# Theory of high-frequency pinched discharge in a dense gas with allowance for electron overheating

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A theory is developed for a high-frequency pinched discharge in a dense gas under conditions of strong skin effect, with allowance for the deviation of the electron temperature from the temperature of the ions and of the neutral atoms under the influence of the electromagnetic field. The structure of the transition layer on the plasma boundary is investigated. The dependence of the plasma temperature on the power input and on the surface resistance of the plasma is obtained. In the presence of strong skin effect, the decrease of the field in the interior of the plasma leads to a spatial inhomogeneity of the electron temperature, causing in turn a thermal-diffusion electron current. This current transfers the electrons from the transition layer both into the interior of the plasma and to the outside. With increasing temperature difference, the electron density inside the plasma decreases at an ever slower rate. There is a critical temperature difference, starting with which the electron density outside the plasma decreases so slowly that the total heat release becomes infinite. This means that no low-temperature pinched discharge can exist in a sufficiently strong field.

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## 1. INTRODUCTION

To ascertain the degree to which Kapitza's experimental results[1–6] offer evidence of the presence of a hot region in a pinched microwave discharge it was necessary to develop in detail a low-temperature theory of a high-frequency discharge in a dense gas. To this end, the equilibrium structure of the transition layer in the high-frequency discharge was calculated,[7,8] a cylindrical discharge was considered without any limitations on the size of the skin effect, and the question of the heat exchange in the equilibrium structure of the high-frequency discharge in a gas stream was investigated.[9]. In[10–12], the deviation from local thermodynamic equilibrium, due to electron diffusion, was considered and a study was made of the role of the inelastic processes that determine the particle balance in a high-frequency discharge.

In[7–12], however, the high-frequency field $E$ was assumed to be small in comparison with the "plasma" field $E_p$:

$$E \ll E_p = [3mTe^{-2}\delta_{el}(\omega^2+\nu^2)]^{1/2}, \tag{1.1}$$

where $e$ and $m$ are the charge and mass of the electron, $T$ is the plasma temperature, $\omega$ is the field frequency, $\nu$ is the collision frequency, and $\delta_{el} = 2m/M$ is the fraction of energy transferred from the electrons to the ions in elastic collisions. The condition (1.1) has made it possible to neglect the difference between the electron temperature and the temperature of the ions and the neutral atoms. Starting with the work of Druyvestein[13] and Davydov,[14] heating of electrons in an electromagnetic field was investigated many times both in a gas-discharge plasma[15–20] and in a solid-state plasma.[21] However, even though the distribution function of the electrons in a strong electromagnetic field is essentially known, no self-consistent theory of a high-frequency discharge with allowance for the temperature difference has been constructed as yet.

The purpose of the present paper is to construct a self-consistent theory of a high-frequency discharge in a dense gas under conditions of a strong skin effect, with allowance for the overheating of the electrons. The situation of strong skin effect can be realized in the outer region ("jacket") of the Kapitza pinch discharge,[1–6] and also in high-pressure induction plasmatrons.[22–24]

Under the conditions of a strong skin effect, when the depth of penetration of the field into the plasma is small in comparison with its linear dimensions, the high-frequency field is in principle inhomogeneous. Under electron overheating conditions, this leads to an inhomogeneity in the electron temperature. A qualitatively new phenomenon that appears in this situation but is absent in the case of local thermodynamic equilibrium is thermal diffusion of the electrons. Thus, besides the diffusion flux, which is proportional to the gradient of the electron density, there is also a thermal diffusion flux of electrons, proportional to the gradient of their temperature. The thermal-diffusion flux moves the electrons from the transition layer both into the interior of the plasma and out of the plasma, as a result of which, with increasing field intensity (with increasing temperature difference) the electron density outside the plasma decreases more and more slowly. There exists a critical temperature difference, starting with which the electron density outside the plasma decreases so slowly that the total Joule heat release becomes infinitely large. This means that no low-temperature pinch discharge can exist in a sufficiently strong field.

Besides influencing the transport processes, the temperature difference leads naturally to a shift of the ionization equilibrium, changing the frequency of atom ionization by electron impact. The relative role of the shift of the ionization equilibrium and thermal diffusion due to overheating of the electrons is analyzed in the Appendix. We shall henceforth neglect the influence of the overheating of the electrons on the ionization-recombination balance; this is justified at not too high a neutral-atom temperature and at a sufficiently high electromagnetic-field frequency.

We consider a high-frequency discharge in a stationary regime in which the Joule heat released in the plasma by the electromagnetic field is transferred by thermal conduction of the gas to the cooled walls of the vessel. The main condition for the applicability of the theory developed below is smallness of the energy-transfer length $l_e = \bar{v}/\nu\delta_{el}^{1/2}$ in comparison with the depth of penetration $\delta = c(8\pi\omega\sigma)^{-1/2}$ of the field into the plasma:

$$l_e \ll \delta. \tag{1.2}$$

Here $\sigma$ is the electric conductivity of the plasma, $v$ is the average thermal velocity of the electrons, and $c$ is the speed of light.

