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Plasma heating during Langmuir collapse

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We solve the three-dimensional problem of the production of accelerated electrons in a plasma during Langmuir collapse. We show that the problem has a scaling solution in which practically the whole energy of the external source of the Langmuir oscillations is put into a small, decreasing with time, group of resonance particles.

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INTRODUCTION

We consider in the present paper the problem of the heating of a plasma in which strong Langmuir turbulence is excited constantly by an external source. This kind of problem is of great interest for the problem of plasma heating by a powerful electron beam or by laser light. The most important property of strong Langmuir turbulence is the location of Langmuir noise in regions with a lower density—cavitons. The characteristic size δ of the cavitons and the density perturbation δn are connected with the local noise energy density W through the relation

$$\delta n/n_0 = r_{De}^2/\delta^2 \lesssim W/nT, \tag{1}$$

where r_{De} is the Debye radius.

Zakharov[1] has shown that such formations are not stationary in the three-dimensional problem: when the energy density is sufficiently high, the cavitons collapse—their size δ vanishes after a finite time, and the quantity W becomes infinite. We shall call such formations in what follows collapsing solitons, without in general having in view any analogy whatever with a stationary Langmuir solitons (see, e.g., [2]).

It is clear that if we take into account kinetic effects, the size δ cannot vanish while condition (1) is retained, since under the condition $\delta \sim r_{De}$ the characteristic phase velocity of the harmonics is $\omega/k \sim v_{Te}$, so that strong Landau damping sets in. This is not the only possible channel for dissipation during the collapse, but Rudakov[3] has shown that under the conditions when the noise is pumped in a stationary way, this mechanism is the most probable one and in that case the whole energy of the collapsing soliton is transferred to a small group of resonance particles with velocities which are appreciably above the thermal one, $v \gg v_{Te}$. Strong Langmuir turbulence leads, therefore, as in the one-dimensional model,[2,4] to the formation of non-Maxwellian tails of hot electrons. This result of Rudakov's[3] has been confirmed by the solution of a model problem about the heating of a plasma by spherical, quasi-planar collapsing solitons.[5,6] Similar statements have been made by Galeev *et al.*[7] who solved the problem of the heating of a plasma during "supersonic" collapse[1] and for that case the spectra of the noise and of the fast electrons were obtained.

We present in the present paper the results of[3,6,7] in correspondence to one another. We also show that the methods for solving the problem in[6,7] are essentially equivalent. We give here the calculations for what is (according to present-day results) the most realistic mode of collapse—the formation of a kind of plane disk.

We dwell in more detail on this model which was first suggested by Rudakov.[3] The dynamics of the collapse in the hydrodynamic approximation is described by the set of equations[1]

$$\operatorname{div}\left(-i\frac{\partial \mathbf{E}}{\partial \tau} + \frac{3}{2}\omega_{pe} r_{De}^2 \nabla^2 \mathbf{E}\right) = \frac{\omega_{pe}}{2n_0}\operatorname{div}\delta n\mathbf{E},$$
$$\left(\frac{\partial^2}{\partial \tau^2} - c_s\nabla^2\right)\delta n = \frac{1}{16\pi M}\nabla^2|\mathbf{E}|^2. \tag{2}$$

Here $c_s^2 = T_e/M$, and $\tau = \tau_0 - t$, t_0 is the moment of collapse.

It was shown already in[1] that any three-dimensional collapsing soliton, described by Eq. (2), ultimately goes into the supersonic regime where $\partial^2/\partial\tau^2 \gg c_s^2\nabla^2$ when the amplitude increases. It is assumed that in that regime the collapsing caviton resembles in form an ellipsoid with a small eccentricity, i.e., with a single characteristic dimension δ. The time-dependence of the basic quantities is then determined from the set (2) as follows:

$$\delta n \sim \delta^{-2}, \quad W/nT \sim \delta^{-3}, \quad \delta \sim \tau^{2/5} \sim k_0^{-1}. \tag{3}$$

