquad (31)$$

In the remaining integral with respect to q_z for the terms with $0 \leqslant n \leqslant \mathcal{N}$, the difference between the Fermi steps is equal to +1 in the interval

$$|V - v_n \sin\Phi| \leqslant \frac{\hbar q_z \sin^2\Phi}{2m} \leqslant V + v_n \sin\Phi \qquad (32)$$

and to −1 if

$$-V - v_n \sin\Phi \leqslant \frac{\hbar q_z \sin^2\Phi}{2m} \leqslant -|V - v_n \sin\Phi|.$$

Outside these intervals, and also for the terms with $n > \mathcal{N}$, the integrand vanishes identically. Using (11), (29), and (32), we obtain for the singular term F_n

$$F_n = \frac{B\pi\hbar\Omega}{8p_F \sin\Phi}(\zeta_{xy}^2 + \zeta_{yz}^2)\ln\frac{V + v_n \sin\Phi}{|V - v_n \sin\Phi|}. \qquad (33)$$

Thus, the choice of the singular term in the sum over n in (11) signifies separation of a term with a number n such that the quantized values of the projection of the electron velocity on the dislocation motion direction $v_n \sin\Phi$ is equal to or close to the dislocation velocity V. The condition $V = v_n \sin\Phi$ denotes precisely that the "dangerous" point $q_z = 0$ falls on the boundary of the integrals (32).

2. We formulate now the necessary conditions under which the result (33) is valid. It is quite obvious that for the existence of a logarithmically large term F_n it is necessary to satisfy the inequality

$$\sin\Phi \geqslant V/v_F. \qquad (34)$$

In addition, it is necessary that the "separation" $\hbar qV$ of the arguments of the Fermi functions in (11) be larger than their temperature broadening T. From the inequality (32) it is seen that the characteristic quantity is $\hbar q \sim mV/\sin^2\Phi$. Thus, the temperature should be quite low:

$$T \ll mV^2/\sin^2\Phi. \qquad (35)$$

In the opposite case, the difference between the Fermi functions in (11) is proportional to the quantity $\hbar q_z V$, which vanishes at $q_z = 0$ and cancels out the singularity of the integral (29).

Electron scattering, like the temperature, also smears out the quantum effect. Indeed, the existence of a singular quantum term is directly connected with the integration with respect to p_H in (11) with the aid of a δ function. If the dependence of p_H in the argument of the δ function vanishes for some reason, then the singularity of the integral (29) at $q_z = 0$ is again eliminated by the vanishing of the product $\mathbf{q}\cdot\mathbf{V}$. Therefore the necessary condition for separating the individual quantum term (33) is the requirement that the relative Doppler frequency shift $p_H q_H/m\Omega$ exceed the characteristic smearing of the δ function ν/Ω. It follows from (31) that $p_H \sim mV/\sin\Phi$. This means that the condition of the rare collisions of the electrons with the scatterers takes the following form:

$$\hbar\nu \ll mV^2/\sin^2\Phi. \qquad (36)$$

Finally, the inequalities (32) must be reconciled with the asymptotic criterion (29). For the singular terms with $n \approx \mathcal{N}$, the self-consistency requirement takes the form

$$\mathcal{N} \ll (v_F/V)\sin^2\Phi. \qquad (37)$$

The conditions (35), (36), and (37) can be combined into a chain of inequalities

$$mV^2/T_{\text{eff}} \gg \sin^2\Phi \gg (V/v_F)\mathcal{N}, \qquad (38)$$

where $T_{\text{eff}} = T + \hbar\nu$ is the effective temperature. The outer inequalities of (38) and relation (34) impose a limitation on the dislocation velocity:

$$\sin\Phi \geqslant V/v_F \gg T_{\text{eff}}/\hbar\Omega. \qquad (39)$$

Thus, if the conditions (38) and (39) are satisfied, and if the vector V lies in the plane of H and y, then it follows from (33) that the drag force can experience "giant" quantum oscillations when the magnetic field, the dislocation velocity, and the inclination angle Φ are varied. Participating in the giant resonance are the electrons of the extremal section of the Fermi surface ($n \lesssim \mathcal{N}$), having small drift velocities along V: $v_n \sin\Phi \approx v_F \sin\Phi(\Delta/\mathcal{N})^{1/2} = V$. The oscillations of the force F_n as functions of the reciprocal field are periodic with a period $\Delta(1/H) = 2\hbar e/cp_F^2$.

If the vector V goes out of the plane $x = \text{const}$, then the giant peaks become strongly smeared out. The reason is that at $V_x \neq 0$ the arguments of the functions in (11) contain both components of the vectors V and q. Then the Fermi steps no longer define the limits of integration with respect to q_z, but the two-dimensional region of variation of the alternating vector q. In other words, the logarithmic singularities (33) are subjected to an additional averaging with respect to q_x. This greatly decreases the magnitude of the periodic changes of F_n. A simple analysis shows that for giant oscillations of the drag force (33) to exist it suffices to satisfy the inequality

$$\delta\psi mV^2/\sin^2\Phi \ll T\hbar\nu/(T + \hbar\nu). \qquad (40)$$

It imposes a limitation on the angular deviation $\delta\psi$ of the vector V from the z direction.

3. The right-hand inequality in (39) is extremely stringent and difficult to satisfy in experiment. A more realistic situation is therefore the inverse of the condition (39). In this case the "singular" terms are those with $n = \mathcal{N}$ and with arbitrary numbers s. For these terms the difference of the Fermi steps in (11) can be replaced by the function $\hbar\mathbf{q}\cdot\mathbf{V}(p_H^2/2m - \Delta\hbar\Omega)$, which can be used to calculate the integral with respect to p_H.