## 2. EQUATIONS OF THE SELF-CONSISTENT STRUCTURE OF THE DISCHARGE

Under the conditions of a strong skin effect, when the depth of penetration of the field into the plasma is small in comparison with the discharge dimensions, all the quantities are functions of a single coordinate $x$ directed along the electron-density gradient. The kinetic equation for the electron distribution function $f_e$ is

$$\frac{\partial f_e}{\partial t} + v_x\frac{\partial f_e}{\partial x} - \frac{e}{m}\left(E_0\frac{\partial f_e}{\partial v_x} + E_y\cos\omega t\frac{\partial f_e}{\partial v_y}\right) = S. \tag{2.1}$$

Here $E_0$ is the static electric field due to the charge separation, $E_y$ is the amplitude of the high-frequency field that maintains the discharge, and $S$ is the collision integral. An electron distribution function satisfying Eq. (2.1) can be represented in the form (see, e.g., [17])

$$f_e = f_0 + \mathbf{v}\mathbf{f}_1/v,$$

where the functions $f_0$ and $\mathbf{f}_1$ depend on $|v|$ (and, of course, on $x$). We assume that the degree of ionization of the gas is so small that the main contribution to the electric conductivity is made by elastic collisions of the electrons with the neutral atoms. (For hydrogen at atmospheric pressure this condition is satisfied at $T_0 < 8000\,^\circ$K.) Under condition (1.2), for a rapidly alternating field ($\omega \gg \nu\delta_{el}$) under stationary conditions, the symmetrical part $f_0$ of the distribution function of the electrons satisfies the equation

$$\frac{\partial}{\partial v} v^2\nu(v)\left[\left(T_0 + \frac{e^2|E_y^2(x)|}{3\delta_{el}mv^2(v)}\right)\frac{1}{mv}\frac{\partial f_0}{\partial v} + f_0\right] = 0, \quad \nu\delta_{el} \ll \omega \ll \nu. \tag{2.2}$$

Here

$$\nu(v) = N_0 v\int q(v,0)(1-\cos\theta)\,d\Omega$$

is the frequency of the collisions of the electron with the neutral atoms and $T_0$ is the neutral-atom temperature. From (2.2), recognizing that $f_0$ is finite at $v=0$, we obtain the well-known expression for the distribution function:

$$f_0 = C(x)\exp\left\{-\int_0^v \frac{mv\,dv}{T_0 + e^2|E_y^2(x)|/3\delta_{el}mv^2(v)}\right\}, \quad \nu\delta_{el} \ll \omega \ll \nu. \tag{2.3}$$

Here $C(x)$ is an arbitrary function of $x$, the form of which is determined by the normalization condition

$$N_e(x) = \int f_0\,d^3v. \tag{2.3'}$$

The electron concentration $N_e(x)$ under stationary conditions satisfies the particle-balance equation, which is obtained from (2.1) by integrating over the velocities:

$$\frac{\partial q_x}{\partial x} = \int S^n\,d^3v. \tag{2.4}$$

The projection of the particle flux along the concentration gradient $q_x$ is expressed in the following form in terms of $f_0$:

$$q_x = \int v_x\frac{\mathbf{v}\mathbf{f}_1}{v}d^3v = -\int\frac{v_x^2}{\nu(v)}\left(\frac{\partial f_0}{\partial x} - \frac{eE_0}{mv}\frac{\partial f_0}{\partial v}\right)d^3v. \tag{2.5}$$

When integrating over the velocities, a nonzero contribution to (2.4) is made only by collisions that do not conserve the number of electrons, that is, accompanied by ionization of the atoms, by recombination, and by sticking of the electrons. We shall assume that the loss of electrons is due mainly to their sticking to the neutral atoms with formation of negative ions. In this case we have[10]

$$\int S^n\,d^3v = -\beta(N_e - N_{e\,\mathrm{eq}}(T_0)), \tag{2.6}$$

$\beta$ is the sticking coefficient, and $N_{e\,\mathrm{eq}}(T_0)$ is the equilibrium electron density determined by the Saha formula. The electromagnetic field satisfies the equation[7]

$$\frac{d}{dx}\frac{1}{\sigma}\frac{d^3|E_y^2|}{dx^3} - \frac{64\pi^2\omega^2\sigma}{c^4}|E_y^2| = 0, \tag{2.7}$$

where the plasma electric conductivity $\sigma$ is expressed in terms of $f_0$ in the following manner:

$$\sigma = -\frac{e^2}{m}\int\frac{v_y^2}{v\nu(v)}\frac{\partial f_0}{\partial v}d^3v, \quad \nu(\bar{v}) \gg \omega. \tag{2.8}$$

Under condition (1.2), the energy acquired by the electrons from the field is transferred by collision to the neutral gas and is given up by thermal conductivity to the cooled walls of the vessel. Under stationary conditions, the thermal-conductivity equation without allowance for radiation is

$$\frac{d}{dx}\varkappa\frac{dT_0}{dx} + \frac{1}{2}\sigma|E_y^2| = 0, \tag{2.9}$$

where $\varkappa$ is the thermal conductivity of the gas.