This model was also used in[7]. However, if we introduce in Eqs. (2) an arbitrarily small damping, it turns out that a collapsing soliton, losing altogether only 5 to 10% of its total energy, leaves the supersonic regime and its further evolution proceeds so that $\delta n \sim E^2$. This important result was obtained numerically for an axially symmetric caviton in[8]. On the other hand for a subsonic collapse when $\partial^2/\partial\tau^2 < c_s^2\nabla^2$ the velocity necessarily increases with time which in the absence of damping leads to a transition to the supersonic regime. The natural approximation for the description of a collapse with damping will, therefore, be a model of sonic collapse, i.e., $\delta \approx c_s\tau$ (δ is the thickness of the disk). The large dimension R is determined by the conditions

$$\frac{\delta n}{n_0} = \frac{r_{De}^2}{\delta^2} = \frac{W}{nT}, \quad \mathscr{E} \sim R^2\delta W \sim \frac{R^2}{\delta}, \tag{4}$$

where $\mathscr{E}$ is the energy of the soliton. So far one can approximately assume it to be constant $\mathscr{E} \equiv \mathscr{E}_i$, whence follows

$$R_0(\tau) = R(\tau)|_{\gamma=0} = \frac{2}{3}\mathscr{E}_i\left(\frac{e}{T}\right)^2 c_s\tau, \tag{5}$$

while if the damping is important, $R(\tau)$ tends to zero faster so that

$$\mathscr{E} = 3\frac{T}{e}\left(\frac{M}{m}\pi nT\right)^{1/2}\frac{R^2(\tau)}{\omega_{pe}\tau}\Big|_{\tau\to 0} \to 0. \tag{6}$$

We note that Rudakov[3] was the first to state the problem of the heating of the plasma in such a collapse and solve it in the form of estimates.

1. SOLUTION OF THE HEATING PROBLEM IN THE FRAMEWORK OF THE SOLITON MODEL

We shall assume that in the plasma solitons with an initial energy $\mathscr{E}_i$ are constantly excited and afterwards collapse. Per unit time a number of solitons $N = Q/\mathscr{E}_i$, where Q is the pumping power per unit volume, are generated. We make the obvious assumption that the characteristic time of collapse is much shorter than the characteristic time for the plasma heating $Q\delta/c_s \ll nT$. The dynamics of the collapsing soliton can then easily be constructed from energy considerations. Under the conditions (4) of sonic collapse the field of the soliton along its smallest dimension is the field of a one-dimensional Langmuir soliton, the spectral expansion of which is well known[2]:

$$E_k = \frac{E_0}{2k_0}\operatorname{ch}^{-1}\frac{\pi k}{2k_0}, \quad k_0 = \frac{eE_0}{6^{1/2}T},$$

whence follows

$$\frac{d\mathscr{E}}{d\tau} = -\int 2\gamma_k W_k\,dS\,dk = \frac{3}{2}\left(\frac{T}{e}\right)^2\pi R^2(\tau)\int_0^\infty dk\frac{2\gamma_k}{\operatorname{ch}^2(\pi k/2k_0)}. \tag{7}$$

If the electron distribution function $f(v)$ is isotropic, we have

$$\gamma_k = -\pi^2\frac{\omega^4}{k^3}f\left(\frac{\omega}{k}\right).$$

Substituting this expression into (7) we get after simple transformations

$$d\mathscr{E}/d\tau = \Gamma\tau\mathscr{E},$$
$$\Gamma = \frac{\pi^2}{2}c_s\omega_{pe}^2\int_{v(\tau)}^\infty\frac{dv\,vf(v)}{\operatorname{ch}^2(\pi v(\tau)/2v)}, \quad v(\tau) = \omega_{pe}c_s\tau. \tag{8}$$

The interaction of a sufficiently fast ($v > l\omega_{pe}$) electron with a single separate caviton resembles the normal acceleration of a charged particle in the field of an almost plane capacitor, since the potential of our soliton in the direction of its smallest dimension is a monotonic function of the coordinate:

$$\varphi = -\sqrt{6}\,Te^{-1}\operatorname{arctg}\operatorname{sh}k_0x.$$

which during the time of flight ($l/v < \omega_{pe}^{-1}$) does not succeed in changing its sign. If, however, in the volume of the plasma there is at each moment of time a set of cavitons, randomly distributed in space, the evolution of the electron distribution function in velocity space is described by a diffusion equation