Formula (11) for the singular term $F_{\mathcal{N}}$ can then be rewritten in the form

$$\mathbf{F}_{\mathcal{N}}\mathbf{V}=\frac{B\hbar}{16m\Omega\sqrt{\mathcal{N}\Delta}}\int d^2\mathbf{q}(\mathbf{q}\mathbf{V})^2\frac{(\zeta_{xy}q_z-\zeta_{yz}q_x)^2}{q^4}$$
$$\times\sum_{\pm}\sum_{s=-\infty}^{\infty}J_s^2\left\{\left[\frac{\hbar q_\perp^2}{2m\Omega}\left(4\mathcal{N}+2s+2-\frac{\hbar q_\perp^2}{2m\Omega}\right)\right]^{1/2}\right\}$$
$$\times\delta\left(s+\frac{\hbar q_H^2}{2m\Omega}-\frac{\mathbf{q}\mathbf{V}}{\Omega}\pm q_H R\sqrt{\Delta/N}\right). \tag{41}$$

Let us analyze first the contribution of the terms with $s\neq 0$. Their sum can be replaced by an integral with respect to s, and the integral can be calculated with the aid of a δ function. If we then use the asymptotic form (14), then the integrals with respect to q are calculated in elementary form and the answer is

$$F_{\mathcal{N}}=F_{\text{qu}}(0)/2\sqrt{\mathcal{N}\Delta}. \tag{42}$$

At the minimum, the singular term is equal to $F_{\text{qu}}(0)/2\sqrt{\mathcal{N}}$. To estimate its value at the maximum ($\Delta\to 0$) it must be recognized that at $T_{\text{eff}}\neq 0$ the parameter Δ has uncertainty on the order of $T_{\text{eff}}/\hbar\Omega\ll 1$. Therefore the maximum attained value of $F_{\mathcal{N}}$ is $F_{\text{qu}}(0)/\hbar\Omega\sqrt{T_{\text{eff}}\varepsilon_F}$. Numerical estimates show that the amplitude of the oscillations of the deceleration force can reach several percent of the principal part, i.e., the changes of F are perfectly observable. We note that the oscillations of F from (42) duplicate the known oscillations of the density of the electronic states. Therefore formula (42) is valid for an arbitrary geometry of the problem; it does not contain the "velocity resonance" like (33), meaning that experimental observation of this effect does not impose stringent conditions on the angles, velocities, etc.

In conclusion, let us discuss the quantum singularities that are contained in the term with $s=0$ of formula (11). Because of its complexity, we shall not analyze in detail the shape of the resonance line. This question will be the subject of a separate communication. We present here only two important results for the quantum increment of F_0. The exact formula for the quantum increment δF_0, due to the term with $n=\mathcal{N}$, coincides with formula (17) if one adds to the integral with respect to μ the function $(1/\mathcal{N})\delta(\mu^2-\Delta/\mathcal{N})$.

In the "low-frequency" region at small Φ, when

$$q_0V\tau\ll 1,\quad q_0l\Phi\sqrt{\frac{\Delta}{\mathcal{N}}}\ll 1,\quad q_0l\Phi^2\ll 1, \tag{43}$$

the resonant increment

$$\delta F_0=\frac{\Omega\tau}{\pi}\frac{F_{\text{qu}}(0)}{2\sqrt{\mathcal{N}\Delta}} \tag{44}$$

is $\Omega\tau/\pi$ times larger than the contribution (42) from the terms with $s\neq 0$. One more limiting case corresponding to the inequalities

$$\frac{v_F}{V}\frac{\Phi}{\sqrt{q_0l}}\quad\frac{v_F}{V}\frac{1}{q_0l}\ll\left|1-\sqrt{\frac{\Delta}{\mathcal{N}}}\frac{v_F\Phi}{V}\right|\ll\frac{\nu R}{V}, \tag{45}$$

leads to the following result:

$$\delta F_0=\frac{B\hbar\Omega}{24\pi p_F\sqrt{\mathcal{N}\Delta}}\frac{\zeta_{yz}^2+2\zeta_{xy}^2}{|1-\sqrt{\Delta/\mathcal{N}}\,v_F\Phi/V|},\quad\psi=0,\pi. \tag{46}$$

A characteristic feature of (46) is that it contains, besides singularities of the static density of states ($\Delta\to 0$), also a dynamic singularity of the type $(\sqrt{\Delta/\mathcal{N}}-V/v_F\Phi)^{-1}$, due to the Landau damping in the magnetic field ($\omega_q=\mathbf{q}\mathbf{V}=q_H v_{\mathcal{N}}$). The maximum value of δF_0 from (46) can be $\Omega\tau(1+\sqrt{q_0l}\Phi)^{-1}$ times larger than the static oscillations from (42).

[1)]We do not consider the particular case when the Fermi surface has flat sections that are perpeneicular to the dislocation axis. The contribution of the electrons from these sections to the drag force is proportional to the mean free path l.[3]

[2)]We emphasize that the existence of the Kohn threshold is due only to the conservation laws and to the Fermi statistics of the conduction electrons, i.e., it is essentially not connected with the concrete model of the electron-phonon interaction.

[3)]In the "high-frequency" limit $q_0V\gg\nu$, elimination of this term calls for an additional justification, which will be presented later on.

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