The smallness of the Debye radius in comparison with the depth of penetration of the field into the plasma leads to quasineutrality. The large value of the coefficient of recombination of the positive and negative ions causes the negative ions to produce as a result of attachment of the electrons to the atoms to recombine rapidly, so that their concentration $N_-$ is small in comparison with the electron density. Taking into account the ambipolar character of the electron diffusion and eliminating from the equations the static field $E_0$ by the same method as

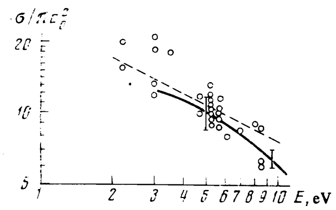


FIG. 1. Measured cross sections for the elastic scattering of electrons by hydrogen atoms. Solid curve—results of[25], the points are from the data of[26]. The dashed line indicates the slope $\sigma \sim v^{-1}$, and $a_0$ is the Bohr radius.

in the Appendix of[10], we obtain the following electron balance equation:

$$\frac{d}{dx} D_i\left\{\frac{dN_e}{dx} - \frac{mN_e}{T_0}\int \frac{v_x^2}{\nu(v)}\frac{\partial f_0}{\partial x} d^3v \Big/ \int \frac{v_x^2}{v\nu(v)}\frac{\partial f_0}{\partial v} d^3v\right\} = \beta[N_e - N_{e\,\mathrm{eq}}(T_0)]. \tag{2.10}$$

Here $D_i$ is the ion diffusion coefficient. Equations (2.7), (2.9), and (2.10) determine in a self-consistent manner the temperature $T_0$ of the neutral gas, the distribution $N_e(x)$ of the electron density, and the electromagnetic field $E_y^2$ with allowance for the overheating of the electrons. The electron distribution function (2.3), with the normalization (2.3′) taken into account, and the electric conductivity (2.8) are functionals of the high-frequency field $|E_y^2(x)|$.

In the interior of the plasma, as $x \to +\infty$, the temperature of the gas $T_0$ tends to a certain limiting value $T_m$. The electron density tends in this case to its equilibrium value $N_m \equiv N_{e\,\mathrm{eq}}(T_m)$, for as $x \to +\infty$ the electromagnetic field $E_y(x)$, which disturbs the equilibrium of the electrons with the ions and atoms, tends exponentially to zero. Outside the plasma, as $x \to -\infty$, the electron density tends to zero and the flux density tends to a specified value equal to the electromagnetic-energy flux $S_0$ into the discharge:

$$\varkappa dT_0/dx = S_0, \quad x \to -\infty. \tag{2.11}$$

## 3. COLLISIONS OF ELECTRONS WITH NEUTRAL ATOMS

The distribution function of the electrons in a high-frequency field (2.3) is very sensitive to the form of the function $\nu(v)$. To continue the analysis we must specify the form of $\nu(v)$. The equations are simplified to the greatest degree if it is assumed that the collision frequency is independent of the velocity, $\nu(v) = \mathrm{const}$. In this case the electron velocity distribution is Maxwellian

$$f_0 = C(x)\exp\{-mv^2/2T_e(x)\} \tag{3.1}$$

with a temperature

$$T_e(x) = T_0 + e^2|E_y^2(x)|/3\delta_{el} m\nu^2, \qquad \nu\delta_{el} \ll \omega \ll \nu. \tag{3.2}$$

We call attention to the fact that the Maxwellian distribution function causes the integral of the electron-electron collisions to vanish. For this reason, allowance for the interelectron collisions does not affect the distribution function (3.1).

A velocity-independent frequency of the collisions between the electrons and the neutrals results from consideration of scattering within the framework of classical mechanics as scattering by a polarization potential that decreases with distance in proportion to $r^{-4}$. The scattering cross section is in this case inversely proportional to the velocity. Figure 1 shows the measured cross sections for elastic scattering of electrons by monotonic hydrogen. The solid line shows the data of[25] and the points are from[26]. The dashed line indicates the slope $\sigma \sim v^{-1}$ corresponding to scattering by a polarization potential. As seen from Fig. 1 in the electron energy range from 2 to 8 eV the assumed approximation $\nu(v) = \mathrm{const}$ does not go beyond the limits of experimental accuracy with which the cross sections $\sigma(v)$ are known. The results of theoretical calculations[27–32] of $\sigma(v)$ in the given energy region are also close to $v^{-1}$ (see[33], p. 150, and the review[34]). We were unable to find in the literature any reliable experimental data for the energy region $\lesssim 1$ eV. In this region, the electron elastic scattering cross sections can have their own singularities, particularly those connected with the Ramsauer effect.[1)] The distribution function (3.1), of course, does not reflect the discharge peculiarities connected with the singularities of the function $\sigma(v)$ for different gases, but describes the general character of the overheating of the electrons by a high-frequency field, including the case of a strong skin effect.

## 4. THERMAL DIFFUSION OF ELECTRONS IN A HIGH-FREQUENCY DISCHARGE

For the electron distribution function (3.1), the integrals in the balance equation (2.10) are equal to

$$\int \frac{v_x^2}{\nu} f_0\, d^3v = \frac{T_e N_e}{m\nu}, \qquad \int \frac{v_x^2}{v\nu}\frac{\partial f_0}{\partial v} d^3v = -\frac{N_e}{\nu},$$

and the usual expression $\sigma = e^2 N_e/m\nu$ is obtained for the electric conductivity of the plasma (2.8). The electron balance equation (2.10) takes the form

$$\frac{d}{dx}(q_x^D + q_x^T) = -\beta(N_e - N_{e\,\mathrm{eq}}(T_0)). \tag{4.1}$$

Here

$$q_x^D = -D_a dN_e/dx \tag{4.2}$$

is the electron diffusion flux, proportional to the electron concentration gradient, and $D_a = 2D_i$ is the ambipolar diffusion coefficient.