$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}v^2D\frac{\partial f}{\partial v}, \quad D \approx \overline{(\Delta v)^2}\nu_{\text{eff}}. \tag{9}$$

Here Δv is the increase in the electron velocity when it passes through a caviton. We can calculate it if we start also from the fact that the field in the caviton is close to the field of a one-dimensional soliton:

$$E(x) = \frac{E_0}{\operatorname{ch}k_0x}, \quad \overline{(\Delta v)^2} = \frac{3\pi^2}{m^2v^2}T_e^2. \tag{10}$$

We can write the effective collision frequency of electrons and cavitons in the form

$$\nu_{\text{eff}} = \pi v\int_{E_{\min}}^\infty R^2(E)F(E)\,dE, \quad E_{\min} = (24\pi nT)^{1/2}\left(\frac{v_{Te}}{v}\right),$$

where $F(E)$ is the amplitude distribution function of the collapsing solitons while $E_{\min}$ is the minimum amplitude of a soliton for which an effective interaction starts between it and an electron of given velocity v.

It is, however, more convenient to use the distribution function of the solitons with respect to the proper time τ:

$$F(\tau)=\text{const}=N=Q/\mathscr{E}_i.$$

In that case

$$\nu_{\text{eff}}=\pi v\int_{\tau(v)}^{0} d\tau\, R^2(\tau)N, \quad \tau(v)=\frac{v}{\omega_{\text{pe}}c_s}. \tag{11}$$

We substitute Eqs. (11) and (10) into (9) and also use Eq. (5). We get

$$D=2^{1/2}\pi^3\left(\frac{M}{m}\right)^{1/2}\frac{Qv_{\text{Te}}}{4\pi nT}\int_0^1 d\chi\,\chi\frac{R^2(\chi)}{R_0^2(\chi)}v, \quad \chi=\frac{\omega_{\text{pe}}\tau c_s}{v}. \tag{12}$$

where $R_0(\tau)$ is the radius of a caviton which collapses without damping. We retained in Eqs. (8) and (9) different notations for the time since they differ with respect to the characteristic size and with respect to their zero-point ($\tau=0$ for each soliton at the moment of collapse).

It is scarcely possible to solve in a general form the set of equations which we have obtained but it allows a self-similar change of variables. We introduce a characteristic hot electron velocity $v_0(t)$ and we consider first of all Eq. (9) in the region $v\gg v_0(t)$ where the damping is small:

$$\frac{\partial f}{\partial t}=\frac{1}{v^2}\frac{\partial}{\partial v}D_1 v^3\frac{\partial f}{\partial v}, \quad D_1=\frac{\pi^3}{2^{1/2}}\left(\frac{M}{m}\right)^{1/2}\frac{Qv_{\text{Te}}}{4\pi nT}. \tag{9'}$$

We make the substitution $t\to D_1 t$ and we shall look for the distribution function in the form

$$f=\varphi(t)\Phi(\xi), \quad \xi=v/t. \tag{13}$$

From the condition that the pumping of the noise is constant

$$Q=\text{const}=\frac{\partial}{\partial t}\int fv^2\,dv$$

we get $\varphi(t)=ct^{-4}$ and also

$$-4\Phi-\xi\Phi'_\xi=\frac{1}{\xi^2}\frac{d}{d\xi}\xi^3\frac{d\Phi}{d\xi}. \tag{14}$$