The electron temperature $T_e(x)$ (3.2), being proportional to the square of the modulus of the electromagnetic-field amplitude, is inhomogeneous under conditions of overheating and skin effect, and changes generally speaking over the same distances as $N_e(x)$. As a result, besides the diffusion flux (4.2), the electron transport equation (4.1) is supplemented by an electron thermal diffusion flux

$$q_x^T = -D_a \frac{d}{dx} \frac{\sigma |E_y^2|}{6\delta_{el} \nu T_0}, \quad (4.3)$$

which is proportional to the gradient of the Joule heat released in the plasma. The heat release $\sigma\,|E_y^2|/2$ reaches a maximum in a transition layer at the plasma boundary and decreases both in the interior of the plasma (owing to the skin effect) and outside the plasma (because of the decrease in the electron density). The thermal-diffusion flux (4.3) vanishes and reverses sign at the maximum of the heat release, and thus moves electrons out of the transition layer both into the interior of the plasma and to its outside.

Eliminating $\sigma\,|E_y^2|$ with the aid of the thermal-conductivity equation (2.9), we can express the thermal-diffusion flux of the electrons in terms of the derivatives of the temperature of the neutral gas:

$$q_x^T = -D_a \frac{d}{dx} \frac{1}{3\delta_{el} \nu T_0} \frac{d}{dx} \varkappa \frac{dT_0}{dx}. \quad (4.4)$$

## 5. DIMENSIONLESS EQUATIONS AND THEIR INVESTIGATION

Just as in[7–12], we change to dimensionless variables. Let $T_m$ be the limiting value of the neutral-gas temperature $T_0$ inside the plasma. It is convenient to introduce the dimensionless electron density $n$, the dimensionless temperature $\Theta$ of the atoms, and the dimensionless coordinate $\zeta$ as follows:

$$n = \frac{N_e}{N_{e\,eq}(T_m)}, \quad \Theta = \frac{I(T_m - T_0)}{2T_m^2}, \quad \zeta = \frac{x}{\delta_m}, \quad (5.1)$$

where $\delta_m = c(8\pi\omega\sigma_m)^{-1/2}$ is the depth of penetration of the field into a plasma with electron density $N_m = N_{e\,eq}(T_m)$, and $I$ is the ionization potential of the atoms. Using the condition $T_m \ll I$, we have

$$N_{e\,eq}(T_0)/N_{e\,eq}(T_m) = e^{-\Theta}.$$

For pinched discharges, in which either the plasma dimensions or the dimensions of the transition layer at the plasma boundary are small in comparison with the distance to the cooled walls of the vessel, the neutral-gas temperature $T_0$ changes slowly in comparison with the electron density $N_e$. The thermal conductivity $\varkappa$ of the neutral gas and the ambipolar diffusion coefficient $D_a$ depend on the temperature $T_0$, which changes insignificantly in the transition layer. This makes it possible to put in the equation $\varkappa = \varkappa_m \equiv \varkappa(T_m)$ and $D_a = D_{am} \equiv D_a(T_m)$. The coefficient $\beta$ of electron sticking to the atoms is a relatively weak function of electron temperature.[10] To simplify the calculations we shall henceforth assume $\beta$ to be constant.

Allowance for the ionization of the atoms by electron impact adds to the right-side of (4.1) a term proportional to $\alpha N_e$, where $\alpha$ is the ionization coefficient. Under these conditions the discharge can have a stationary state at $\beta > \alpha$. If the atom ionization by electron impact must be taken into account, then $\beta$ in our equations should be taken to mean the difference between the adhesion and ionization coefficients. A detailed analysis of questions involved in ionization-recombination kinetics is contained in the review by Biberman, Vorob'ev, and Yakubov.[37]

We introduce the dimensionless parameters $\gamma = \delta_m/d$—the ratio of the depth of penetration of the field into the plasma to the diffusion length $d = (D_{am}/\beta)^{1/2}$ and $\lambda = 2\varkappa_m T_m/3I\delta_{el}\nu N_m \delta_m^2$.

In the dimensionless variables (5.1), the thermal-diffusion flux (4.4) is equal to

$$q_x^T \delta_m / N_m D_{am} = -\lambda d^3\Theta/d\zeta^3, \quad (5.2)$$

and the equations describing the charge with allowance for the electron overheating take the form

$$\frac{d^3}{d\zeta^3} \frac{1}{n} \frac{d^2\Theta}{d\zeta^2} - n\frac{d\Theta}{d\zeta} = 0, \quad (5.3)$$

$$\frac{d^2 n}{d\zeta^2} + \lambda \frac{d^4\Theta}{d\zeta^4} = \gamma^2 (n - e^{-\Theta}). \quad (5.4)$$

The electron temperature (3.2) in terms of the variables (5.1) is

$$T_e = T_0(1 + 2\lambda n^{-1} d^2\Theta/d\zeta^2).$$

The parameter $\lambda$, which characterizes the overheating of the electrons and the thermal diffusion produced by them, can be rewritten in the form

$$\lambda = \frac{16\pi}{3} \frac{e^2}{mc^2} \frac{\omega\varkappa_m T_m}{I\delta_{el}\nu^2} = 2.93 \cdot 10^7 \frac{\omega\varkappa_m T_m}{I\delta_{el}\nu^2}, \quad (5.5)$$

where $\omega$ and $\nu$ are in sec$^{-1}$, $\varkappa_m$ in W/cm °K, $T_m$ in °K, $I$ in eV, $\delta_{el} = 2m/M$, where $M$ is the mass of the ion or atom. For example, for hydrogen at atmospheric pressure, a temperature $T_m = 6 \cdot 10^3$ °K and a field frequency $\omega = 10^{10}$ sec$^{-1}$ we have $\varkappa_m = 0.04$ W/cm °K, $\nu = 2.3 \cdot 10^{11}$ sec$^{-1}$, $I = 13.6$ eV, $\delta_{el} = 10^{-3}$, and for the temperature-difference parameter we obtain $\lambda = 0.1$. With increasing gas pressure, the number of collisions at a given temperature increases rapidly. Therefore the overheating of the electrons decreases rapidly with increasing gas pressure at a fixed temperature. With decreasing pressure $p$, the overheating parameter $\lambda$ increases rapidly (in proportion to $p^{-2}$, if we disregard the weak dependence of the thermal conductivity of the gas on the pressure), and can become of the order of and larger than unity. In this case the electron overheating leads to a strong thermal diffusion, which is decisive significance for the transport of the electrons in the discharge. At a fixed temperature and a fixed gas pressure, the electron overheating in the considered frequency interval $\delta_{el}\nu \ll \omega \ll \nu$ increases in proportion to the field frequency $\omega$.

Outside the plasma, as $\zeta \to -\infty$, the electron density tends to zero and the heat flux tends to a specified finite value (2.11), equal to the electromagnetic-energy flux into the discharge. In terms of the dimensionless variables (5.1), these conditions are

$$n \to 0, \; d\Theta/d\zeta < \infty, \; \zeta \to -\infty. \quad (5.6)$$

Inside the plasma, as $\zeta \to +\infty$, the gas temperature approaches exponentially its maximum value, and the electron density tends to the equilibrium value:

$$\Theta\to e^{-(\zeta-\zeta_0)},\quad n\to 1,\quad \zeta\to+\infty. \tag{5.7}$$

Equations (5.3) and (5.4) with boundary conditions (5.6) and (5.7) describe a stationary high-frequency discharge with allowance for the overheating of the electrons in the case of a strong skin effect. The coordinate $\zeta$ does not enter explicitly in expressions (5.3) and (5.4). Therefore the dimensionless electron density $n$ and the dimensionless gas temperature $\Theta$ are functions of $\zeta-\zeta_0$ ($\zeta_0$ is an arbitrary constant) and depend on $\gamma$ and $\lambda$, which serve as parameters. Outside the plasma, as $\zeta\to-\infty$, the dimensionless heat flux $d\Theta/d\zeta$ assumes a constant value that depends on the parameters $\gamma$ and $\lambda$:

$$d\Theta/d\zeta=-F(\gamma,\lambda),\ \zeta\to-\infty. \tag{5.8}$$

Comparing (2.11) with (5.8) we obtain the electromagnetic-energy flux density $S_0$ needed to heat the gas to the temperature $T_m$

$$S_0(T_m)=2F(\gamma,\lambda)\varkappa_m T_m^2/I\delta_m. \tag{5.9}$$

The functions $F(\gamma,\lambda)$, $n(\zeta-\zeta_0)$ and $\Theta(\zeta-\zeta_0)$ can be obtained only by solving the equations.

In the limiting case $\lambda\ll 1$, when the overheating of the electrons is negligible, Eq. (5.4) goes over into Eq. (2.11) of[10], where a study was made of a high-frequency discharge with allowance for the deviation, due to diffusion of the electrons, from the local thermodynamic equilibrium, characterized by the value of the parameter $\gamma$.

The electron balance equation (5.4) describes the deviation of the electron density $n$ from equilibrium as a result of transport processes (diffusion—the term $d^2n/d\zeta^2$—and thermal diffusion—the term $\lambda d^4\Theta/d\zeta^4$). Outside the plasma, as $\zeta\to-\infty$, the dimensionless gas temperature $\Theta$ increases linearly with increasing $|\zeta|$, so that the term $\gamma^2e^{-\Theta}$, which describes in our model the thermal ionization of the gas, tends exponentially to zero. As shown in[10], the diffusion of the electrons in conjunction with their sticking to the neutral atoms leads to an exponential decrease of the electron density with increasing distance outside the discharge. We shall show that thermal diffusion in conjunction with formation of negative ions leads to a slower, power-law decrease of the electron density with increasing distance outside the discharge. Omitting from (5.4) the terms that decrease exponentially outside the discharge, we have

$$n=(\lambda/\gamma^2)\,d^4\Theta/d\zeta^4,\ \zeta\to-\infty. \tag{5.10}$$

The term $n d\Theta/d\zeta$ in (5.3) tends to zero together with the electron density $n$. Therefore

$$\frac{d^3}{d\zeta^3}\frac{1}{n}\frac{d^2\Theta}{d\zeta^2}=0\quad\text{as}\quad\zeta\to-\infty. \tag{5.11}$$

Integrating (5.11), we obtain

$$d^2\Theta/d\zeta^2={}^1/_2[c_0(\zeta-\zeta_0)^2+c_1]n,\ \zeta\to-\infty. \tag{5.12}$$

The integration constants $c_0$ and $c_1$ are functions of the parameters $\gamma$ and $\lambda$, which can be obtained only by solving Eqs. (5.3) and (5.4) at all values of $\zeta$. Naturally, at $\gamma\sim 1$ and $\lambda\sim 1$ the constants $c_0$ and $c_1$ are also of the order of unity. As $|\zeta-\zeta_0|\to\infty$, the constant $c_1$ of (5.12) can be omitted. Eliminating the electron density $n$ from (5.10) and (5.12), we obtain the following equation for the coordinate dependence of the Joule heat release $\Theta''=d^2\Theta/d\zeta^2$ outside the discharge:

$$d^2\Theta''/d\zeta^2=[2\gamma^2/c_0\lambda(\zeta-\zeta_0)^2]\Theta'',\ \zeta\to-\infty. \tag{5.13}$$