Equation (14) has the exact solution:

$$\Phi=(\xi-3)e^{4-\xi}. \tag{15}$$

We now consider the velocity range $v\ll v_0(t)$. The diffusion coefficient is here exponentially small as the solitons give up all their energy to particles with $v\sim v_0(t)$: the distribution function in this region must, nonetheless, satisfy the scaling substitution (13) as it was established as a result of a scaling process. This condition is satisfied by the power-law function

$$f_\infty(v)=C/v^4\equiv\varphi(t)\xi^{-4}. \tag{16}$$

It is convenient to match up the solutions (15) and (16) in the point $\xi=4$ in order to guarantee a zero current from the non-resonance to the resonance region. We have thereby established the approximate form of the self-similar solution:

$$f(v)=\begin{cases} C/v^4 \\ Cv^{-4}(v/D_1t-3)\exp(4-v/D_1t). \end{cases} \tag{17}$$

$v_0(t)=4D_1t$. The number of resonance particles decreases with time:

$$n_R=4n_0\pi C/v_0(t)\sim 1/t.$$

The width of the interaction region for the function (17) $\Delta v\sim v_0/4$. The exact form of the solution of the set (8), (9) in the region $v\sim v_0$ can be established only numerically but we can give a more exact value for Δv from the condition that the number of particles is conserved.

$$\int_{v_0}^{\infty} f_\infty(v)\,dv=\varphi(t)\int_{\xi_0}^{\infty}\Phi(\xi)\,d\xi,$$

whence

$$\Delta v\sim v_0.$$

One must, however, prove that the scaling substitution of variables found by us is valid not only for Eq. (9′) but also for the exact set (8), (9) in the region $v\sim v_0$.

We turn to Eq. (8). We make the substitution of variables: $x=\omega_{\text{pe}}c_s\tau/D_1t$. The initial condition $\mathscr{E}(\tau\to-\infty)=\mathscr{E}_i$ changes to a completely similar one while Eq. (8) takes the form

$$\frac{d\mathscr{E}}{dx}=\Gamma_1\mathscr{E}x\int_x^{\infty}\frac{d\xi\,\xi\Phi(\xi)}{\text{ch}^2(x/\xi)}. \tag{18}$$

The self-similarity of Eq. (8) has thereby been proved. We show that the self-similar substutition (13) is valid for Eq. (9). In the expression (12) for the diffusion coefficient $\chi=x/\xi$ while the ratio $R^2(\chi)/R_0^2(\chi)\approx\mathscr{E}(x)/\mathscr{E}_i$. Therefore

$$D(v)=\frac{2D_1v}{\xi^2}\int_0^{\xi} dx\,x\frac{\mathscr{E}(x)}{\mathscr{E}_i}. \tag{19}$$

Equation (9) with the diffusion coefficient (19) indeed satisfies the self-similar substitution (13).

Finally, we determine the constant C in Eq. (17) for the distribution function. It is uniquely determined by the condition that the soliton is completely damped in the interaction region $v_0/2<v<v_0$. Equation (18) allows us to make the estimate $C=\alpha c_s$, $\alpha\sim 1$. It has no sense to determine this quantity with greater accuracy because of the simplifications which we have made. It is important to note that in the heating process there are at all times two sections which differ in principle in their characteristics of the distribution function of the accelerated electrons (this follows from the solutions given above). To wit, to the left of $v_0(t)$ (i.e., when $v<v_0(t)$) a stationary distribution function is formed $f_\infty(v)\sim 1/v^4$ which contains the main number of particles which increases with time because of the motion of the boundary $v_0(t)$ but which does not contribute to the total energy of the tail. On the other hand, to the right of $v_0(t)$ ($v>v_0(t)$) an exponentially decreasing front of the distribution function is formed $f(v)\sim\exp(-v/D_1t)$ with a number of particles in it which decreases with time as follows

$$\frac{n_R}{n_0}\sim\left(\frac{m}{M}\right)^{1/2}\frac{v_{Te}}{v_0}\sim\frac{1}{t},$$

but it is just in this part that the whole energy of the tail is concentrated and it is only into this small group of particles that the whole energy of the external source of the Langmuir oscillations is put.