Its solution, which tends to zero as $\zeta\to-\infty$, is of the form

$$\Theta''=A|\zeta-\zeta_0|^{-b},\ \zeta\to-\infty,$$

where the degree of decrease of the heat release with increasing distance outside the discharge is equal to

$$b=({}^1/_4+2/x)^{1/2}-{}^1/_2,\quad x=\lambda c_0/\gamma^2.$$

Thus, thermal diffusion of the electrons in conjunction with their sticking to the neutral atoms causes the heat release $\Theta''$, and consequently also the electron density $n\sim d^4\Theta/d\zeta^4$, to decrease outside the plasma in power-law fashion. The dependence of the exponent $b$ on $x=\lambda c_0/\gamma^2$ is shown in Fig. 2. With increasing $x$, the exponent $b$ decreases. The condition (5.6) that the heat flux be finite at $\zeta\to-\infty$ means that the heat release $\Theta''$ must decrease as $\zeta\to-\infty$ in integrable fashion, from which it follows that $b>1$ or

$$x=\lambda c_0/\gamma^2<1. \tag{5.14}$$

The condition (5.14) limits the region of the physically attainable values of the parameter $\lambda$ characterizing the overheating of the electrons and the thermal diffusion caused by it.

In the limiting case $\lambda\ll 1$ and $\gamma\sim 1$, the overheating of the electrons does not play any role, and thermal diffusion can be neglected in comparison with diffusion. This case is investigated in detail in[10].

## 6. ELECTRON OVERHEATING IN THE LIMIT OF WEAK DIFFUSION. CONTRACTION OF HIGH-FREQUENCY DISCHARGE

We consider below the limiting case of weak diffusion

$$\gamma\gg 1,\ \lambda\ \text{arbitrary}, \tag{6.1}$$

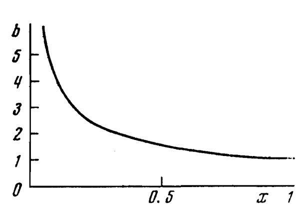


FIG. 2. Plot of the function $b(x)$.

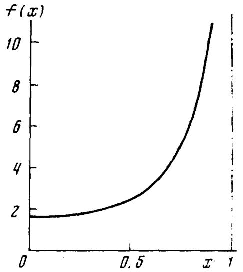


FIG. 3. Plot of $f(x)$ (6.5).

when thermal diffusion in conjunction with the sticking of the electrons to the atoms dominates over all the transport processes. This case is of greatest interest from the point of view of overheating of electrons in a high-frequency discharge. Neglecting in the transport equation (5.4) the term $d^2n/d\zeta^2$, which describes electron diffusion, we obtain the following expression for the dimensionless density

$$n=e^{-\Theta}+(\lambda/\gamma^2)\,d^4\Theta/d\zeta^4,\quad \gamma\gg 1. \tag{6.2}$$

In the weak-diffusion limit $\gamma \ll 1$, the deviation from local thermodynamic equilibrium is determined by the parameter $\lambda/\gamma^2$. Therefore the deviation from local equilibrium due to overheating of electrons has a strong influence, in the limit of weak diffusion, on the discharge characteristics only in the case of strong overheating, when $\lambda \approx \gamma^2 \gg 1$.

Substituting (6.2) in (5.2) and noting that $nd\Theta/d\zeta$ is a total differential, we obtain the first integral of (5.3):

$$\frac{d^2}{d\zeta^2}\left(\frac{\Theta''}{n}\right)=1-e^{-\Theta}+\frac{\lambda}{\gamma^2}\left(\Theta'''\Theta'-\frac{\Theta''^2}{2}\right). \tag{6.3}$$

Equation (6.3) takes into account the boundary conditions (5.7). Outside the plasma, as $\zeta \to -\infty$, the right-hand side of (6.3) tends to unity and thus the integration constant $c_0$ in formulas (5.12) and (5.14) is equal to unity in the limit of weak diffusion: $c_0=1$, $\gamma \gg 1$. In this case $x=\lambda/\gamma^2$, so that the region of the physically attainable values of the electron overheat parameters is given by

$$x=\lambda/\gamma^2<1,\quad \gamma\gg 1. \tag{6.4}$$

The total heat release at $x \geq 1$ becomes infinite because the region of the Joule heat release outside the plasma increases without limits because of the insufficiently rapid decrease of the electron density. This means that the distance to the cooling walls of the vessel cannot be regarded as large compared with the dimension of the heat-release region. Thus, a critical value $\lambda_{cr}=\gamma^2$ exists for the electron overheat parameter and determines the limit of the contraction of a high-frequency discharge. At $\lambda < \lambda_{cr}$ the above-described state of the discharge detached from the walls of the cooled vessel and occupying a small fraction of its volume is possible. At $\lambda \geq \lambda_{cr}$, the region of the Joule heat release is limited by the dimensions of the vessel, so that its entire volume can be occupied by the discharge. For an unpinched discharge, at $\lambda \geq \lambda_{cr}$, in contrast to a pinched one, an important role is played by the concrete mechanisms of the interaction of the electrons and the ions of the plasma with the vessel walls, mechanisms not considered by us.

The function $F(\gamma, \lambda)$ (5.8), which determines the dependence (5.9) of the temperature on the power, depends in the weak diffusion limit $\gamma \gg 1$ on $\lambda$ and $\gamma$ in the combination $x=\lambda/\gamma^2$:

$$F(\gamma,\lambda)=f(x),\quad x=\lambda/\gamma^2,\quad \gamma\gg 1. \tag{6.5}$$

The function $f(x)$ (6.5), obtained by numerically integrating (5.3) and (6.2) with the boundary conditions (5.6) and (5.7), is shown in Fig. 3. For the pinched discharge considered by us, the region where it is defined is limited to the interval (0, 1). At $x \ll 1$, the electron overheating does not play any role and we have $f(x)=1.57\ldots$, $x \ll 1$, as should be the case in the presence of local thermodynamic equilibrium in a discharge charge.[7] Near the contraction region, as $x \to 1$, the function $f(x)$ is proportional to $(1-x)^{-1}$, so that the energy flux density (5.9) increases without limit.

The spatial distributions of the heat release $\Theta''$, of the electron density $n$, and of the thermal-diffusion flux $-x\Theta'''$ (see (5.2)) are shown in Fig. 4 in the limit of weak diffusion $\gamma \gg 1$. The numbers on the curves indicate the corresponding values of the parameter $x$.

By the same method as in[7], we obtain for the surface resistance of the plasma in the limit of weak diffusion

$$R=(4\pi\omega\delta_m/c)\,f(\lambda/\gamma^2),\quad \gamma\gg 1, \tag{6.6}$$

where $f(x)$ is the same function as in Fig. 3. If the overheat of the electrons is small, then $f(x)=1.57\ldots$,

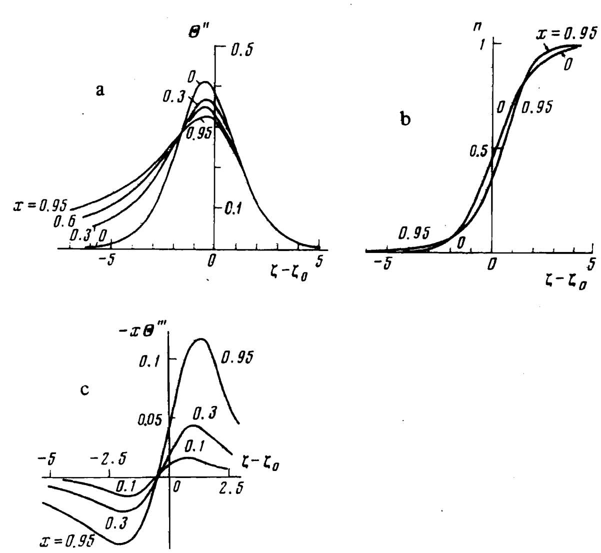


FIG. 4. Coordinate dependences of the heat release $\Theta''$ (Fig. a), of the electron density $n$ (Fig. b) and of the thermal-diffusion flux $x\Theta'''$ (Fig. c) in the transition layer at the discharge boundary in the limit of weak electron diffusion. The parameter $x$ characterizes the overheating of the electrons. Its values are indicated by the numbers on the curves.

$x \ll 1$ and formula (6.6) yields the surface resistance of an equilibrium discharge. In the opposite limiting case of strong overheating near the contraction boundary ($x \to 1$), the surface resistance increases without limit in proportion to $(1-x)^{-1}$, this being due to the Joule absorption of a large amount of electromagnetic energy in the external region of the discharge.

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## APPENDIX

Let us analyze the conditions under which the overheating of the electrons influences the transport (thermal-diffusion) processes more strongly than the ionization equilibrium. The thermal diffusion changes $\partial N_e/\partial t$ by an amount

$$\delta(\partial N_e/\partial t)_T = dq_x^T/dx.$$

In terms of the dimensionless variables (5.1) we have

$$\delta(\partial N_e/\partial t)_T = \lambda\Theta^{(4)} N_m D_{am} \delta_m^{-2}. \quad (A.1)$$

The change of $\partial N_e/\partial t$ due to the shift of the ionization equilibrium, with allowance for the overheating, is

$$\delta\left(\frac{\partial N_e}{\partial t}\right)_{\text{ion}} = N_a \int \sigma_{\text{ion}} v [f_0(T_e) - f_0(T_0)] d^3v,$$

where $f_0(T_e)$ is the distribution of the overheated electrons (3.1), $f_0(T_0)$ is the distribution function of the electrons in the absence of overheating, and $\sigma_{\text{ion}}$ is the cross section for the ionization of the atom by electron impact. For hydrogen, with accuracy sufficiency for our estimates, the cross section of the ionization by electron impact near the threshold is equal to $\sigma_{\text{ion}} \approx \pi a_0^2(1 - I/E)$, where $a_0$ is the Bohr radius and $E$ is the energy of the ionizing electron. In the approximation linear in the overheat parameter $\lambda$ we have

$$\delta\left(\frac{\partial N_e}{\partial t}\right)_{\text{ion}} \approx \frac{8}{\sqrt{\pi}} \lambda\Theta'' \pi a_0^2 N_a N_m \bar{v}_{T_0} \frac{I}{T_0} e^{-I/T_0}, \quad I \gg T_0. \quad (A.2)$$