2. THE THEORY OF PLASMA HEATING IN THE k-REPRESENTATION

A problem similar to the one considered above was solved earlier[7] by the Fourier transform method. The soliton model of the collapse seems more physical to us but we consider our problem also in the k-representation in order to establish the correspondence between the models. When deriving the basic equations it was assumed[7] that each soliton corresponds to only a single harmonic with $k\sim k_0$. Furthermore the quasi-linear diffusion equation and the equation for the energy flux in k-space, taking damping into account, were used. In the three-dimensional problem these equations look as follows:

$$\frac{\partial f}{\partial t}=\frac{1}{v^2}\frac{\partial}{\partial v}\frac{D}{v}\frac{\partial f}{\partial v},$$
$$\frac{\partial W_k}{\partial t}=\frac{1}{k^2}\frac{\partial}{\partial k}\left(k^2W_k\frac{dk}{\partial\tau}\right)+2\gamma_kW_k, \tag{20}$$

where

$$D=16\pi^3\left(\frac{e}{m}\right)^2\omega_{pe}\int\frac{W_k\,dk}{k},\qquad \gamma_k=-\pi^2\omega_{pe}v^3f(v)\,|_{v=\omega/k},$$

$dk/d\tau$ is a function of k given by Eqs. (2). We have already mentioned above that the most probable mode when dissipation is present is a purely sonic collapse when $\delta\approx c_s\tau$, i.e., $dk/d\tau=-k^2c_s$.

We consider this case in the framework of the model of Galeev *et al.*[7] Making the self-similar substitution

$$f(v,t)=\frac{1}{2\pi^2c_s^3(\omega_{pe}t)^4}\Phi(\xi),\qquad \xi=\frac{v}{\omega_{pe}c_st},$$
$$W_k=\frac{m}{M}\frac{nT}{4\pi^2}\omega_{pe}c_s^3t^4w(\eta),\qquad \eta=\frac{1}{kc_st}, \tag{21}$$

we get the set of Eqs. (20) in self-similar form

$$4\Phi+\xi\Phi_\xi'+\frac{1}{\xi^2}\frac{d}{d\xi}\frac{1}{\xi}\int_0^\xi w\frac{d\eta}{\eta}\frac{d\Phi}{d\xi}=0, \tag{22}$$

$$\eta w_\eta'+\eta^4\frac{\partial}{\partial\eta}\frac{w(\eta)}{\eta^4}-4w(\eta)-\eta^3\Phi(\eta)w(\eta)=0. \tag{23}$$

It follows from Eq. (23) that

$$w(\eta)\sim\exp\int d\eta\frac{4+4\eta+\eta^4\Phi}{\eta(\eta+1)}. \tag{24}$$

We substitute expression (24) into Eq. (22):

$$4\Phi+\xi\Phi_\xi'+\frac{1}{\xi^2}\frac{d}{d\xi}\frac{1}{\xi}\left(\int_0^\xi d\eta\frac{w_0}{\eta}\int_0^\eta d\zeta\frac{4+4\zeta+\zeta^4\Phi(\zeta)}{\zeta(\zeta+1)}\right)\frac{d\Phi}{d\xi}=0. \tag{25}$$

If we assume that the function $\Phi(\xi)$ decreases exponentially as $\xi\to\infty$ we have

$$w(\eta)\,|_{\eta\to\infty}\sim\eta^4.$$

Equation (25) simplifies then considerably and when $\xi\gg w_0$ has the simple solution:

$$\Phi=\text{const}\,(\xi/w_0-3)\exp(-\xi/w_0),$$

which is, apart from the notation, exactly identical with the solution (15). As $\xi\to0$, assuming the quantity $w(\xi)$ to be exponentially small we get $\Phi=\Phi_0\xi^{-4}$ (see (16)) where

$$w(\eta)=w_1\eta^{4+\Phi_0},\qquad w_1\ll w_0. \tag{26}$$

Unfortunately, when $\xi\sim\xi_0=w_0$ the solution of Eq. (25) cannot be obtained in analytic form as in the soliton model and we can only estimate the constant Φ_0 from the condition that the noise density decreases exponentially in the interval $(\xi_0/2,\xi_0)$. We must then recognize that in this interval the function $\Phi(\xi)$ will not decrease as fast as ξ^{-4} as can be seen from Eq. (25). As to order of magnitude $\Phi_0\sim1$, i.e., in the physical variables

$$f_\infty(v)=\Phi_0c_sv^{-4}/2\pi^2,\qquad v\ll v_0(t)=\beta\omega_{pe}tc_sw_0, \tag{27}$$

$\Phi_0,\ \beta\sim1$.