$\bar{v}_{T_0}$ is the average thermal velocity of the electrons in the absence of overheating. The relative role of the shift of the ionization in comparison with thermal diffusion is determined by the parameter

$$\Delta = \delta\left(\frac{\partial N_e}{\partial t}\right)_{\text{ion}} \Big/ \delta\left(\frac{\partial N_e}{\partial t}\right)_T = \frac{8}{\sqrt{\pi}} \frac{\Theta''}{\Theta^{(4)}} \pi a_0^2 \frac{N_a \bar{v}_{T_0} \delta_m^2}{D_{am}} \frac{I}{T_0} e^{-I/T_0}, \quad (A.3)$$

the value of which does not depend on the degree of overheating $\lambda$. Noting that the ambipolar diffusion coefficient is

$$D_{am} = 2D_i \approx \frac{2}{3}\left(\frac{m}{M}\right)^{1/2} \frac{\bar{v}_{T_0}^2}{\nu},$$

we obtain for $\Delta$ (A.3) the estimate

$$\Delta \sim \frac{I}{T_0}\left(\frac{M}{m}\right)^{1/2} \frac{\delta_m^2}{l_{fp}\, l_{\text{ion}}},$$

where $l_{fp}$ is the free path length, and $l_{\text{ion}}$ is the electron path in which it acquires an energy sufficient to ionize the atom. Recognizing that $\delta_m^2 = c^2(8\pi\omega\sigma_m)^{-1} \sim \omega^{-1} e^{I/2T_m}$, we can easily see that the parameter $\Delta$ is inversely proportional to the field frequency $\omega$, and its temperature dependence is determined mainly by the factor $\exp(-I/2T_m)$. For hydrogen at atmospheric pressure, a field frequency $\omega = 10^{10}$, and a neutral-atom temperature $T_m = 6 \cdot 10^3$ °K, the ratio (A.3) is small: $\Delta \approx 2.5 \cdot 10^{-4} \ll 1$.

Thus, in the range of parameters considered by us, the influence of electron overheating on the ionization-recombination balance can be neglected in comparison with thermal diffusion.

1)As indicated in[35] (see also[36]), in the absence of skin effect these singularities can influence the character of the electron overheating.


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# Plasma heating during Langmuir collapse


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We solve the three-dimensional problem of the production of accelerated electrons in a plasma during Langmuir collapse. We show that the problem has a scaling solution in which practically the whole energy of the external source of the Langmuir oscillations is put into a small, decreasing with time, group of resonance particles.


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## INTRODUCTION

We consider in the present paper the problem of the heating of a plasma in which strong Langmuir turbulence is excited constantly by an external source. This kind of problem is of great interest for the problem of plasma heating by a powerful electron beam or by laser light. The most important property of strong Langmuir turbulence is the location of Langmuir noise in regions with a lower density—cavitons. The characteristic size $\delta$ of the cavitons and the density perturbation $\delta n$ are connected with the local noise energy density $W$ through the relation

$$\delta n/n_0 = r_{\mathrm{De}}^2/\delta^2 \lesssim W/nT, \tag{1}$$

where $r_{\mathrm{De}}$ is the Debye radius.

Zakharov[1] has shown that such formations are not stationary in the three-dimensional problem: when the energy density is sufficiently high, the cavitons collapse—their size $\delta$ vanishes after a finite time, and the quantity $W$ becomes infinite. We shall call such formations in what follows collapsing solitons, without in general having in view any analogy whatever with a stationary Langmuir solitons (see, e.g., [2]).

It is clear that if we take into account kinetic effects, the size $\delta$ cannot vanish while condition (1) is retained, since under the condition $\delta \sim r_{\mathrm{De}}$ the characteristic phase velocity of the harmonics is $\omega/k \sim v_{\mathrm{Te}}$, so that strong Landau damping sets in. This is not the only possible channel for dissipation during the collapse, but Rudakov[3] has shown that under the conditions when the noise is pumped in a stationary way, this mechanism is the most probable one and in that case the whole energy of the collapsing soliton is transferred to a small group of resonance particles with velocities which are appreciably above the thermal one, $v \gg v_{\mathrm{Te}}$. Strong Langmuir turbulence leads, therefore, as in the one-dimensional model,[2,4] to the formation of non-Maxwellian tails of hot electrons. This result of Rudakov's[3] has been confirmed by the solution of a model problem about the heating of a plasma by spherical, quasi-planar collapsing solitons.[5,6] Similar statements have been made by Galeev *et al.*[7] who solved the problem of the heating of a plasma during "supersonic" collapse[1] and for that case the spectra of the noise and of the fast electrons were obtained.

We present in the present paper the results of[3,6,7] in correspondence to one another. We also show that the methods for solving the problem in[6,7] are essentially equivalent. We give here the calculations for what is (according to present-day results) the most realistic mode of collapse—the formation of a kind of plane disk.

We dwell in more detail on this model which was first suggested by Rudakov.[3] The dynamics of the collapse in the hydrodynamic approximation is described by the set of equations[1]

$$\operatorname{div}\left(-i\frac{\partial \mathbf{E}}{\partial \tau} + \frac{3}{2}\omega_{\mathrm{pe}} r_{\mathrm{De}} \nabla^2 \mathbf{E}\right) = \frac{\omega_{\mathrm{pe}}}{2n_0}\operatorname{div}\delta n\mathbf{E},$$
$$\left(\frac{\partial^2}{\partial \tau^2} - c_s \nabla^2\right)\delta n = \frac{1}{16\pi M}\nabla^2 |\mathbf{E}|^2. \tag{2}$$