In the k-representation we obtain thus a solution of the same form as in the soliton model.

One can easily generalize the results to the case $dk/d\tau\sim k^n$ for any $1<n<3$. In particular when $n=5/2$ (supersonic collapse) we can perform in the set (20) the change of variables

$$f=\varphi(\xi)v^{-9/2},\qquad W_k=v^{9/2}U(\xi),\qquad \xi=v/v_0(t), \tag{28}$$

the system is then reduced to the self-similar form and one can obtain a solution which in all respects is similar to the one obtained in the previous section or in[7]. One can estimate the characteristic velocity $v_0(t)$ from the energy conservation law:

$$\int_0^\infty fv^4\,dv=v_0^{1/2}\int_0^\infty\frac{\varphi(\xi)}{\xi^{1/2}}d\xi=\int_0^t Q(t')\,dt'. \tag{29}$$

We shall not give the details of the calculations but only the main results:

$$f_\infty(v)=\text{const}/v^{9/2},\qquad \varphi(\xi)\sim\exp(-\xi^{1/2}),\qquad \xi\gg1,$$
$$n_R(t)=\left(\frac{m}{M}\right)^{1/4}n\left(\frac{v_{Te}}{v_0}\right)^{1/2},\qquad v_0(t)=\frac{(Qt)^2}{n^2v_{Te}^3(m/M)^{1/2}}. \tag{30}$$

These results were obtained from the equations of[7] for $dk/d\tau\sim k^{5/2}$ which gives us the possibility to compare them with the results of that paper. To wit, in[7] a solution of the set (20) with a constant particle flux was given for the supersonic collapse case $dk/d\tau\sim k^{5/2}$:

$$\frac{1}{k^2}\frac{\partial}{\partial k}k^{9/2}W_k=\text{const}\cdot v^3 f(v)\,W_k,$$
$$\frac{D}{v}\frac{\partial f}{\partial v}=\text{const}, \tag{31}$$

which gave the result

$$f(v)\sim v^{-9/2},\qquad W_k\sim k^{-11/2}. \tag{32}$$

The solution (30) differs from the particular solution for f, shown in[7], although it can be seen by comparing expressions (30) and (32) that they are formally the same in the range $v<v_0(t)$. However, the meaning of the self-similar solution (30) and also the solution (17) for $n=2$ (sonic collapse) is different.

If in the heating regime in the solution of[7] new particles (a constant flux of particles through the interaction region) get involved in our self-similar solution there remain increasingly fewer particles in the heating region:

$$\frac{n_R}{n_0}=\left(\frac{m}{M}\right)^{1/2}\frac{v_{Te}}{v_0}\sim\frac{1}{t},$$

and those which leave the heating regime form to the left of $v_0(t)$ the distribution function $f_\infty(v)\sim 1/v^{n+2}$ so that the total number of particles in the tail is constant and independent of the generation power. The whole energy of the external source is then, as we have already shown, injected at once into a small group of resonance particles $n_R\sim 1/t$ to the right of $v_0(t)$ and those also determine the energy conservation of the plasma.

Moreover, the existence of the stationary solution (32) with a constant particle flux into the high-velocity region requires the introduction of a sink of particles at the right-hand boundary of the distribution $f(v)$ as in the opposite case there would occur an accumulation of particles in the region where the noise is generated which contradicts the stationarity condition. In our view the self-similar solution of the time-dependent problem is more physical as it conserves the total number of particles and does not require the inclusion of additional physical mechanisms of a sink. Even if a stationary regime were possible, its establishment requires an appreciably larger energy contribution as the energy of the tail is determined by the upper velocity boundary. The process of establishing a stationary regime is in this case also described by the solutions obtained in the present paper.

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